Feedback Control Using The Laplace Transform

Course Feedback Control and Real-time SystemsI

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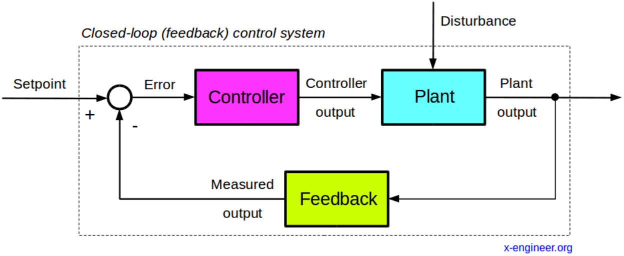
Stabilization by Feedback

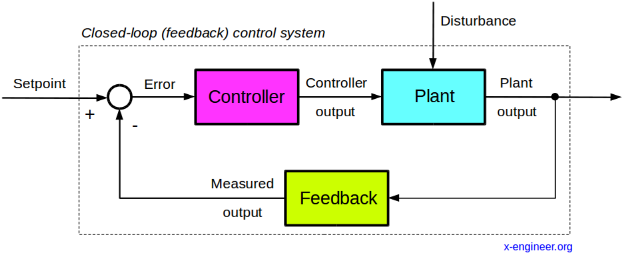
- Stabilization by feed-back
- Pole Placement

Consideration of Disturbance



- disturbance $w(t)$ represents perturbations or modelling error
- the output $y(t)$ and the setpoint/reference $r(t)$
- the controller C and the system S are supposed to be rational fractions $C(s), S(s)$

Transfer Function of the Closed-Loop System



We now compute the closed-loop transfer function:

$$Y(s) = S(s)(C(s)(R(s) - Y(s)) + W(s))$$

$$(1 + S(s)C(s))Y(s) = S(s)(C(s)R(s) + W(s))$$

where $W(s)$, $Y(s)$, $R(s)$ are the Laplace transforms of the disturbance $w(t)$, the output $y(t)$ and the reference $r(t)$.

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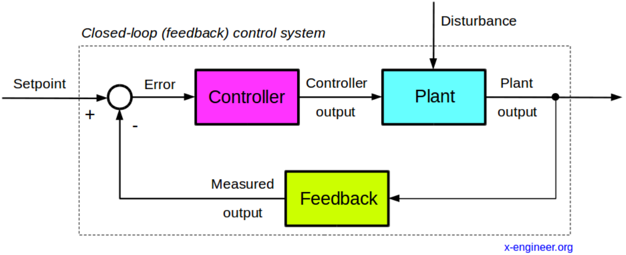

We obtain closed-loo thep transfer function:

$$Y(s) = \frac{S(s)C(s)}{1 + S(s)P(s)} R(s) + \frac{S(s)}{1 + S(s)C(s)} W(s)$$

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1. $\dfrac{S(s)C(s)}{1+S(s)C(s)}$ is stable et close to the identity (fidelity)
2. $\dfrac{S(s)}{1+S(s)P(s)}$ is small (robustness or disturbance rejection)

Exemple: PID controllers

We consider the system $S(s) = \dfrac{1}{s^2}$ (double integrator) and a proportional controller $C(s) = a$

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$D = s^2(cs+d) + as + b = cs^3 + ds^2 + as + b$

This is a third-order polynomial with 3 roots that we can fix as we want. We choose stable roots

$(s+1)(s-e^{\frac{3i\pi}{4}})(s-e^{\frac{5i\pi}{4}}) = (s+1)(s^2+\sqrt{2}s+1) =$
$s^3 + 2.4s^2 + 2.4s + 1$

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We identify : $c = 1, d = 2.4, a = 2.4, b = 1$

Homework

Use Simulink to simulate this PI controller and the system. Add some disturbance (by using the block named “Band-Limited White Noise”). Is the result satisfactory? If not, modify the controller to reject the disturbance.