Modeling and simulation of the evolution of a population

Part I: Modeling

In 1978, Malthus proposed a model to study the evolution of a population of individuals. The main assumption of this model is that the growth rate is constant. The rate of increase is the ratio of the derivative of the number of individuals to the number of individuals.

1. Express mathematically this hypothesis. Let r_0 denote the rate of increase. To which other measurable magnitudes on a population is this rate of increase related?
2. Deduce the law of evolution over time of the size of the population, following the above model. What are the parameters of this model? Trace this temporal evolution. What remark can you make?
3. This model is applied to a concrete case corresponding to adding reindeers in the Bering Islands. In 1911, 21 individuals were added. In 1938, there are 2000. Calculate the parameters of the model.
4. In the following years, we observe that the number of individuals is more and more inferior to that estimated by the model. What can be the reasons?
5. An extension of the model makes it possible to account for this deviation: the growth rate is no longer constant and it decreases as the population increases. This is the logistic "brake" phenomenon that leads to the logistic law. We consider that the rate of increase r follows the following law:

$$r(N) = (N^{*} - N) \gamma$$

   where N is the population size; N^{*} and γ are 2 parameters of the model. Then deduce the differential equation describing the model. Is it linear or nonlinear?
6. Show that if the population follows a temporal evolution such that:

$$N(t) = N^{*} [1 + K \exp(- \gamma N^{*} t)]^{-1}$$

   (where N^{*} and γ are 2 positive constants), then this evolution is the solution of the equation defined in Question 5. Give the expression of N(0) and N(∞). What is the approximate expression of N(t) when t is small, considering that K >> 1? Rewrite the equation by parametrizing it only with $N(0) = N_0$, N^{*} and r_0. Then trace this new temporal evolution.
7. Depending on the biotope of the islands, the maximum capacity of nutrition is estimated at 3000 individuals. Ddetermine all the parameters of the model.
8. What could be other natural phenomena that are not taken into account in this new model?

Part II: Simulation (Bonus questions)

1 °) From the differential equations found in part 1 (Questions 1 and 5), draw the functional block diagrams modelling these equations.

2) In the Simulink environment, implement the first model and simulate it. Do the same for the second model.