Proving termination using dependent types: the case of xor-terms

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Abstract

We study a normalization function in an algebra of terms quotiented by an associative, commutative and involutive operator (logical xor). This study is motivated by the formal verification of cryptographic systems, where a normalization function for xor-terms turns out to play a key role. Such a function is easy to define using general recursion. However, as it is to be used in a type theoretic proof assistant, we also need a proof termination of this function. Instead of using a clever mixture of various rewriting orderings, we follow an approach involving the power of Type Theory with dependent types. The results are to be applied in the proof of the security API described in [CM06].

1 INTRODUCTION

In the course of the formal verification of cryptographic systems using symbolic approaches, one deals with algebras of terms whose constructors include $\oplus$, denoting the binary bitwise exclusive or and O, denoting a bitstring consisting only of zeros. Bitwise exclusive or is often used in cryptographic systems and many (potential or effective) attacks are based on its algebraic properties [YAB+05, Bon04, CKRT05, CLC03].

Dealing with the congruence generated by the usual arithmetic laws on $\oplus$ and O is therefore necessary in order to successfully verify these systems: in the following we consider an algebra of terms $\mathcal{T}$ built up using a number of constructors, where two of them, denoted by $\oplus$ and O, enjoy the following algebraic properties.

Commutativity:	$x \oplus y \simeq y \oplus x$	(1)
Associativity:	$(x \oplus y) \oplus z \simeq x \oplus (y \oplus z)$	(2)
Neutral element:	$x \oplus \mathsf{O} \simeq x$	(3)
Involutivity:	$x \oplus x \simeq \mathsf{O}$	(4)

Formally, $\simeq$ denotes the least congruence generated by equations (1) to (4). In order to reason on terms of $\mathcal{T}$ up to $\simeq$, a standard technique is to define a canonicalization function over $\mathcal{T}$. One also actually needs such a function to give minimal terms with respect to simplification as one also needs a subterm relation $\preceq$ which takes into account equalities such as $u \simeq u \oplus x \oplus x$:

$$x \preceq y \quad \text{if } x \simeq y$$
$$x \preceq t \quad \text{if } t \simeq x \oplus y_0 \ldots \oplus y_n \text{ and } x \not\preceq y_i \text{ for all } i,\ 0 \leq i \leq n$$

Turning equations (1) to (4) into a convergent and strongly normalizing AC-rewriting system is straightforward. Therefore, the existence of a normalization function can be proven easily. Moreover, in any decent programming language, defining a normalization function on $\mathcal{T}$ is quite easy, using general recursion.

However, formally giving such a normalization function in Type Theory and formally proving its correction is much more challenging. Using standard rewriting arguments to define such a function is surprisingly difficult in a proof assistant such as Coq [The05, BC04].

- Although some theoretical works address the addition of rewriting to the Calculus of Constructions [Bla01], these works are yet to be implemented.

- Some works provide ways to define tactics for reasonning over associative-commutative theories [AN00], but they only provide ways to normalize given terms, not to define a normalization function.

We therefore tried to define our own specific rewriting relation corresponding to the defining equations of $\simeq$, but found this approach really costly:

- A well-founded ordering has to be given. As no rpo or lpo ordering library is available in Coq, we used the lexicographic combination of a partial ordering $\leq_1$ with a total ordering $\leq_2$, where $\leq_1$ is a polynomial ordering, and $\leq_2$ is a lexicographic ordering. Although $\leq_2$ is not well-founded, the set of terms having a given weight for the polynomial defining $\leq_1$ is finite, therefore we could prove in Coq the lexicographic combination of $\leq_1$ and $\leq_2$ to be finite.

- Then we defined a rewriting relation. The difficult part here is to take into account commutativity and associativity. In order to avoid AC-matching issues, we decided to throw in associativity and to add commutativity as a conditional rule ($x \oplus y$ would rewrite to $y \oplus x$ if and only if x is smaller than y). Moreover, we had to complete our rewriting system in order to close critical pairs such as $x \oplus x \oplus y$, which could be rewritten to y or to $x \oplus (x \oplus y)$.

- A normalization function has to be given. The definition of such a function using well-founded induction in Coq is uneasy. (Some tools or methodology such as those developed by Bertot and Balaa, or Bove and Capretta [BB00, BC05] may help a bit here.) Therefore we stopped there and used an other approach instead.

- Once this would be done, we would still have to prove that the transitive closure of our rewriting relation is irreflexive, and that our normalization function is sound with respect to it and computes normal forms. Essentially, the main results to prove here would be $\forall t, t \not\triangleright^+ t$, $\forall t\ t \triangleright^\star norm(t)$ and $\forall t_1 \forall t_2\ \ t_1 \triangleright t_2 \Rightarrow norm(t_1) = norm(t_2)$.

Instead we experimented an ad-hoc approach involving typical features of Type Theory. The intuition behind our approach is very simple. In a first stage, the term to be normalized is first layered, in such a way that each level is built up from terms belonging to the previous level. These levels alternate between layers built up using only $\oplus$ constructors and layers built up using only other constructors, as lasagnas alternate between pasta-only layers and sauce layers (mixed up to your taste of tomato, meat, and cheese – in fact anything but pasta). In a second stage, layers are normalized bottom-up. Normalizing a $\oplus$-layer roughly boils down to sorting, while normalization of a non-$\oplus$-layer is just identity.

As may be expected, the second stage is almost trivial. However the first stage requires more work. In particular, we need the full power of programming with dependent types.

The approach we describe in this paper was designed and implemented using the Coq proof assistant. Its results are to be applied in the proof of security properties of an API described in [CM06].

2 FORMALIZATION

2.1 Splitting the type of terms

Let $\{\oplus, \mathsf{O}\} \uplus \mathcal{C}$ be the set of constructors of $\mathcal{T}$. For instance, in our case, we have $\mathcal{C} = \{\mathtt{PC}, \mathtt{SC}, \mathtt{E}, \mathtt{Hash}\}$ with

$$\mathtt{PC} : \mathtt{public\_const} \to \mathcal{T} \qquad \mathtt{E} : \mathcal{T} \to \mathcal{T} \to \mathcal{T}$$
$$\mathtt{SC} : \mathtt{secret\_const} \to \mathcal{T} \qquad \mathtt{Hash} : \mathcal{T} \to \mathcal{T} \to \mathcal{T}$$

where `public_const` and `secret_const` are suitable enumerated types.

We introduce two polymorphic inductive types $\mathcal{T}_x(\alpha)$ and $\mathcal{T}_n(\alpha)$ respectively called the pasta layer type and the sauce layer type. The constructors of $\mathcal{T}_x(\alpha)$ are (copies of) $\oplus$ and O while the constructors of $\mathcal{T}_n(\alpha)$ are (copies of) those belonging to $\mathcal{C}$. Moreover, $\mathcal{T}_x(\alpha)$ (respectively $\mathcal{T}_n(\alpha)$) has an additional constructor $I_x : \alpha \to \mathcal{T}_x(\alpha)$ (respectively $I_n : \alpha \to \mathcal{T}_n(\alpha)$).

It is then clear that any term t in $\mathcal{T}$ can be recasted into either the type $\mathcal{T}_x(\mathcal{T}_n(\mathcal{T}_x(\ldots(\emptyset))))$ or the type $\mathcal{T}_n(\mathcal{T}_x(\mathcal{T}_n(\ldots(\emptyset))))$, according to the top constructor of t.

Normalizing t can then be defined as bottom-up sorting in the following way.

We say that a type X is *sortable* if it is equipped with a decidable equality and a decidable total irreflexive and transitive relation—equivalently, we could take a decidable total ordering but the above choice turns out to be more convenient. If X is a sortable, then

- $\mathcal{T}_n(X)$ is sortable;

- the multiset of X-leaves of any inhabitant t of $\mathcal{T}_x(X)$ can be sorted (with deletion of duplicates) into a list $N_X(t)$, such that $t_1 \simeq t_2$ iff $N_X(t_1)$ is syntactically equal to $N_X(t_2)$;
- `list`(X) is sortable (*i.e.* can be equipped with the suitable .equality and comparison relation).

Then we can normalize any term of type $\ldots\mathcal{T}_x(\mathcal{T}_n(\mathcal{T}_x(\emptyset))))$ by induction on the number of layers. Note that thanks to polymorphism, we deal with each layer in a pleasant modular way.

We now have to handle types such as $\ldots\mathcal{T}_x(\mathcal{T}_n(\mathcal{T}_x(\emptyset))))$ in a formal way.

2.2 Formalized stratified types

2.2.1 Defining pasta and sauce layers

A layer is said trivial when it consists only in a term $I_x(a)$ (resp $I_n(a)$). In order to unfold sequences of $\oplus$, we want to avoid artificial separation of $\oplus$ layers like $x \oplus I_x(I_n(y \oplus z))$. Therefore, we want to be able to forbid constructions like $I_x(I_n(a))$. Hence we distinguish between potentially trivial layers and non-trivial layers, by adding to the pasta layer type $\mathcal{T}_x$ a boolean parameter telling us whether trivial layers are included:

Section *sec_x*.
Variable A : *Set*.
Inductive $\mathcal{T}_x$: *bool* $\rightarrow$ *Set* :=
  | *X_Zero* : $\forall$ *b*, $\mathcal{T}_x$ *b*
  | *X_ns* : $\forall$ *b*, *Is_true b* $\rightarrow$ *A* $\rightarrow$ $\mathcal{T}_x$ *b*
  | *X_Xor* : $\forall$ *b*, $\mathcal{T}_x$ *true* $\rightarrow$ $\mathcal{T}_x$ *true* $\rightarrow$ $\mathcal{T}_x$ *b*
.
Definition I_x := *X_ns true I*.
End *sec_x*.

Likewise the inductive sauce layer type $\mathcal{T}_n$ (non-xor terms) is parameterized by a boolean telling whether trivial layers are included.

Section *sec_nx*.
Variable A : *Set*.
Inductive $\mathcal{T}_n$: *bool* $\rightarrow$ *Set* :=
  | *NX_PC* : $\forall$ *b*, *public_const* $\rightarrow$ $\mathcal{T}_n$ *b*
  | *NX_SC* : $\forall$ *b*, *secret_const* $\rightarrow$ $\mathcal{T}_n$ *b*
  | *NX_sum* : $\forall$ *b*, *Is_true b* $\rightarrow$ *A* $\rightarrow$ $\mathcal{T}_n$ *b*
  | *NX_E* : $\forall$ *b*, $\mathcal{T}_n$ *true* $\rightarrow$ $\mathcal{T}_n$ *true* $\rightarrow$ $\mathcal{T}_n$ *b*
  | *NX_Hash* : $\forall$ *b*, $\mathcal{T}_n$ *true* $\rightarrow$ $\mathcal{T}_n$ *true* $\rightarrow$ $\mathcal{T}_n$ *b*
.
Definition I_n := *NX_sum true I*.
End *sec_nx*.

2.2.2 Maps over lasagnas

Given a function f from $A : Set$ to $B : Set$, one can easily define a map_x (resp. map_n) function lifting f to functions from $\mathcal{T}_x(A)$ to $\mathcal{T}_x(B)$ (resp. from $\mathcal{T}_n(A)$ to $\mathcal{T}_n(B)$).

Moreover, given an evaluation function $f : A \rightarrow \mathcal{T}$, one can extend it to the domain $\mathcal{T}_x(A)$ (resp. $\mathcal{T}_n(A)$) by interpreting copies of $\oplus$ and O (resp. of constructors belonging to $\mathcal{C}$) as the corresponding constructors of $\mathcal{T}$ and I_x (resp. I_n) as the identity over $\mathcal{T}$.

2.2.3 Stacking layers

Building a stack of k layers now essentially amounts to building the type $(\mathcal{T}_x \circ \mathcal{T}_n)^{k/2}(\emptyset)$ or $\mathcal{T}_n(\mathcal{T}_x \circ \mathcal{T}_n)^{k/2}(\emptyset)$, depending on the parity of k. In a more type-theoretic fashion, we defined two mutually inductive types alt_{even} and alt_{odd} respectively denoting even and odd natural numbers: the constructors of alt_{even} are 0_e and $S_{o \rightarrow e}$, the successor function from odd to even numbers, whereas alt_{odd} has only one constructor, $S_{e \rightarrow o}$, the successor function from even to odd numbers. We also define *parity* as either P_e or P_o. One can then build the function

$$\begin{array}{rrcl} alt\_of\_parity : & parity & \rightarrow & Set \\ & P_e & \mapsto & alt_{even} \\ & P_o & \mapsto & alt_{odd} \end{array}$$

2.3 Stratifying a term

2.3.1 Lifting a lasagna

The intuitive idea we have about lasagnas is somewhat misleading, because the number of pasta and sauce layers is uniform in a whole lasagna dish, while the number of layers of subterms which are rooted at the same depth of a given term are different in the general case. However, any lasagna of height n can be lifted to a lasagna of height $n+e$, where e is even, because the empty type at the bottom of types such as $\mathcal{T}_x(\mathcal{T}_n(\mathcal{T}_x(\ldots(\emptyset))))$ can be replaced with any type. Formally, the lifting is defined by structural mutual induction as follows, thanks to map combinators.

Fixpoint $lift\_lasagna\_x\ e_1\ e_2\ \{struct\ e_1\} : \mathcal{L}_x\ e_1 \rightarrow \mathcal{L}_x\ (e_1 + e_2)$:=
  match e_1 return $\mathcal{L}_x\ e_1 \rightarrow \mathcal{L}_x\ (e_1 + e_2)$ with
  $|\ 0_e \Rightarrow \lambda\ emp \Rightarrow$ match emp with end
  $|\ S_{o \rightarrow e}\ o_1 \Rightarrow map_x\ (lift\_lasagna\_n\ o_1\ e_2)\ false$
  end
with $lift\_lasagna\_n\ o_1\ e_2\ \{struct\ o_1\} : \mathcal{L}_n\ o_1 \rightarrow \mathcal{L}_n\ (o_1 + e_2)$:=
  match o_1 return $\mathcal{L}_n\ o_1 \rightarrow \mathcal{L}_n\ (o_1 + e_2)$ with
  $|\ S_{e \rightarrow o}\ e_1 \Rightarrow map_n\ (lift\_lasagna\_x\ e_1\ e_2)\ false$
  end.

2.3.2 Counting layers of a $\mathcal{T}$-term

Given a $\mathcal{T}$-term t, the type of the corresponding lasagna depends on the number $l(t)$ of its layers, which has to be computed first.

At first sight, we may try to escape the problem by computing a number $u(t)$ which is known to be greater, or equal to, $l(t)$ (a suitable u is the height). However we would then have to handle proofs that the proposed number $u(t)$ does provide an upper bound on $l(t)$. Such proofs have to be constructive, because they provide a bound on the number of recursive calls in the computation of the layering of a $\mathcal{T}$-term. Then they embark the difference between $u(t)$ and $l(t)$, in a more or less hidden way. So it is unclear that $u(t)$ would really help us to simplify definitions, and we chose to stick to an accurate computation of $l(t)$ as follows.

The lifting functions explained in section 2.3.1 are basically used in the following way. We define the maximum of two natural numbers n and m as $n - m + m$. It is easy to check that this operation is commutative, hence the lasagnas of two immediate subterms of a $\mathcal{T}$-term can be lifted to lasagnas of the same height.

A further difficulty is that the arguments of a constructor occurrence in t are heterogeneous, i.e. some of them can be $\oplus$ and the others can be in $\mathcal{C}$. We then may use appropriate injections I_x or I_n. However, recall that their use is controlled (see section 2.2.1): they can be used only at the separation line between two different layers.

The trick is that, in general, we do not compute the lasagna of height n of a given term, that is, a $\mathcal{T}_x(X, \texttt{false})$ or a $\mathcal{T}_n(X, \texttt{false})$, where X is a lasagna of the opposite kind and of height $n-1$ but only a *lasagna candidate of height* $n-1$, that is, a function which yields a $\mathcal{T}_x(X, b)$ or a $\mathcal{T}_n(X, b)$ for any Boolean b.

Similarly, the definition of the height for a lasagna candidate (called *alt_allpar_of_term*) depends on a given parity p.

Definition $inj\_odd\_parity\ p : alto \rightarrow alt\_of\_parity\ p :=$
  match p return $alto \rightarrow alt\_of\_parity\ p$ with
  $\mid P_e \Rightarrow S_{o \rightarrow e}$
  $\mid P_o \Rightarrow \lambda\ o \Rightarrow o$
  end.

Similarly for inj_even_parity

Fixpoint $alt\_allpar\_of\_term\ (t{:}\mathcal{T}) : \forall\ p,\ alt\_of\_parity\ p :=$
 match t return $\forall\ p,\ alt\_of\_parity\ p$ with
 $\mid Zero \Rightarrow \lambda\ p \Rightarrow inj\_odd\_parity\ p\ (S_{e\rightarrow o}\ 0_e)$
 $\mid Xor\ x\ y \Rightarrow$
  let $o_1 := alt\_allpar\_of\_term\ x\ P_o$ in
  let $o_2 := alt\_allpar\_of\_term\ y\ P_o$ in
  $\lambda\ p \Rightarrow inj\_odd\_parity\ p\ (max\_oo\ o_1\ o_2)$
 $\mid PC\ x \Rightarrow \lambda\ p \Rightarrow inj\_even\_parity\ p\ 0_e$
 $\mid E\ x\ y \Rightarrow$
  let $e_1 := alt\_allpar\_of\_term\ x\ P_e$ in

let e_2 := $alt\_allpar\_of\_term\ y\ P_e$ in
λ $p \Rightarrow inj\_even\_parity\ p\ (max\_ee\ e_1\ e_2)$
[Similarly for other constructors]
end.

The lifting functions of section 2.3.1 are easily generalized to lasagna candidates.

2.3.3 Computing the lasagna

The main recursive function computes a true lasagna candidate. In other words, the type of its result depends on the desired parity.

Definition $kind\_lasagna\_cand\_of\_term\ (t{:}\mathcal{T})\ (p{:}\ parity) : Set :=$
match p with
| $P_e \Rightarrow lasagna\_cand\_n\ (alt\_allpar\_of\_term\ t\ P_e)\ true$
| $P_o \Rightarrow lasagna\_cand\_x\ (alt\_allpar\_of\_term\ t\ P_o)\ true$
end.

Its body introduces injections as required. Here is its definition.

Fixpoint $lasagna\_cand\_of\_term\ (t{:}\mathcal{T})$:
$\forall\ p,\ kind\_lasagna\_cand\_of\_term\ t\ p :=$
match t return $\forall\ p,\ kind\_lasagna\_cand\_of\_term\ t\ p$ with
| $Zero \Rightarrow$
λ $p \Rightarrow$ match p return $kind\_lasagna\_cand\_of\_term\ Zero\ p$ with
| $P_e \Rightarrow I_n\ (X\_Zero\ false)$
| $P_o \Rightarrow X\_Zero\ true$
end
| $Xor\ t_1\ t_2 \Rightarrow$
let $l_1 := lasagna\_cand\_of\_term\ t_1\ P_o$ in
let $l_2 := lasagna\_cand\_of\_term\ t_2\ P_o$ in
λ $p \Rightarrow$ match p return $kind\_lasagna\_cand\_of\_term\ (Xor\ t_1\ t_2)\ p$ with
| $P_e \Rightarrow I_n\ (bin\_xor\ X\_Xor\ l_1\ l_2)$
| $P_o \Rightarrow bin\_xor\ X\_Xor\ l_1\ l_2$
end

| $PC\ x \Rightarrow$
[similarly for constructors in $\mathcal{C}$].

The above definition requires a function called *bin_xor* which maps a constructor of $\mathcal{T}_x$ to an operation on lasagna candidates of arbitrary height. This is the place where lifting is used. Note the essential use of the conversion rule in its typing.

Definition *bin_xor*
$(bin : \forall\ A\ b,\ \mathcal{T}_x\ A\ true \rightarrow \mathcal{T}_x\ A\ true \rightarrow \mathcal{T}_x\ A\ b)\ o_1\ o_2\ b$
$(l_1 : lasagna\_cand\_x\ o_1\ true)\ (l_2 : lasagna\_cand\_x\ o_2\ true)$:
$lasagna\_cand\_x\ (max\_oo\ o_1\ o_2)\ b :=$

$bin\ (\mathcal{L}_n\ (max\_oo\ o_1\ o_2))\ b$
$(lift\_lasagna\_cand\_x\ true\ o_1\ (o_2 - o_1)\ l_1)$
$(coerce\_max\_comm\ (lift\_lasagna\_cand\_x\ true\ o_2\ (o_1 - o_2)\ l_2))$.

Finally, the function *lasagna_of_term* is defined on top of *lasagna_cand_of_term*. In contrast with the latter, we force the parity to depend on the constructor at the root:

Definition $alt\_of\_term\ t := alt\_allpar\_of\_term\ t\ (parity\_of\_term\ t)$.

Definition $lasagna\_of\_parity\ p : alt\_of\_parity\ p \rightarrow Set :=$
match p return $alt\_of\_parity\ p \rightarrow Set$ with
| $P_e \Rightarrow \mathcal{L}_x$
| $P_o \Rightarrow \mathcal{L}_n$
end.

Definition $lasagna\_of\_term\ (t{:}\mathcal{T})$:
$lasagna\_of\_parity\ (parity\_of\_term\ t)\ (alt\_of\_term\ t) :=$
match t return $lasagna\_of\_parity\ (parity\_of\_term\ t)\ (alt\_of\_term\ t)$ with
| $Zero \Rightarrow X\_Zero\ false$
| $Xor\ t_1\ t_2 \Rightarrow$
let $l_1 := lasagna\_cand\_of\_term\ t_1\ P_o$ in
let $l_2 := lasagna\_cand\_of\_term\ t_2\ P_o$ in
$bin\_xor\ X\_Xor\ l_1\ l_2$

| $PC\ x \Rightarrow NX\_PC\ false\ x$
[similarly for constructors in $\mathcal{C}$].

2.4 Normalizing

We define a new pair of types $\mathcal{S}_x$ and $\mathcal{S}_n$ along the same lines as for $\mathcal{L}_x$ and $\mathcal{L}_n$, where $\mathcal{T}_x(\alpha)$ is replaced with $list(\alpha)$. Then we define a pair of normalization functions $N_x : \forall\, e, \mathcal{L}_x\, e \rightarrow \mathcal{S}_x\, e$ and $N_n : \forall\, o, \mathcal{L}_n\, o \rightarrow \mathcal{S}_n\, o$. The latter does essentially nothing, while the core of the former is
$\lambda x.\ fold\_insert\ (map\_xor\ (N_n\ o)\ false\, x)\ [\,]$.

3 CONCLUSION

The Epigram project [AMM05] already advocates the definition of functions using dependent types. They mostly aim at ensuring partial correctness properties (such as a balancing invariant in the case of *mergesort*).

The present paper shows how dependent types can help for ensuring termination too. We showed that an alternate path to termination orderings can be followed in some situations. While our approach is certainly less general, it relies on more elementary arguments. As a consequence, we can get a better insight on the reasons that make the normalization process terminate: they boil down to a (mutual) induction on the implicit structure of terms. As for approaches advocated by Epigram, the whole game consists in finding dependent types that render this implicit structure explicit.

Our development is available at http://www-verimag.imag.fr/˜monin/.

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