Self-Stabilizing Leader Election in Polynomial Steps[1]

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Anaïs Durand Franck Petit

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[1] This work has been partially supported by the LabEx PERSYVAL-Lab (ANR-11-LABX-0025-01) and the AGIR project DIAMS.

Problem

- **Silent Self-stabilizing Leader Election**
- Model:
  - **Locally shared memory** model
  - Read/write atomicity
  - Distributed **unfair** daemon
- Network:
  - Any connected topology
  - Bidirectional
  - **Identified**
- **No global knowledge** on the network

State of the Art

Model	Paper	Knowledge			Daemon	Complexity			Silent
		D	N	B		Memory	Rounds	Steps	
Message Passing	Afek, Bremler, 1998			x		$\Theta(\log n)$	$O(n)$	?	✓
	Awerbuch *et al*, 1993	x				$\Theta(\log D \log n)$	$O(\mathcal{D})$	?	✓
	Burman, Kutten, 2007	x				$\Theta(\log D \log n)$	$O(\mathcal{D})$	?	✓
Locally Shared Memory	Dolev, Herman, 1997		x		Fair	$\Theta(N \log N)$	$O(\mathcal{D})$	?	
	Arora, Gouda, 1994	x			Weakly Fair	$\Theta(\log N)$	$O(N)$	?	✓
	Datta *et al*, 2010				Unfair	unbounded	$O(n)$	?	✓
	Kravchik, Kutten, 2013				Synchronous	$\Theta(\log n)$	$O(\mathcal{D})$	?	✓
	Datta *et al*, 2011				Unfair	$\Theta(\log n)$	$O(n)$	?	✓

$\mathcal{D}$: Diameter
$D \geq \mathcal{D}$: Upper bound on the diameter

n: Number of nodes
$N \geq n$: Upper bound on the number of nodes

B: Upper bound on the link-capacity

Our Contribution

Algorithm $\mathcal{LE}$

- Memory requirement asymptotically optimal: $\Theta(\log n)$ bits/process
- Stabilization time (worst case):
  - $3n + \mathcal{D}$ rounds
  - Lower Bound: $\frac{n^3}{6} + \frac{5}{2}n^2 - \frac{11}{3}n + 2$ steps,
    Upper Bound: $\frac{n^3}{2} + 2n^2 + \frac{n}{2} + 1$ steps

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Analytical Study of Datta *et al*, 2011[2]

- Stabilization time not polynomial in steps:
  - $\forall \alpha \geq 3$, $\exists$ networks and executions in $\Omega(n^{\alpha+1})$ steps.

[2] Datta, Larmore, and Vemula. Self-stabilizing Leader Election in Optimal Space under an Arbitrary Scheduler. 2011

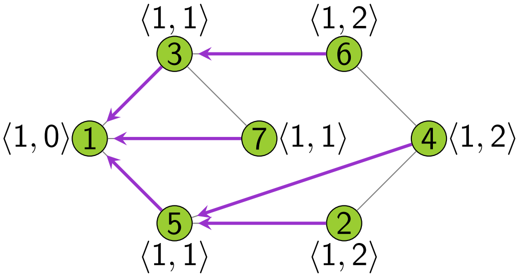
Design of the Leader Election Algorithm

Simplified Algorithm (Non Self-stabilizing)

Join a Tree

3 variables per process p

- $p.idR \in \mathbb{N}$: ID of the root
- $p.par \in \mathcal{N}_p \cup \{p\}$: Parent pointer
- $p.level \in \mathbb{N}$: Level



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Initial Configuration

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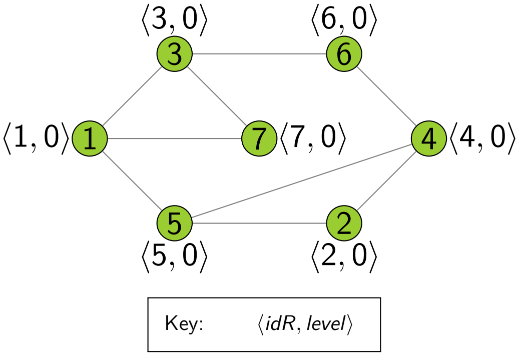

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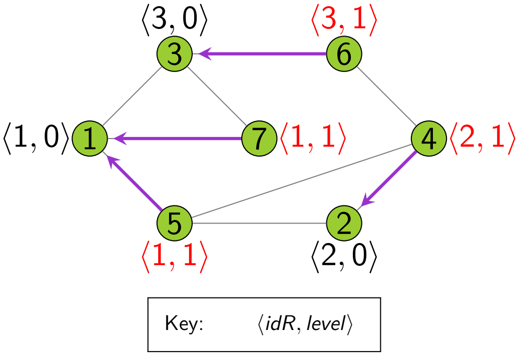

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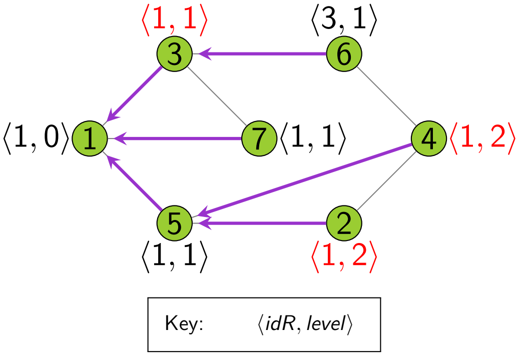

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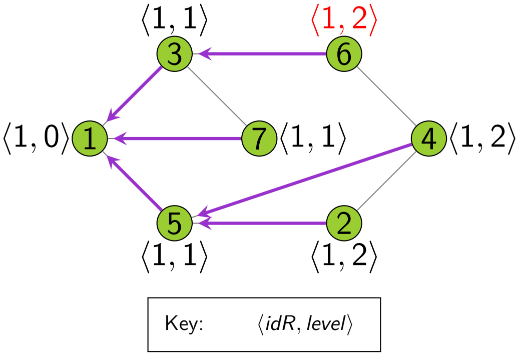
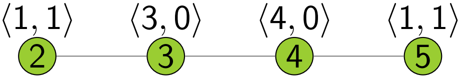


Simplified Algorithm (Non Self-Stabilizing)

Self-stabilization $\Longrightarrow$ Arbitrary initialization



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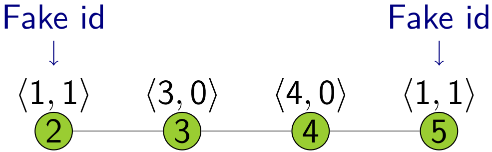
Simplified Algorithm (Non Self-Stabilizing)

Self-stabilization $\Longrightarrow$ Arbitrary initialization $\Longrightarrow$ Fake ids



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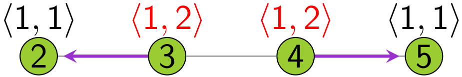

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Simplified Algorithm: Removal of Fake Ids

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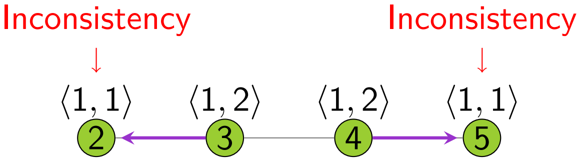

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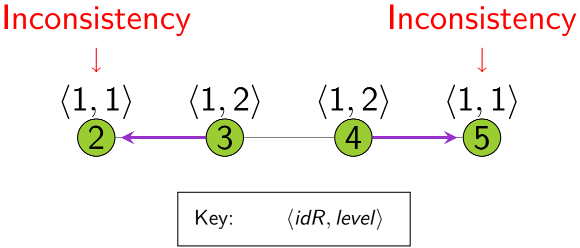

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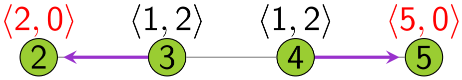

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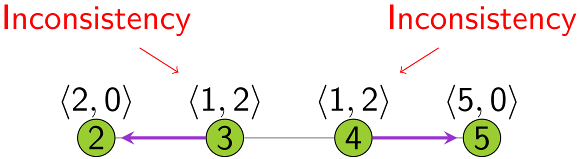

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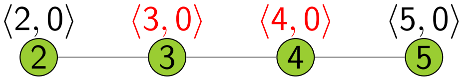

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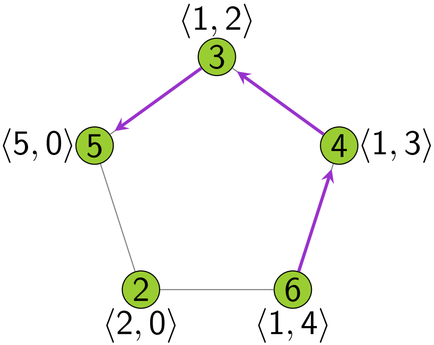

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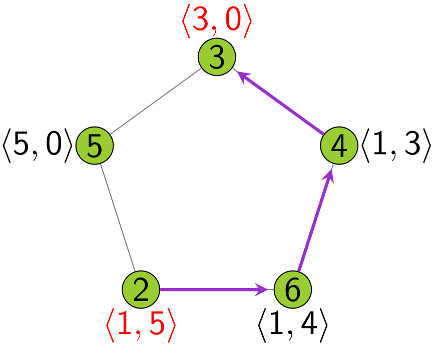

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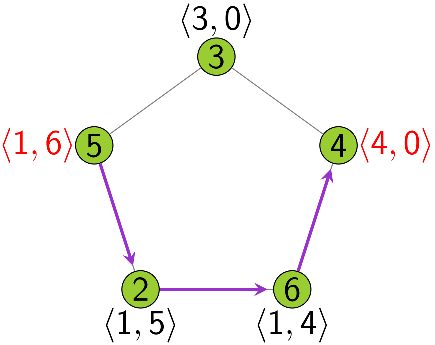

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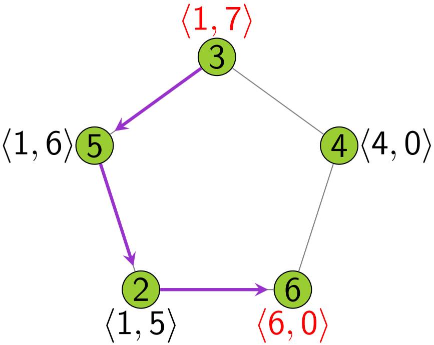

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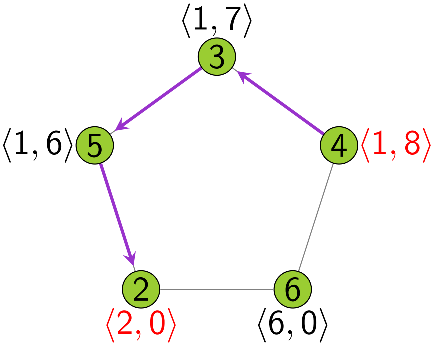

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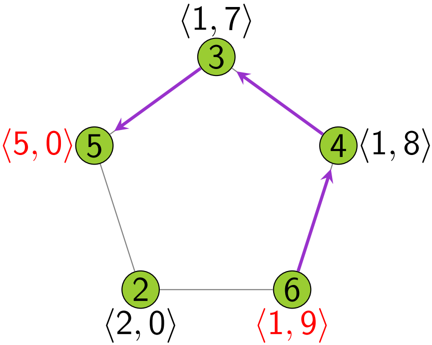

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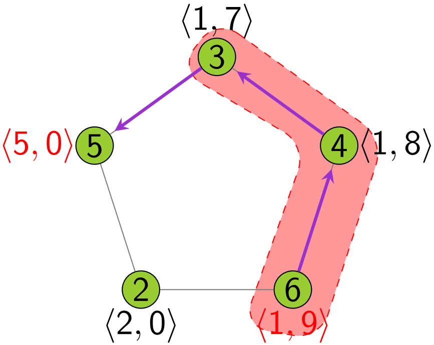

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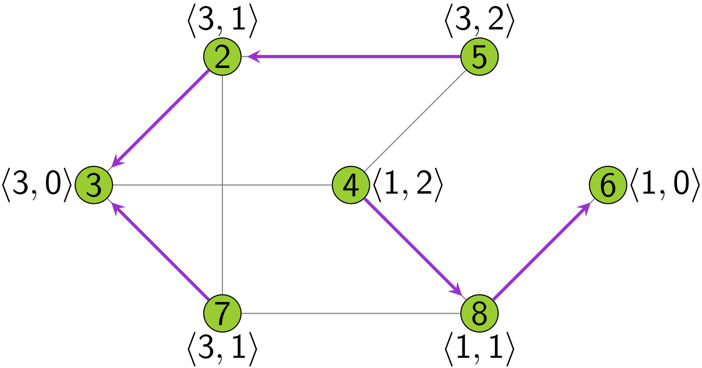
Abnormal Trees



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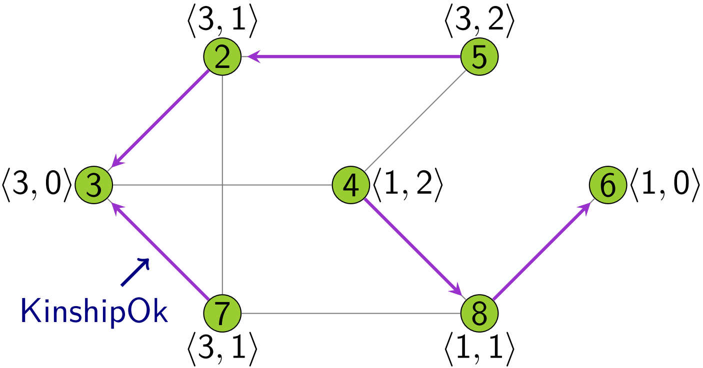

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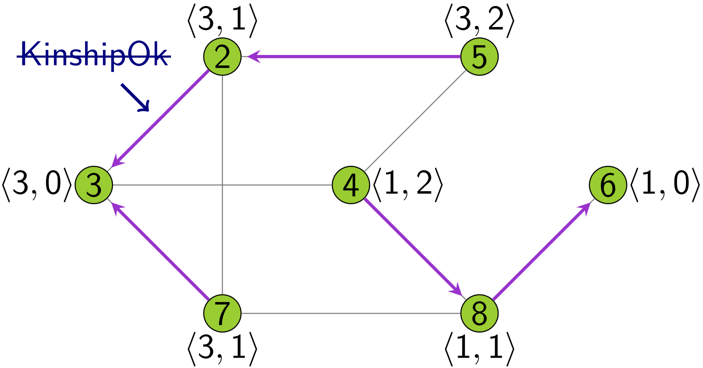

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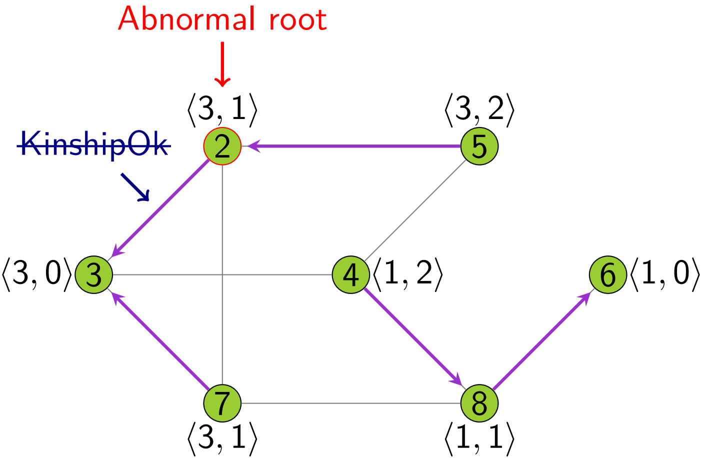

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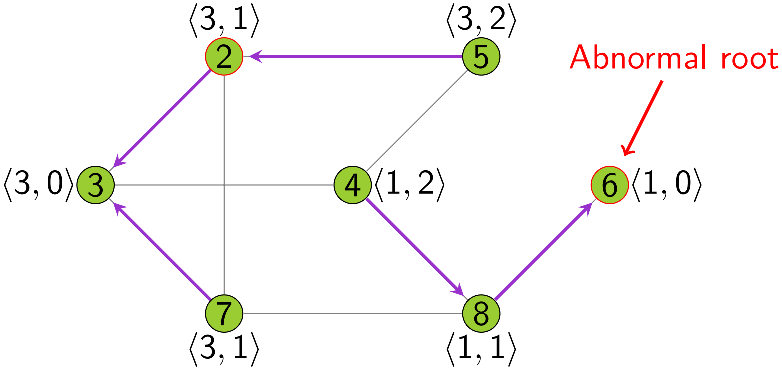

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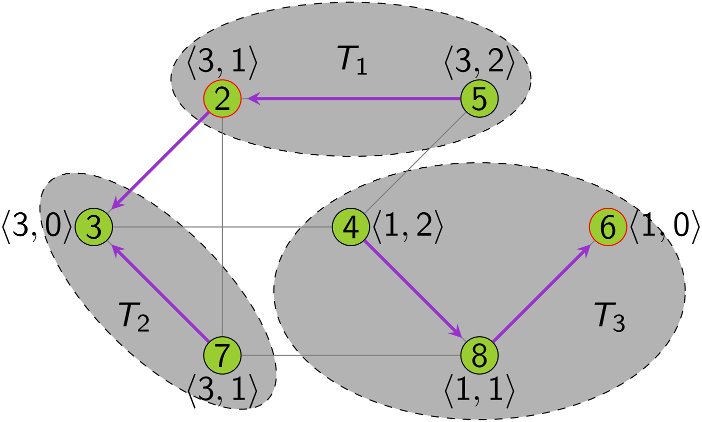

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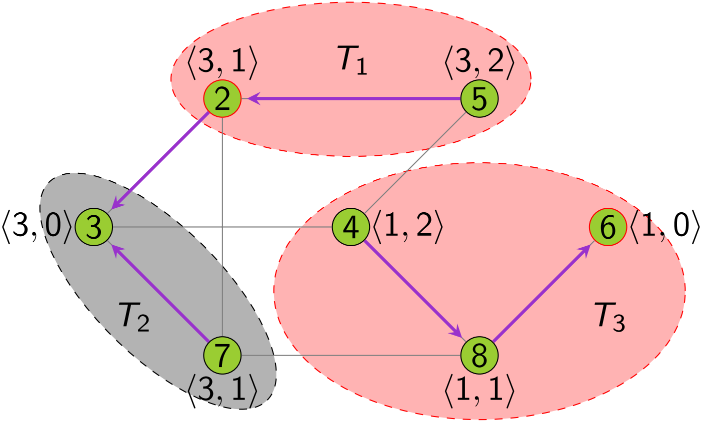

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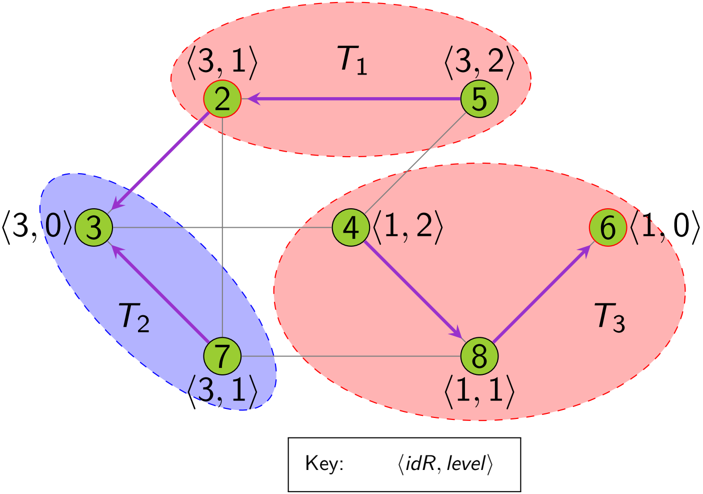

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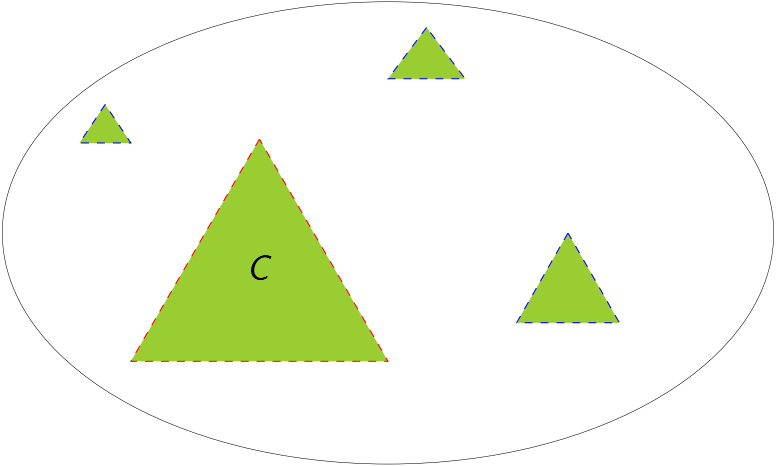

Key: $\langle idR, level\rangle$ $Clean$ $EBroadcast$ $EFeedback$

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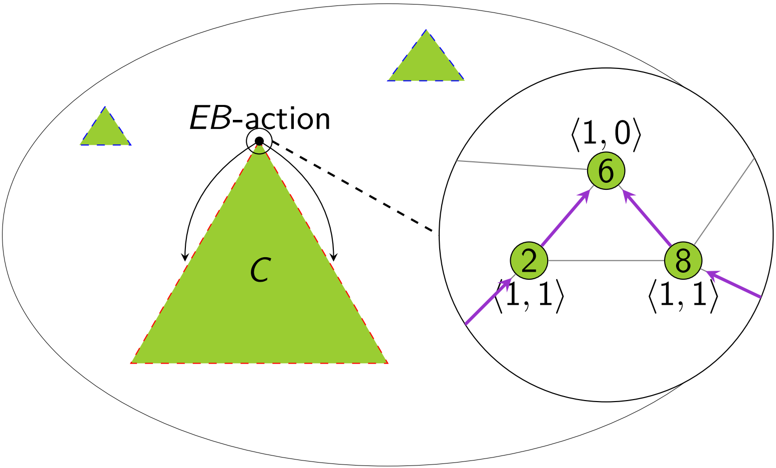

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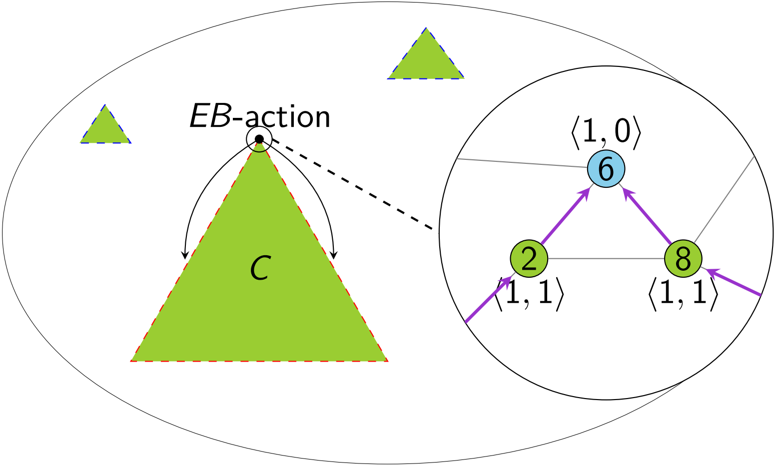

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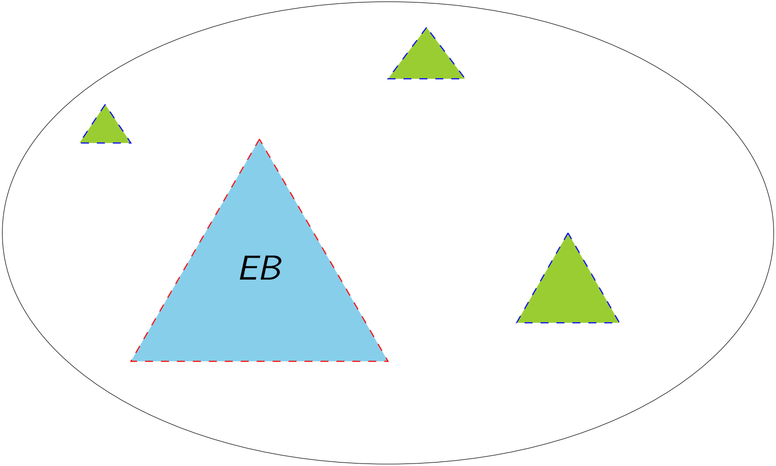

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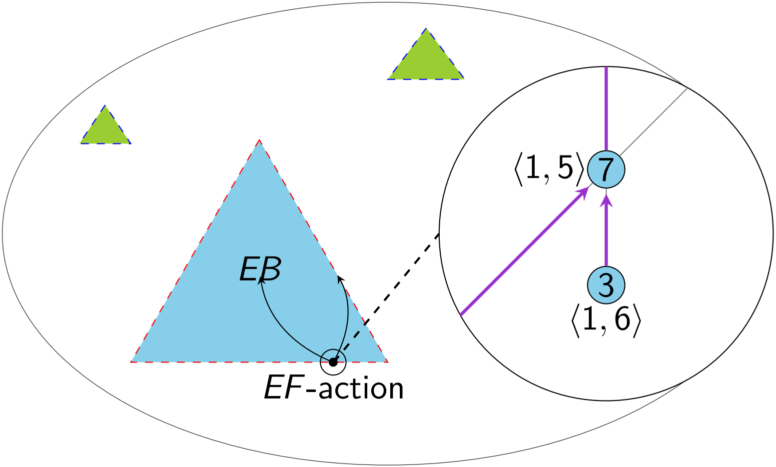

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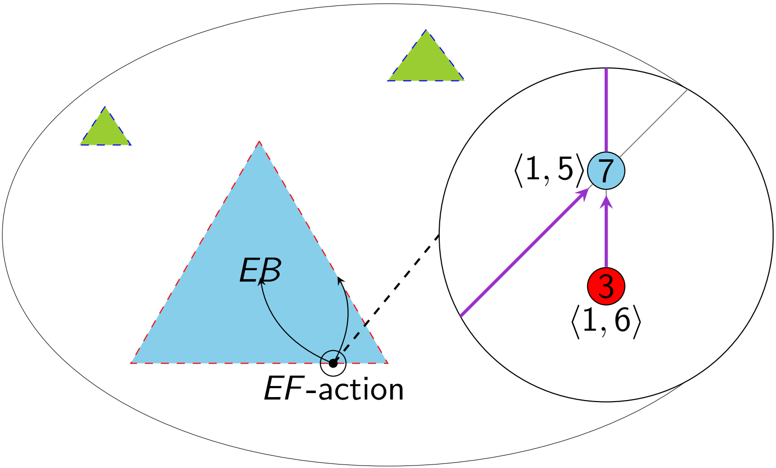

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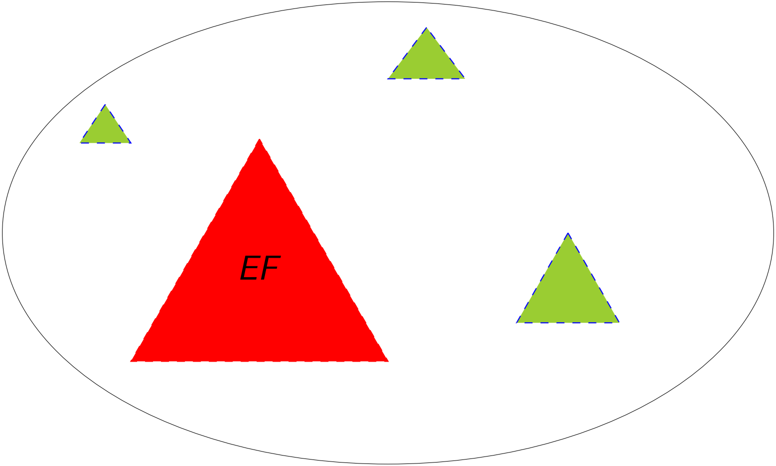

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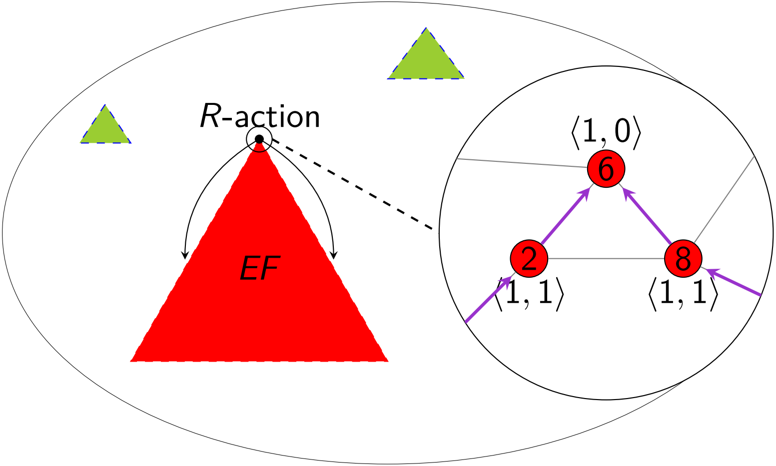

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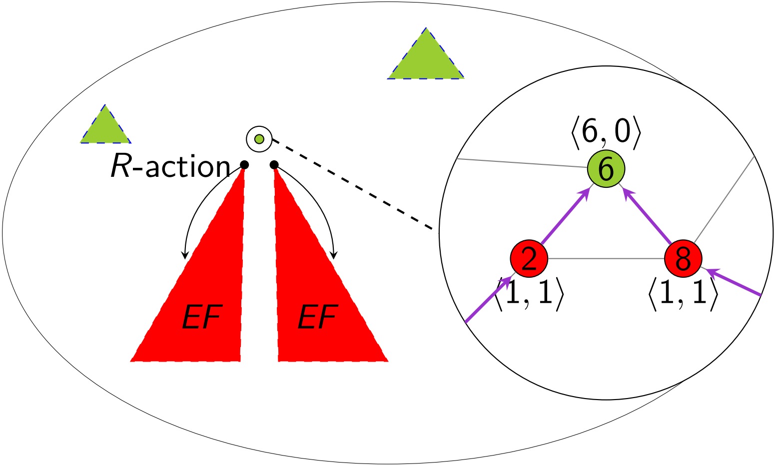

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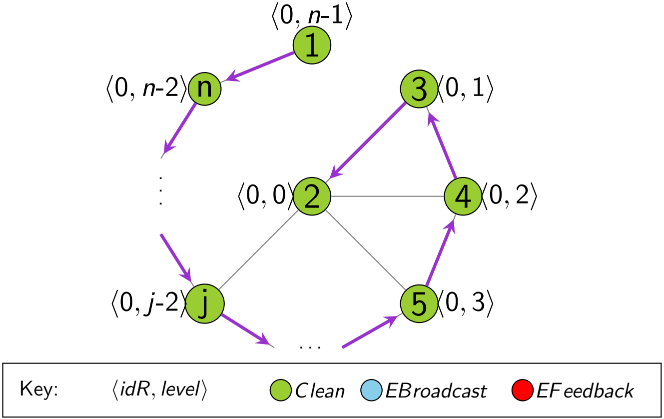
$O(3n + \mathcal{D})$ **rounds**

Lower Bound on the Worst Case Stabilization Time in Rounds

- k links
- $j = k + 3$
- $\mathcal{D} = n - k$



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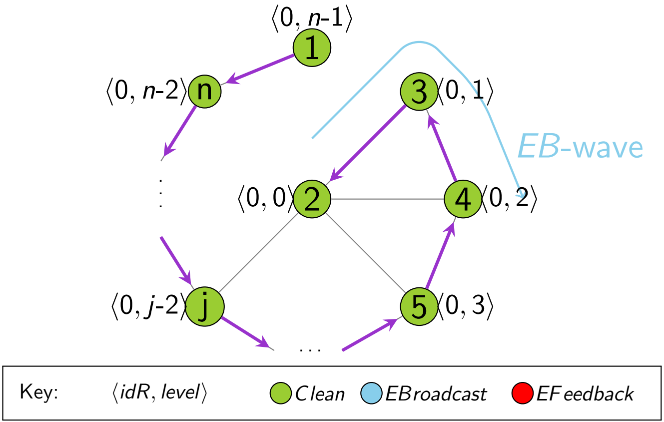

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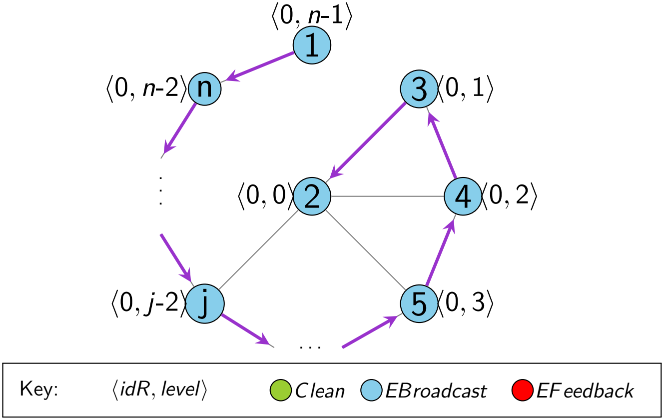

n

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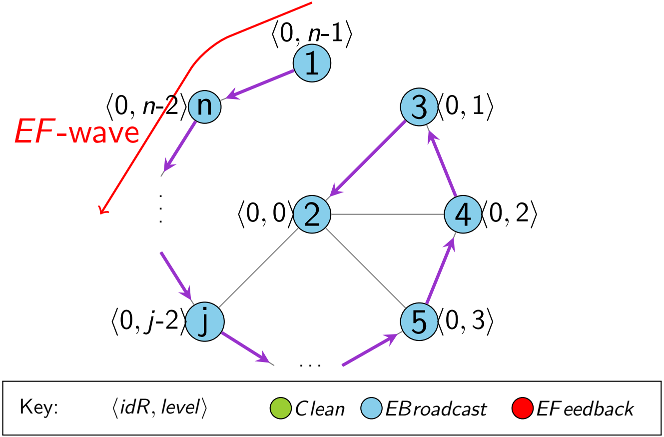

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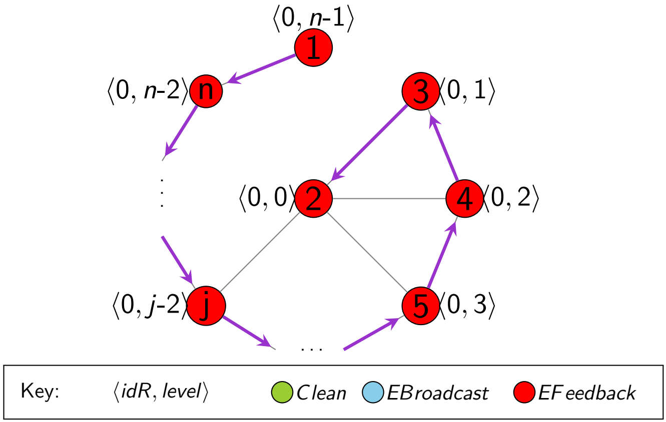

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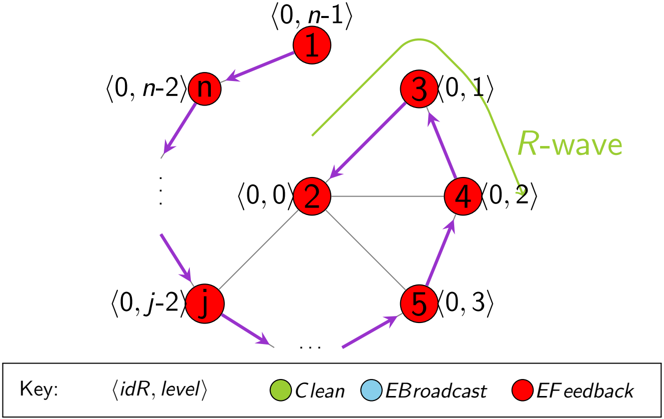

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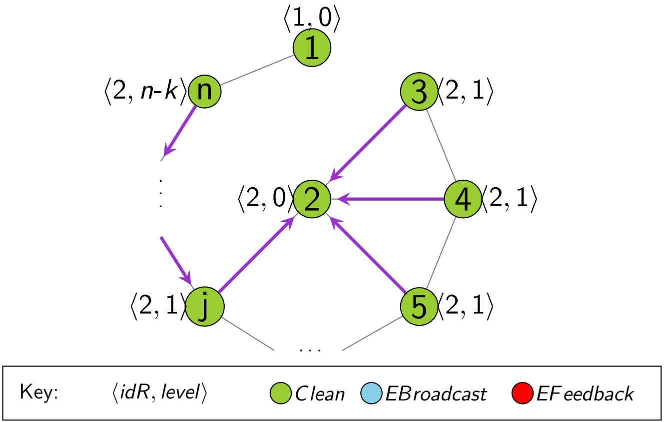

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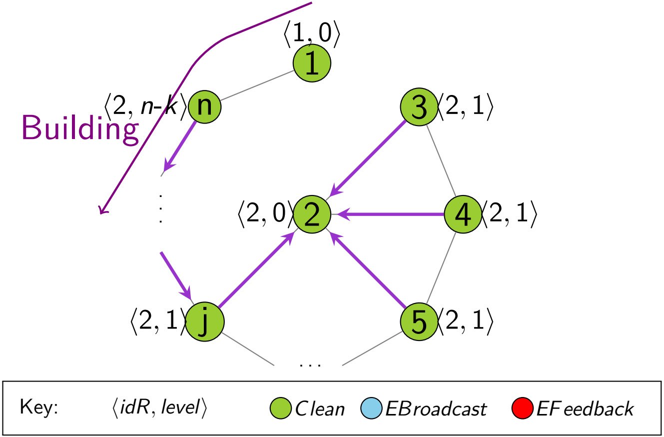

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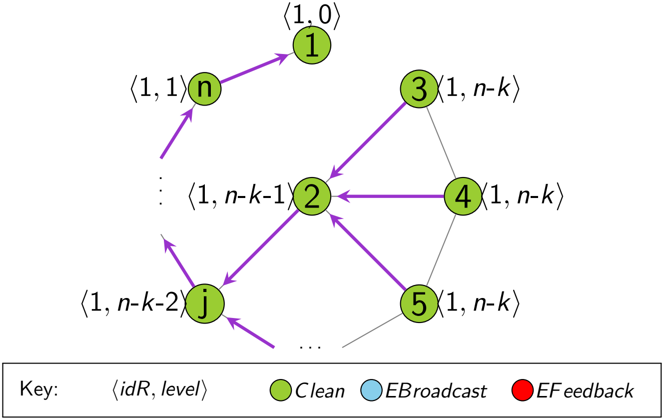

$n + n + n + (n - k)$

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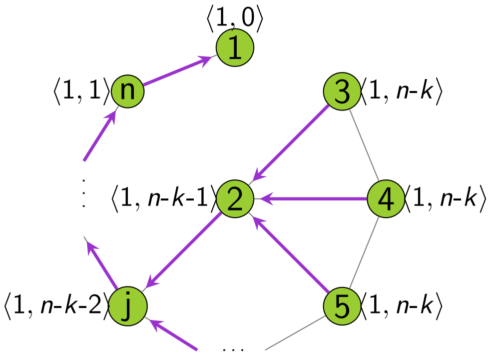
- $j = k + 3$
- $\mathcal{D} = n - k$

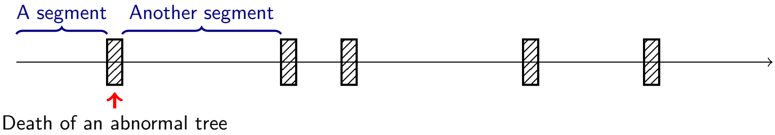


Key: ⟨idR, level⟩ Clean EBroadcast EFeedback

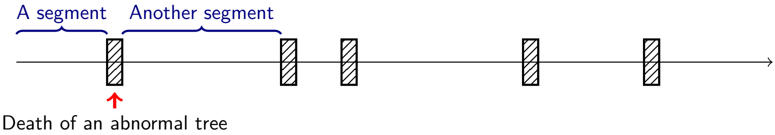
$n + n + n + (n - k)$

$=$ exactly $3n + \mathcal{D}$ rounds

Stabilization Time in Steps

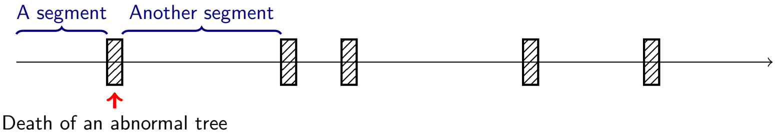


Stabilization Time in Steps



At most *n* alive abnormal trees + No alive abnormal tree created

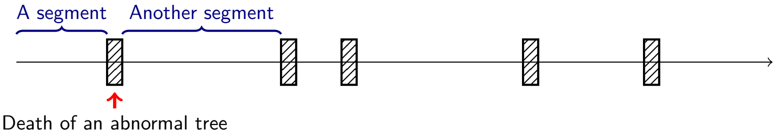
Stabilization Time in Steps



At most n alive abnormal trees + No alive abnormal tree created

$\longrightarrow$ At most $n+1$ segments

Stabilization Time in Steps



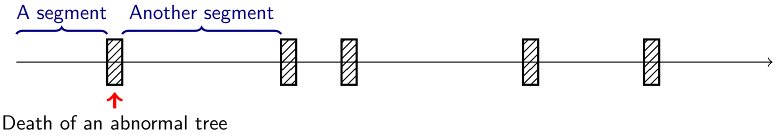
At most n alive abnormal trees $+$ No alive abnormal tree created

$\longrightarrow$ At most $n+1$ segments

In a segment

$idR : 7 \xrightarrow{J\text{-action}} 5 \xrightarrow{J\text{-action}} 3 \xrightarrow{J\text{-action}} 2 \underbrace{\xrightarrow{EB\text{-action}} \xrightarrow{EF\text{-action}} \xrightarrow{R\text{-action}}}_{\text{Death of an abnormal tree = End of the segment}} 7 \xrightarrow{J\text{-action}} 3$

Stabilization Time in Steps



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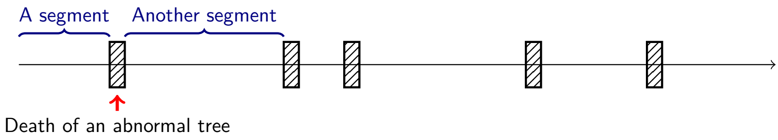
- $n-1$ *J*-actions
- 1 *EB*-action
- 1 *EF*-action
- 1 *R*-action

Stabilization Time in Steps



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$\longrightarrow$ At most $n+1$ segments

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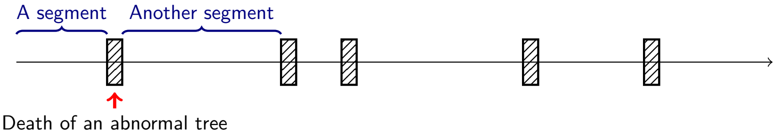
- $n-1$ J-actions
- 1 EB-action
- 1 EF-action
- 1 R-action

$\Rightarrow O(n)$ actions per process

Stabilization Time in Steps



At most n alive abnormal trees $+$ No alive abnormal tree created

$\longrightarrow$ At most $n+1$ segments

In a segment

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Death of an abnormal tree = End of the segment

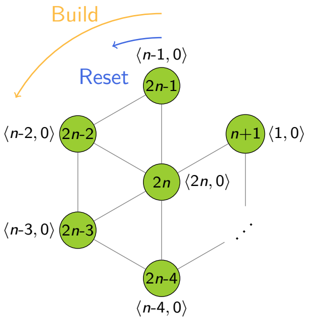
- $n-1$ J-actions
- 1 EB-action
- 1 EF-action
- 1 R-action

$\Rightarrow O(n)$ actions per process

$O(n^3)$ steps

Lower Bound: $\frac{n^3}{6} + \frac{5}{2}n^2 - \frac{11}{3}n + 2$ steps Upper Bound: $\frac{n^3}{2} + 2n^2 + \frac{n}{2} + 1$ steps

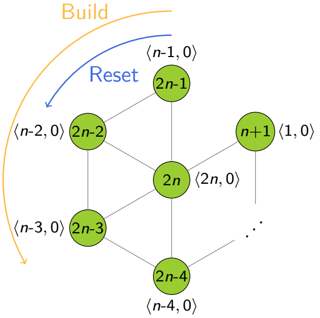
Lower Bound on the Worst Case Stabilization Time in Steps



Key: ⟨*idR*, *level*⟩

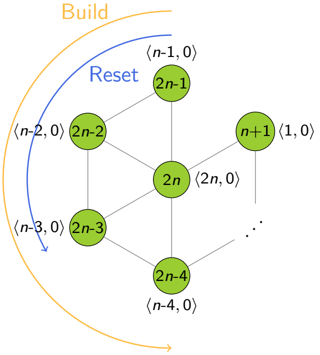
Clean *EBroadcast* *EFeedback*

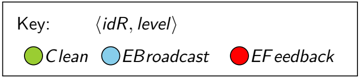
Lower Bound on the Worst Case Stabilization Time in Steps



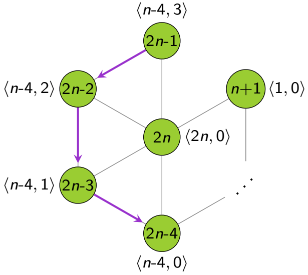
Key:	$\langle idR, level \rangle$		
	Clean	*EBroadcast*	*EFeedback*

Lower Bound on the Worst Case Stabilization Time in Steps





Lower Bound on the Worst Case Stabilization Time in Steps



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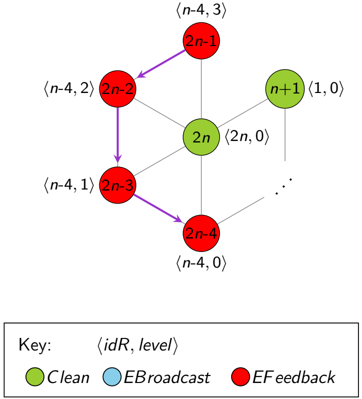
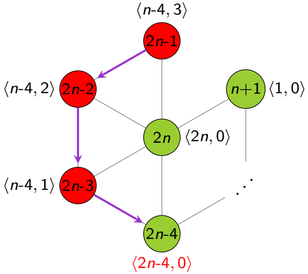
Clean *EBroadcast* *EFeedback*

Lower Bound on the Worst Case Stabilization Time in Steps



Lower Bound on the Worst Case Stabilization Time in Steps



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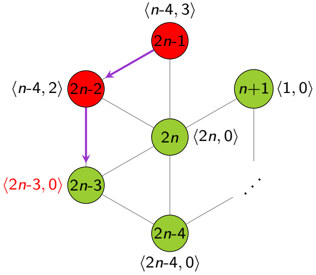
Lower Bound on the Worst Case Stabilization Time in Steps



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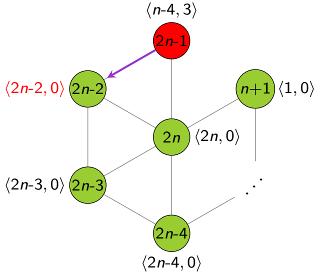

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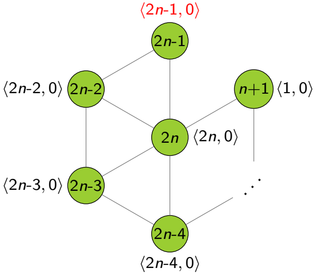
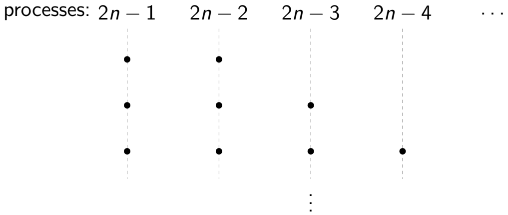


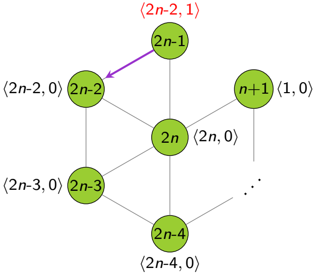
Key: ⟨*idR*, *level*⟩

Clean *EBroadcast* *EFeedback*

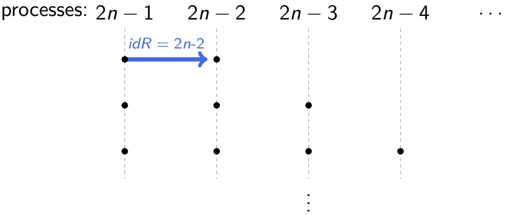
Case of the reset of $2n - 4$



Lower Bound on the Worst Case Stabilization Time in Steps



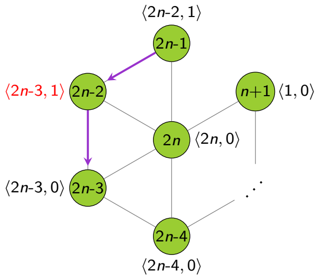
Case of the reset of $2n - 4$

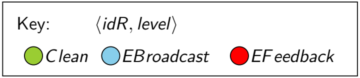


Key: ⟨*idR*, *level*⟩

Clean *EBroadcast* *EFeedback*

Lower Bound on the Worst Case Stabilization Time in Steps





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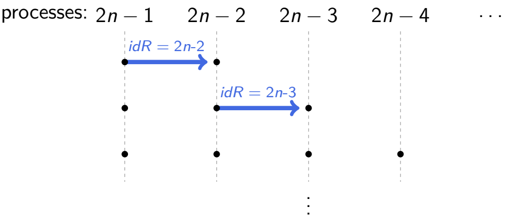
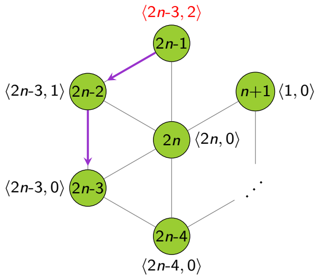
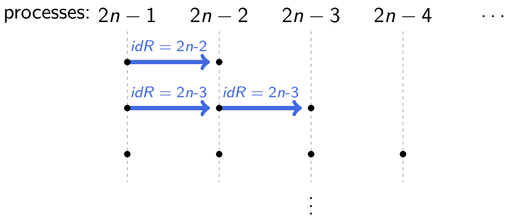


Lower Bound on the Worst Case Stabilization Time in Steps



Case of the reset of $2n-4$



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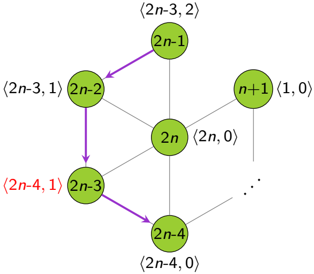

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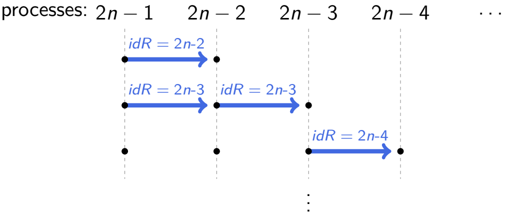
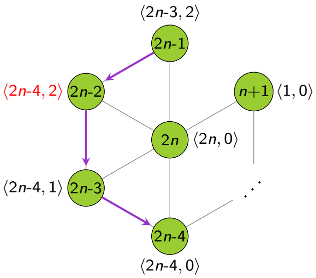


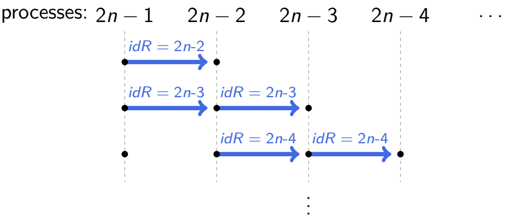
Key: $\langle idR, level \rangle$

Clean *EBroadcast* *EFeedback*

Lower Bound on the Worst Case Stabilization Time in Steps

Case of the reset of $2n-4$





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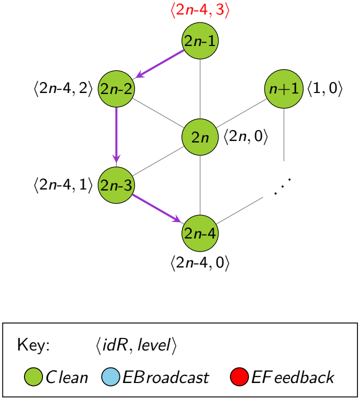
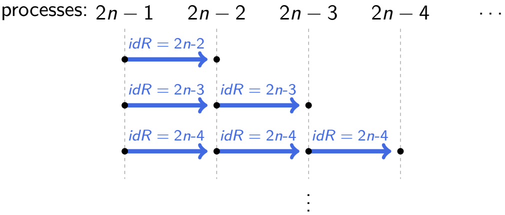
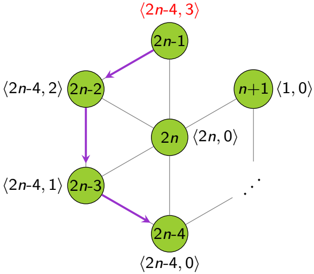


Case of the reset of $2n - 4$



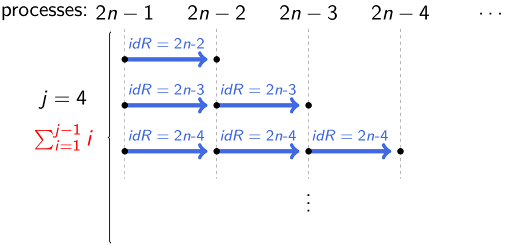
Lower Bound on the Worst Case Stabilization Time in Steps



Key: $\langle idR, level\rangle$

Clean *EBroadcast* *EFeedback*

Case of the reset of $2n-4$



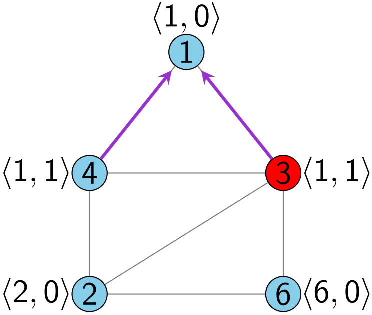
$$\Theta(n) \text{ reset } \Rightarrow \sum_{j=1}^{n} \sum_{i=1}^{j-1} i \Rightarrow \Theta(n^3) \text{ steps}$$

Analytical Study of Datta *et al*, 2011[3]

[3] Datta, Larmore, and Vemula. Self-stabilizing Leader Election in Optimal Space under an Arbitrary Scheduler. 2011

Principles

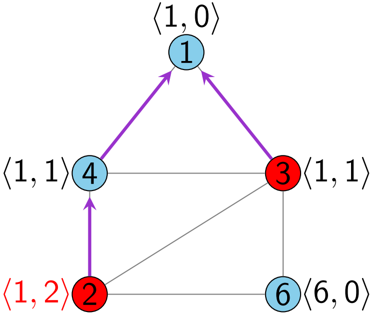
Join a tree



Key: $\langle \mathit{idR}, \mathit{level} \rangle$ Can be joined Cannot be joined

Principles

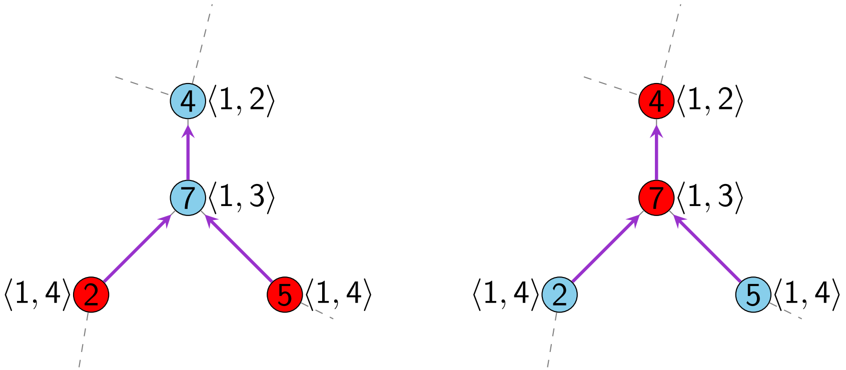
Join a tree



Key: ⟨*idR*, *level*⟩ Can be joined Cannot be joined

Principles

Change of color



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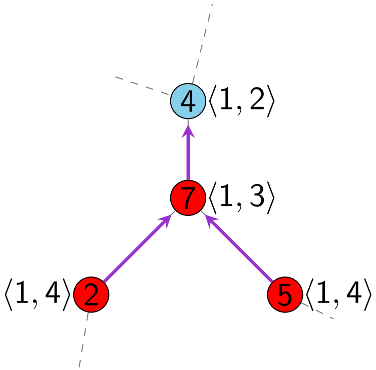

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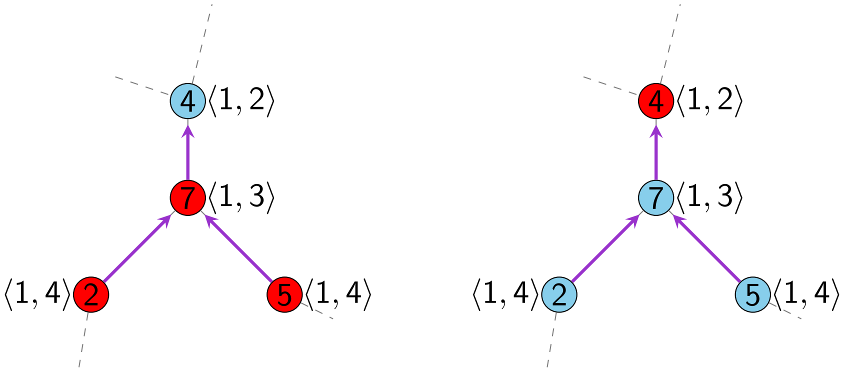

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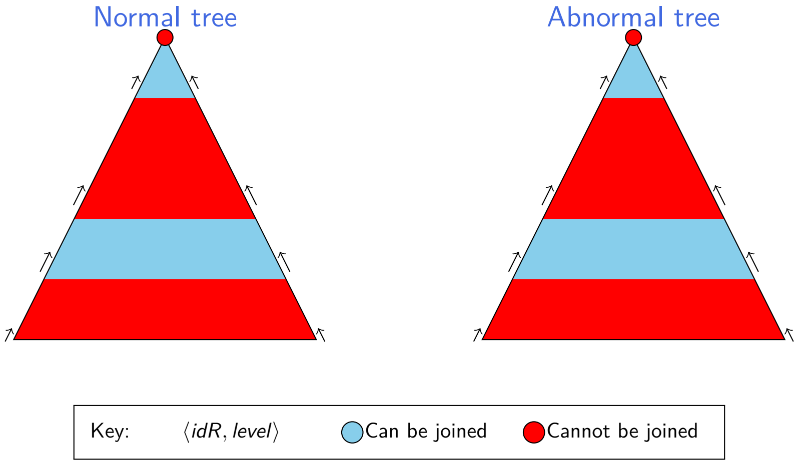
Principles

Color Waves Absorption



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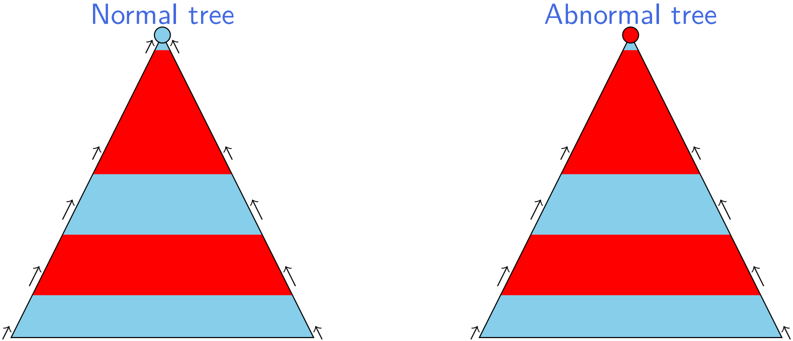

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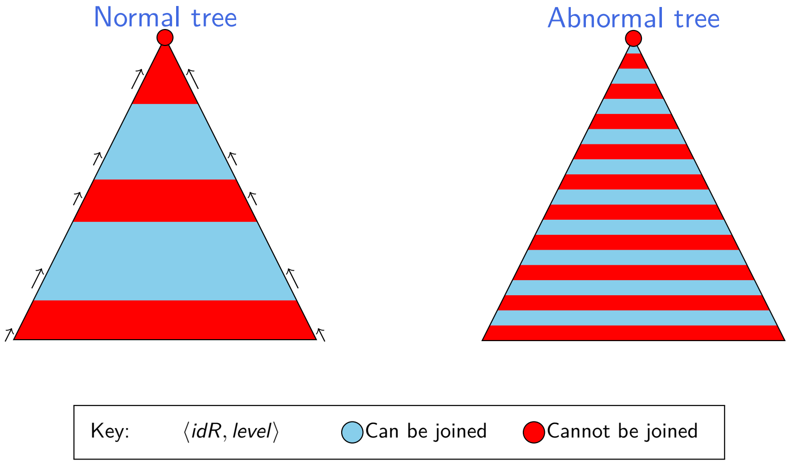


Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

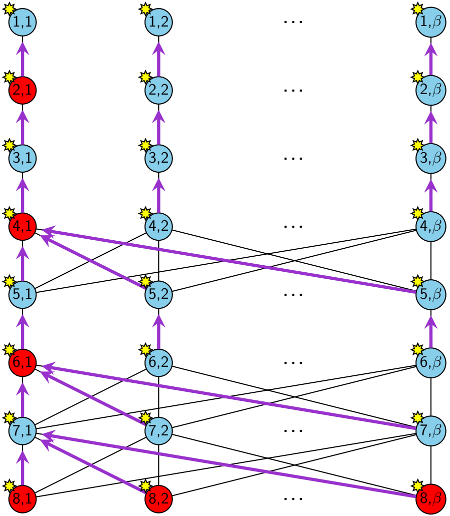
Key:

$(i, j).ID = (i-1)\beta + j$

$(i, j).idR = 0$

Can be joined

Cannot be joined



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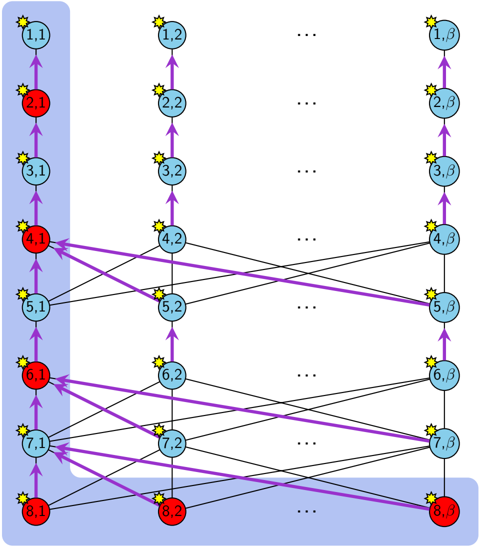

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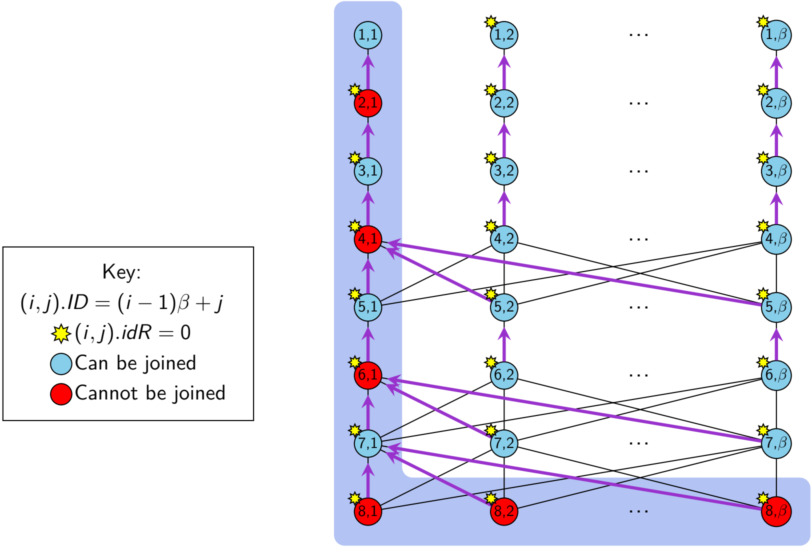

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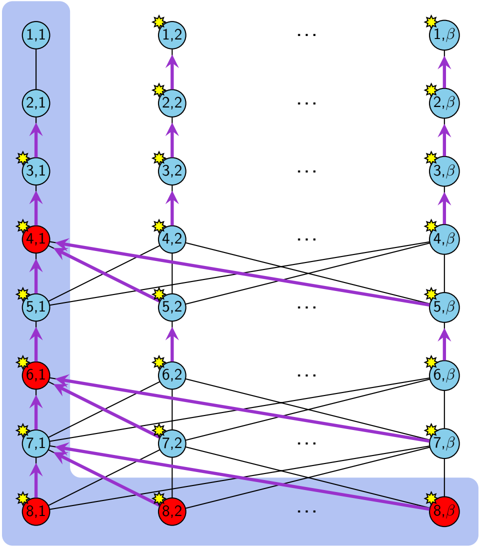

Datta et al, 2011

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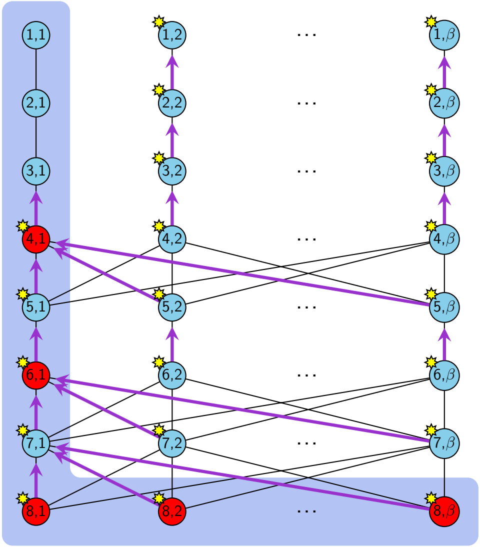

Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$ (star)

Can be joined (blue)

Cannot be joined (red)

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i,j).ID = (i-1)\beta + j$

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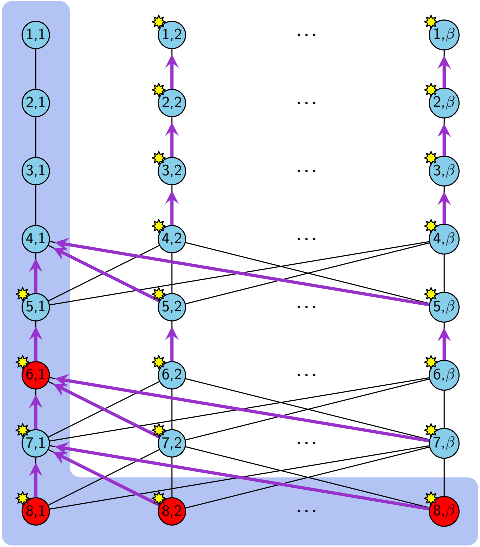

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

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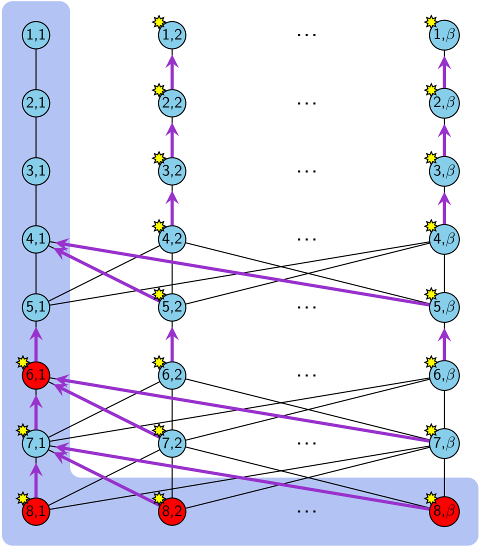
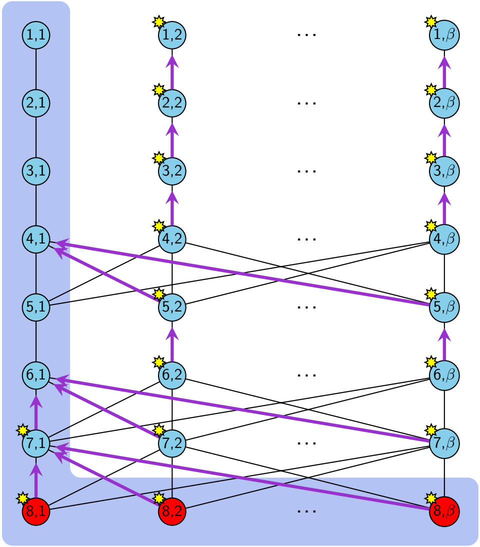


Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$



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Datta et al, 2011

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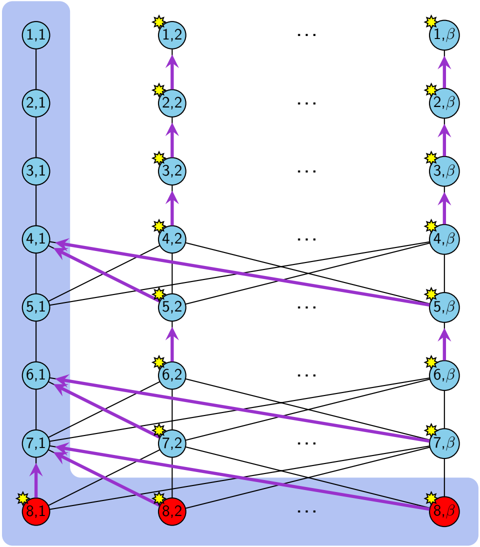

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β

Key:
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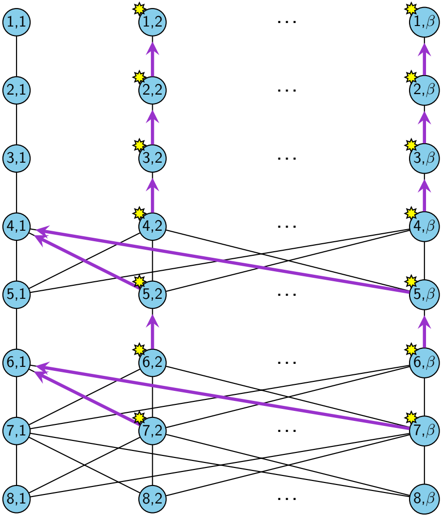
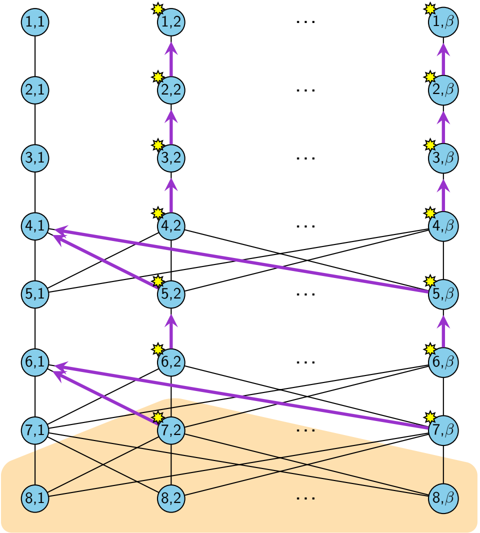
$(i,j).idR = 0$
Can be joined
Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β



Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$

Can be joined

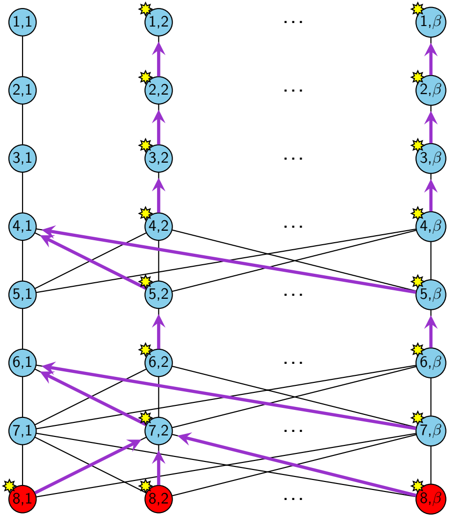
Cannot be joined

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β

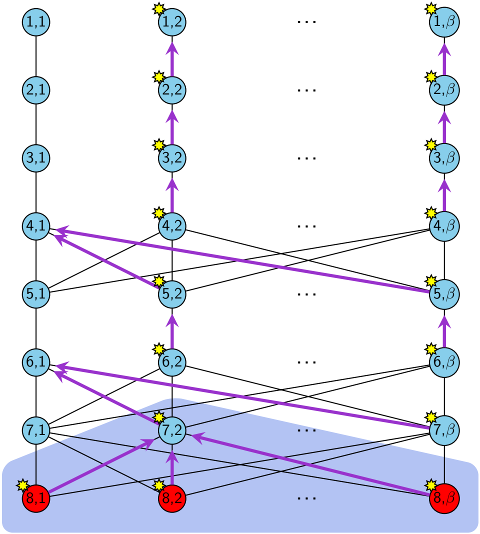
Key:
$(i,j).ID = (i-1)\beta + j$
$(i,j).idR = 0$
Can be joined
Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β



Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$

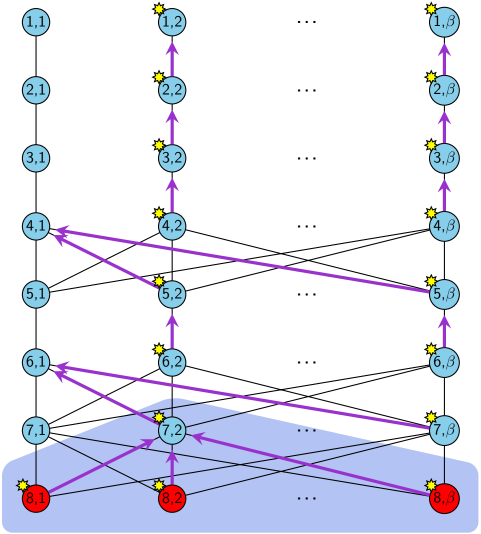
Can be joined

Cannot be joined

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β



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$(i,j).idR = 0$

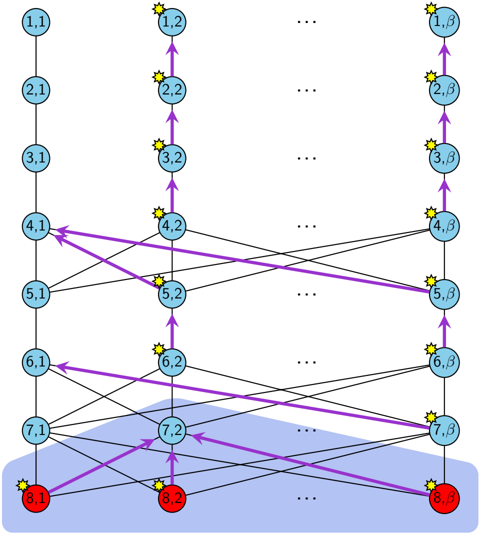
Can be joined

Cannot be joined

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β

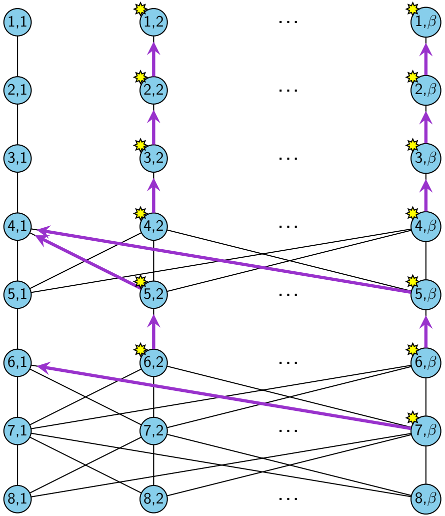


Key:
$(i,j).ID = (i-1)\beta + j$
$(i,j).idR = 0$
Can be joined
Cannot be joined

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β



Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$

Can be joined

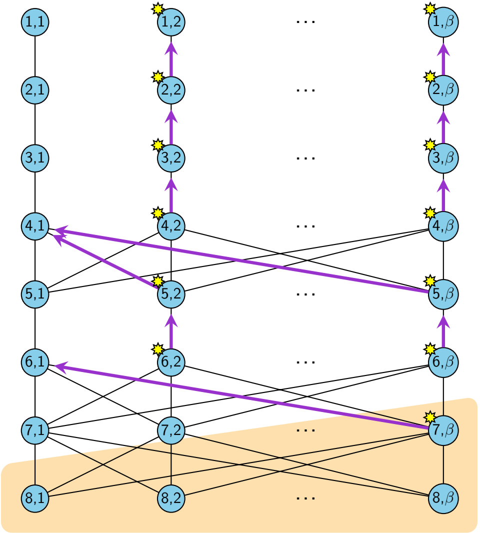
Cannot be joined

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β

Key:
$(i,j).ID = (i-1)\beta + j$
$(i,j).idR = 0$
Can be joined
Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β

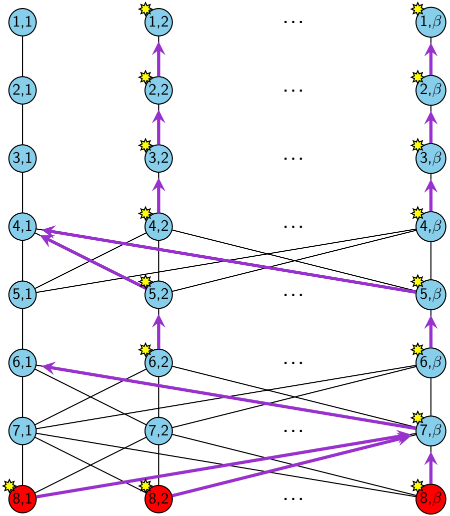
Key:

$(i, j).ID = (i-1)\beta + j$

$(i, j).idR = 0$ (star)

Can be joined (blue)

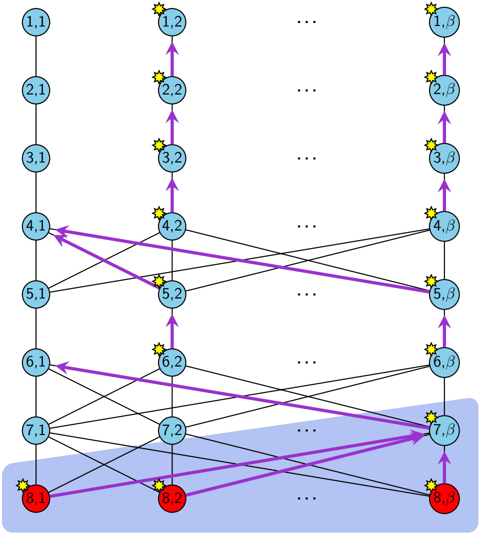
Cannot be joined (red)



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β



Key:

$(i,j).ID = (i-1)\beta + j$

★ $(i,j).idR = 0$

● Can be joined

● Cannot be joined

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β

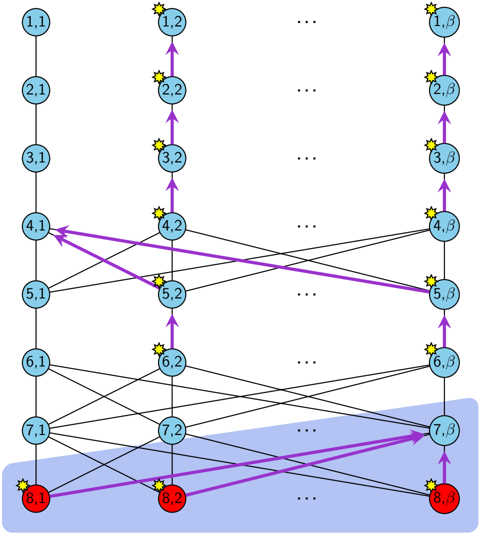
Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$

Can be joined

Cannot be joined

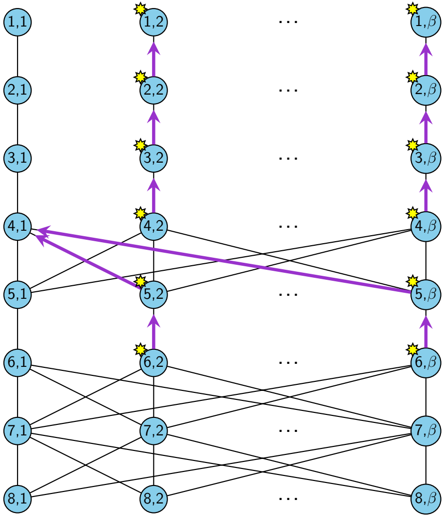


Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

Key:
$(i,j).ID = (i-1)\beta + j$
$(i,j).idR = 0$
Can be joined
Cannot be joined

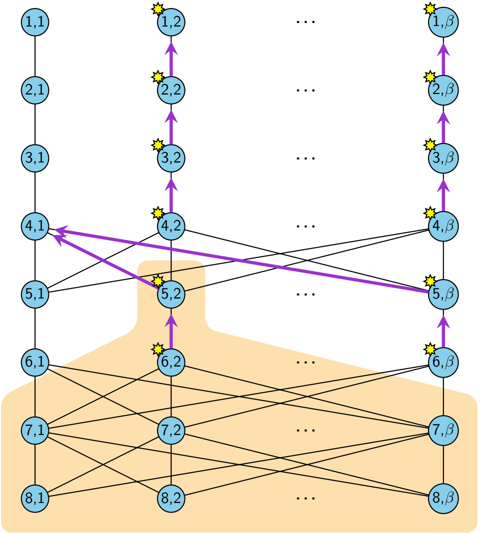


Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

Key:
$(i,j).ID = (i-1)\beta + j$
$(i,j).idR = 0$
Can be joined
Cannot be joined

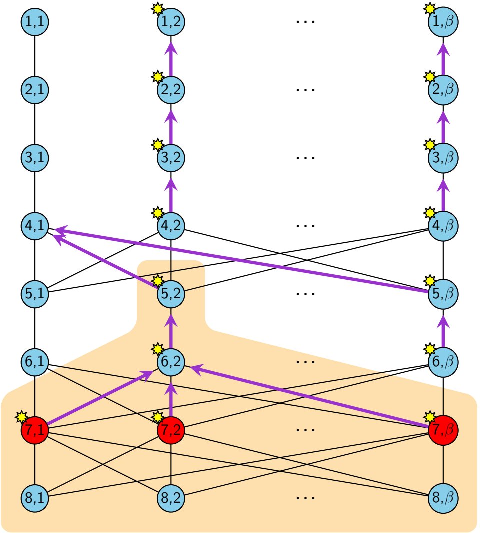


Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

Key:
$(i,j).ID = (i-1)\beta + j$
$(i,j).idR = 0$
Can be joined
Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

Key:

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$(i,j).idR = 0$

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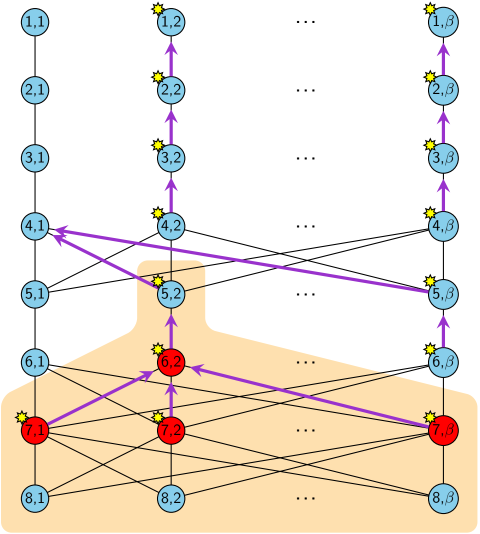
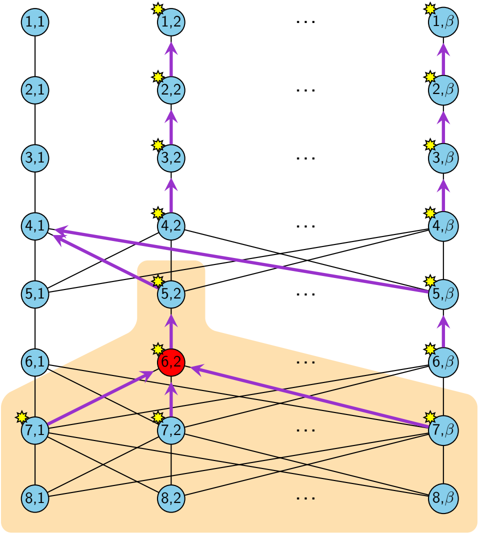


Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2



Key:

$(i, j).ID = (i-1)\beta + j$

$(i, j).idR = 0$

Can be joined

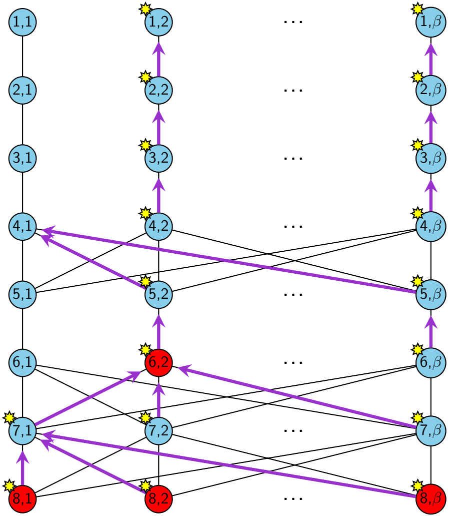
Cannot be joined

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

Key:
$(i,j).ID = (i-1)\beta + j$
$(i,j).idR = 0$
Can be joined
Cannot be joined

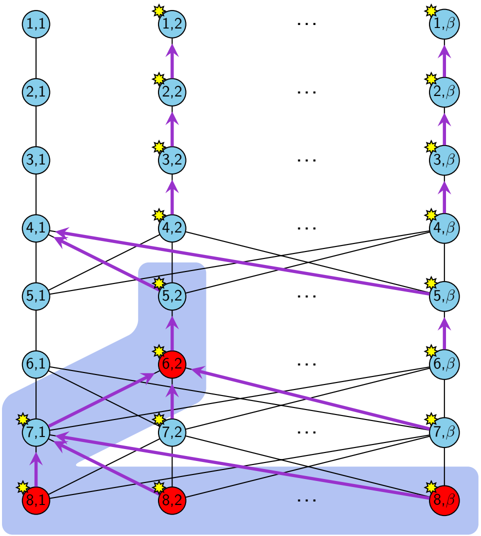


Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

Key:
$(i,j).ID = (i-1)\beta + j$
$(i,j).idR = 0$
Can be joined
Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

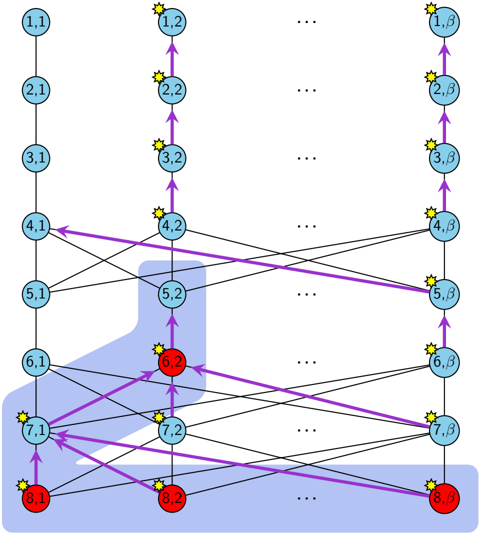
Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$ (star)

Can be joined (blue)

Cannot be joined (red)

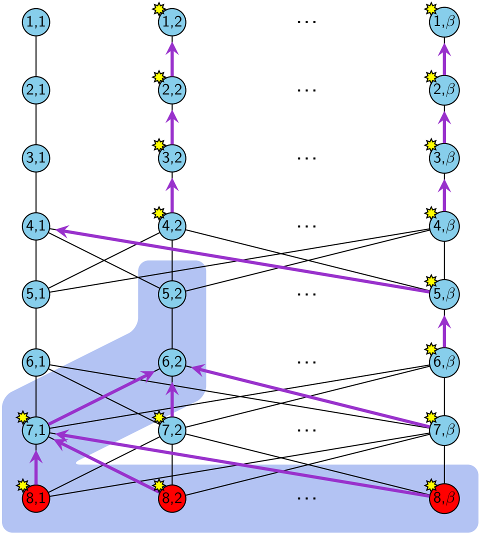


Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

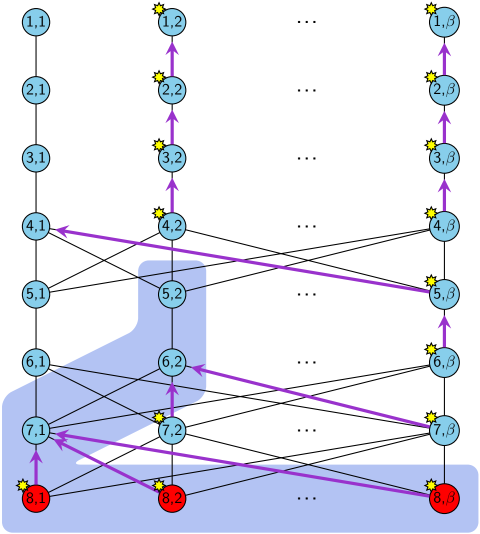
Key:
$(i,j).ID = (i-1)\beta + j$
$(i,j).idR = 0$
Can be joined
Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2



Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$

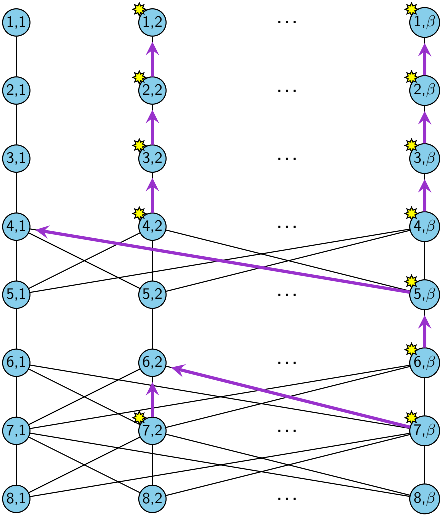
Can be joined

Cannot be joined

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2



Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$

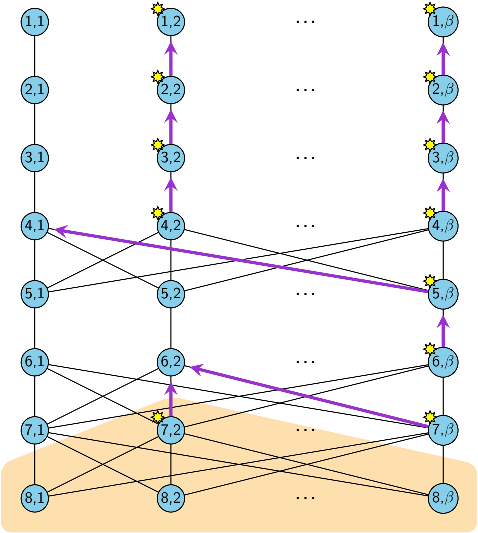
Can be joined

Cannot be joined

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2



Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$ (star)

Can be joined (blue)

Cannot be joined (red)

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

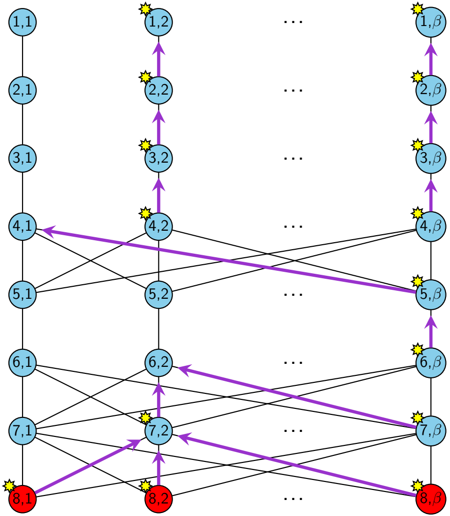
Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$

Can be joined

Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

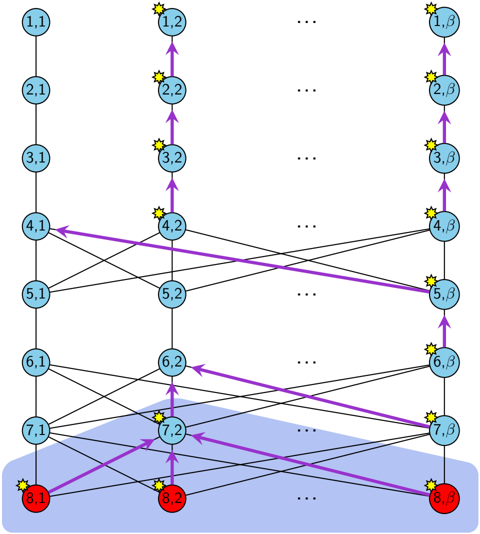
Key:

$(i, j).ID = (i-1)\beta + j$

$(i, j).idR = 0$ (marked by a star)

Can be joined (blue)

Cannot be joined (red)



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

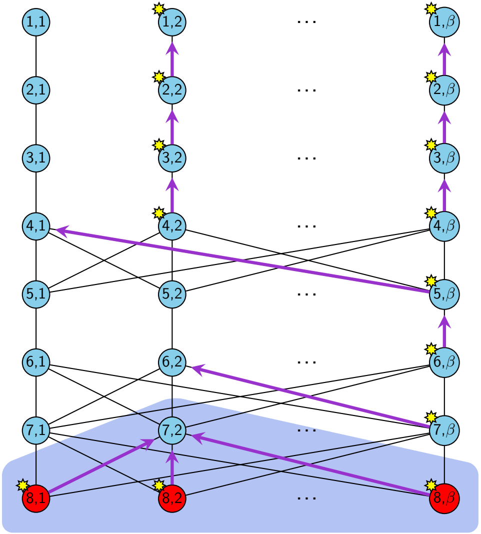
Key:

$(i, j).ID = (i-1)\beta + j$

$(i, j).idR = 0$

Can be joined

Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

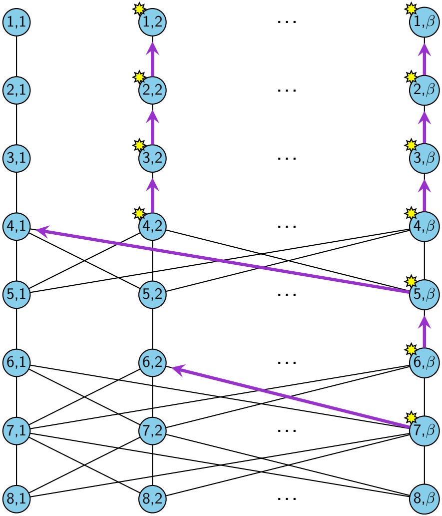
Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$ (star)

Can be joined (blue)

Cannot be joined (red)



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

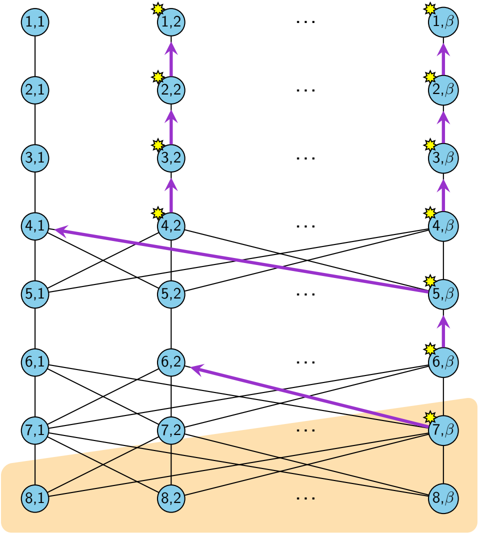
Key:

$(i, j).ID = (i-1)\beta + j$

$(i, j).idR = 0$

Can be joined

Cannot be joined

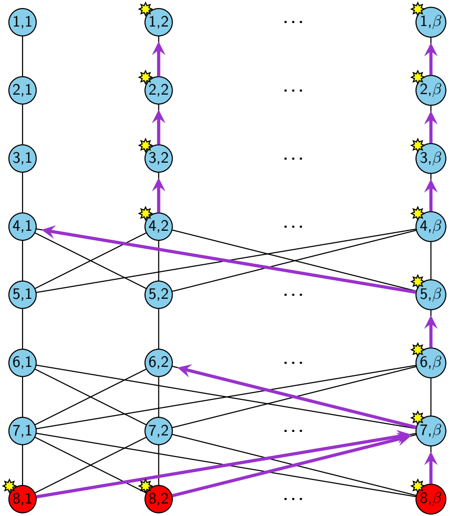


Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

Key:
$(i, j).ID = (i-1)\beta + j$
$(i, j).idR = 0$
Can be joined
Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

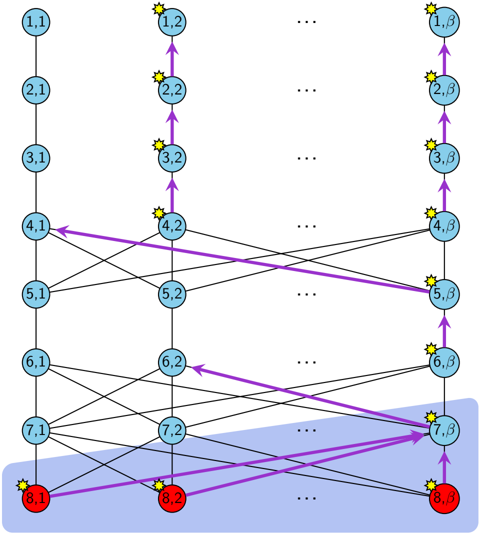
Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$

Can be joined

Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

Key:

$(i,j).ID = (i-1)\beta + j$

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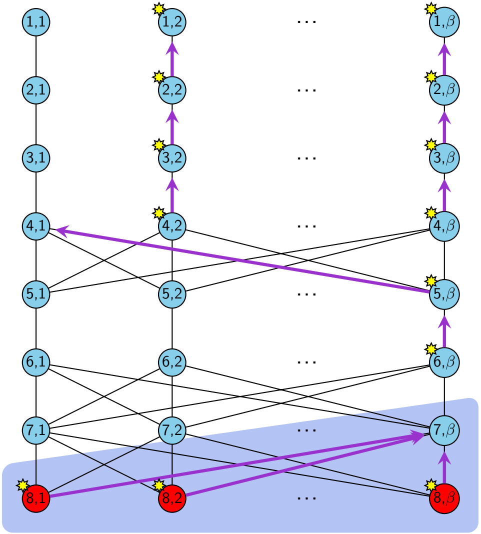
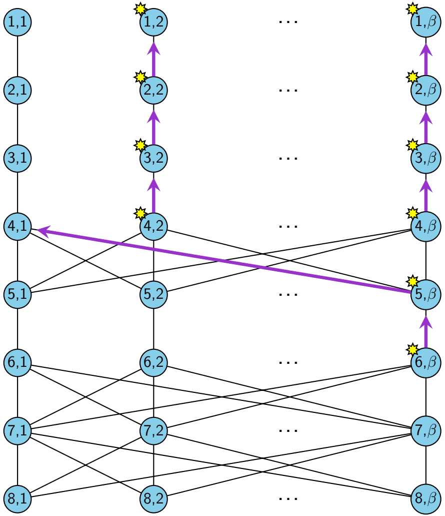
Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2



Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$

Can be joined

Cannot be joined

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

Key:

$(i,j).ID = (i-1)\beta + j$

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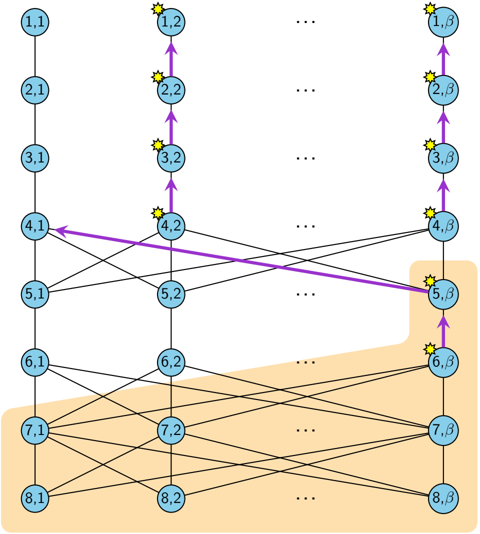

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

Key:

$(i,j).ID = (i-1)\beta + j$

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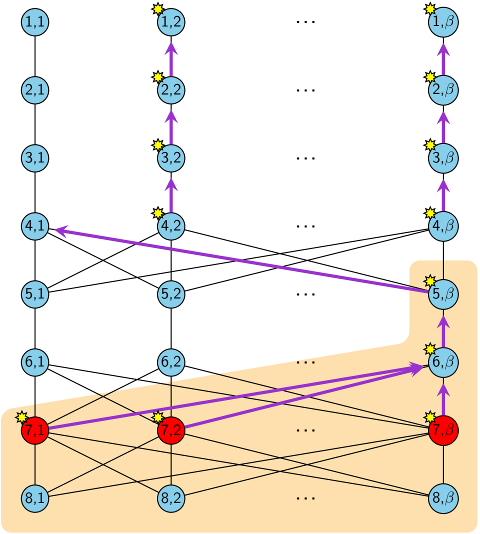


Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

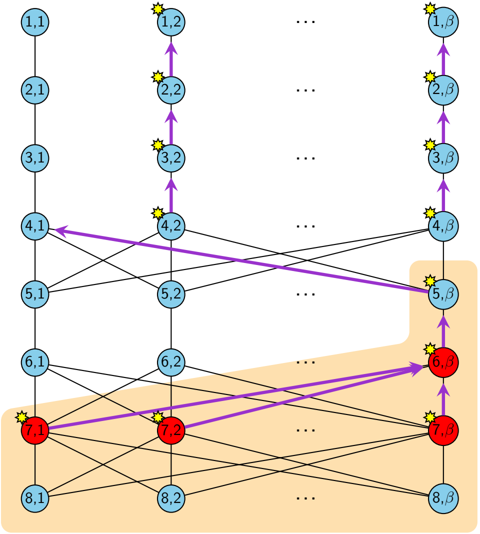
Key:

$(i,j).ID = (i-1)\beta + j$

✸ $(i,j).idR = 0$

● Can be joined

● Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

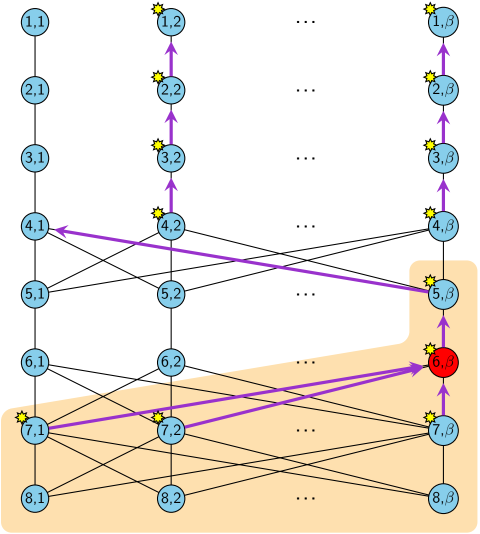
Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$

Can be joined

Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

Key:

$(i,j).ID = (i-1)\beta + j$

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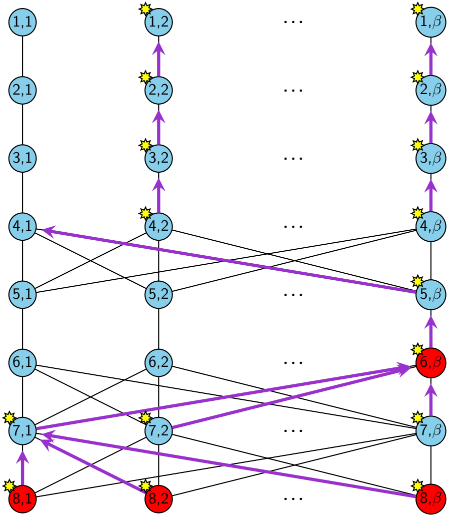

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

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$(i,j).ID = (i-1)\beta + j$

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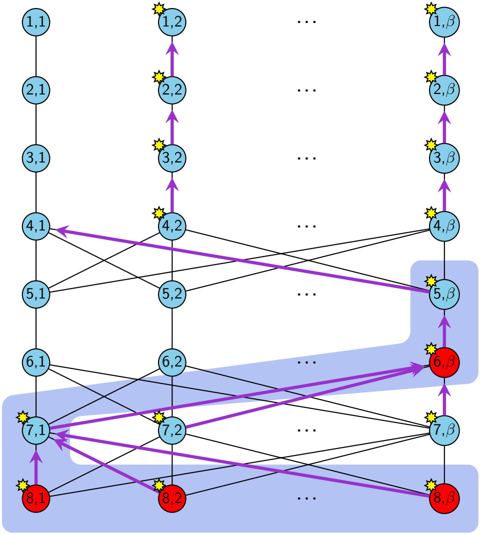

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

Key:

$(i,j).ID = (i-1)\beta + j$

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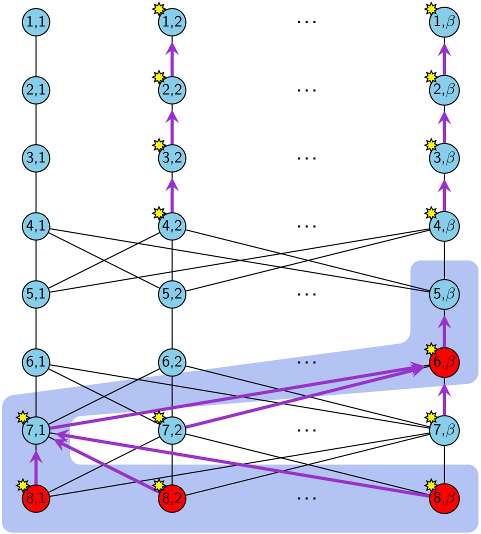
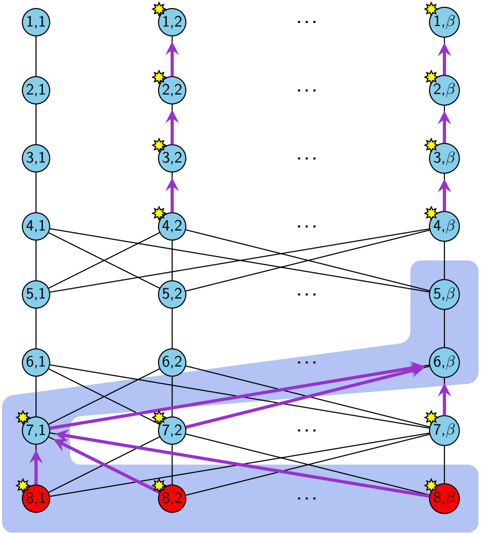


Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2



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Datta et al, 2011

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β^2

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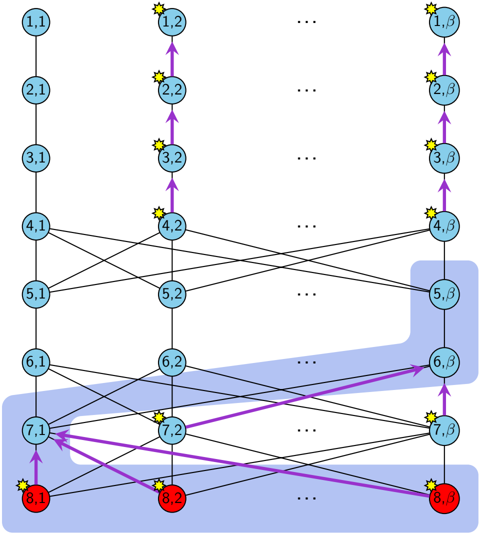
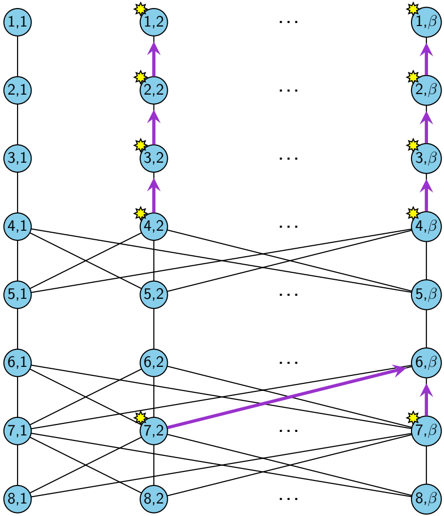


Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2



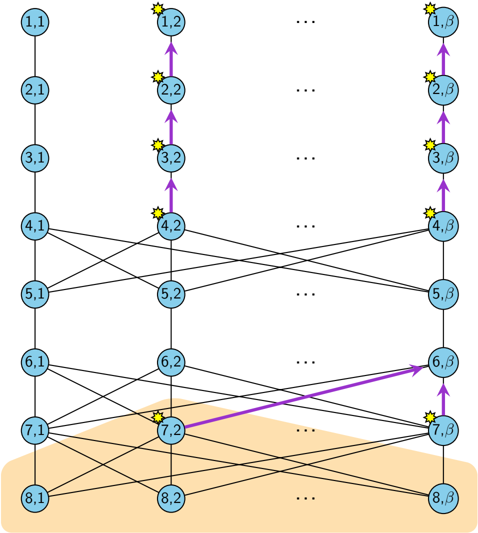
Key:
$(i,j).ID = (i-1)\beta + j$
$(i,j).idR = 0$
Can be joined
Cannot be joined

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

Key:
$(i,j).ID = (i-1)\beta + j$
$(i,j).idR = 0$
Can be joined
Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

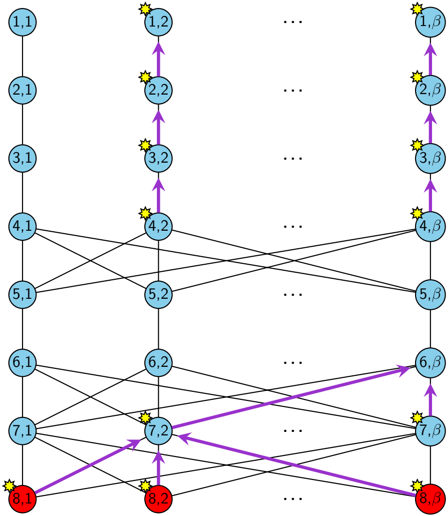
Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

Can be joined

Cannot be joined

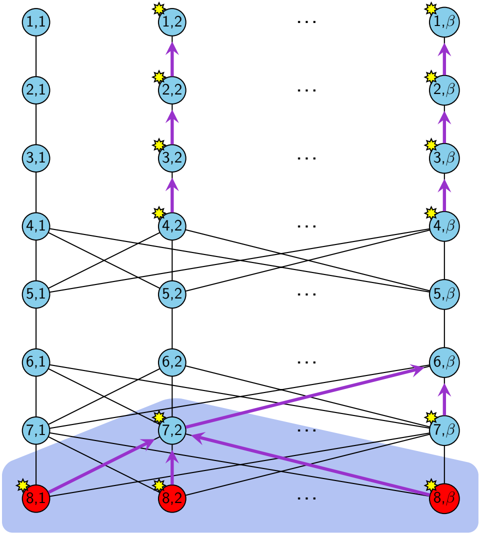


Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

Key:
$(i,j).ID = (i-1)\beta + j$
$(i,j).idR = 0$
Can be joined
Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

Key:
$(i,j).ID = (i-1)\beta + j$
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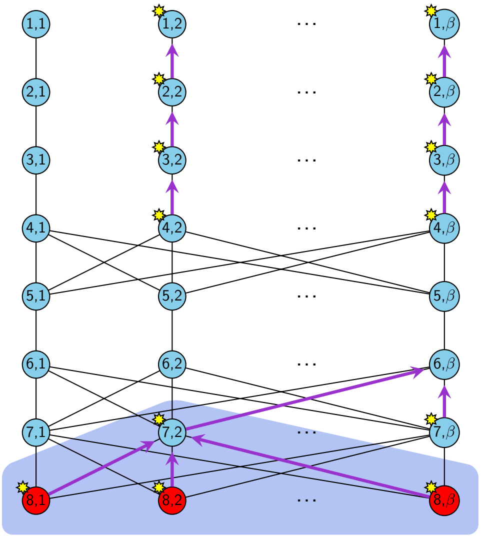
Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

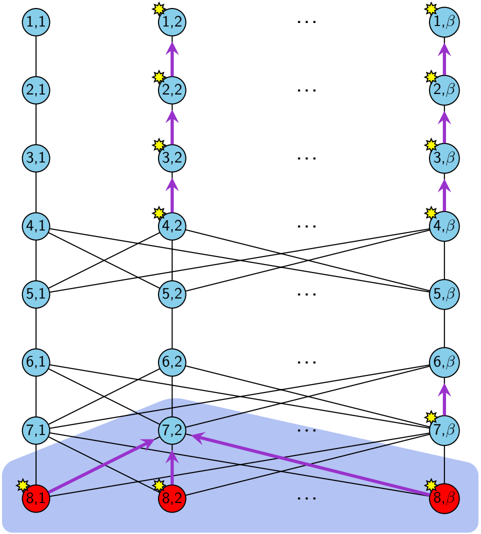
Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$ (star)

Can be joined (blue circle)

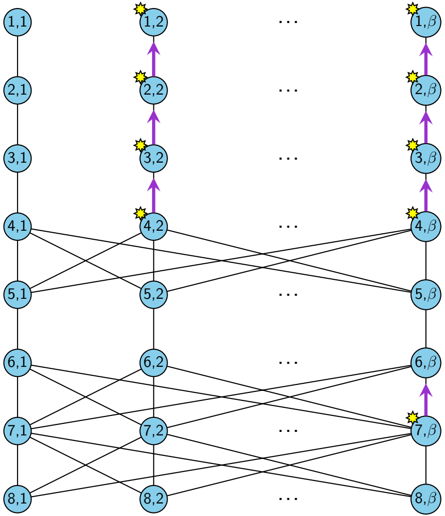
Cannot be joined (red circle)



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2



Key:

$(i,j).ID = (i-1)\beta + j$

✸ $(i,j).idR = 0$

● Can be joined

● Cannot be joined

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

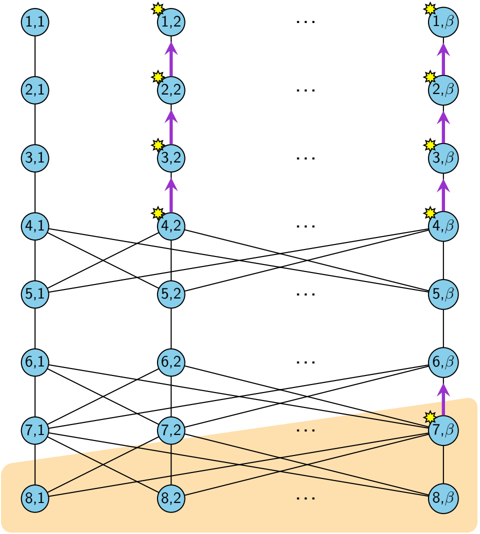
Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$

Can be joined

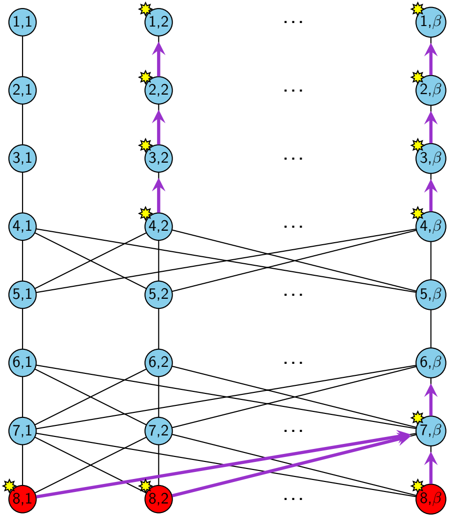
Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2



Key:
$(i, j).ID = (i-1)\beta + j$
$(i, j).idR = 0$
Can be joined
Cannot be joined

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

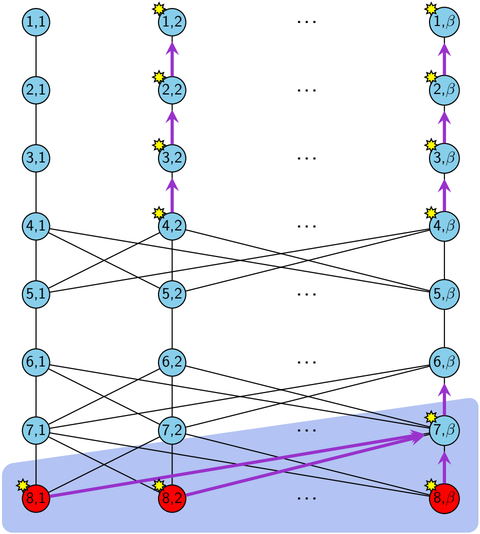
Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$

Can be joined

Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^2

Key:

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Can be joined

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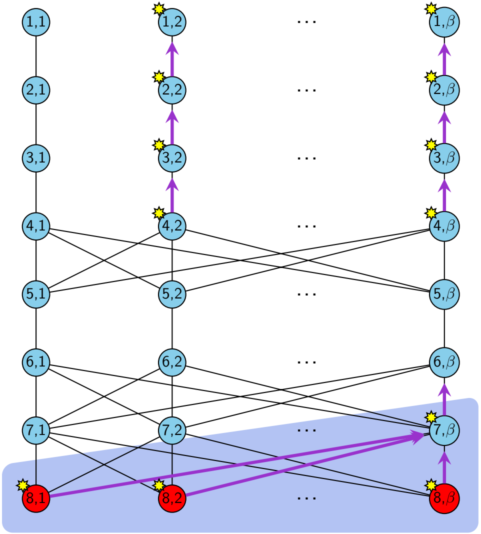
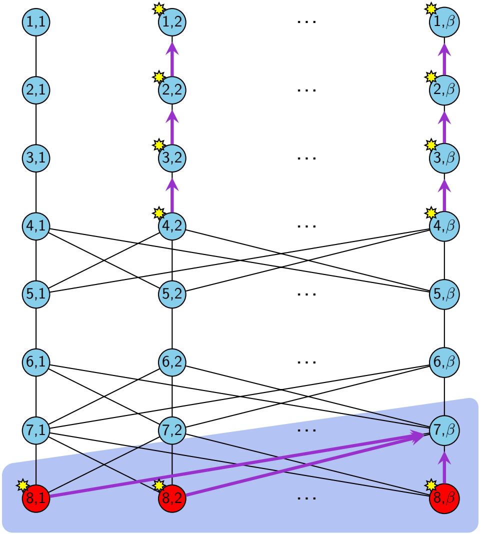


Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

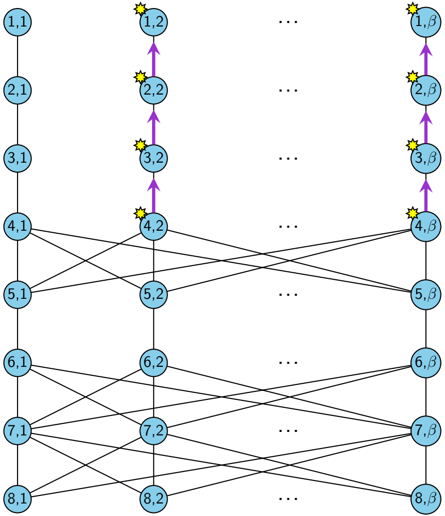
β^2

Key:
$(i, j).ID = (i-1)\beta + j$
$(i, j).idR = 0$
Can be joined
Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$



β^3

Key:

$(i, j).ID = (i-1)\beta + j$

$(i, j).idR = 0$

Can be joined

Cannot be joined

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^3

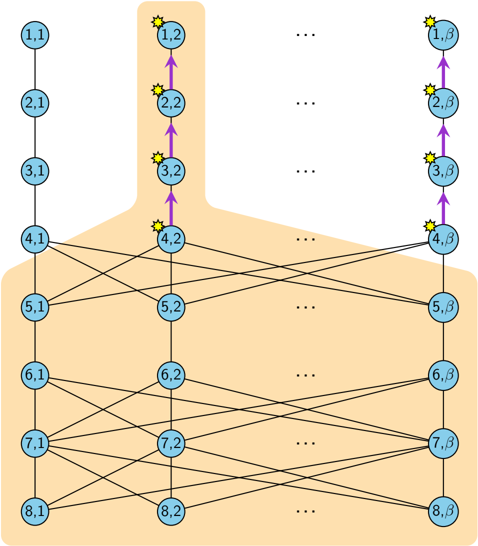
Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$

Can be joined

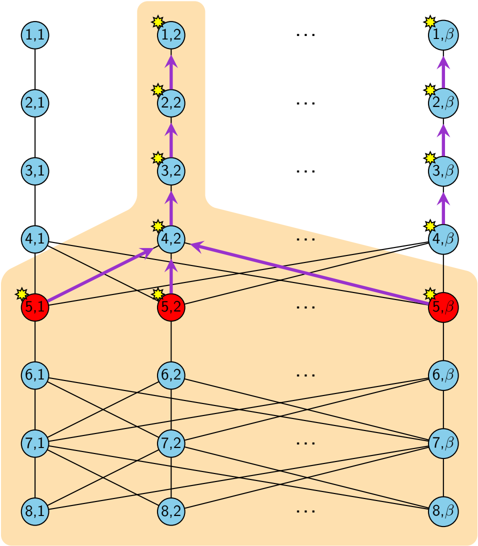
Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^3



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$(i,j).idR = 0$

Can be joined

Cannot be joined

Datta et al, 2011

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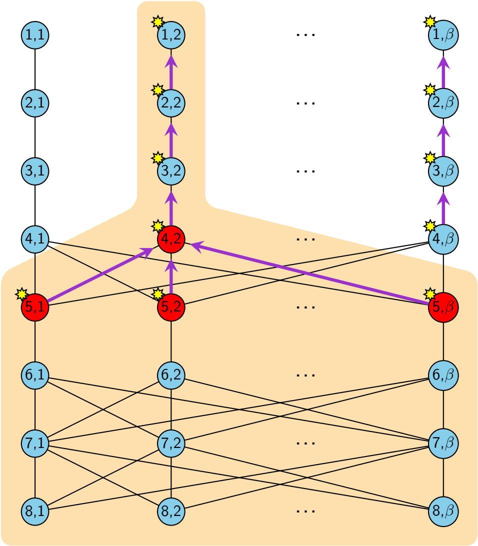
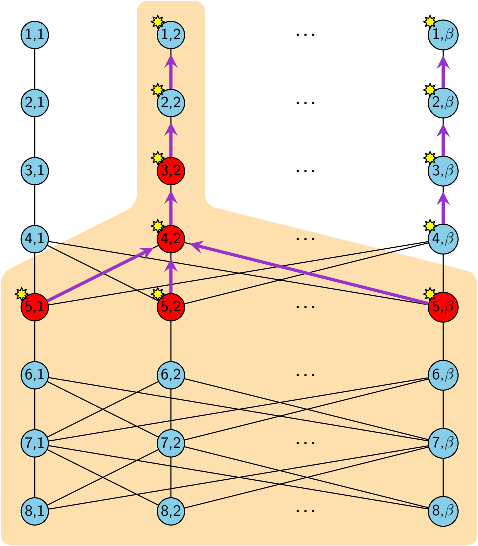


Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^3



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Datta et al, 2011

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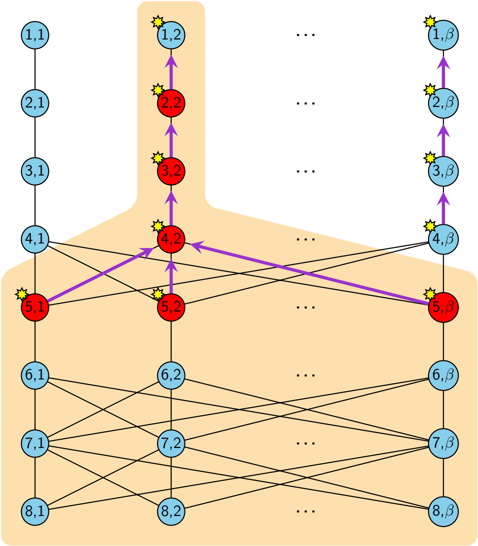

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Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

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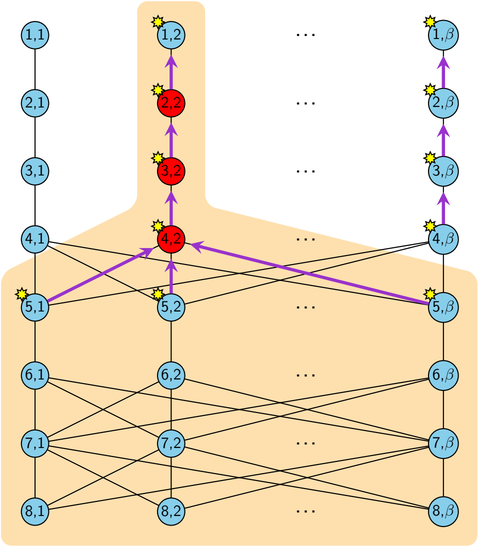
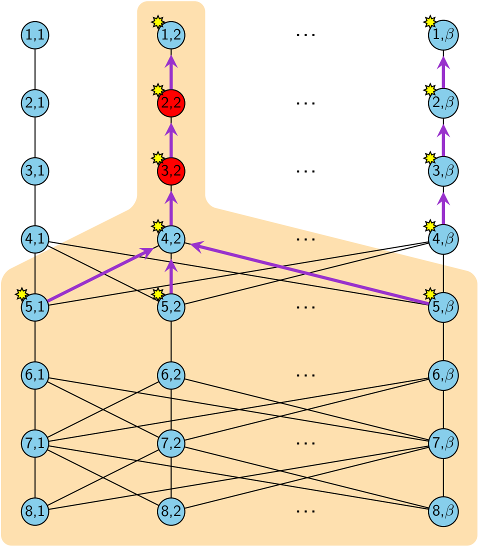


Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^3

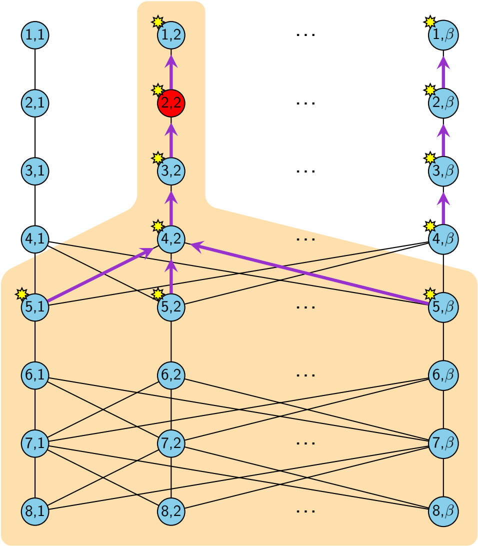


Key:
$(i,j).ID = (i-1)\beta + j$
$(i,j).idR = 0$
Can be joined
Cannot be joined

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^3



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$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$

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Cannot be joined

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^3

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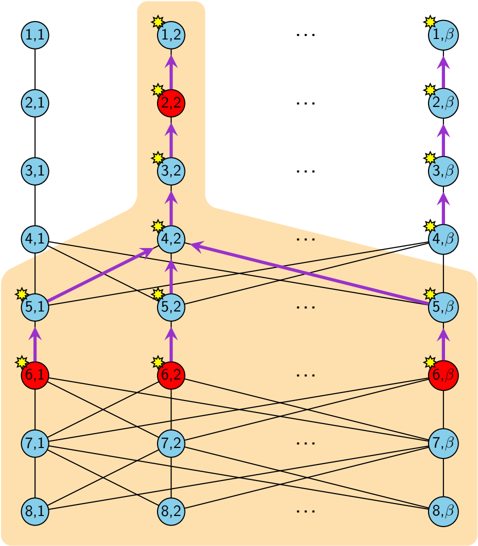
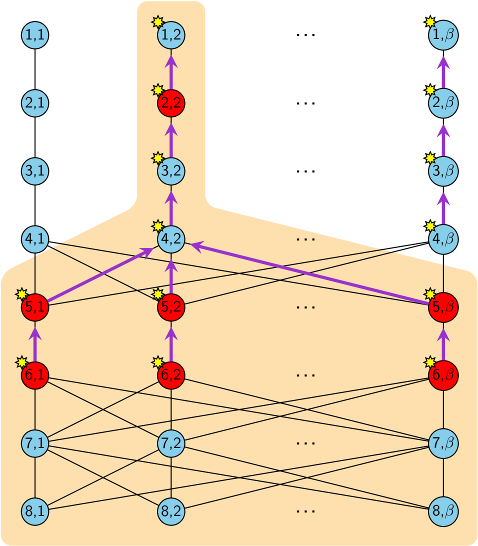
Can be joined
Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^3



Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

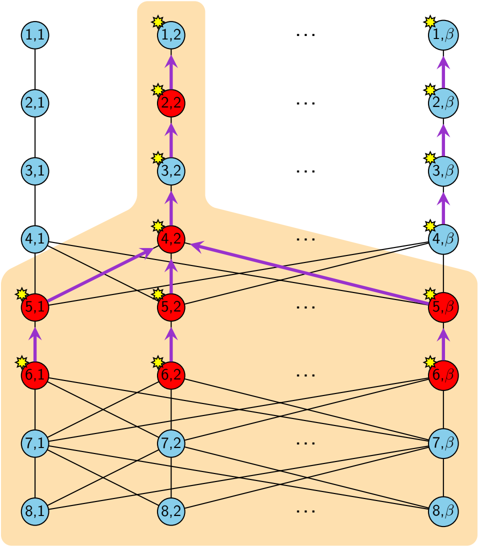
Can be joined

Cannot be joined

Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^3



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Datta et al, 2011

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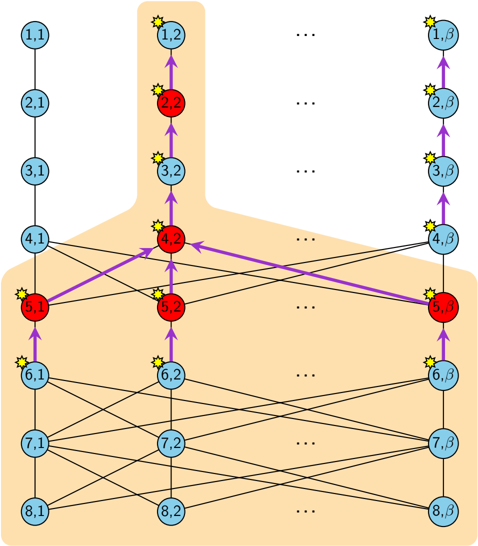

Key:

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$(i,j).idR = 0$

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Cannot be joined

Datta et al, 2011

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β^3

Key:
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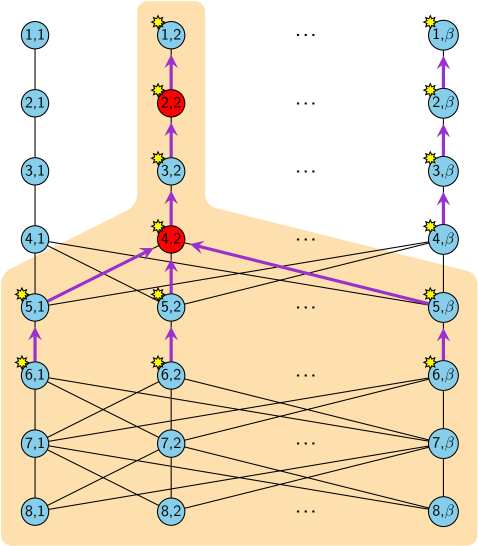
✸ $(i,j).idR = 0$
● Can be joined
● Cannot be joined



Datta et al, 2011

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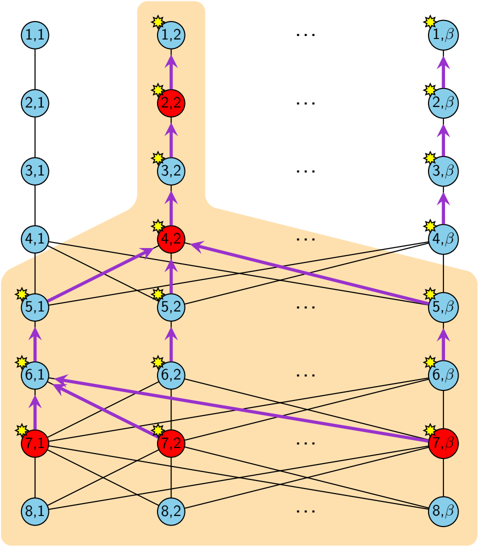
β^3



Key:

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Can be joined

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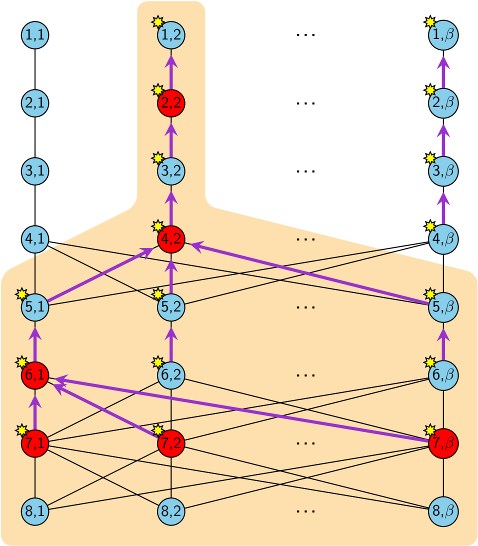
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^3



Key:
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Datta et al, 2011

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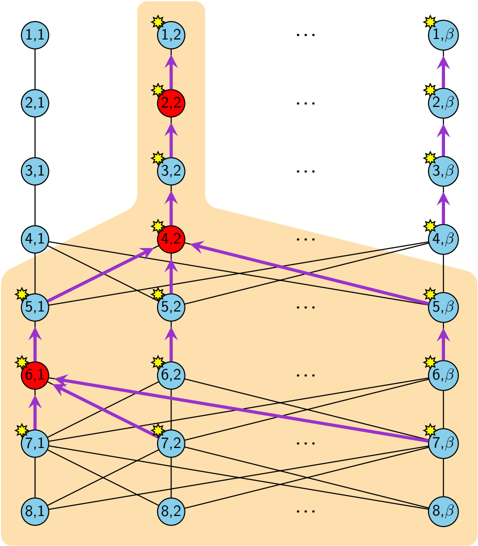
β^3



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Can be joined
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Datta et al, 2011

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β^3

Key:

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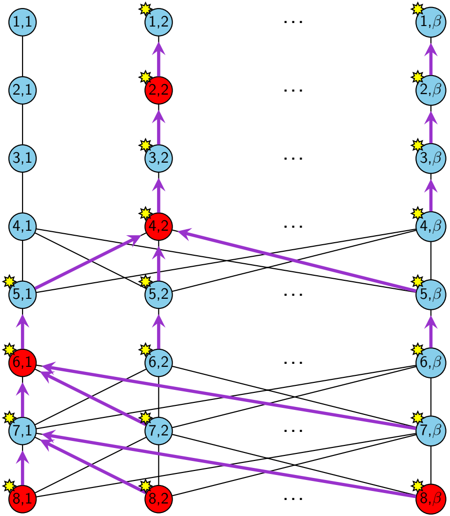
Can be joined

Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^3

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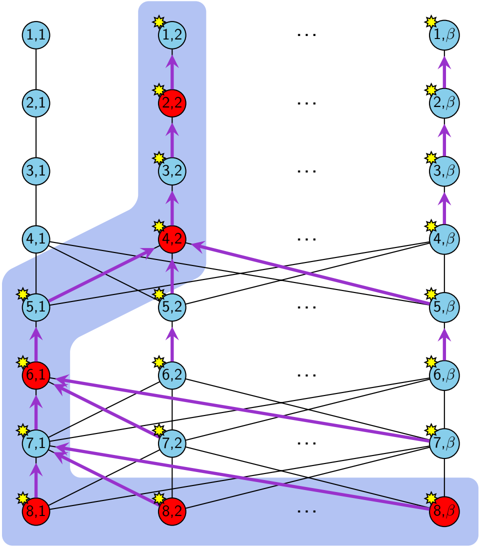


Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

β^4

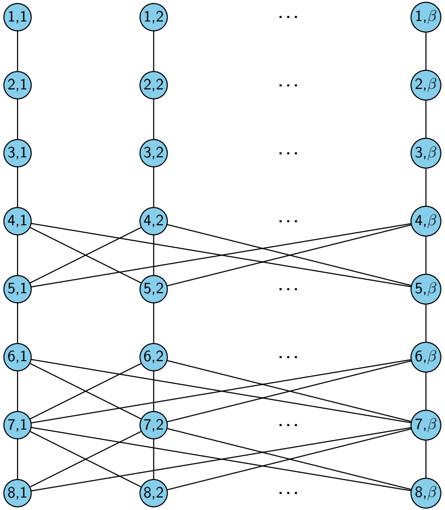
Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$

Can be joined

Cannot be joined



Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$$\beta = \Omega(n) \Rightarrow \Omega(n^4)$$

β^4

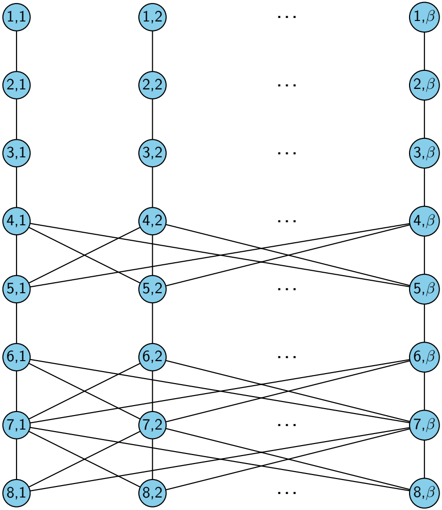
Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$

Can be joined

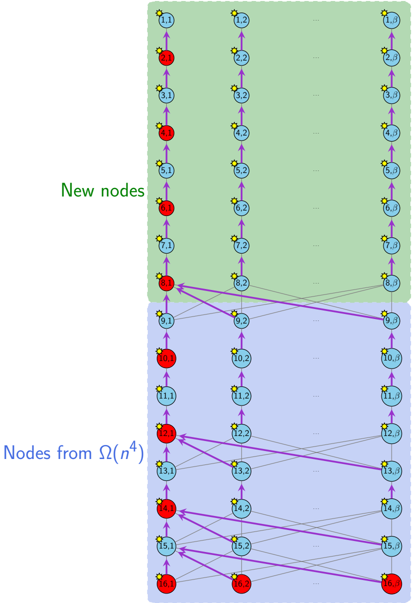
Cannot be joined



Datta et al, 2011

Network for $\Omega(n^5)$ steps

$\forall \alpha \geq 3$, $\exists$ networks and executions in $\Omega(n^{\alpha+1})$ steps.



Perspectives

Goal

Design a self-stabilizing leader election algorithm that stabilizes in $O(\mathcal{D})$ rounds.

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Hypotheses

- Unfair daemon
- Memory requirement of $\Theta(\log n)$ bits/process

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Design a self-stabilizing leader election algorithm that stabilizes in $O(\mathcal{D})$ rounds.

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- With the knowledge of $D \geq \mathcal{D}$, $(D = O(\mathcal{D}))$: ✓

Perspectives

Goal

Design a self-stabilizing leader election algorithm that stabilizes in $O(\mathcal{D})$ rounds.

Hypotheses

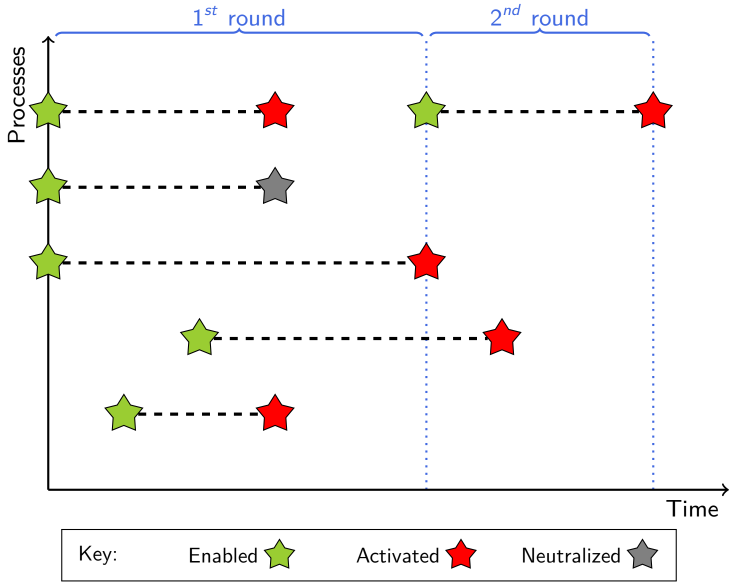
- Unfair daemon
- Memory requirement of $\Theta(\log n)$ bits/process

- With the knowledge of $D \geq \mathcal{D}$, $(D = O(\mathcal{D}))$: ✓
- Without any global knowledge : **??**

Thank you for your attention.

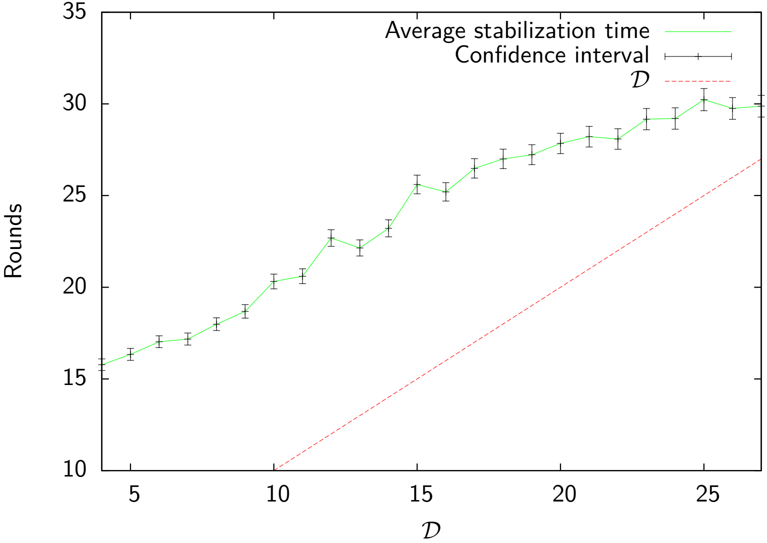
Do you have any questions ?

Rounds



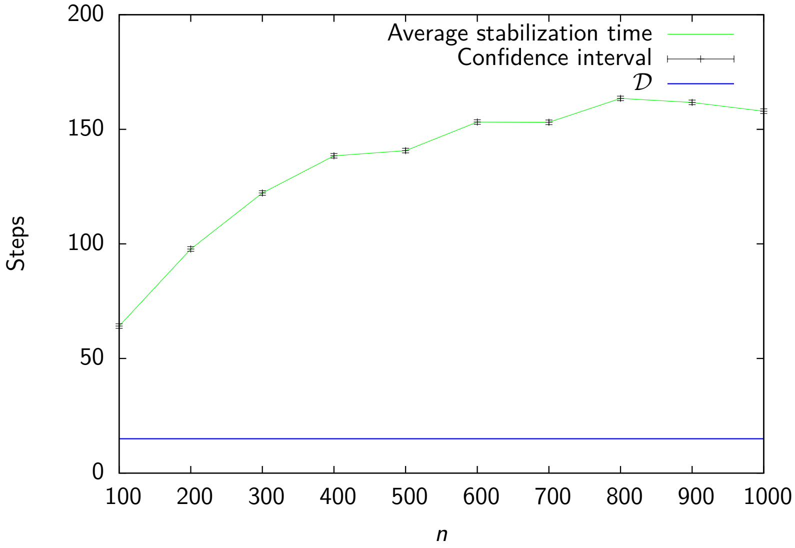
1st round
2nd round
Processes
Time
Key:
Enabled
Activated
Neutralized

Experimental Results



Average stabilization time in rounds in UDGs ($n = 1000$)

Experimental Results



Average stabilization time in steps in UDGs ($\mathcal{D} = 15$)