Acyclic Strategy for Silent Self-Stabilization in Spanning Forest

Karine Altisen[1] Stéphane Devismes[1] Anaïs Durand[2]

[1] Univ. Grenoble Alpes, CNRS, Grenoble INP, VERIMAG, 38000 Grenoble, France
[2] IRISA, Université de Rennes, 35042 Rennes, France

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Roadmap

Introduction

Trees/Forests: Ubiquitous in Self-Stabilization

Composition is a popular way to design self-stabilizing algorithms (modular approach, simplicity of the design and proofs)

Numerous self-stabilizing algorithms [Arora et al., 1990, Blin et al., 2010, Datta et al., 2016] are made as **a composition of**

- **a spanning directed treelike construction** and
- some other **algorithms specifically designed for directed tree/forest topologies**.

Many solutions are silent

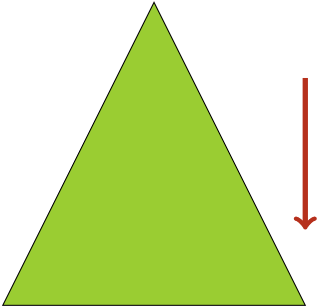
Spanning Directed Treelike Constructions

Many (silent) self-stabilizing spanning tree constructions are available, *e.g.*:

- **Arbitrary Tree:** [Chen et al., 1991]
- **DFS:** [Collin and Dolev, 1994]
- **BFS:** [Cournier et al., 2009, Cournier et al., 2011]
- **Shortest-Path:** [Glacet et al., 2014]
- ...
- **(Efficient) General Scheme:** [Devismes et al., 2019]

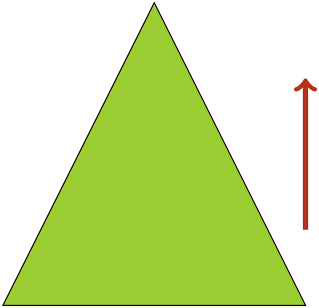
Self-stabilizing Algorithms for Directed Spanning Tree/Forest

Classical design pattern based on top-down (broadcast) and bottom-up (convergecast) computations:



Top-Down

Computations are propagated from parents to nodes



Bottom-Up

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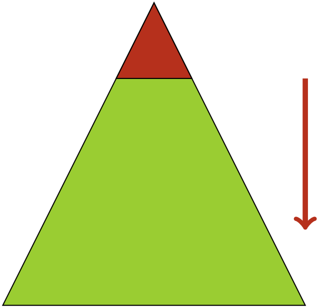

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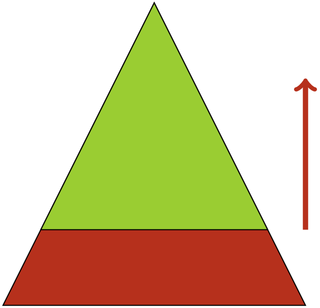

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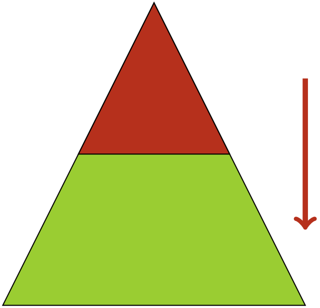

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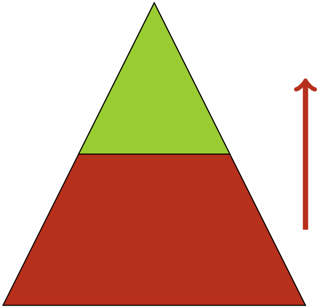

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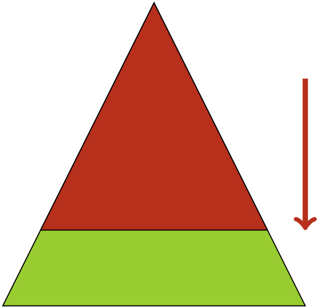

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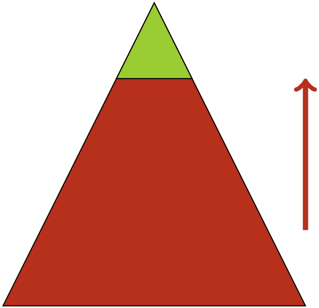

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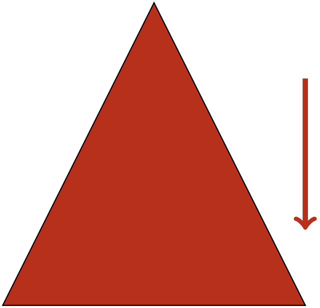

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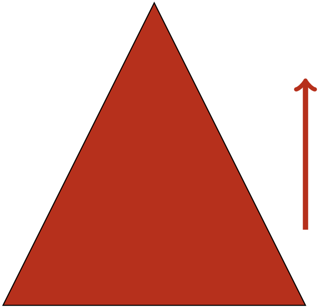

Bottom-Up

Computations are propagated from children to nodes

Our Goal

Define a class of algorithms for networks endowed with a spanning forest (*e.g.*, a spanning tree) based the notions of **top-down** and **bottom-up** computations.

- The definition should be **simple** to check (*i.e.*, quasi-syntactic)
- Algorithms of the class should be **(silent) self-stabilizing**
- Algorithms of the class should be **efficient** in stabilization time

Challenge: Trade-off efficiency/versatility

Context

- **Locally shared memory model** with **composite atomicity**
  - Distributed unfair daemon
    (the most general scheduling assumption)
- **Silent self-stabilizing** algorithms
- **Sense of direction** defining spanning forest
  - *p.par*: *p.par* is either a neighbor (its *parent*), or $\perp$ (for a root).
  - *p.chldrn*: the set of *children*
- Time complexity in **moves** and **rounds**

Contribution

Acyclic Strategy

Definition 1.

A distributed algorithm $\mathcal{A}$ follows an **acyclic strategy** if

- it is well-formed,
- its graph of actions' causality **GC** is (directed) acyclic, and
- for every A_i in its families' partition, A_i is
  - correct-alone and
  - either bottom-up or top-down.

syntactic condition / semantic condition

(An illustrative example in few slides ...)

Main Result

Theorem 1.

Let $\mathcal{A}$ be a distributed algorithm. If

- *$\mathcal{A}$ follows an acyclic strategy,*
- *every terminal configuration of $\mathcal{A}$ satisfies SP*
- *$\mathcal{A}$ is locally mutually exclusive*

then

- *$\mathcal{A}$ is silent and self-stabilizing for SP in G under* **the distributed unfair daemon**
- **its stabilization time** *is at most $(1 + \mathbf{d} \cdot (1 + \Delta))^{\mathfrak{H}} \cdot k \cdot n^{\mathfrak{H}+2}$ moves*
- **its stabilization time** *is at most $(\mathfrak{H} + 1) \cdot (H + 1)$ rounds*

(Δ is the degree of the network, k is the number of families of $\mathcal{A}$, $\mathbf{d}$ is the in-degree of **GC**, $\mathfrak{H}$ the height of **GC**, H is the height of the spanning forest)

(typically, k, $\mathbf{d}$, and $\mathfrak{H}$ are constants)

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Acyclic Strategy

Toy Example

Compute a sum of inputs and broadcast the result

- Each process p holds a constant integer input $p.in \in \mathbb{N}$
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  - $p.sub \in \mathbb{N}$ (to compute the sum of input values in the subtree of p)
  - $p.res \in \mathbb{N}$ (to broadcast the result).

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- Every process p has two variables:
  - $p.sub \in \mathbb{N}$ (to compute the sum of input values in the subtree of p)
  - $p.res \in \mathbb{N}$ (to broadcast the result).
- Legitimacy predicate: $\texttt{SumOfInputs} \equiv \forall p \in V, p.res = \sum_{q \in V} q.in$

Toy Example

Algorithm $\mathcal{TE}$

For every process p

$S(p) :: p.sub \neq (\sum_{q \in p.chldrn} q.sub) + p.in \rightarrow p.sub \leftarrow (\sum_{q \in p.chldrn} q.sub) + p.in$

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For process r

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For every process $p \neq r$

$R(p) :: p.res \neq \max(p.par.res, p.sub) \rightarrow p.res \leftarrow \max(p.par.res, p.sub)$

Families and Well-Formedness

Definition 1.

A distributed algorithm $\mathcal{A}$ follows an **acyclic strategy** if

- it is well-formed,
- its graph of actions' causality C is (directed) acyclic, and
- for every A_i in its families' partition, A_i is
  - correct-alone and
  - either bottom-up or top-down.

- **Family of actions:** set of n actions, one per process
- **Well-Formed:** Actions partitioned into families s.t. **each variable is written in exactly one family**

(Well-formedness is rather a guideline to simplify the analysis.)

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$\mathcal{TE}$ **is well-formed:** two families S and R

- $S = \{S(p) \ : \ p \in V\}$
- $R = \{R(p) \ : \ p \in V\}$

$$S(p) :: p.sub \neq (\sum_{q \in p.chldrn} q.sub) + p.in \rightarrow \mathbf{p.sub} \leftarrow (\sum_{q \in p.chldrn} q.sub) + p.in$$

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- $A_1, \ldots, A_k$: families' partition of $\mathcal{A}$.
- $A_j \prec_{\mathcal{A}} A_i$ iff
  - $i \neq j$ and
  - $\exists p, q$ s.t. $A_j(p)$ writes in variables "read" by $A_i(q)$.
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$$S \prec_{\mathcal{TE}} R$$

$\mathbf{GC} : S \longrightarrow R$ is acyclic

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Result for $\mathcal{TE}$ (1/2)

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-
  - $\mathbf{d} = 1$ (in-degree of **GC**)
  - $k = 2$ (number of families)
  - $\mathfrak{H} = 1$ (height of **GC**)

- $\mathcal{TE}$ is silent and self-stabilizing for `SumOfInputs` in G under **the distributed unfair daemon**
- **its stabilization time** is at most $(4 + 2\Delta) \cdot n^3$ moves, where Δ is the degree of G and n the number of processes

Result for $\mathcal{TE}$ (2/2)

The complexity of $\mathcal{TE}$ can be refined to at most $n^2(3+2H)$ moves where n the number of processes and H is the height of the spanning tree using the following technical lemma

Lemma 1.

Let A_i be a family of actions and p be a process. For every execution e of the algorithm $\mathcal{A}$ on G, $\#m(e, A_i, p) \leq \left(n \cdot \left(1 + \mathbf{d} \cdot \left(1 + maxO(A_i)\right)\right)\right)^{\mathfrak{H}(A_i)} \cdot |Z(p, A_i)|$.

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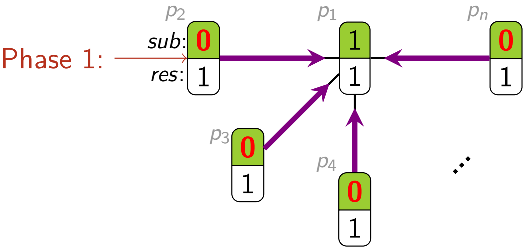
This bound is tight: there is an execution of $\mathcal{TE}$ containing $O(H \cdot n^2)$ moves

Round Complexity

Lower Bound in Rounds for Algorithm $\mathcal{TE}$

$\forall i \in \{1, \ldots, n\},\ p_i.in = 1$



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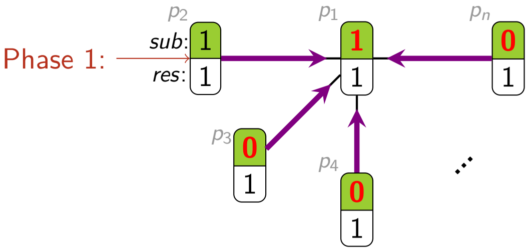

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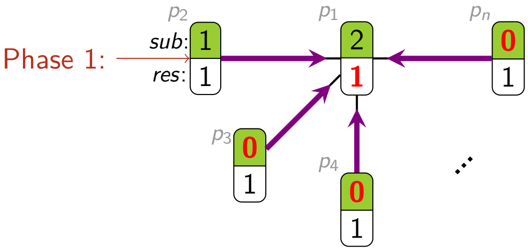

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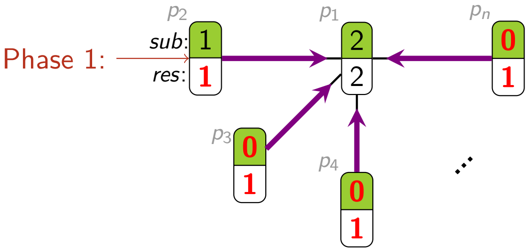

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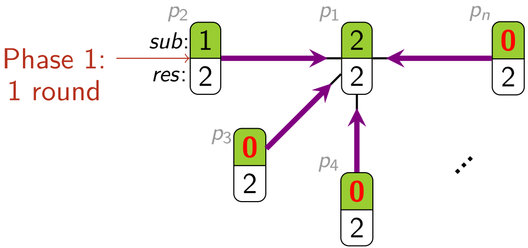

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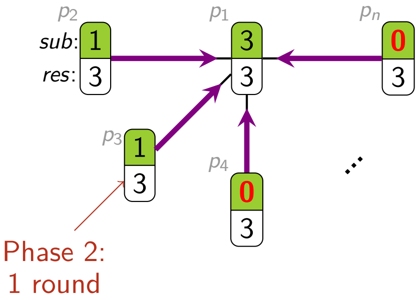

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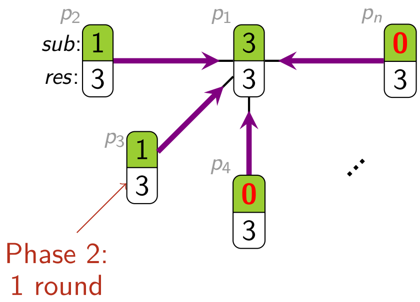

$n - 1$ phases $\Rightarrow \Omega(n)$ rounds

Condition for a Stabilization Time in $O(H)$ Rounds

Round complexity of $\mathcal{TE} = \Omega(n)$ rounds $\Rightarrow$ **not optimal!**

Why?

Condition for a Stabilization Time in $O(H)$ Rounds

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Why? S and R of $\mathcal{TE}$ are not **mutually exclusive**

Condition for a Stabilization Time in $O(H)$ Rounds

Round complexity of $\mathcal{TE} = \Omega(n)$ rounds $\Rightarrow$ **not optimal!**

Why? S and R of $\mathcal{TE}$ are not **mutually exclusive**

Theorem 2.

Let $\mathcal{A}$ be a distributed algorithm. If $\mathcal{A}$

- *follows an acyclic strategy and*
- *is locally mutually exclusive*

then every execution of A reaches a terminal configuration within at most at most $(\mathfrak{H} + 1) \cdot (H + 1)$ rounds

($\mathfrak{H}$ is the height of **GC** and H is the height of the spanning forest)

(typically, $\mathfrak{H}$ is a constant)

Transformation into Locally Mutually Exclusive Algorithm

- **Idea:** use a strict total order compatible with the partial order $\prec_{\mathcal{A}}$ to implement priorities on actions locally at each process

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- **Application on the Toy Example:**

$\mathcal{TE}$

- For every process p:
  $S(p) :: p.sub \neq (\sum_{q \in p.chldrn} q.sub) + p.in$
  $\quad \rightarrow p.sub \leftarrow (\sum_{q \in p.chldrn} q.sub) + p.in$
- For process r:
  $R(r) :: r.res \neq r.sub$
  $\quad \rightarrow r.res \leftarrow r.sub$
- For every process $p \neq r$:
  $R(p) :: p.res \neq \max(p.par.res, p.sub)$
  $\quad \rightarrow p.res \leftarrow \max(p.par.res, p.sub)$

Transformation into Locally Mutually Exclusive Algorithm

- **Idea:** use a strict total order compatible with the partial order $\prec_{\mathcal{A}}$ to implement priorities on actions locally at each process

- **Application on the Toy Example:**

$\mathtt{T}(\mathcal{TE})$ using $S < R$, *i.e.* $S(p)$ as priority over $R(p)$, $\forall p$

- For every process p:
  $S(p) :: p.sub \neq (\sum_{q \in p.chldrn} q.sub) + p.in$
  $\rightarrow p.sub \leftarrow (\sum_{q \in p.chldrn} q.sub) + p.in$
- For process r:
  $R(r) :: (r.sub = (\sum_{q \in r.chldrn} q.sub) + r.in) \wedge r.res \neq r.sub$
  $\rightarrow r.res \leftarrow r.sub$
- For every process $p \neq r$:
  $R(p) :: (p.sub = (\sum_{q \in p.chldrn} q.sub) + p.in) \wedge p.res \neq \max(p.par.res, p.sub)$
  $\rightarrow p.res \leftarrow \max(p.par.res, p.sub)$

Transformation into Locally Mutually Exclusive Algorithm

Theorem 3.

Let $\mathcal{A}$ be a distributed algorithm. If

- *$\mathcal{A}$ follows an acyclic strategy and*
- *$\mathcal{A}$ is silent and self-stabilizing for SP in G under* **the distributed unfair daemon**

then

- *$\mathcal{T}(\mathcal{A})$ is silent and self-stabilizing for SP in G under* **the distributed unfair daemon**
- **its stabilization time** *is at most $(\mathfrak{H}+1)\cdot(H+1)$ rounds*
- **its stabilization time in moves** *is less than or equal to the one of $\mathcal{A}$*

($\mathfrak{H}$ the height of **GC** and H is the height of the spanning forest)

(typically, $\mathfrak{H}$ is a constant)

Result for $\mathcal{TE}$

Since $\mathfrak{H} = 1$, $(\mathfrak{H} + 1) \cdot (H + 1)$ gives

$$2H + 2$$

- $\mathtt{T}(\mathcal{TE})$ is silent and self-stabilizing for `SumOfInputs` in G under **the distributed unfair daemon**
- **its stabilization time** is at most $2H + 2$ rounds (asymptotically optimal)
- **its stabilization time in moves** is at most $O(H \cdot n^3)$ moves

Conclusion

Application to the Literature

- ★ (blue) Same results
- ★ (green) More general daemon
- ★ (purple) New/better complexity

- [Turau and Köhler, 2015] ★ (purple)
- [Chaudhuri and Thompson, 2005] ★ (green) ★ (purple)
- [Chaudhuri and Thompson, 2011] ★ (blue)
- [Chaudhuri, 1999a] ★ (purple)
- [Chaudhuri, 1999b] ★ (purple)
- [Karaata, 1999] ★ (green) ★ (purple)
- [Karaata and Chaudhuri, 1999] ★ (purple)
- [Devismes, 2005] ★ (blue)

Conclusion

- **Contribution:** General scheme to prove and analyze silent self-stabilizing algorithms designed for networks endowed with a spanning forest.

- **Future work:** How to compose those algorithms **carefully** with (silent) self-stabilizing spanning tree construction?

  i.e., to obtain efficient composite algorithms

Thank you for your attention

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