Self-Stabilizing Silent Disjunction in an Anonymous Network

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Join Work

Ajoy K. Datta

Lawrence L. Larmore

Self-Stabilization

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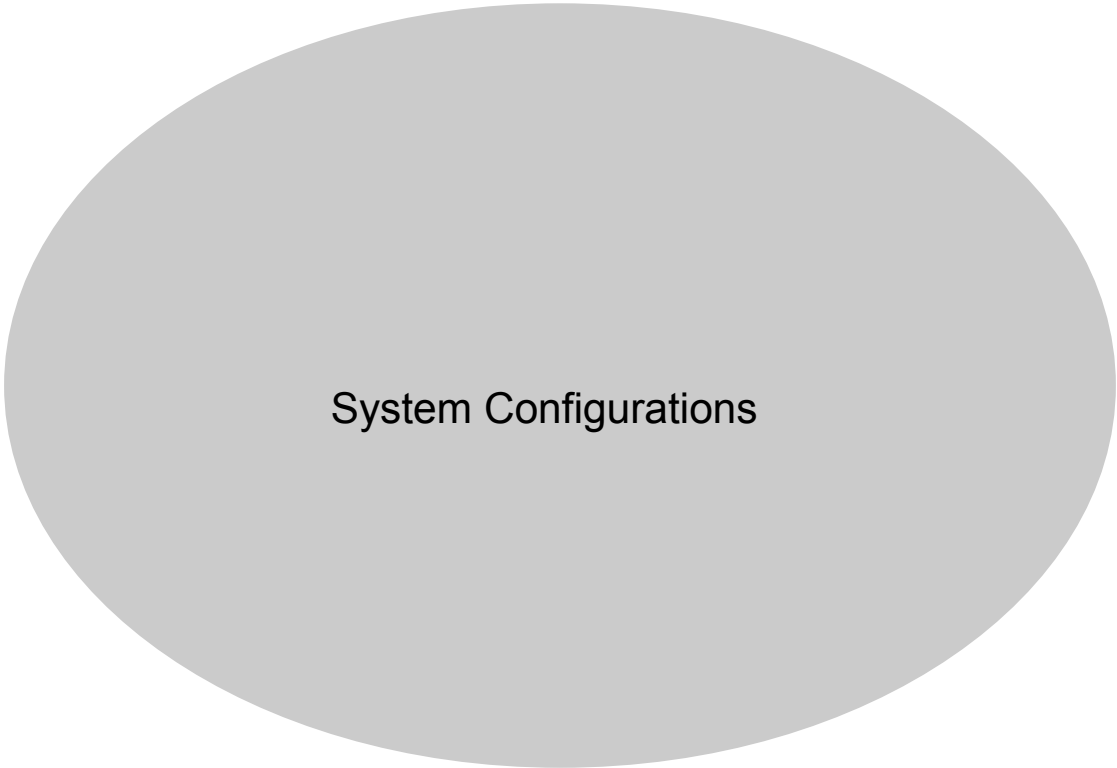

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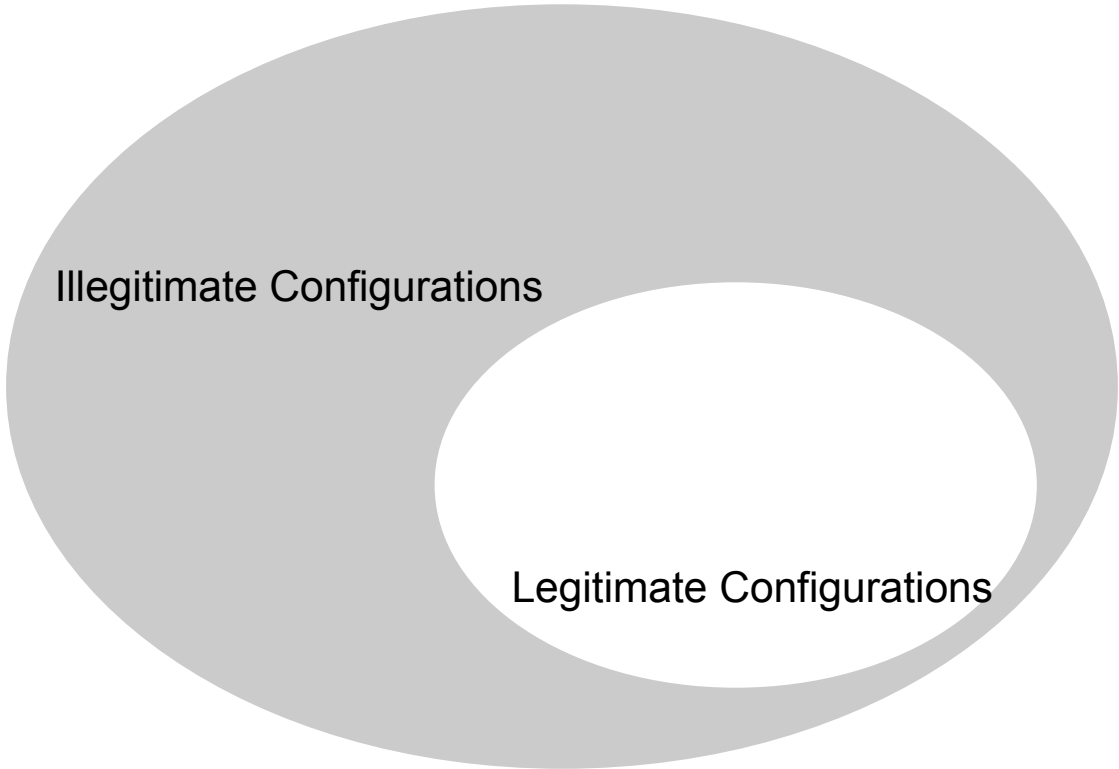

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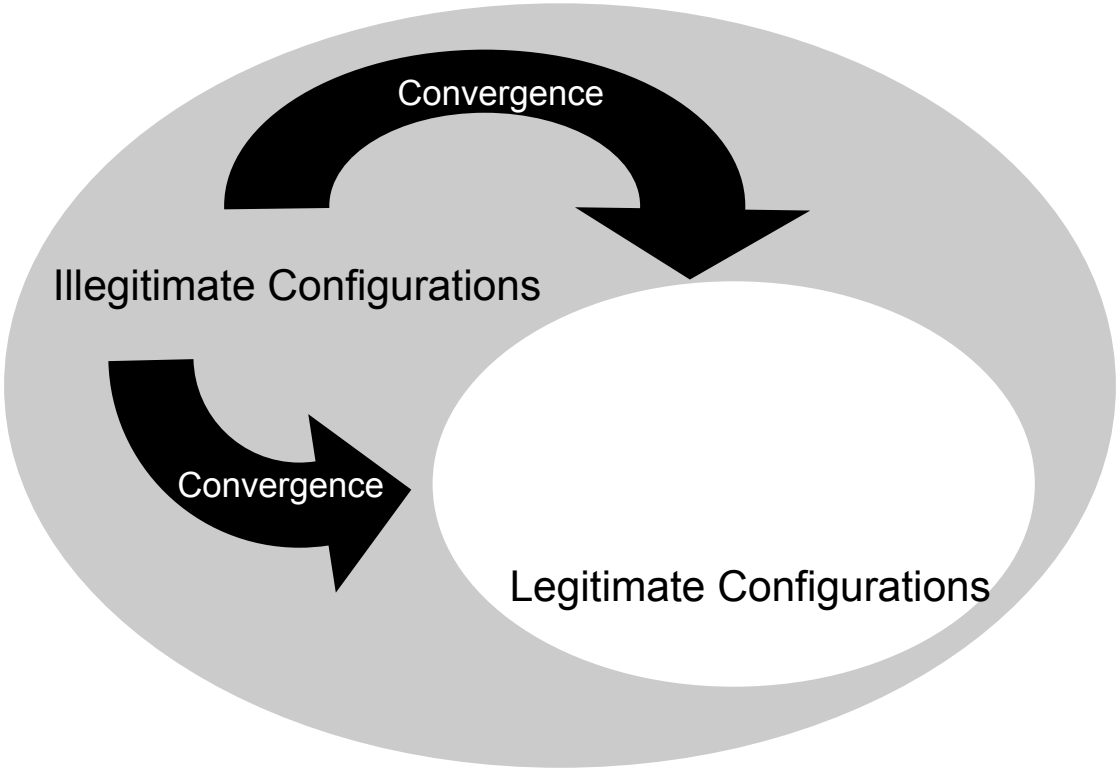

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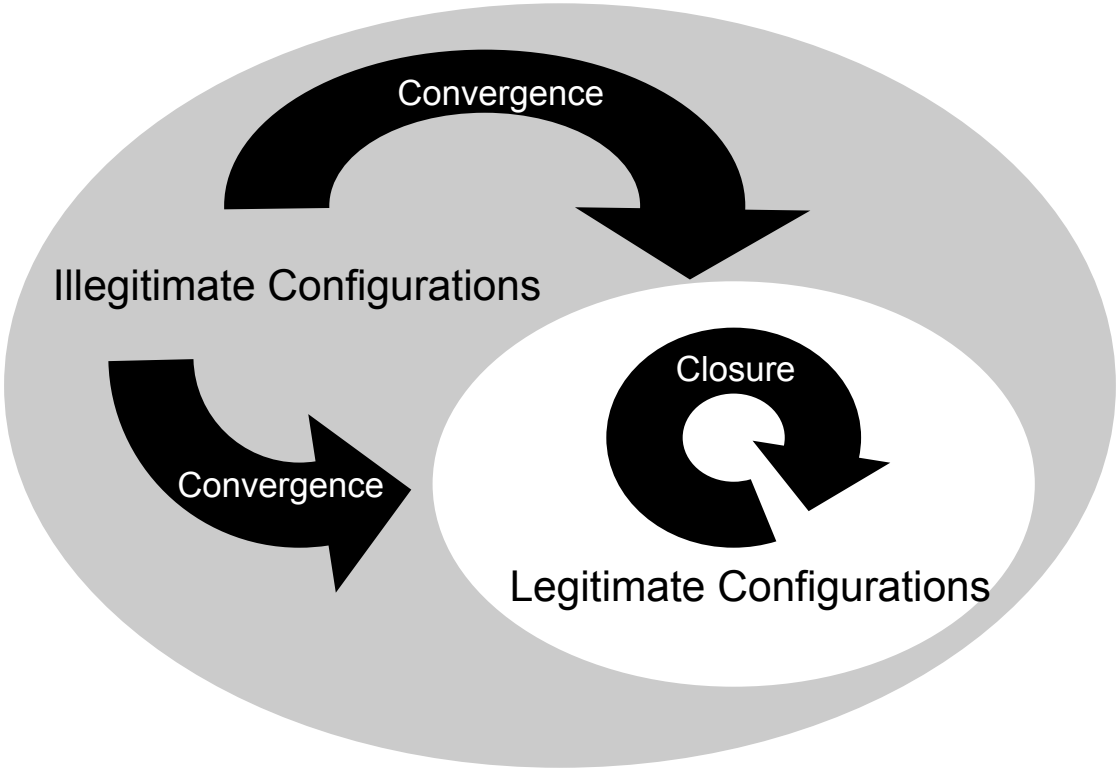
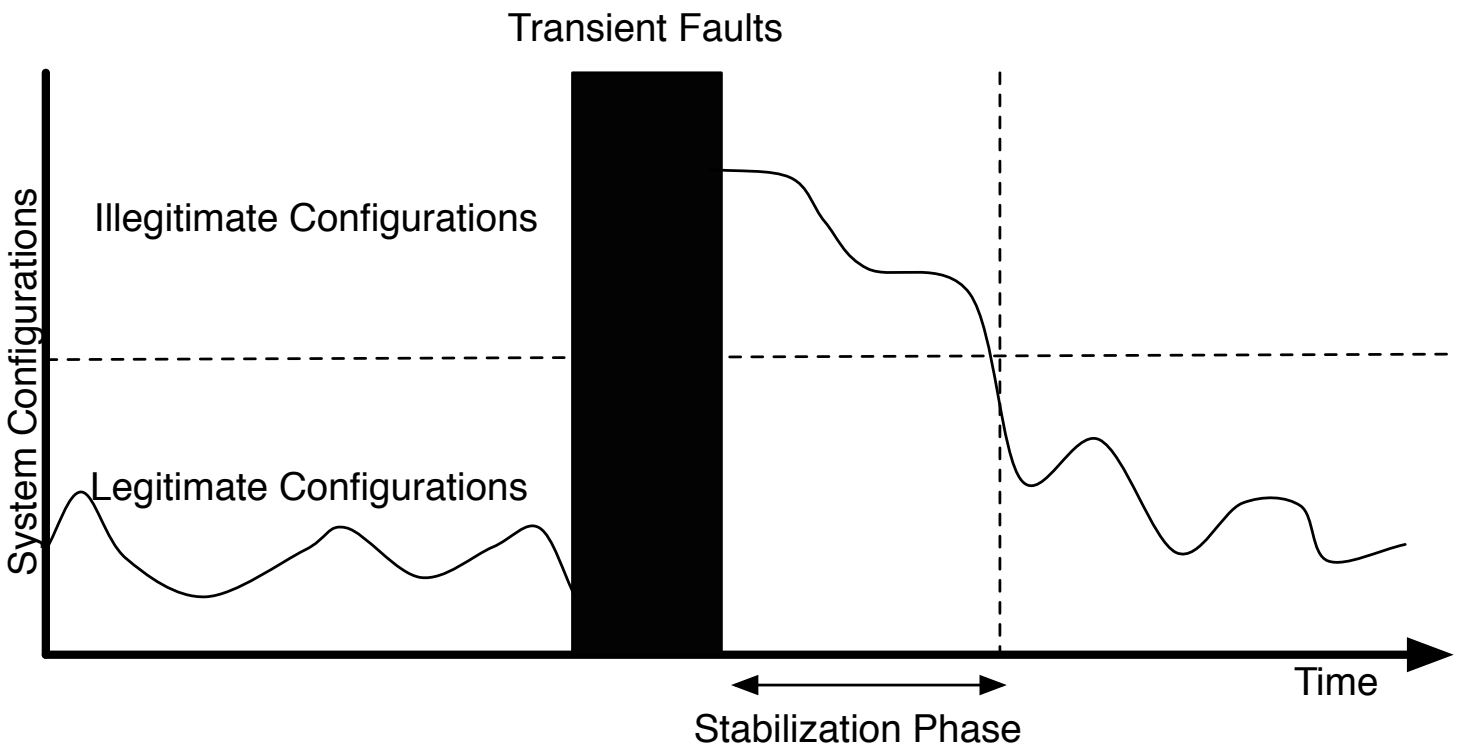
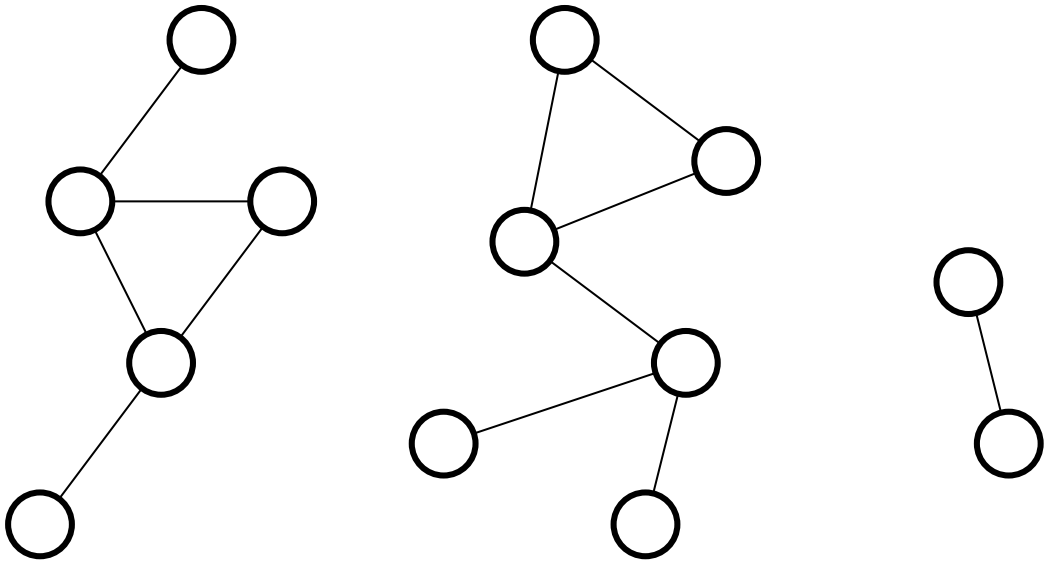


Tolerance to Transient Faults

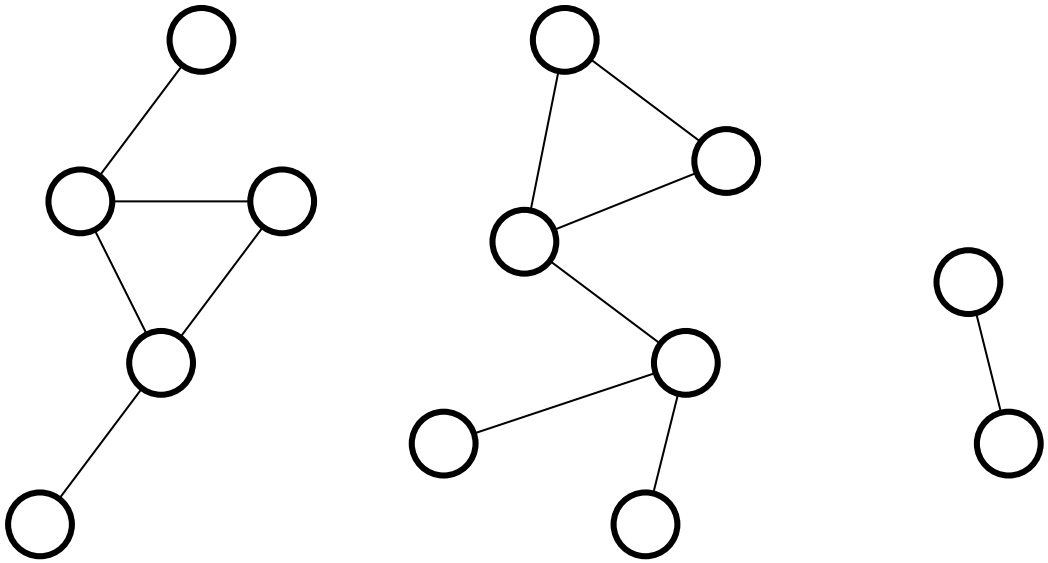


The Disjunction Problem



The Disjunction Problem

- **Anonymous** Network of **Processes**.



The Disjunction Problem

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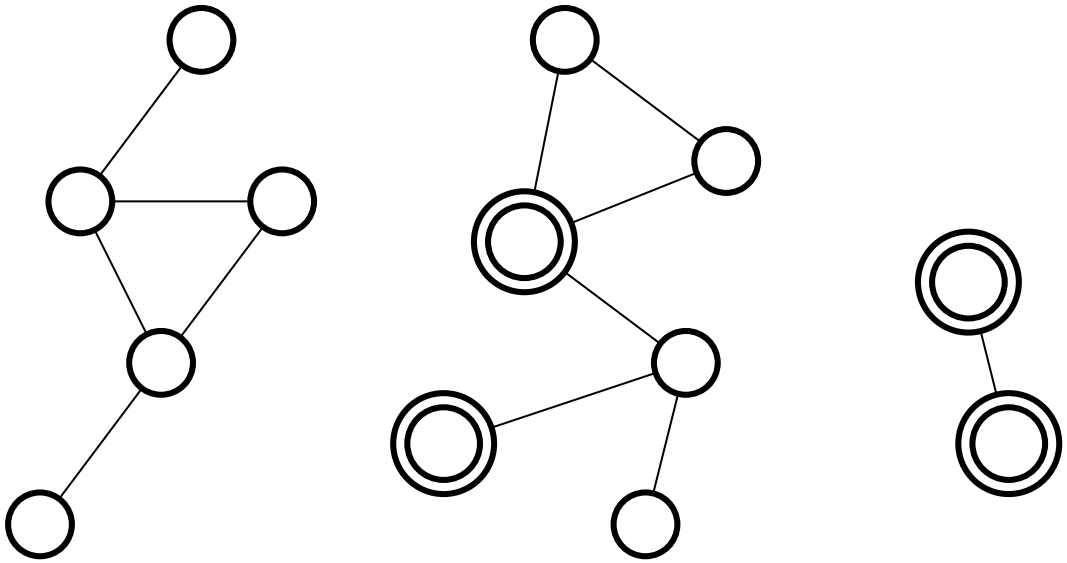
  - Each Process has **Input Bit**.



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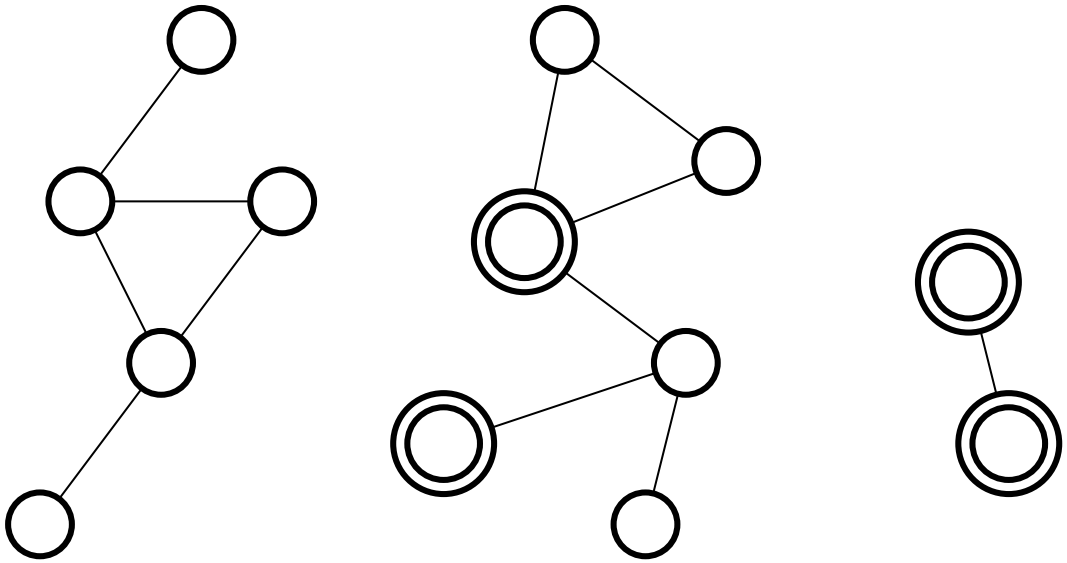
  - **Double Circle** $\Longleftrightarrow$ Input Bit = 1. Others are 0.



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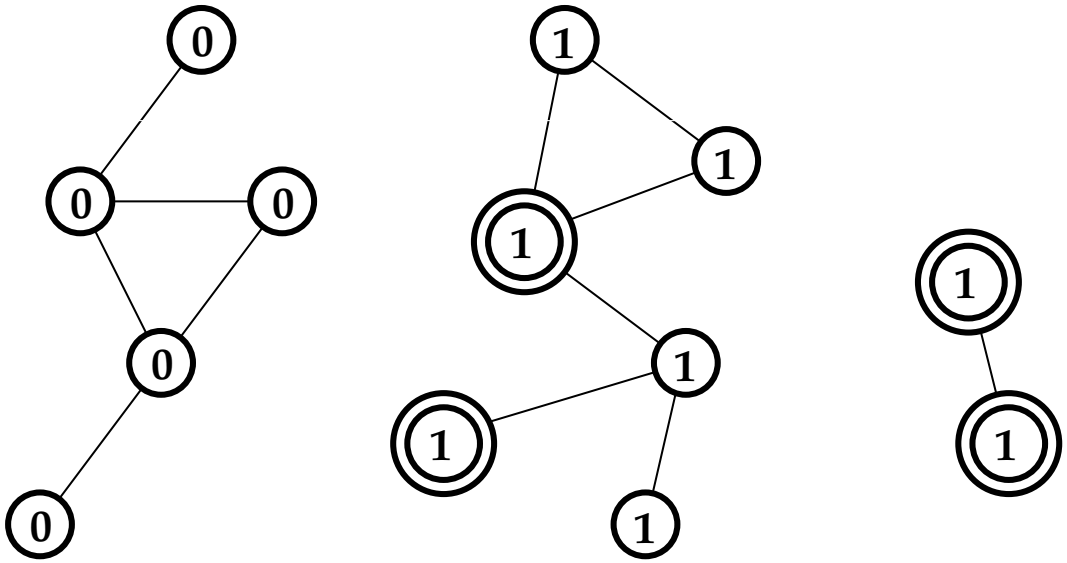

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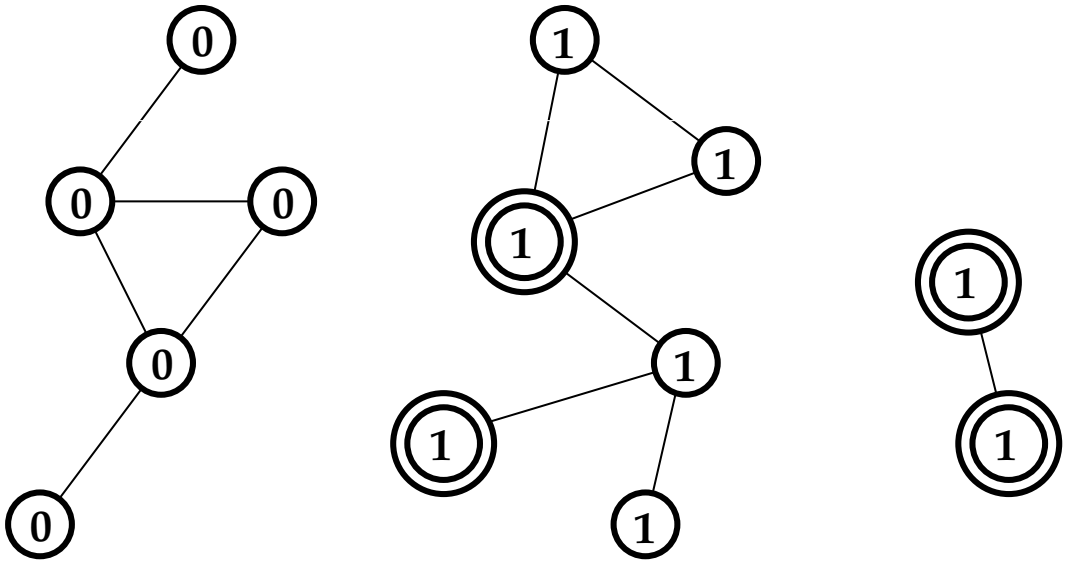

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  - Double Circle $\Longleftrightarrow$ Input Bit = 1. Others are 0.
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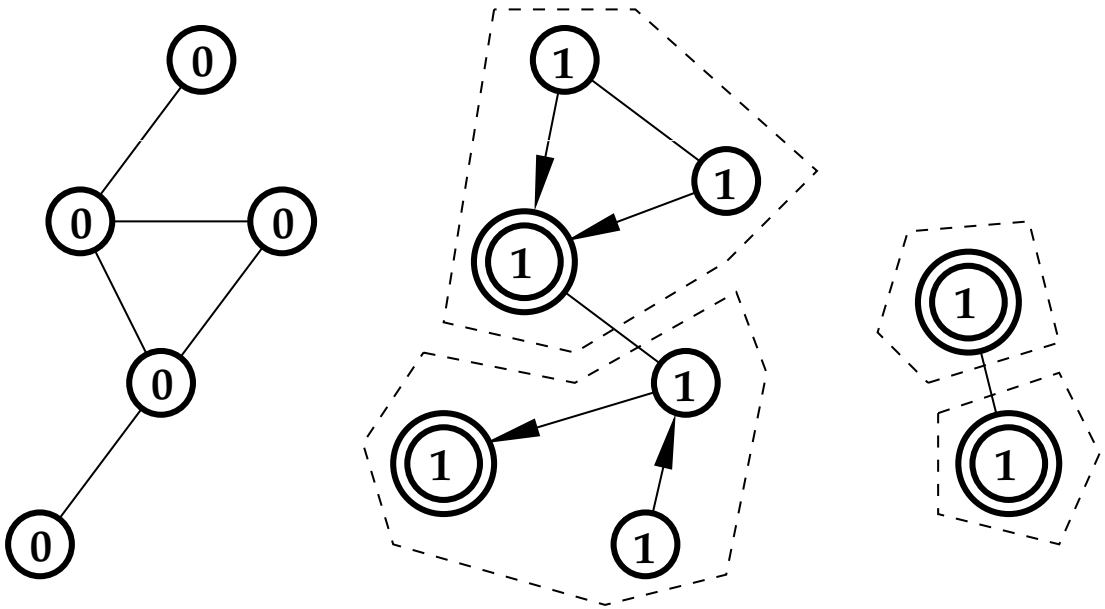
- (Construct **BFS Forest** Rooted at Processes with Input 1.)



Simple Flooding

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- Works if Clean Start.
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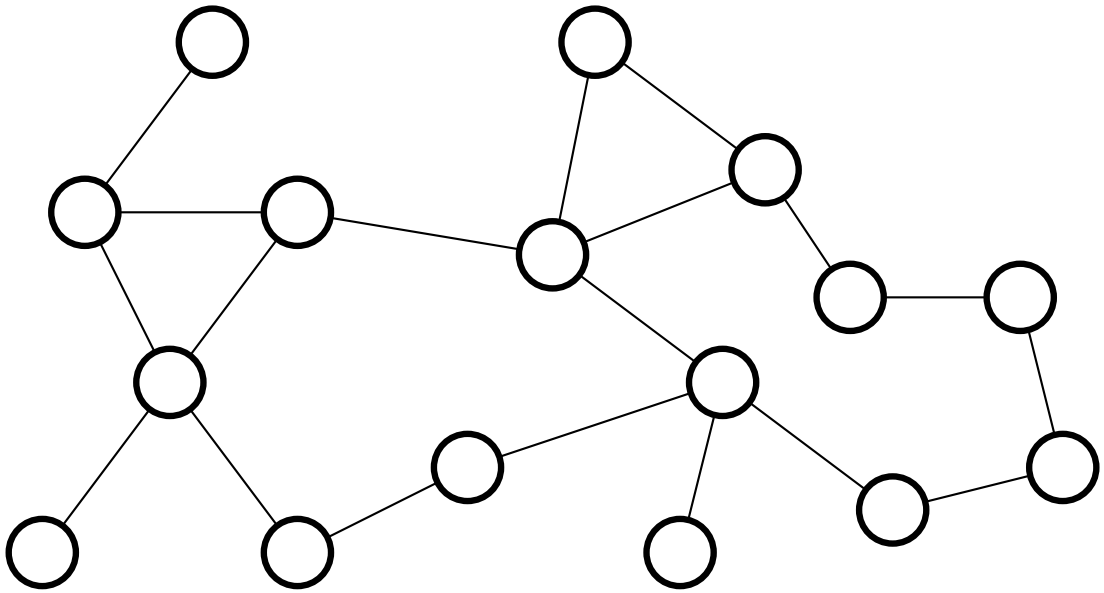
  - All Processes have **Input 0:** might go into **Endless Loop!**

Clean Start

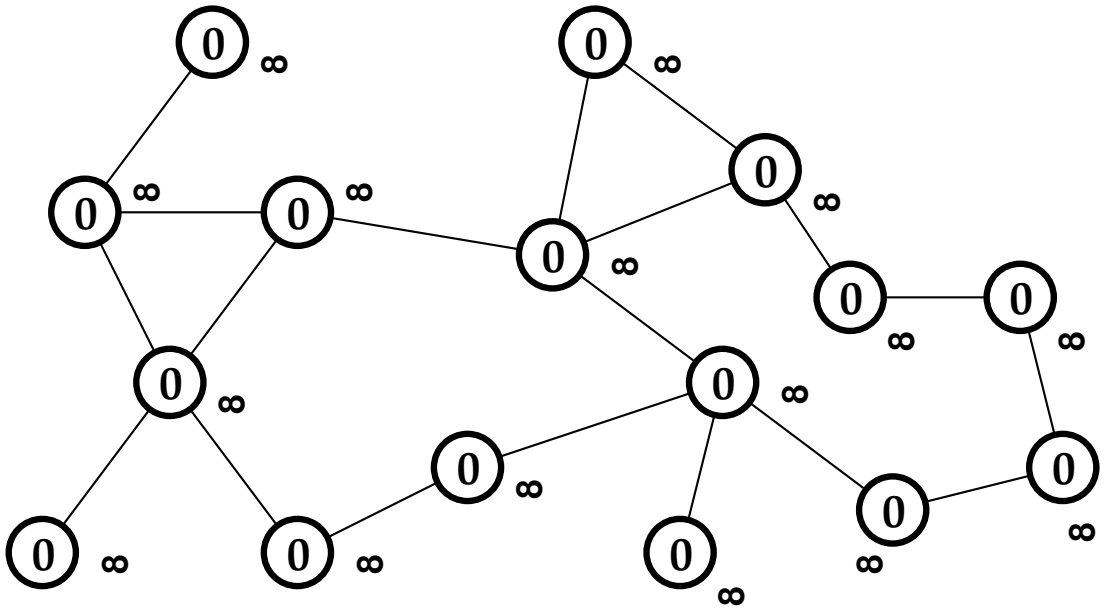
Clean Start

- **All Input Bits 0.**



Clean Start

- All Input Bits 0.
- Clean Configuration is **Final.**



Clean Start

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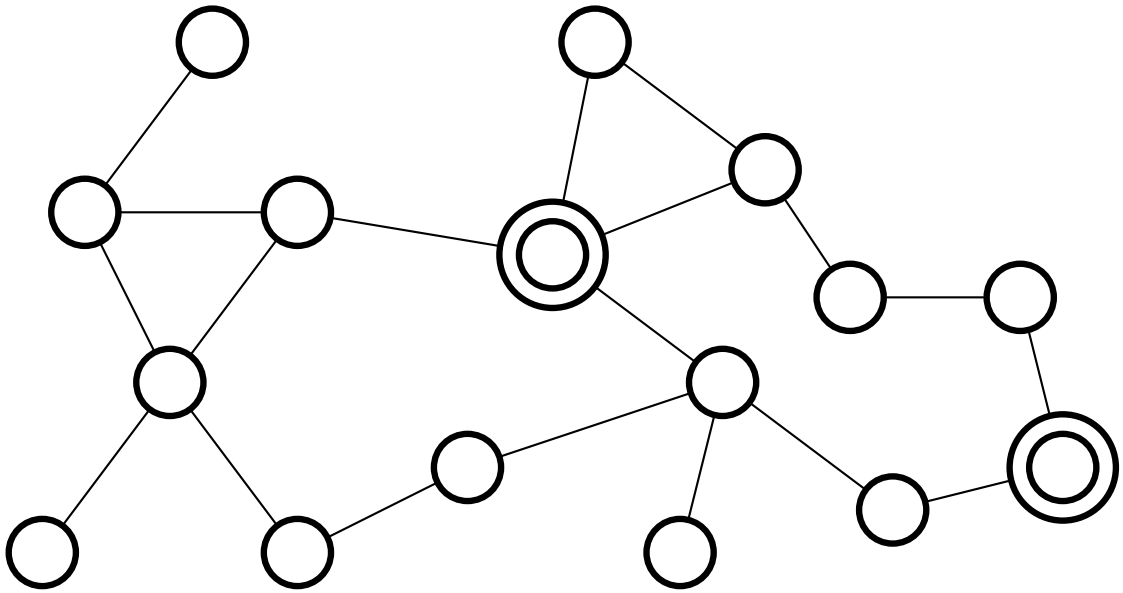
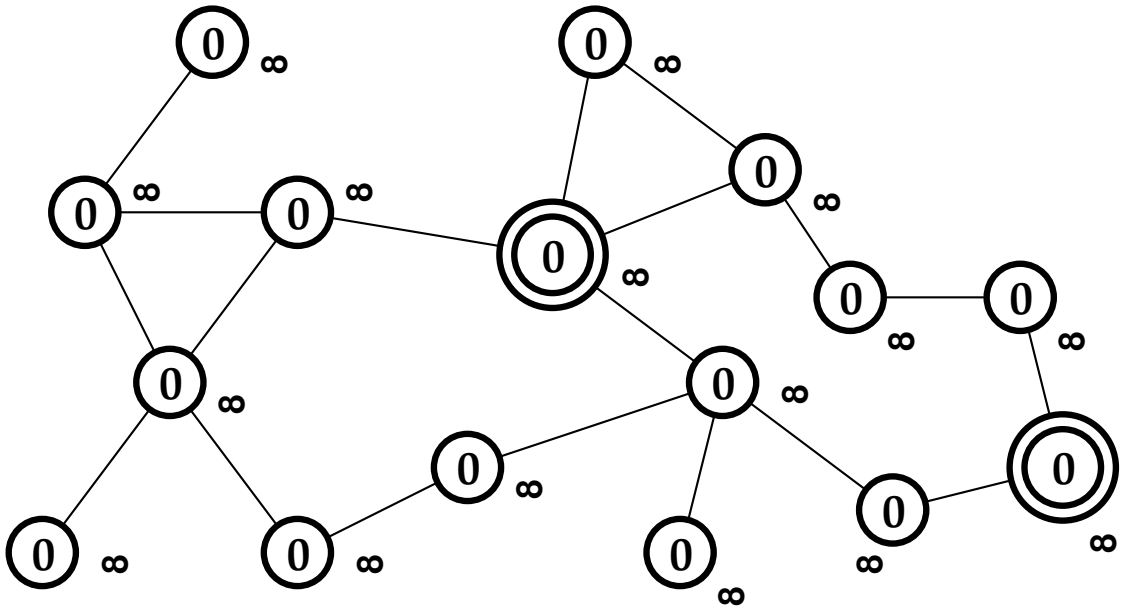


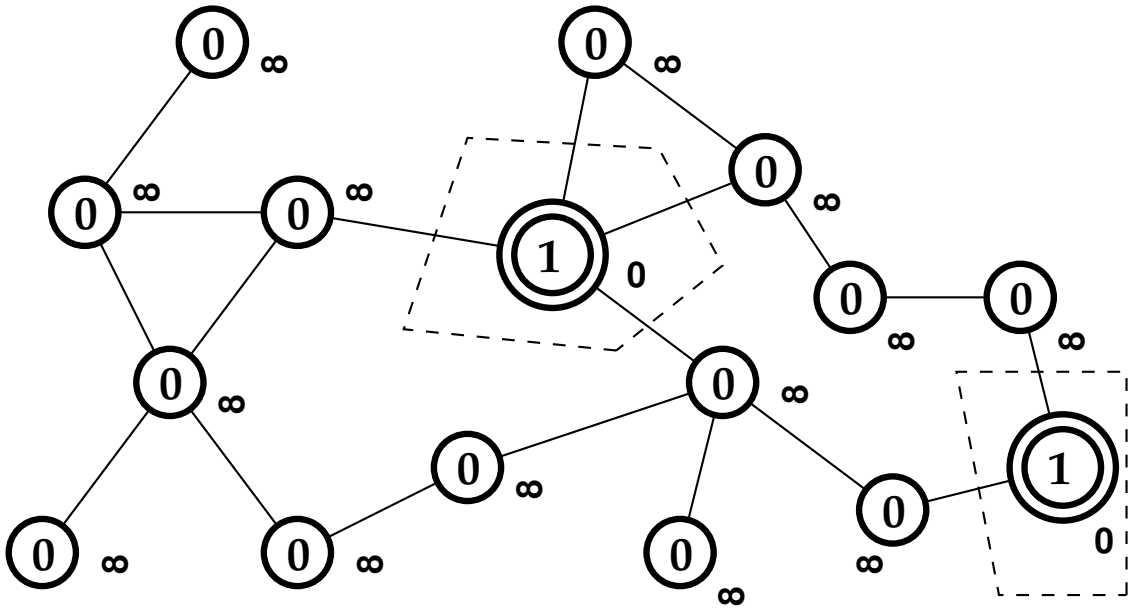
Clean Start

- Some Input Bits = 1.
- Clean Configuration is **Not Final.**



Clean Start

- Some Input Bits = 1.
- Clean Configuration is not Final.
- Clusterheads Execute **Reset.**



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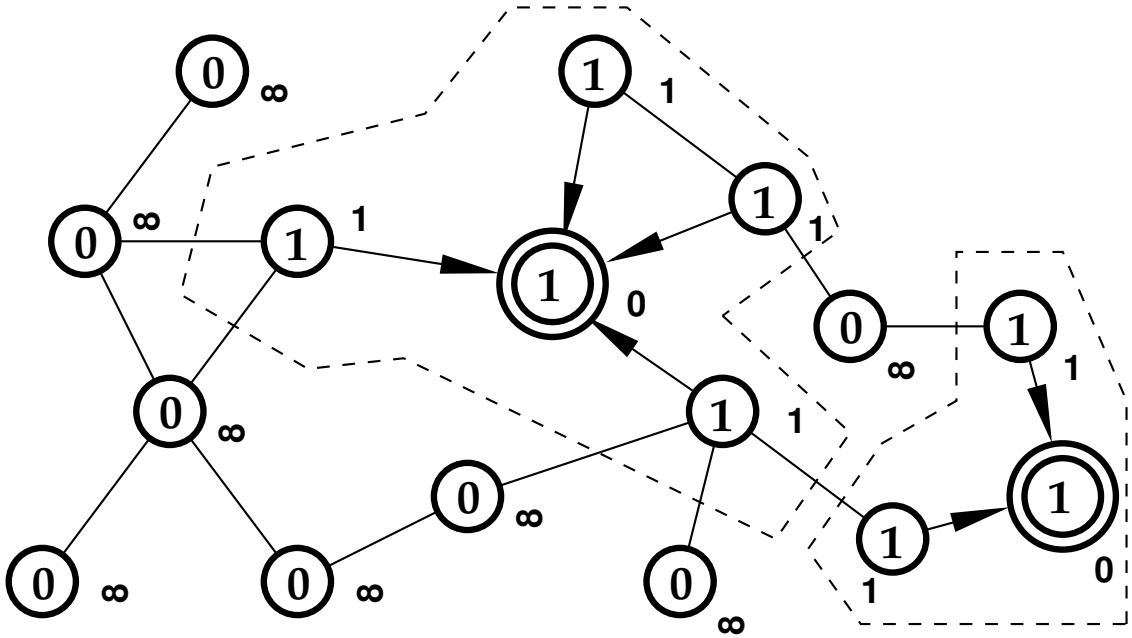

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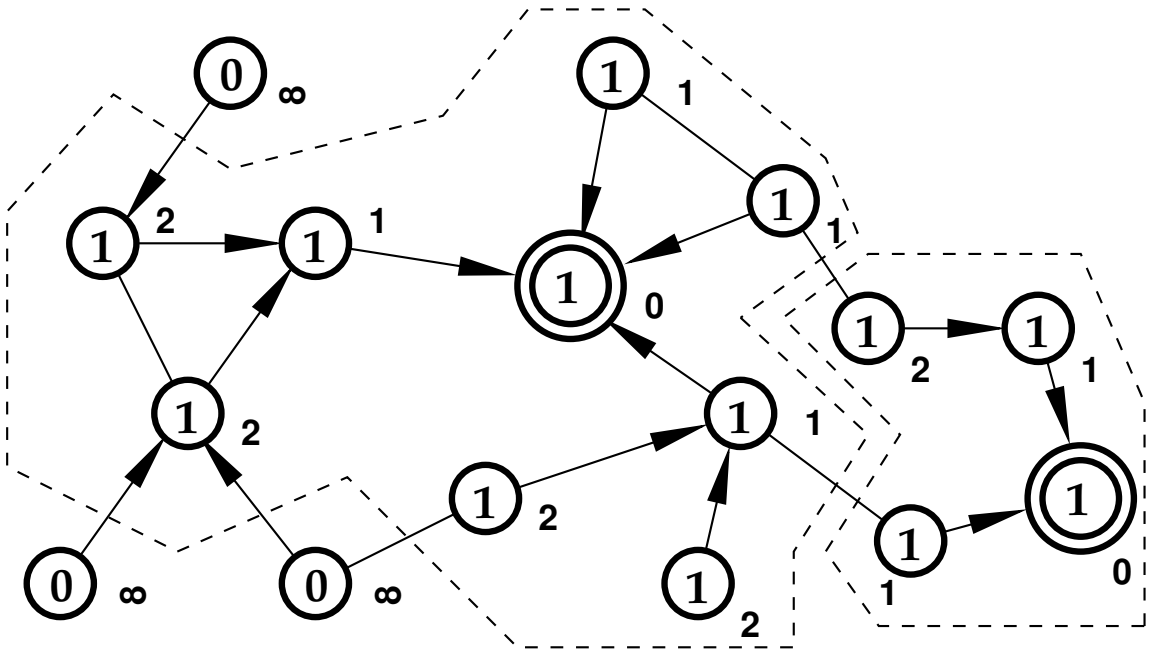
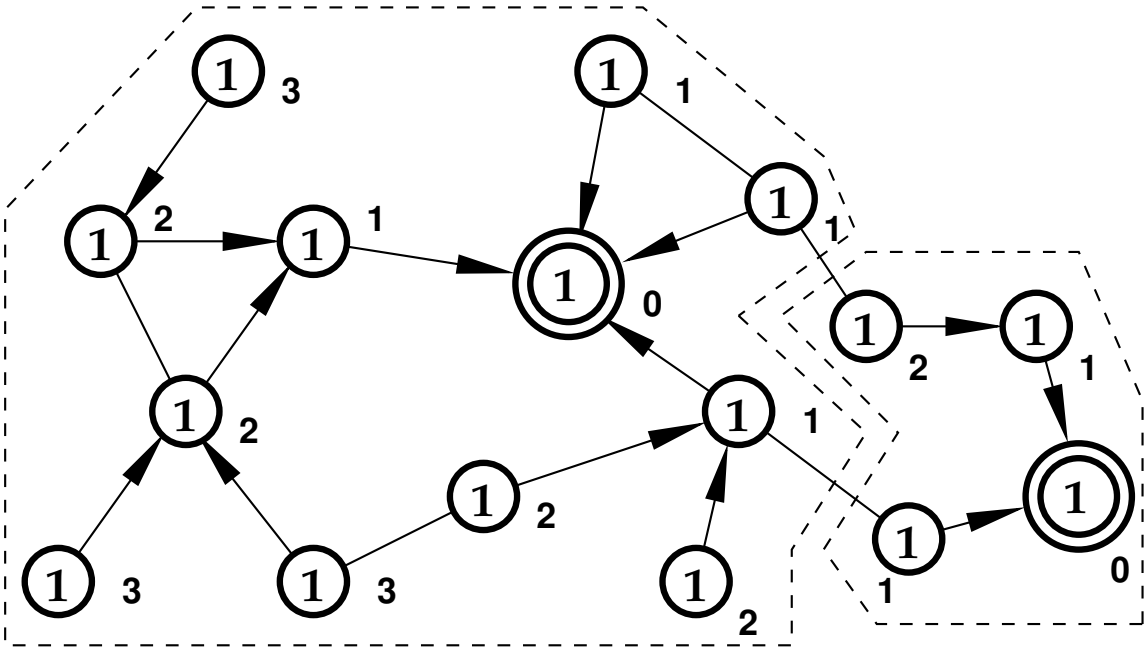


Clean Start

- Some Input Bits = 1.
- Clean Configuration is not Final.
- Clusterheads Execute Reset.
- Adjacent Processes Execute Join.
- **Flooding:** All Parent Pointers and Levels are Computed in at Most $(1 + Diam)$ **Rounds.**



Simple Flooding Is Not Self-Stabilizing!

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- **Arbitrary Initial Configuration?**

Simple Flooding Is Not Self-Stabilizing!

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- No Problem if Some Process has **Input 1.**

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- Arbitrary Initial Configuration.
- No Problem if Some Process has Input 1.
- Otherwise, might go into **Endless Loop.**

Arbitrary Start: Output = 1: No Problem!

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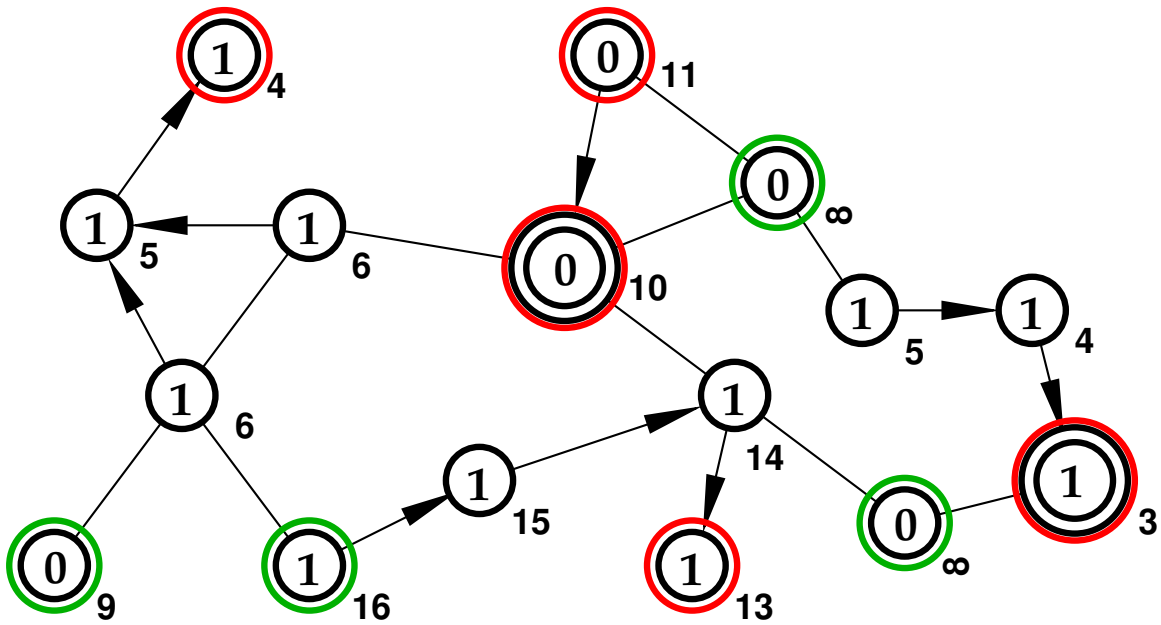

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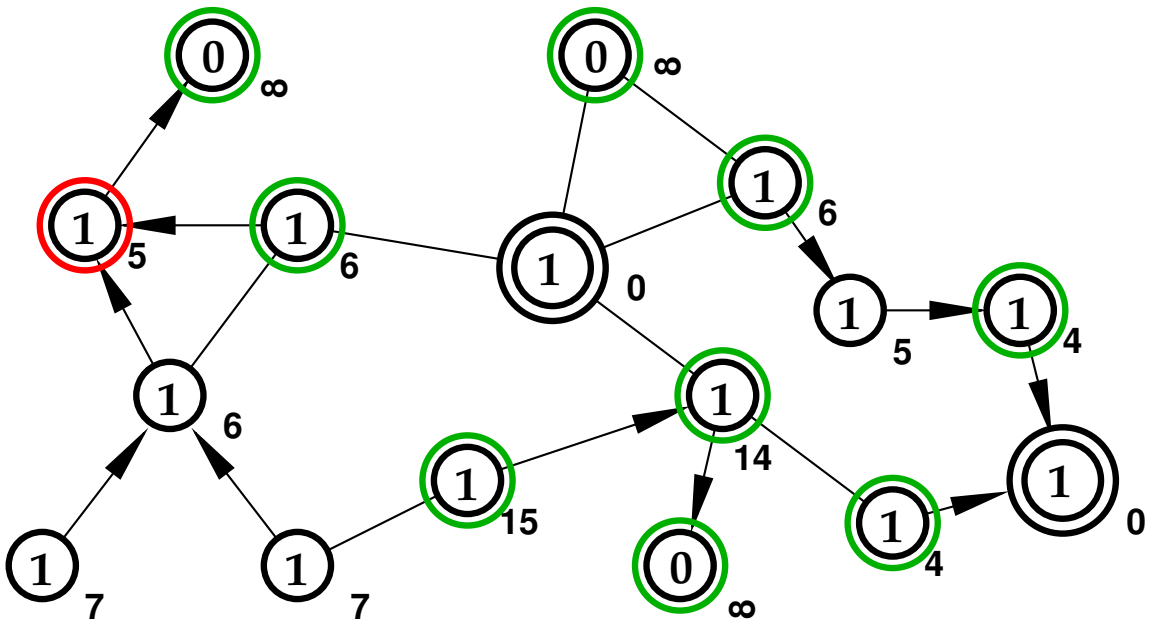
- **Clusterheads Reset**.



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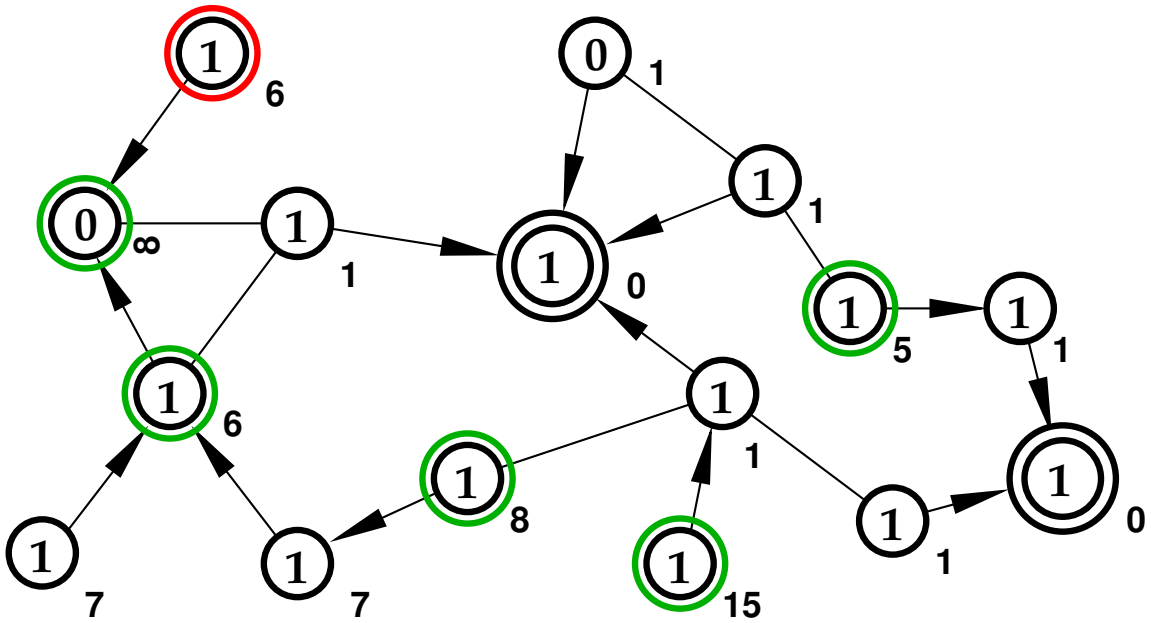

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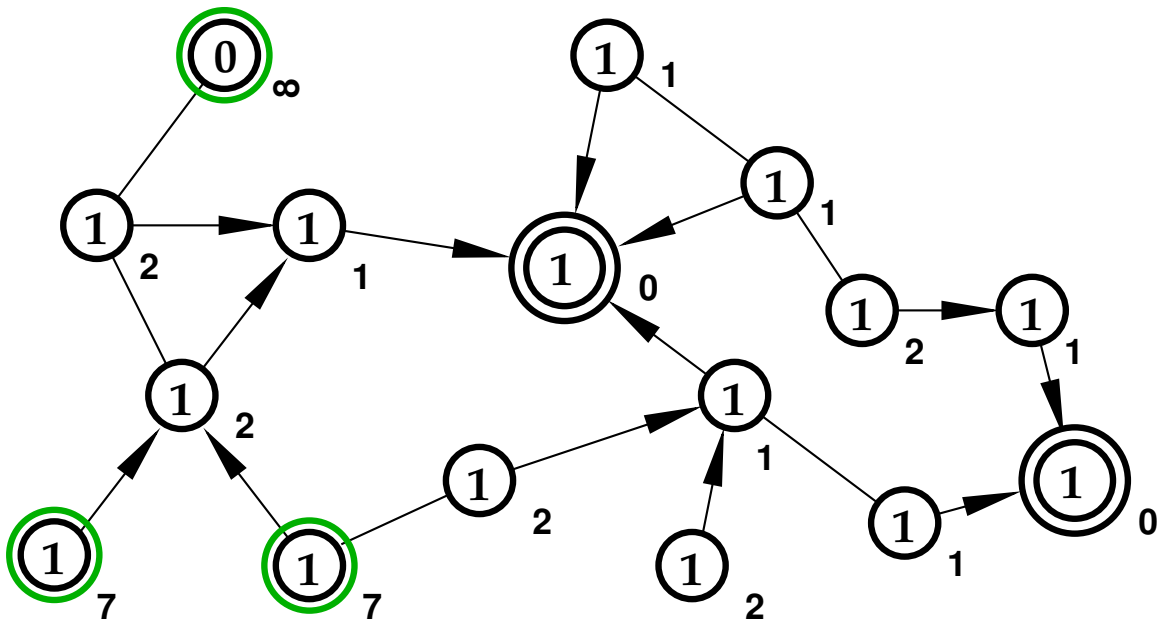

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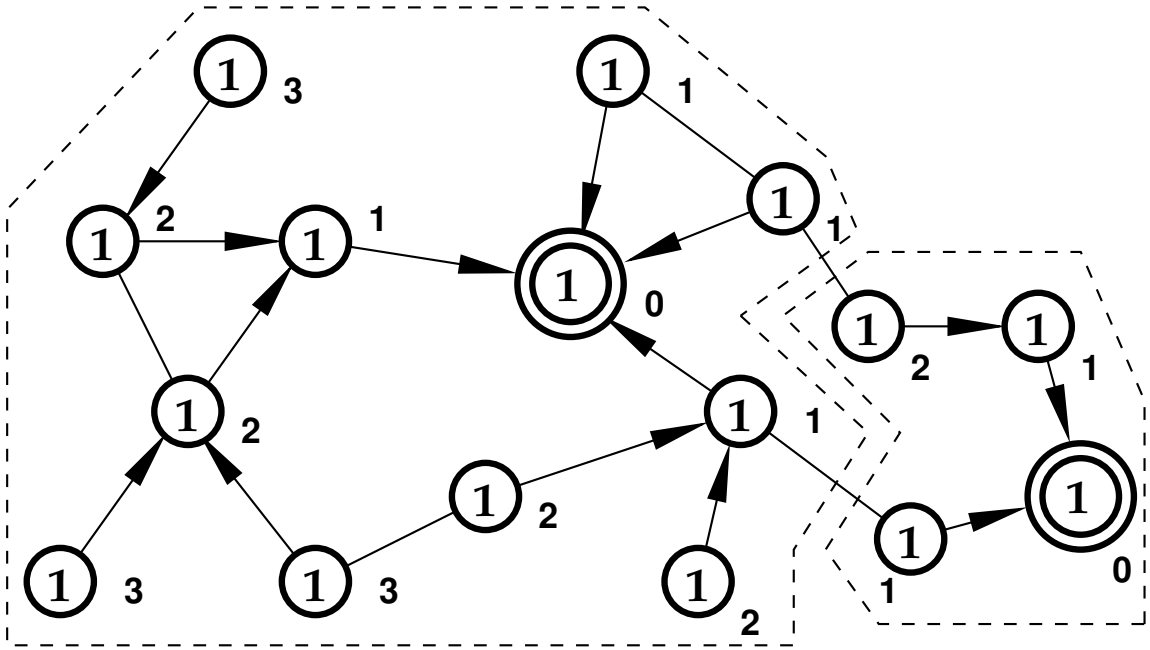
- **Convergence** within $1 + Diam$ **Rounds**.



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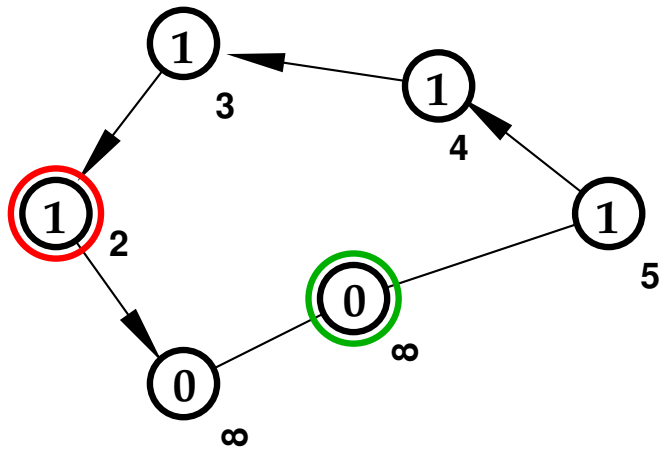

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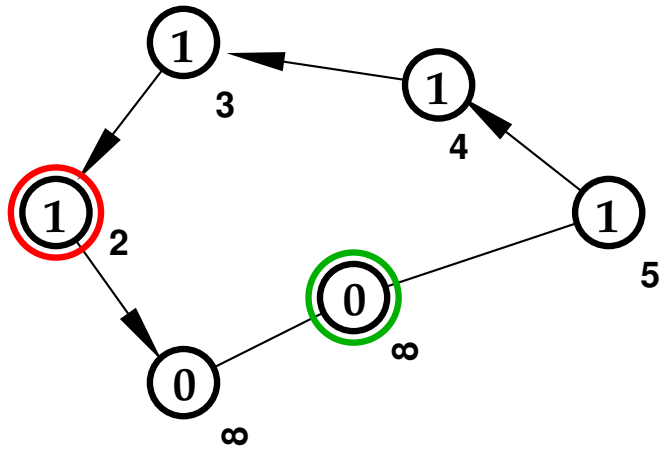

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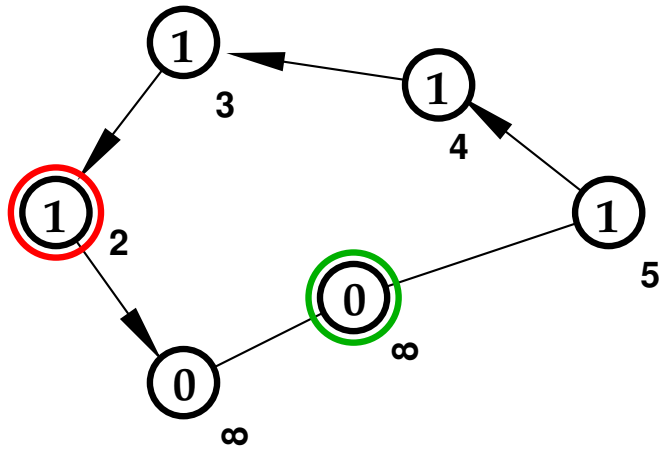

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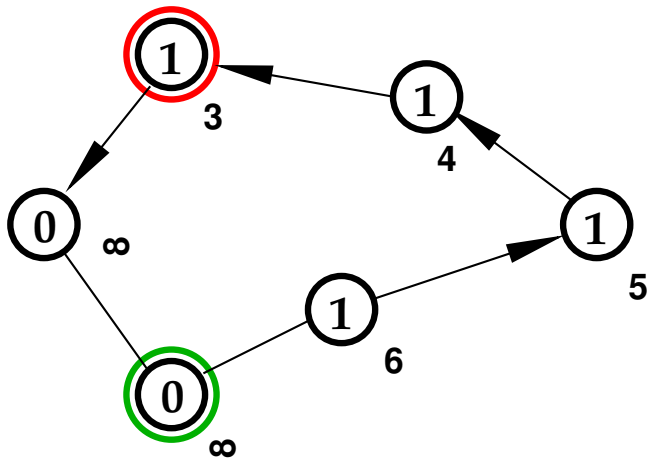

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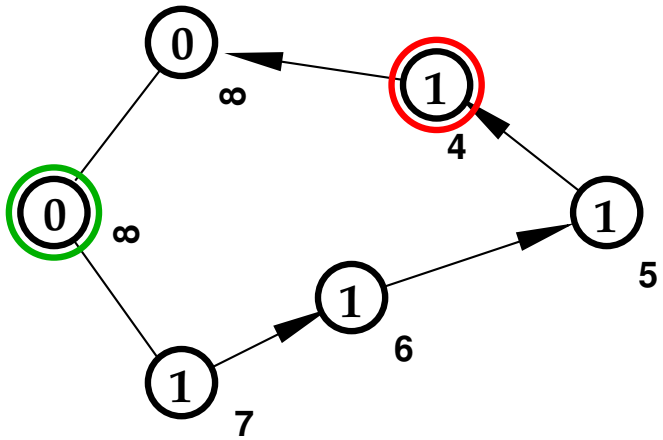

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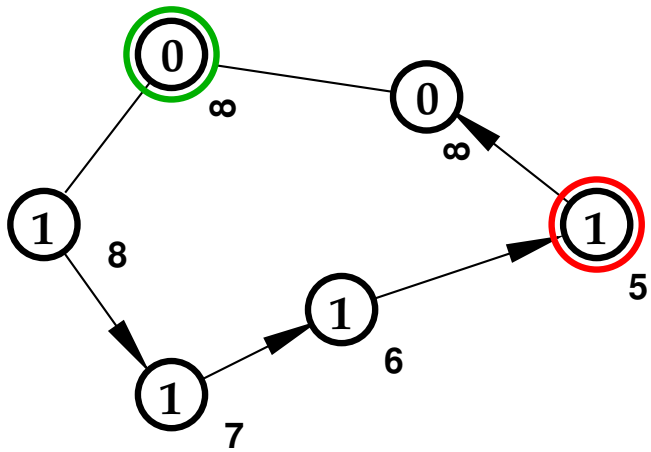

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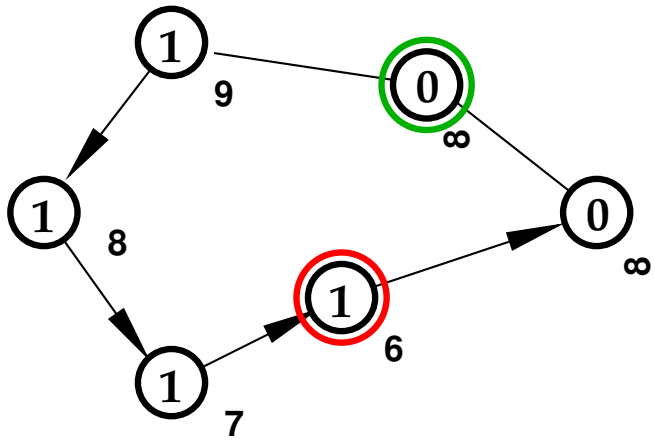

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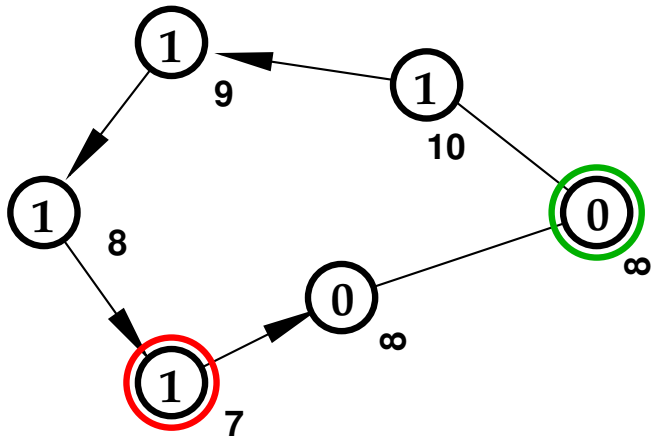

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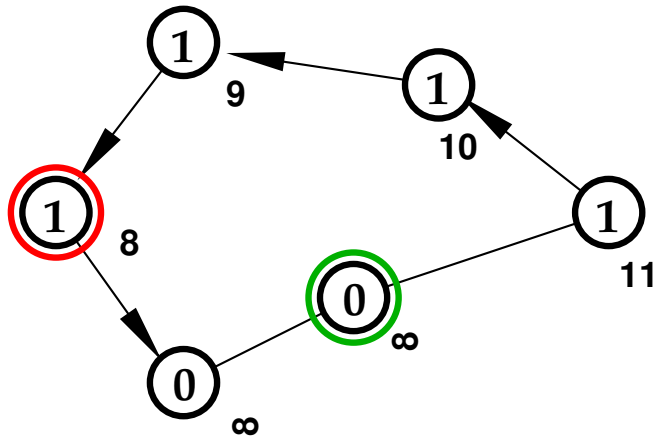
- **Return** to **First Configuration**, Except for Levels.



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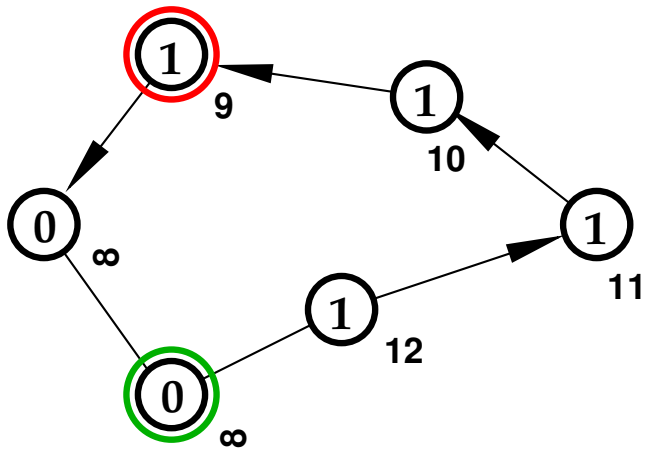
- Return to First Configuration, Except for Levels.
- **Endless!**



Purpose of Color Waves

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- **Prevent Indefinite Growth of Fictitious Trees.**

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Side Effects of Color Waves

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- **Slow Down Algorithm by a Factor of Three**

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- Slow Down Algorithm by a Factor of Three
- **After Legitimacy, Color Waves could Run Forever.**

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Counteract Effect with Done Waves

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Counteract Effect with Done Waves

- **Convergecast: Leaves Detect Algorithm Done**

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- Slow Down Algorithm by a Factor of Three
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Counteract Effect with Done Waves

- Convergecast: Leaves Detect Algorithm Done
- **Root (Clusterhead) Color Freezes.**

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- **Color Lock Results Within $O(Diam)$ Rounds.**

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- Slow Down Algorithm by a Factor of Three
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Counteract Effect with Done Waves

- Convergecast: Leaves Detect Algorithm Done
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- Color Lock Results Within $O(Diam)$ Rounds.
- **Silence.**

Color Wave Details:

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- Process with **Output Bit = 1** has **Color:** 0 = ①, 1 = ①.

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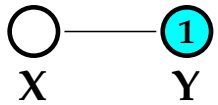

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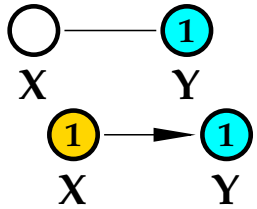

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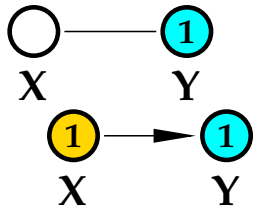

- Process **X** can **Change Color** if the **Following Conditions Hold:**

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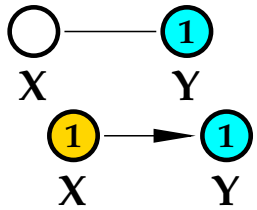

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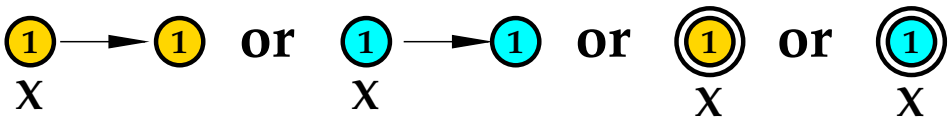

Color Wave Details:

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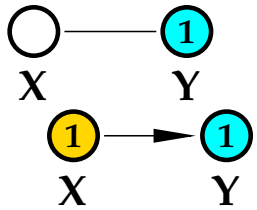

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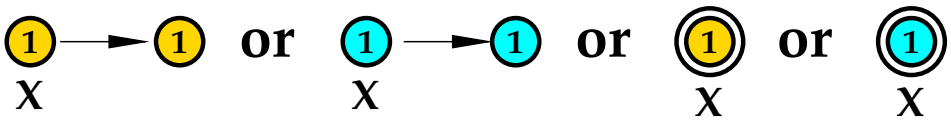
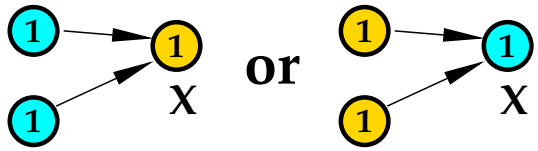


  - **Children** have **Opposite Color**:



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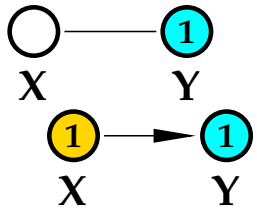

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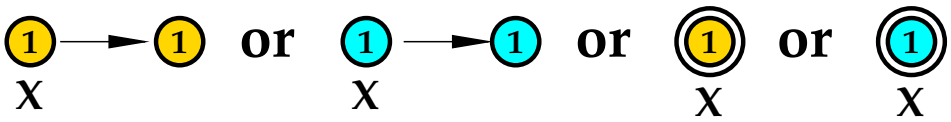

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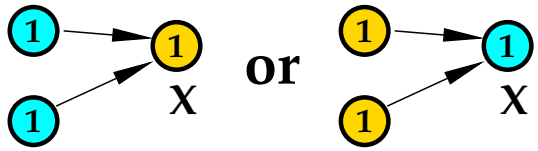

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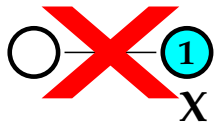

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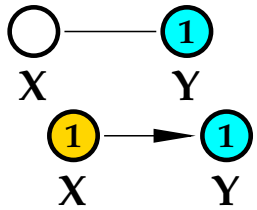

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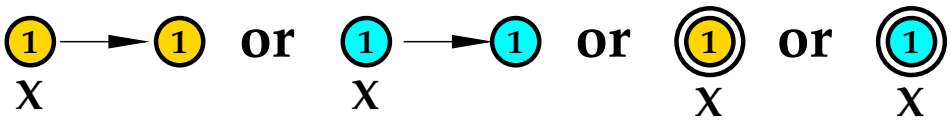

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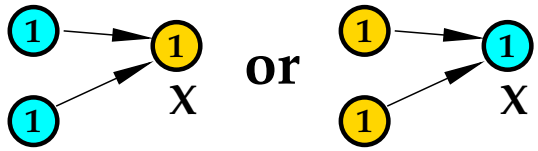

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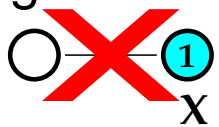

- When **Clusterhead** Changes Color, a **Color Wave** is **Absorbed.**

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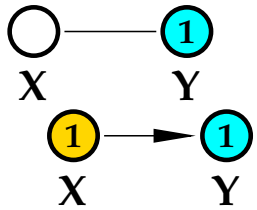

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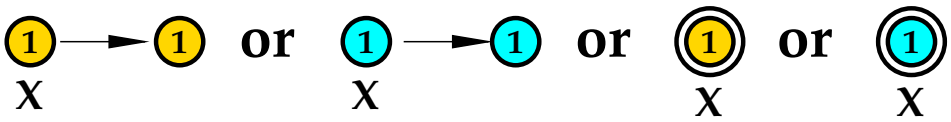

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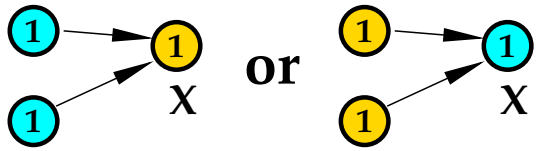

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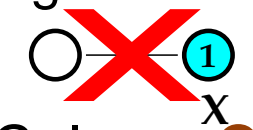

- When **Clusterhead** Changes Color, a **Color Wave** is **Absorbed**.
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Color Waves:

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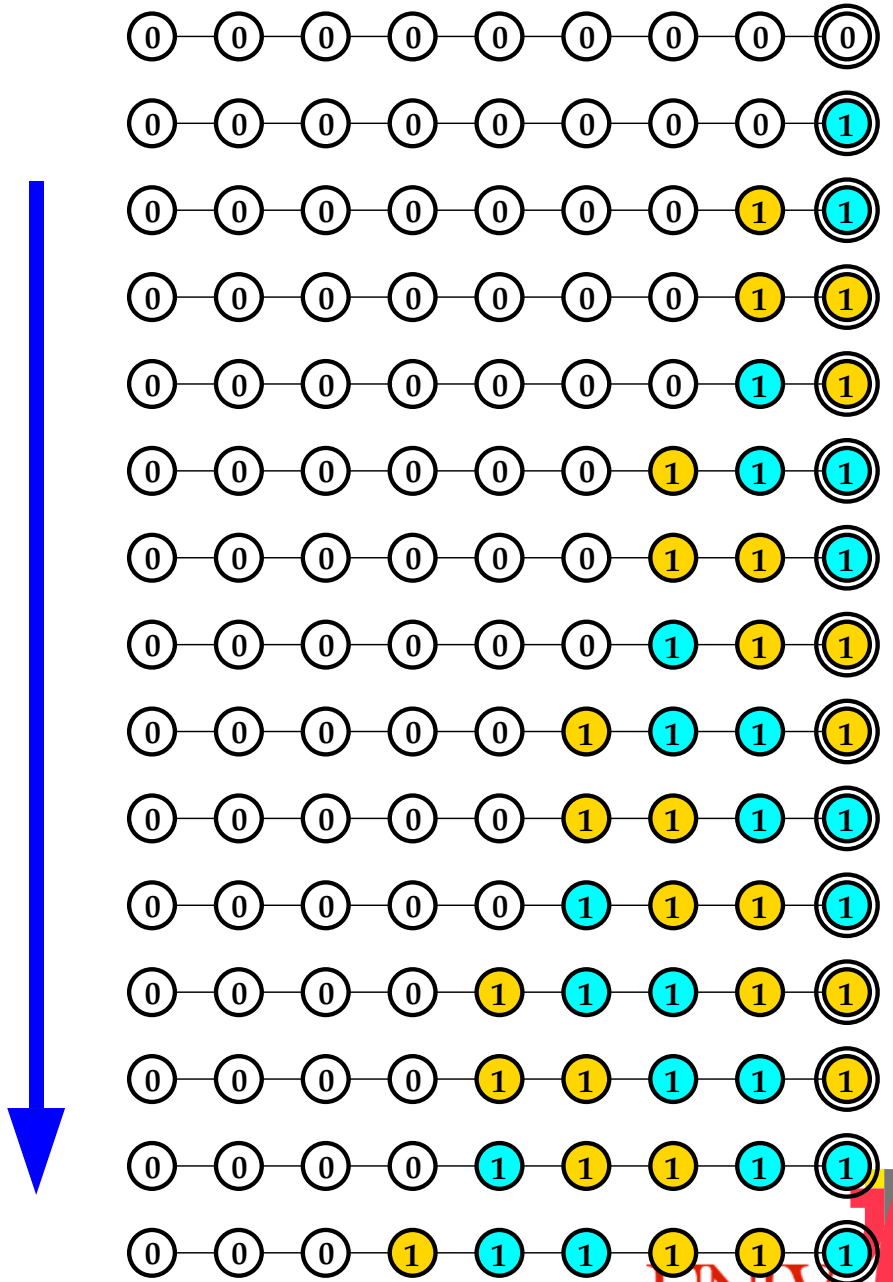
- **Chain Example.**



Color Waves:

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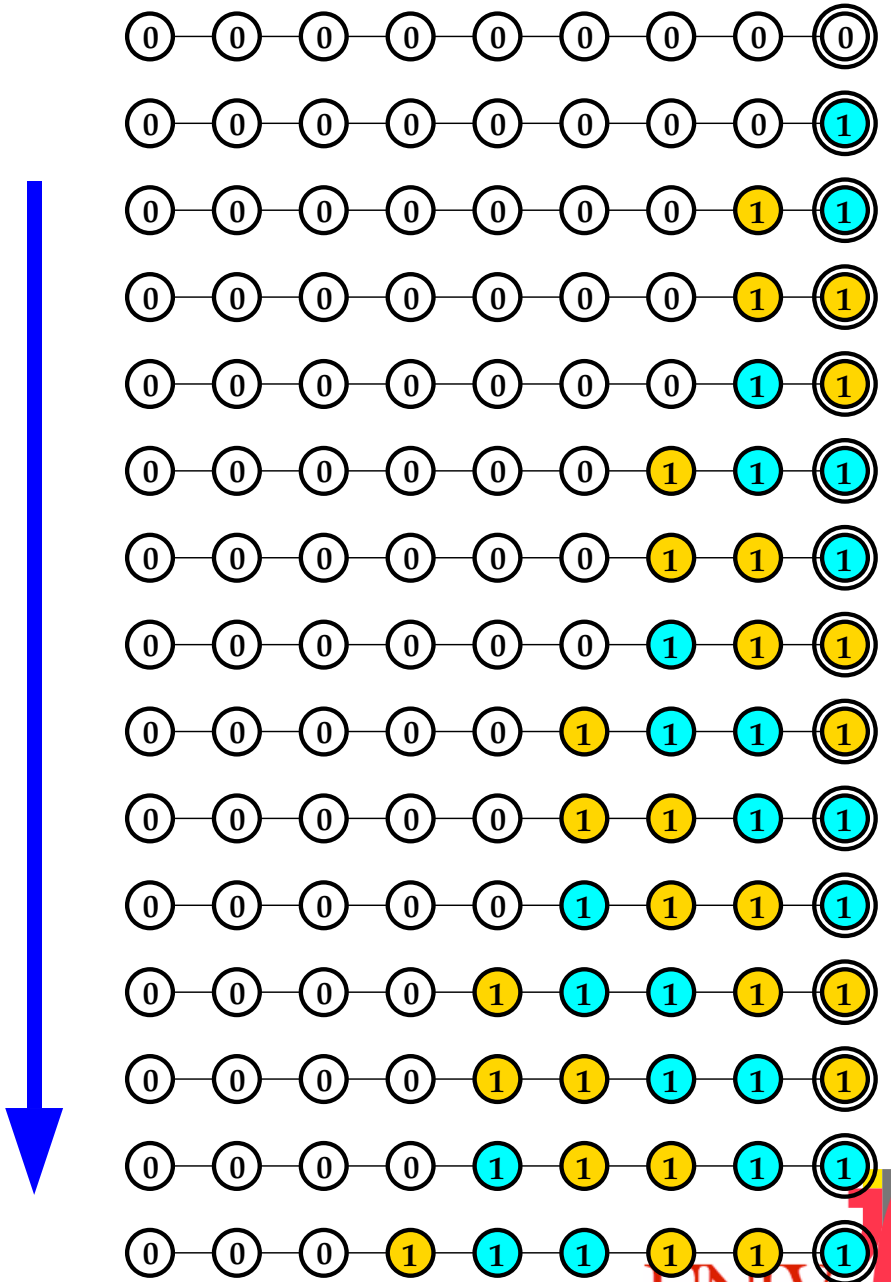

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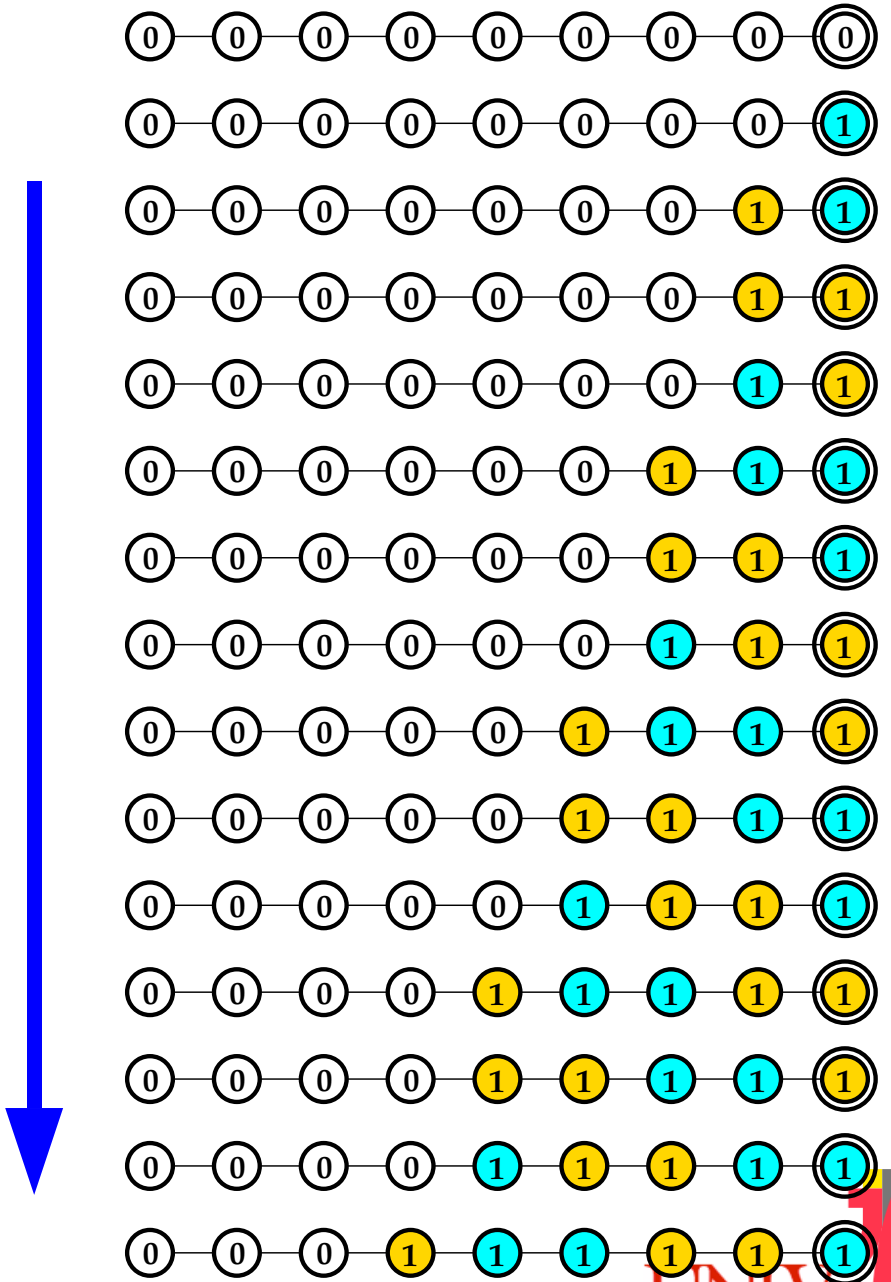

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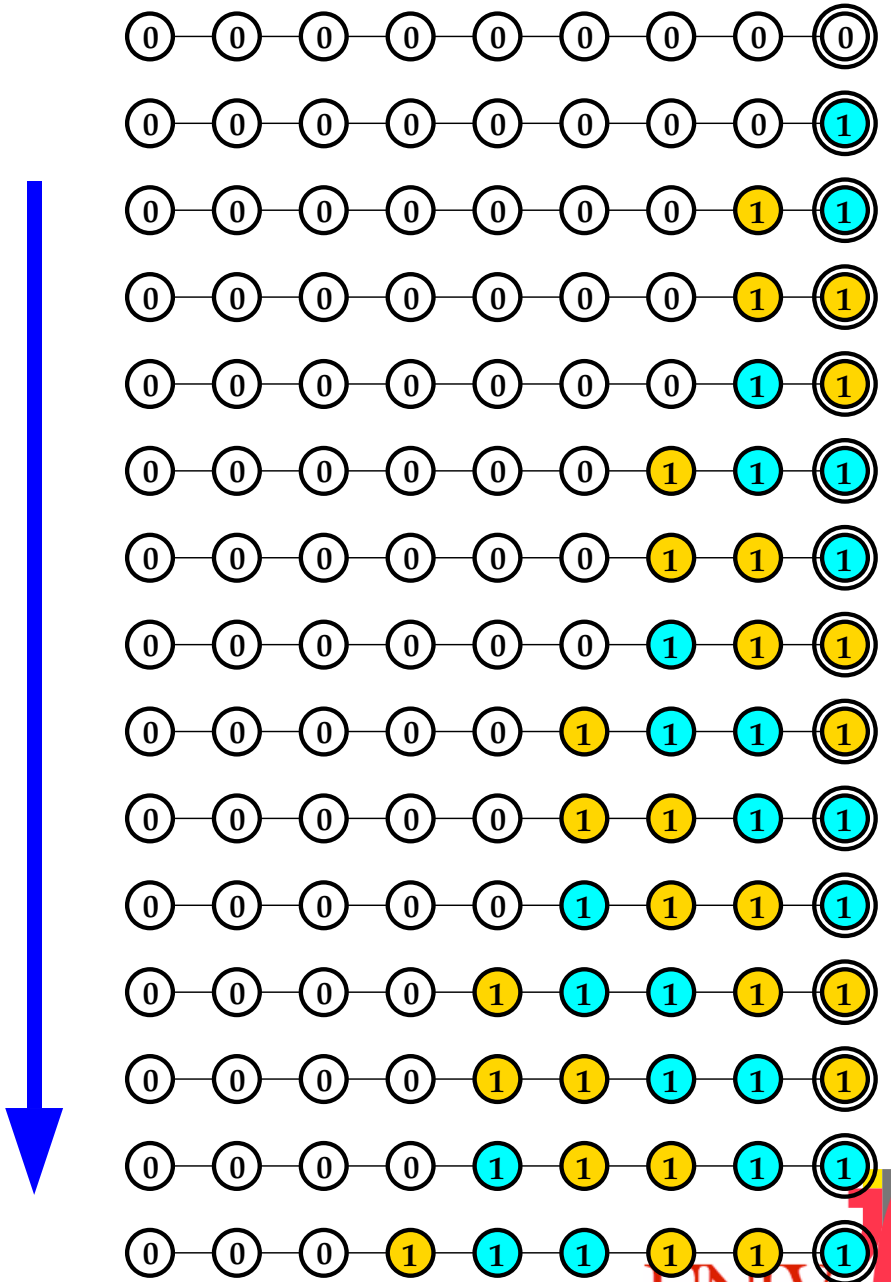

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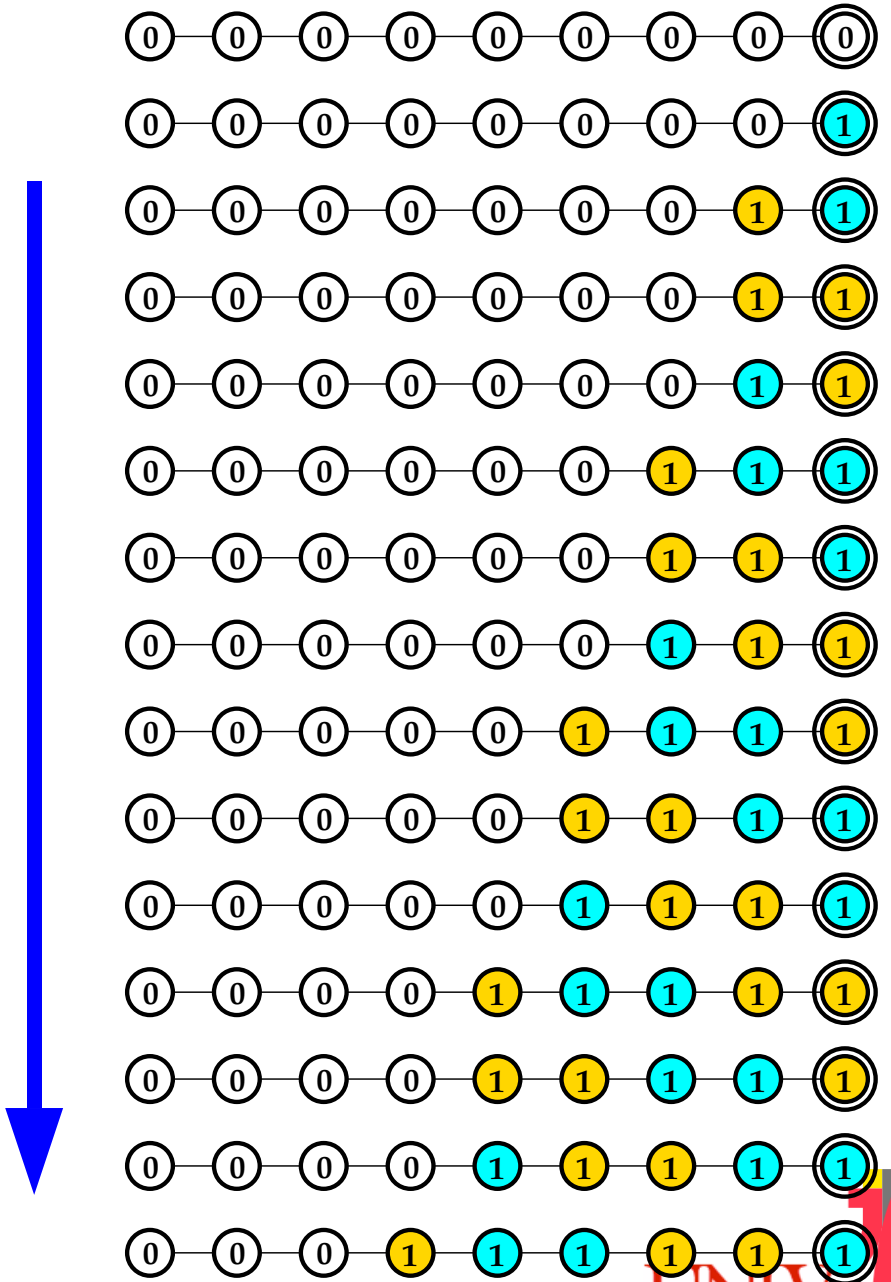

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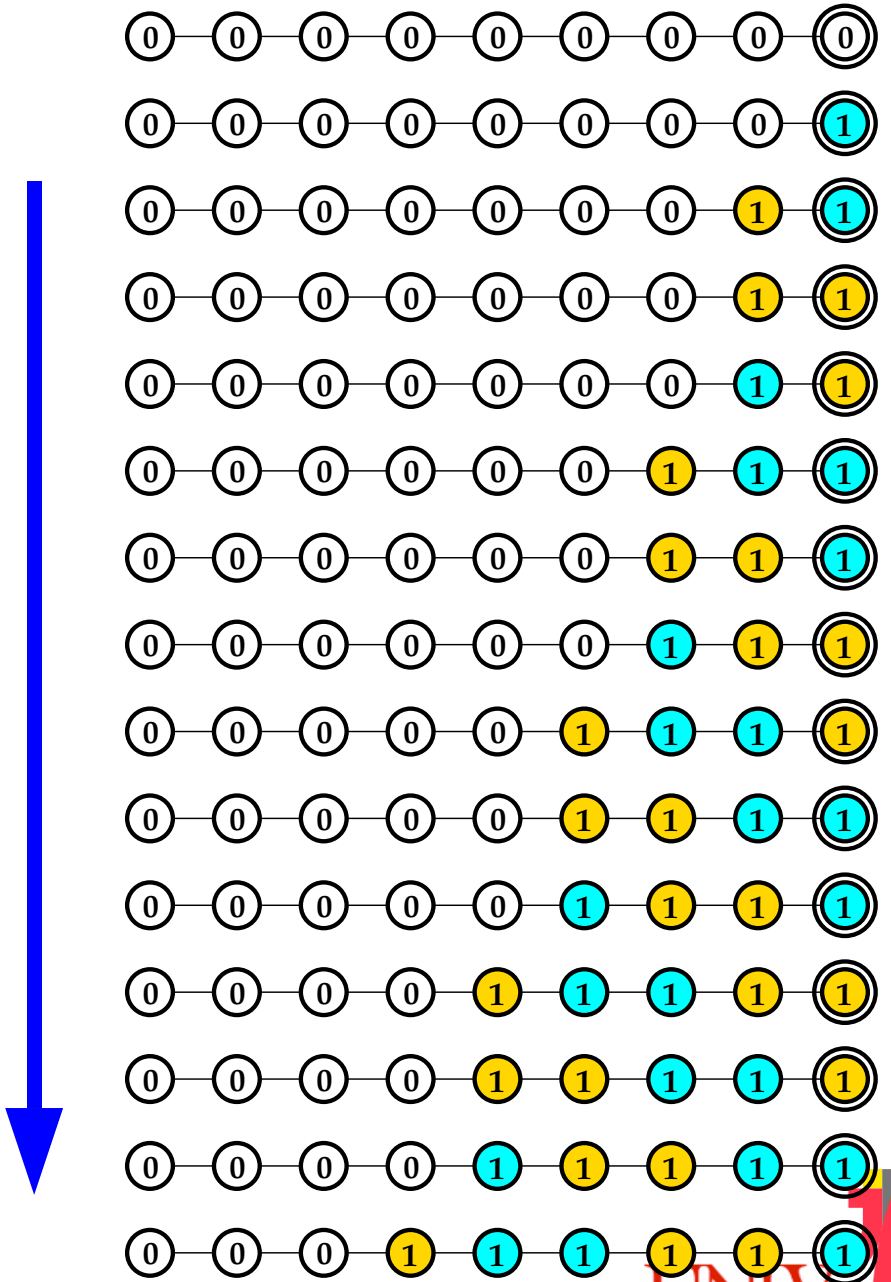

Getting Rid of False Roots:

- Consider a **False Root**, **R**.
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- Tree Rooted at **R** Cannot Grow Forever.
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  - **Color Lock:** Within $O(Diam)$ Rounds: **Silence** is Achieved.

Arbitrary Start: Output = 0: Problem Solved!

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- **Asynchronous Example Computation**

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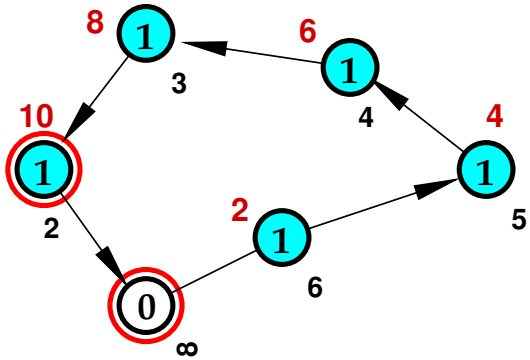

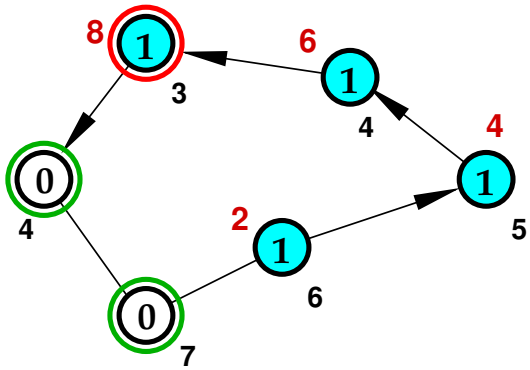

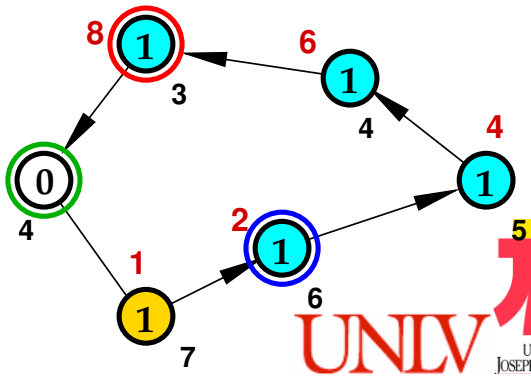

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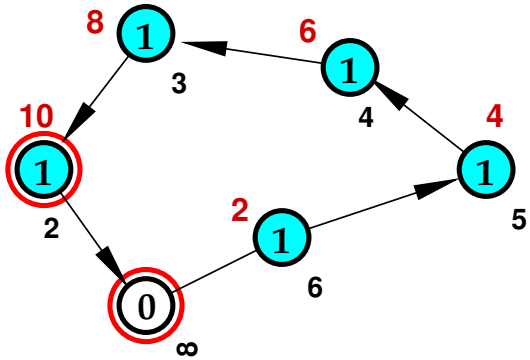

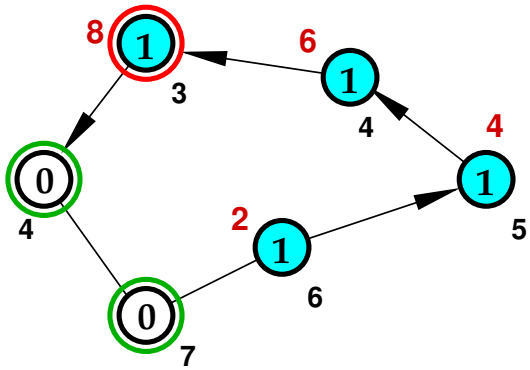

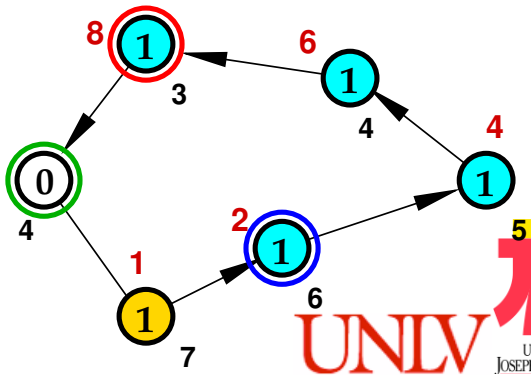

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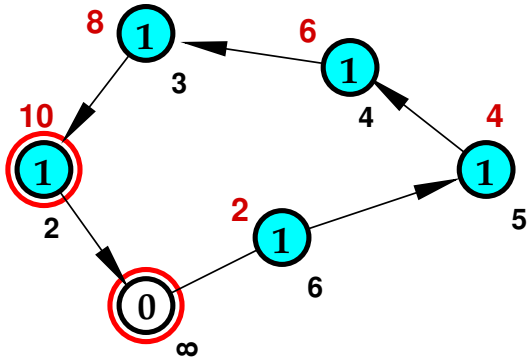

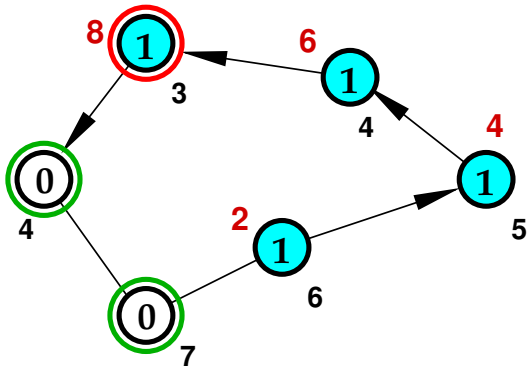

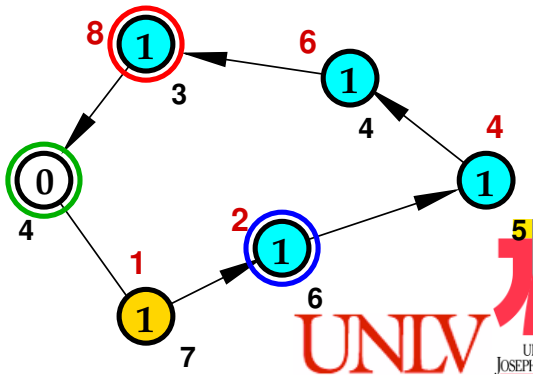

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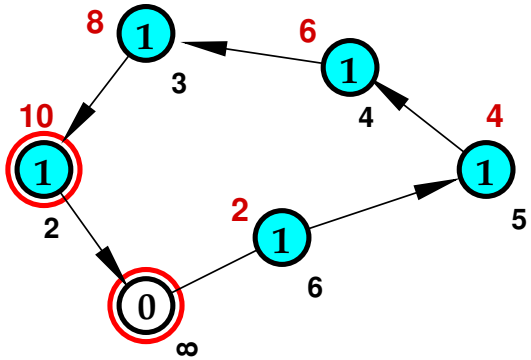

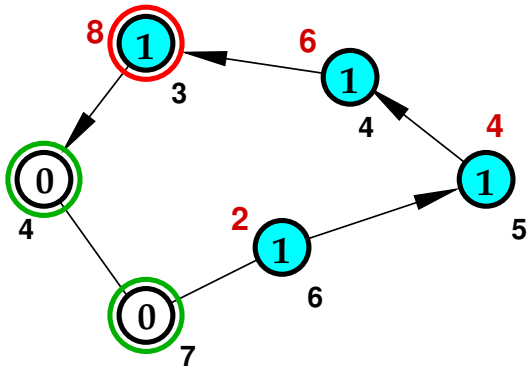

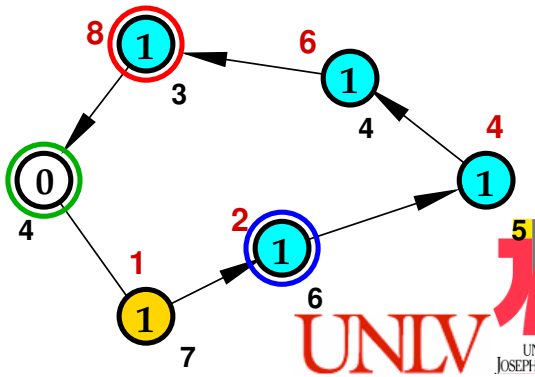

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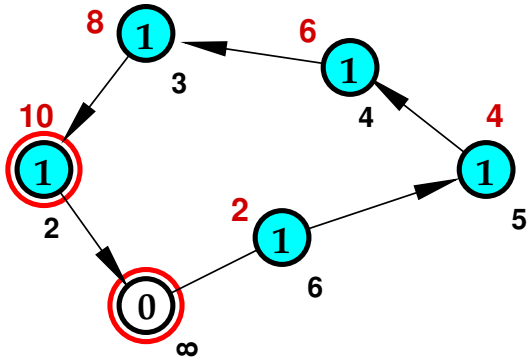
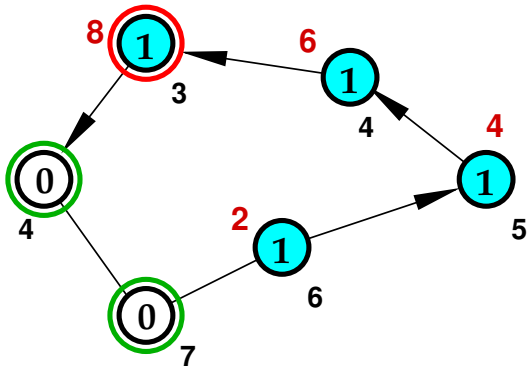
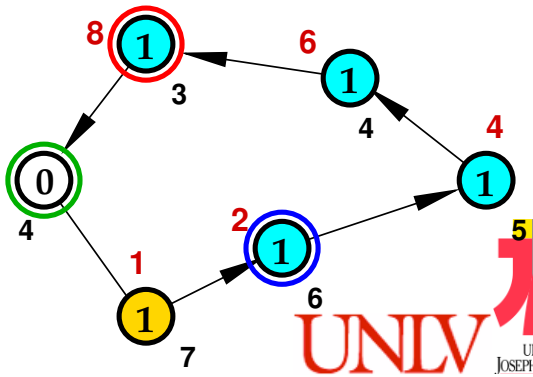
- Maximum **Energy** Initially = 10.







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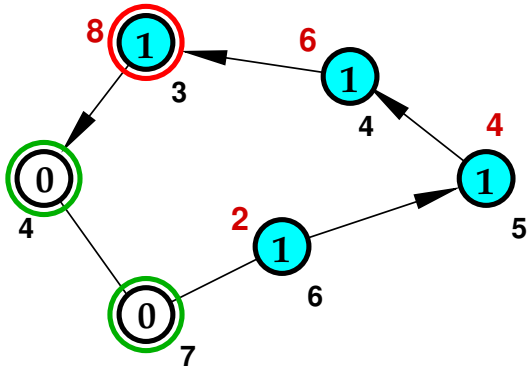

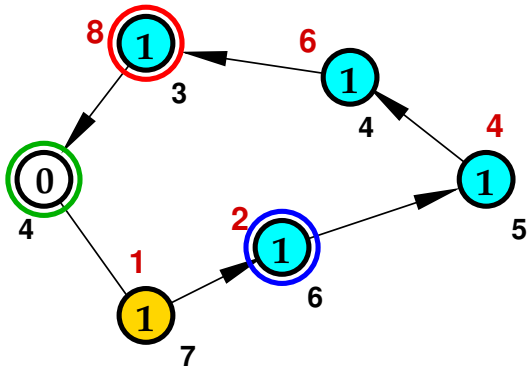

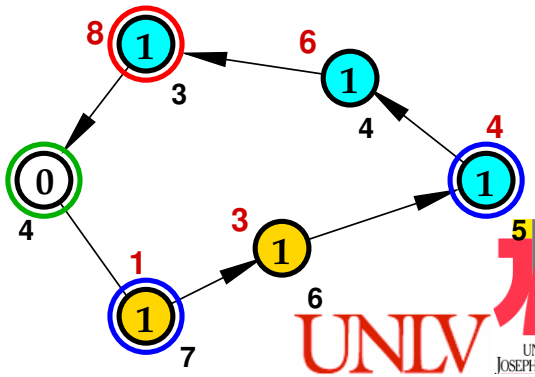

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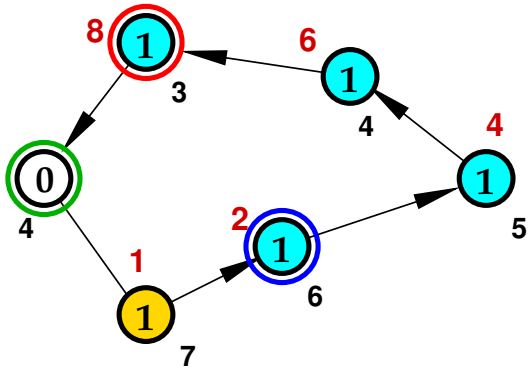

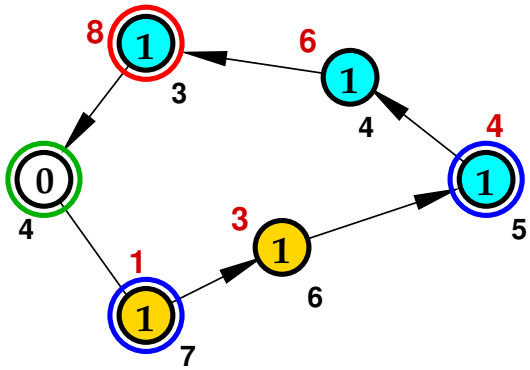

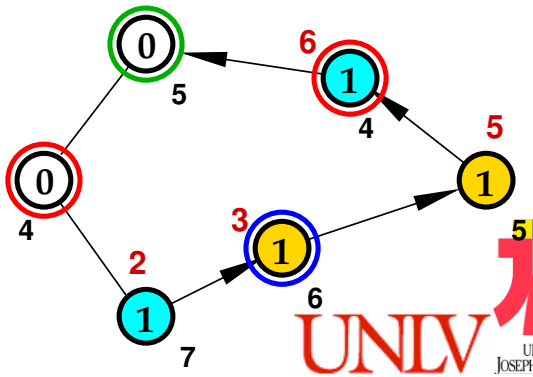

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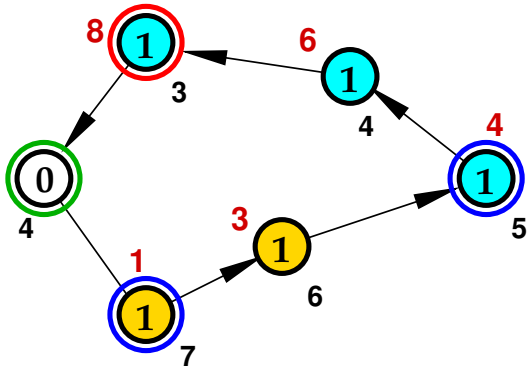

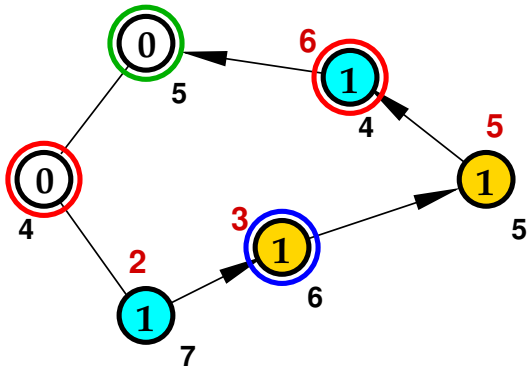

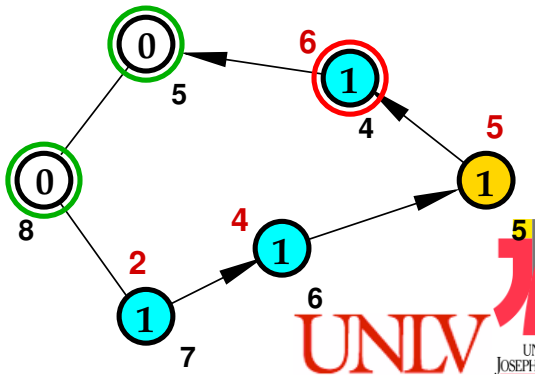

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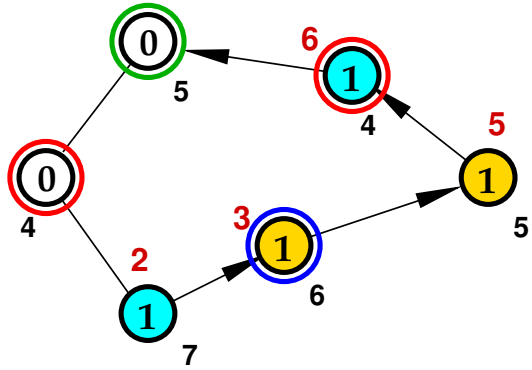

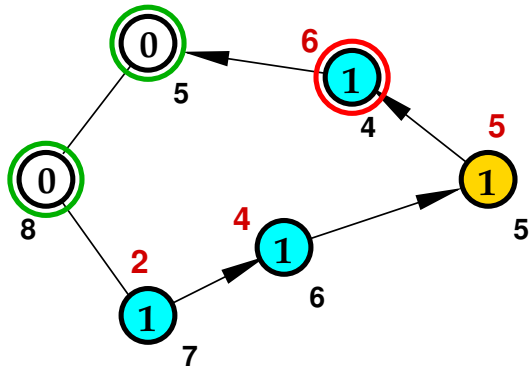

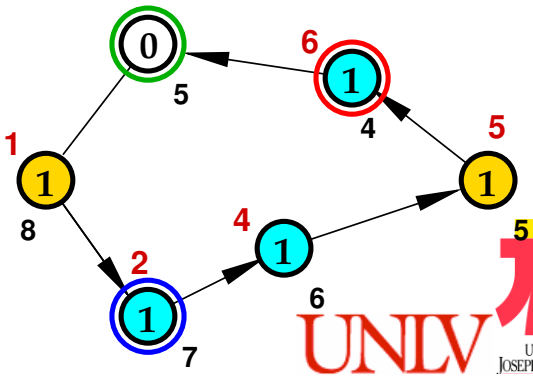

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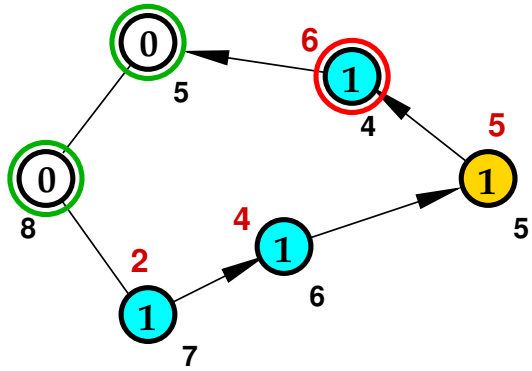

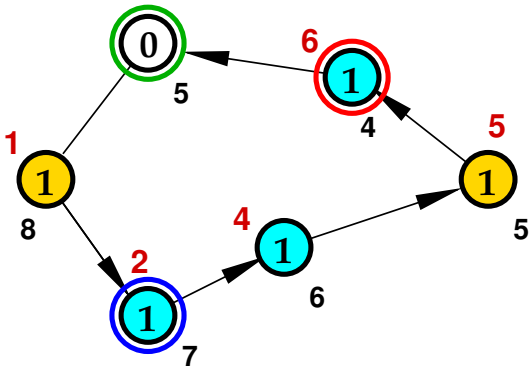

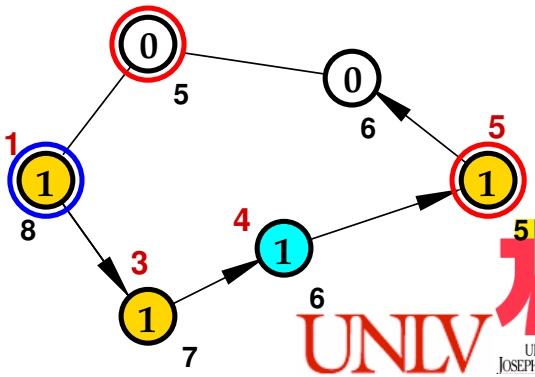

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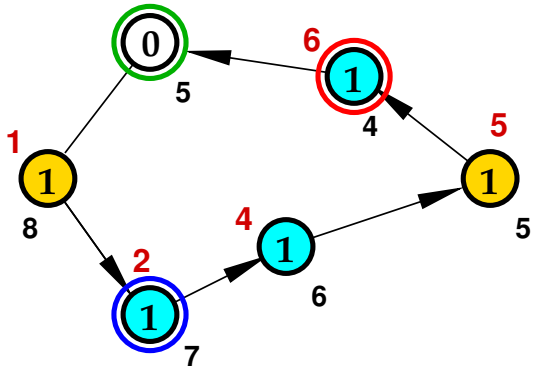

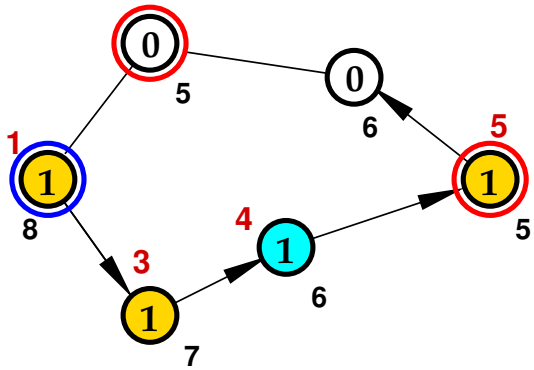

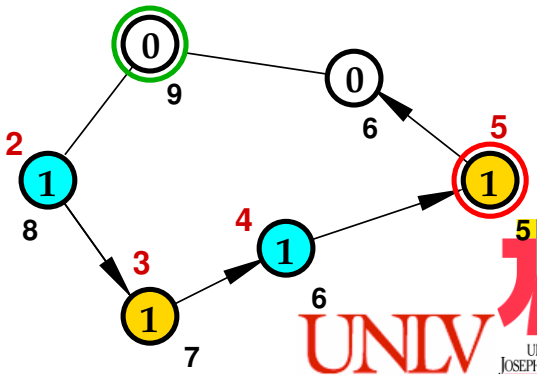

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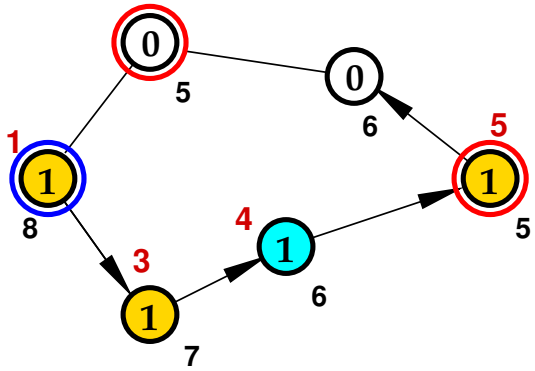

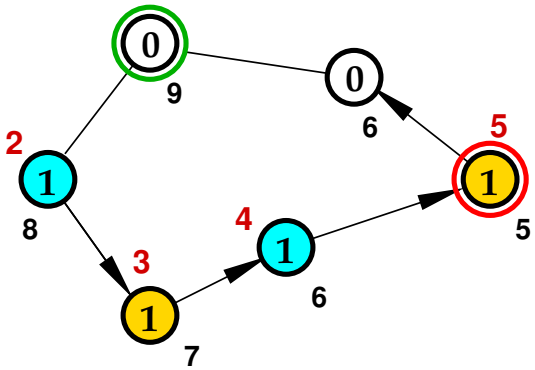

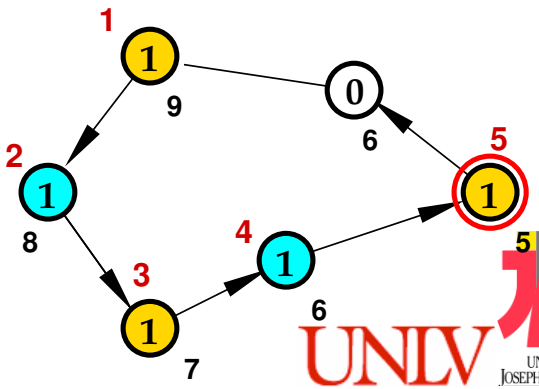

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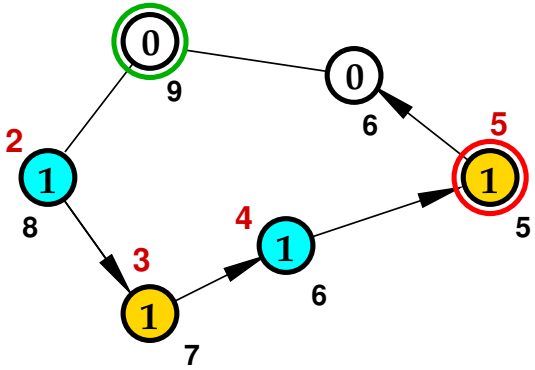

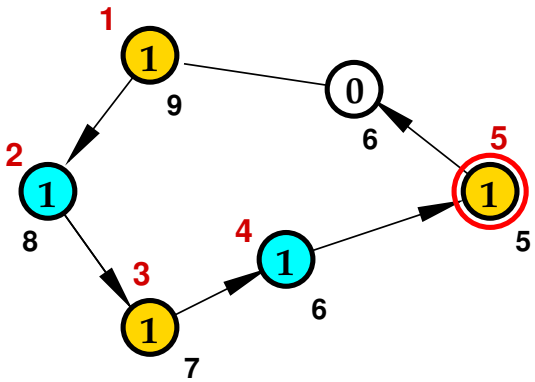

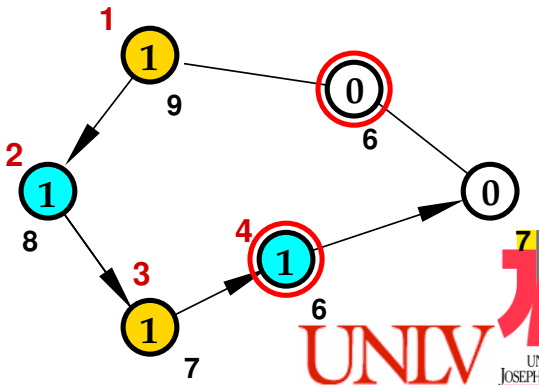

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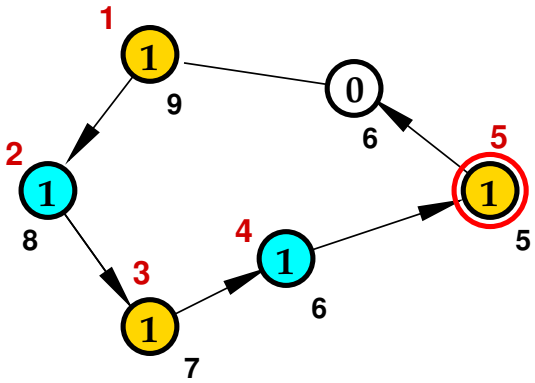

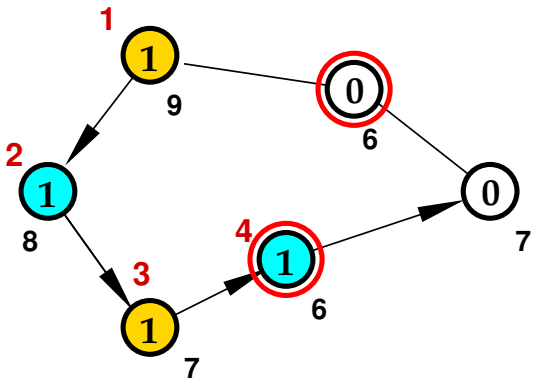

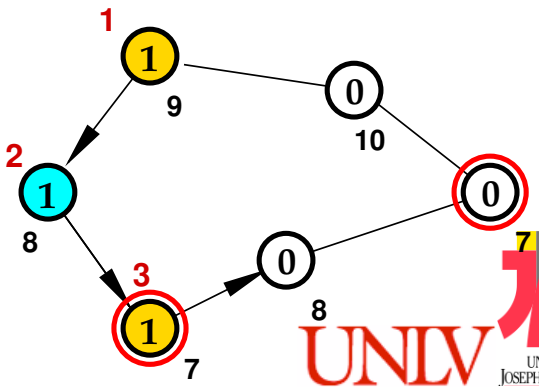

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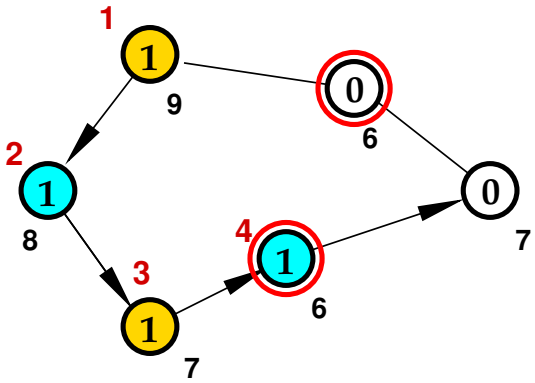

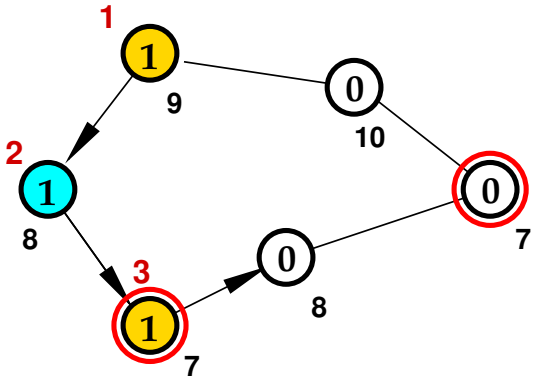

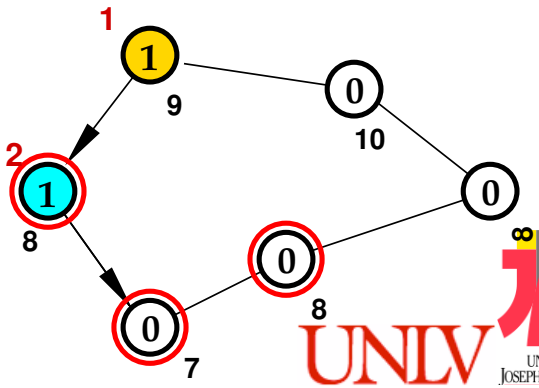

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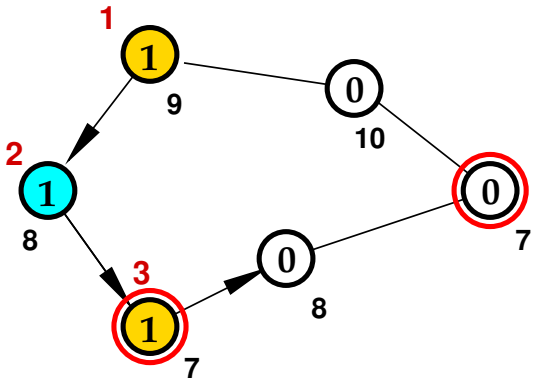

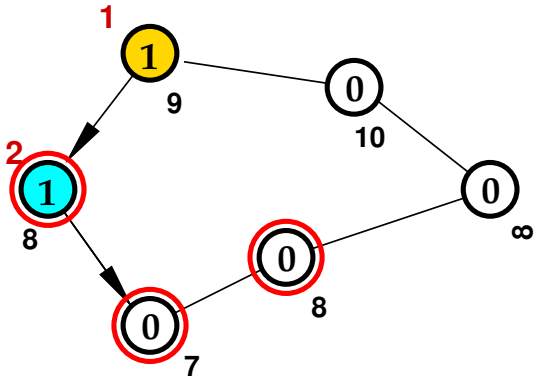

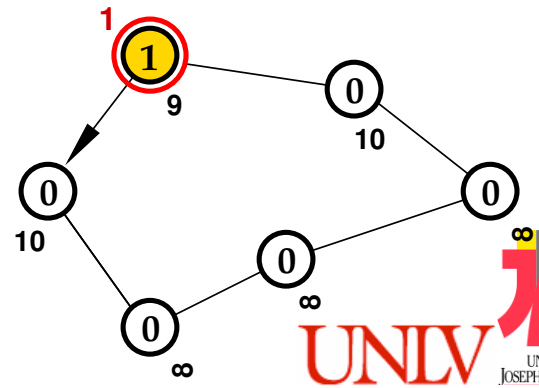

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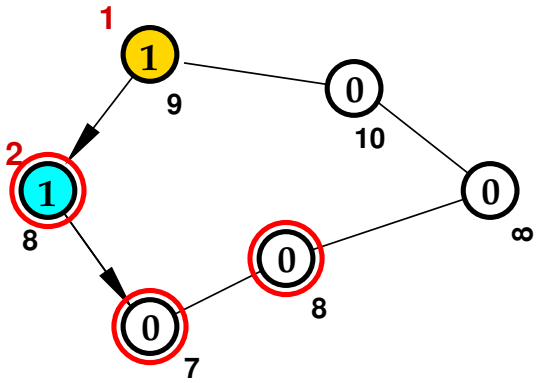

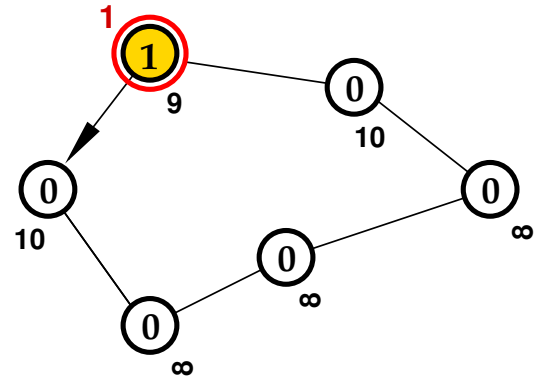

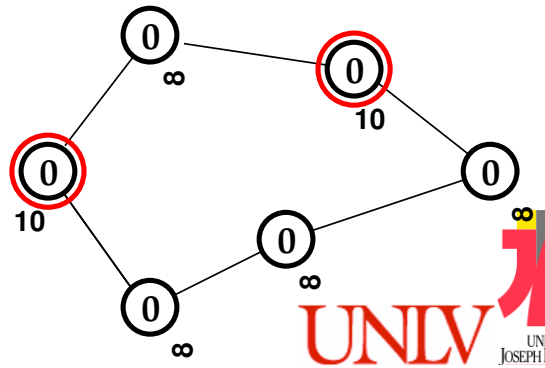

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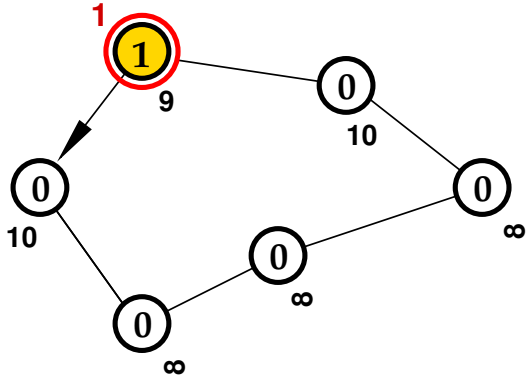

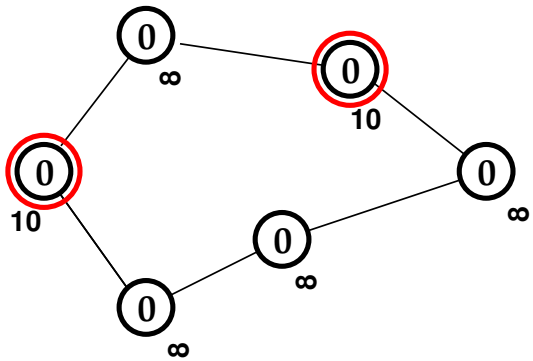

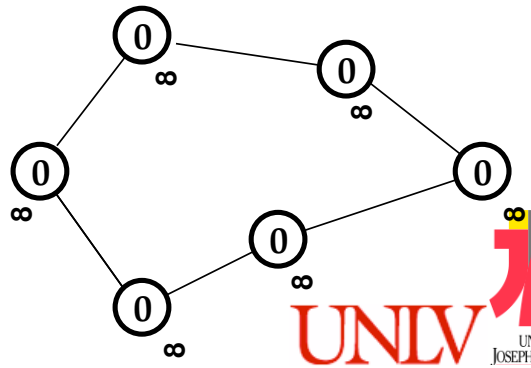

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  - $O(\log Diam + \log \Delta)$ space per process
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Thank You!