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Research Group
Formal Proofs

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Scientific Context
Automated reasoning is a widely-used technique for system and program verification, where problems are translated into logical formulas. These formulas can be checked by automated theorem provers. However, an automated prover may fail to answer a specific problem because of undecidability.

By contrast, interactive proof assistants allow the user to manually guide the proof process. This allows for more expressive proof techniques to be used at the cost of more user time and expertise. In order to reduce the burden on the expert user, it is desirable to provide suitable automation, especially for the seemingly trivial parts of interactive proofs.

Congruence closure[3] and equivalence graphs (E-graphs) are common tools used in Satisfiability Modulo Theory (SMT) automated provers. They have also been successfully applied to small-scale automation in the Coq interactive proof assistant [1]: indeed the congruence tactic is commonly used by Coq users to deal with proof problems involving equality and difference[2].

The final goal of the proposed research is to provide a decision procedure for the Coq proof assistant [1] that would extend equality-based reasoning to heterogeneous problems where equalities are expressed using multiple equivalence relations.

Scientific Problem
Equality reasoning is a very common paradigm for proofs both in the field of automated theorem proving and when using interactive proof assistants such as Coq [1]. The Coq proof assistant comes with a natural notion of equality which encompasses the notion of computation within the language (mostly typed λ-calculus). When reasoning over functions, the Coq equality captures equality of the programming code (as a λ-term) of the function rather than pointwise equality (for all inputs). However, most properties about functions are extensional, i.e., they are in fact properties of the images of the function.

A common approach to circumvent the problem is to work in a setoid (that is, a set equipped with an equivalence relation): instead of using Coq equality, one can define an ad hoc user-defined equality. In our current research prototype, we have designed an extended E-graph data structure capable of handling multiple equivalence and partial equivalence relations and we have implemented the corresponding (heterogeneous) congruence-closure algorithm.

In order to give our solver additional deductive power, an E-matching algorithm allows to find instances of patterns within the E-graph, and to perform actions such as adding new terms and equivalences to the E-graph. In particular, this allows to perform deductions and simplifications of propositions modulo logical equivalence. Preliminary tests look promising.

The next step is to evaluate different strategies for expanding the E-graph during the proof-search process. In order to evaluate such strategies, we plan to instrument Coq in order to massively test our proof-search procedure on a properly identified proof corpus.

(see next page)
**Proposed Work**  In this internship, we propose to work on the following tasks:

- Study & implement the addition of user-specified deduction rules to the existing procedure.
- Conduct tests and identify a suitable proof corpus where the procedure can be tested.
- Implement a repeatable testing infrastructure.
- Study aspects of proof-search strategies such as:
  - Criteria for instantiating quantified equivalence lemmas
  - Priorities and bounding strategies for the search space
  - Structural rules for reasoning with logical propositions
  - Structural rules for dealing with tuples / constructors

**Required Skills**  The main goal of the internship is to study automated reasoning rather than Coq itself, so no prior knowledge of Coq is required. We expect the candidate to be familiar with basic first-order logic. Typed $\lambda-$calculus is a plus. Development skills in Objective Caml are mandatory.

**Bibliography**


*Possible extension into a PhD thesis.*