1-synchronous clocks, underspecified clocks and non-determinism

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ENS - PARKAS

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Context of the presentation

- Link with the previous presentation:
  - Front-end in the previously presented compilation chain
  - Based on the synchronous compiler *Heptagon*
  - Orthogonal to the architecture used

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  - Front-end in the previously presented compilation chain
  - Based on the synchronous compiler *Heptagon*
  - Orthogonal to the architecture used
- In relation to Lopht:
  - Manage the harmonic multi-periodic aspect
  - Normalization of the input Lustre program + annotations
- Other motivations:
  - Make specification easier to write manually in Lustre
  - Using more information which could be specified

Background - Synchronous language

- Manipulate infinite flow of values
- Global tick synchronize the production of values
- Point-to-point operators
- Accessing past values possible ("fby" ≈ memory)

x	0	1	1	2	...
y	4	-2	1	4	...
42	42	42	42	42	...
$x + y$	4	-1	2	6	...
42 fby y	42	4	-2	1	...

Background - Clocks

- A stream might have no value on a tick
- **Clock**: $x :: clk$
    - Encode the presence of a value
    - Can be an arbitrary boolean stream
- Temporal operators: sub-sampling (when) and fusion (merge)
- Clocking analysis: check coherency of clocks

x	$:: c$	0	1	1	2	...
b	$:: c$	t	f	t	t	...
$z = x \text{ when } b$	$:: c \text{ on } b$	0	–	1	2	...
y	$:: c \text{ on not } b$	–	42	–	–	...
$\text{merge } b\ z\ y$	$:: c$	0	42	1	2	...

Background - Lustre

- Equational language for synchronous programs (similar languages: Scade, Heptagon, ...)

```
node accumulator(i : int) returns (o : int)
var x : int
let
   x = 0 fby o;
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- **Code generation:**
  - "reset" and "step" functions
  - Infinite "while" loop (1 iteration = 1 base tick)
  - Clocks: encoded using "if" conditions

Background - N-synchronous model

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  - Ultimately periodic clocks
  - Example: 101(1001)
  - Strictly periodic: no initialization phase

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- **buffer**: Communication between variables on two different clocks
  - Clocks must be compatible (adaptability relation: $<:$)

  ⇒ Able to compute the size of a buffer

1-synchronous clocks

- Consider **integration program**:
  Top-level node, orchestrating all tasks of an application
  - Multiple harmonic periods (ex: 5 ms / 10 ms / 20 ms / ...)
  - Tasks are present only once per period

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- Integration program: only 1-synchronous clocks are used
  $\rightsquigarrow$ Can use that condition to do more inside a compiler

In this talk

Three incremental modifications on top of Lustre:

1. Restriction of the clock calculus to 1-synchronous clocks
   - Specialization of the N-synchronous clocks
   - Associated specialized clocking rules
   - Code generation possibilities (Hyperperiod Expansion)

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3. Non-deterministic computation
   - Don't mind which instance of a value used
   - Neither semantics are satisfied
   - More freedom for phase selection
   - Go back to deterministic program

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$$\frac{H \vdash \mathrm{a} :: (0^k 1 0^{n-k-1}) \quad 0 \leq d < n - k}{H \vdash \mathrm{delay(d)\ a} :: (0^{k+d} 1 0^{n-(k+d)-1})}$$

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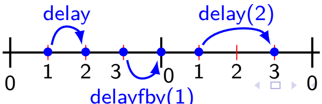
- **delayfby(d):** (initialization required / ≈ "short fby")

$$\frac{H \vdash a :: (0^k 1 0^{n-k-1}) \quad H \vdash i :: (0^{k+d-n} 1 0^{n-(k+d-n)-1}) \quad 0 \leq k + d - n < n}{H \vdash \text{i delayfby(d) a} :: (0^{k+d-n} 1 0^{n-(k+d-n)-1})}$$



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⇒ Harmonicity condition

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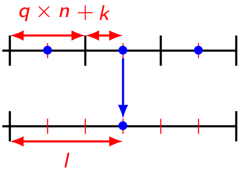

y = x when (FTF)

$$\frac{H \vdash a :: (0^k 10^{n-k-1}) \quad m = pn \quad l = qn + k}{H \vdash a \text{ when } (F^q TF^{p-1-q}) :: (0^l 10^{m-l-1})}$$

Toward faster periods (merge/current)

Clocks must be 1-synchronous + subclock condition:

⇒ Harmonicity condition

- **merge**: one branch per instance of fast period
- **current** (repetition of a value, with eventual updates)
  - Argument (when the update occurs) must be "$(F^k T F^{n-k-1})$"
  - Initialization needed ("i")

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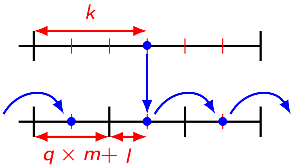

y = current((FTF), 0, x)

$$\frac{H \vdash a :: (0^k 10^{n-k-1}) \quad H \vdash i :: (0^l 10^{m-l-1}) \quad n = pm \quad l = k - mq}{H \vdash \mathrm{current}((F^q TF^{p-1-q}), i, a) :: (0^l 10^{m-l-1})}$$

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- Three code generation schemes:
  - Classical step function (base clock)
    - If conditions
  - One step function per phase (base clock)
    - No if conditions / while loop looping on them in order
  - One step function for the whole period (slowest clock)

  $\Rightarrow$ Hyperperiod expansion transformation

Hyperperiod expansion - Example

Idea: change base period to a slower one (ex: scm of all periods)
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Example:

```
Input:   x :: (1)
Local:   a :: (1), b :: (10)
     a   =   f(x);                 // f stateless
     b   =   g(a when (10));       // g stateless
         ...
```

```
Input:   x0, x1 :: (1)
Local:   a0, a1, b :: (1)
    a0   =   f(x0);
    a1   =   f(x1);
     b   =   g(a0);
         ...
```

Hyperperiod expansion - More details

Transformed equation gives a set of equations. Intuitions:

- $r(Var) \in \mathbb{N}^*$: ratio between Var's period and slowest period
- Variable duplication: $Var \rightsquigarrow Var_0, \ldots, Var_{r(Var)-1}$
- Applied on a normalized program
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- $\mathbf{a = op(b1, \ldots, bm)} \Rightarrow \mathrm{a}_i = \mathrm{op}(\mathrm{b1}_i, \ldots, \mathrm{bm}_i)$ for $0 \leq i < r$

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- **a = current(($\mathbf{F}^p$ T $\mathbf{F}^{n-p-1}$), init, b)**

$$\Rightarrow \begin{cases} a_i = init_i \text{ fby } b_{r(b)-1} & \text{for } 0 \leq i < p \\ a_i = b_{\lfloor \frac{i-p}{n} \rfloor} & \text{for } p \leq i < r(a) \end{cases}$$

Hyperperiod expansion - Discussion

- **Positive points:**
  - Get rid of the multi-periodic aspect
  - Natural way to manage long tasks (with no cutting)
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  - Natural way to manage long tasks (with no cutting)
  - Decouple the phases of different instances of a variable
- **Negative points:**
  - Stateless functions needed
    (If stateful, need to expose the internal state and pass it
    + reset function to get initial state
    + at annotation to reuse the memory of states)
  - Additional real-time constraints needed on inputs/outputs (release/deadline)

The problem with phases

- Phases = large-grain schedule across the periods
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- **Modification proposed:**
  - Option to only define the period of some local variables
  - Implicit buffers operator (clock of rhs $<:$ clock of lhs)
- **Compilation flow:**
  - Clocking analysis gathers the constraints on phase
  - Solver finds a solution (given cost function)
  - Use this solution to explicit phases and buffer (→ delay)

Extracting constraints from clocking rules

- **buffer:** delay of an unknown length
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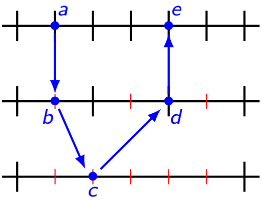
- **bufferfby:** additional initialization (period crossed)
- Variations of buffer with other constraints:
  - buffer which fixes its phase (ex: $p \leq 3$)
  - buffer which constraint the latency (ex: $p_B - p_A \leq 3$)

Example of clock extraction

a,e :: period(1);
b,d :: period(2);
c :: period(6);
b = buffer f_1(a when (FT));
c = buffer f_2(b when (TFF));
d = buffer f_3(current((FFT), 0, c))
e = buffer f_4(current((TF), 0, d))



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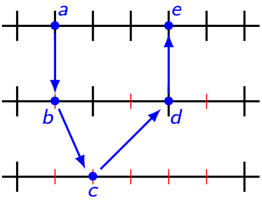

- Bounds from variable declaration:
  $0 \le p_a, p_e < 1$ / $0 \le p_b, p_d < 2$ / $0 \le p_c < 6$
- Constraints from buffer:
  $p_a + 1 \le p_b$ / $p_b \le p_c$ / $p_c - 4 \le p_d$ / $p_d \le p_e$
- Solutions:
  $p_a = p_e = 0$ / $p_b = 1$ / $p_d = 0$ / $1 \le p_c \le 4$

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- **Use case:** flight control application (6k nodes, 30k data, 4 harmonic periods)
  - Sequential case: load balancing across phases (task weight = its WCET)
  - Direct ILP formulation of the problem tricky possible (Introduce boolean variable $\delta_{T,k}$ for the phases)

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    $\Rightarrow$ Does not scale...
  - ILP formulation with only boolean variable

    $\Rightarrow$ First integral solution found after 40 mins
    - Good solution, non-optimal, but takes too mush time

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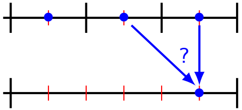
- **Reinjection step:**
  - Complete the clocks of local variables
  - Replace all buffer with delay (or remove them)

Non-deterministic computation

- Physical values with low temporal variability
  - Ex: outside temperature
  - Want last value, but not strict requirement (older one ok)
  - Constraint on phase can be relaxed

$\Rightarrow$ Express and use ND to give more freedom to the compiler



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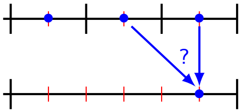

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- How to express notion in a minimal way in the language?

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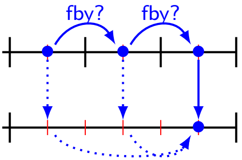
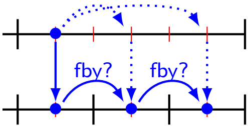
  - `expr`
  - or (`i fby`^{k} `expr`) (with $0 \leq k \leq n$)
- **Determinization pass:** Replace all fby? by a possible value (in our case: fix that depending on its phase)

Constraint extraction with non-determinism



y = (i fby?^{2} x) when (FFT)



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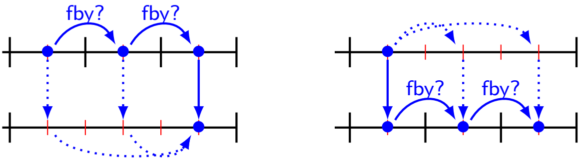

y = (i fby?^{2} x) `when` (FFT)   y = i `fby?`^{2} `current`((TFF), 0, x)

- Typing analysis: rule for `fby?` doesn't give any constraint
  - Recognize fby? under a `when` & above a `current`
  - → Typing rules for these specific situations
- Other option: defining `when?` and `current?` operators

In summary. . .

- 3 incremental extensions:
  - 1-synchronous clocks
  - . . . with unknown phases
  - . . . with non-deterministic computation
- Hyperperiod expansion transformation
- Constraints on phase can be inferred from the clocking rules
- Non-deterministic operator & adaptation of constraints

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- Thank you for listening, . . .
  - . . . Do you have any questions?