Symmetry Breaking for Multi-Criteria Mapping and Scheduling on Multicores

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Context

- Typical in parallel programming: spawn multiple identical tasks
  - data parallelism
  - obtain hyperperiod of a multi-periodic system
  - duplicate tasks for fault-tolerance

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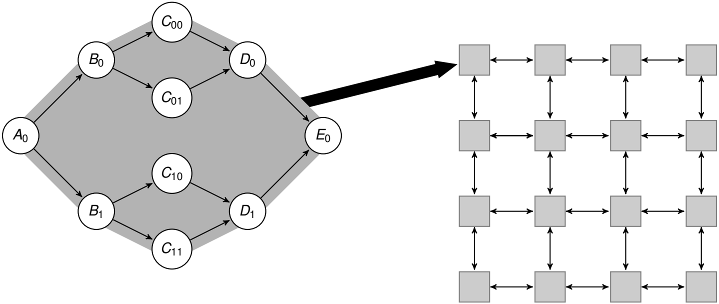
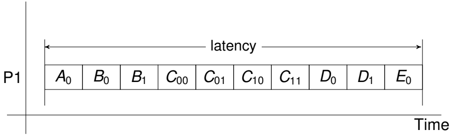
  - obtain hyperperiod of a multi-periodic system
  - duplicate tasks for fault-tolerance
- Often the platform have multiple identical processors.
- Hence, symmetry in the solution space.

Multi-criteria Optimization

minimize latency using minimal number of processors





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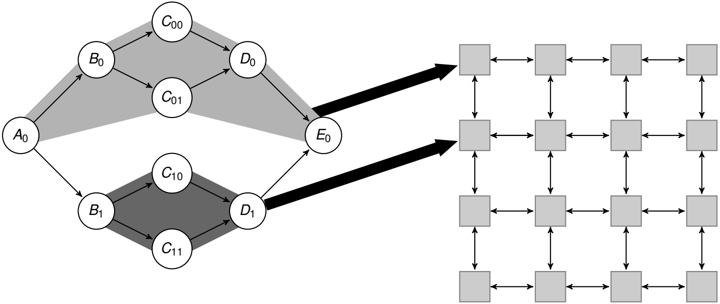

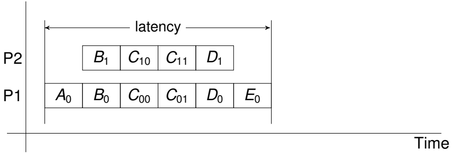

Contribution

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static mapping and scheduling for programs with data-parallelism

multi-criteria optimization using SMT solvers

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goal: increase the tractable problem size of SMT solvers

experiments : problem size increase from 20 to 50 tasks

Outline

1. Motivation
2. Application Model
3. Problem Formulation - SMT
4. Symmetry Breaking
5. Cost Space Exploration
6. Experiments and Results
7. Conclusions

Outline

Model of Computation

synchronous dataflow graphs (SDF)

- by E. Lee and D. Messerschmitt in 1987
- task graph + symbolic representation of data parallelism
- signal-processing, video-coding applications

a ‘standard’ in academic multicore compilers:

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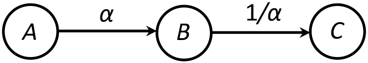
we introduce split-join graphs : restriction of SDF

- still covering perhaps 90% of use cases

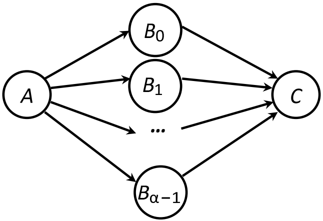
Split-Join Graphs

a simple split-join graph example:



α : spawn and split

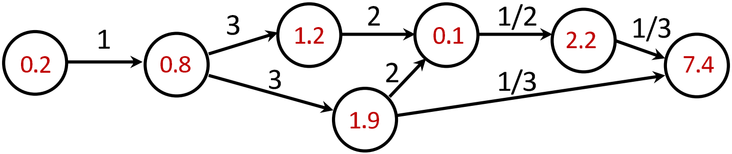
$1/\alpha$: wait and join



Split-Join Graphs

Definition (Split-Join Graph)

$S = (V, E, d, \alpha)$, (V, E) : DAG, V:actors, E:channels
$d : V \to \mathbb{R}_+$: actor execution time,
$\alpha : E \to \mathbb{Q}$: channel counter: split (> 1), join (< 1) or neutral ($= 1$)



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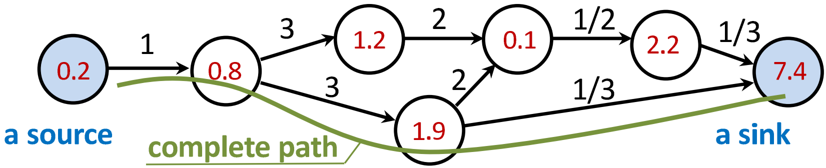

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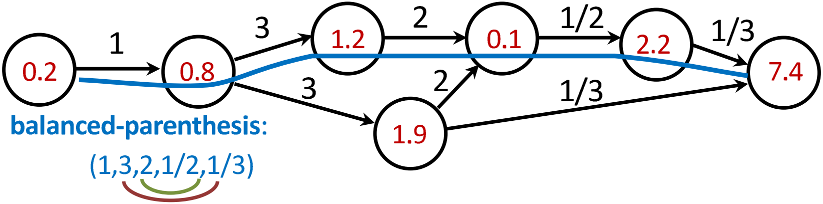


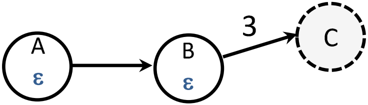
Well-behaved Graphs

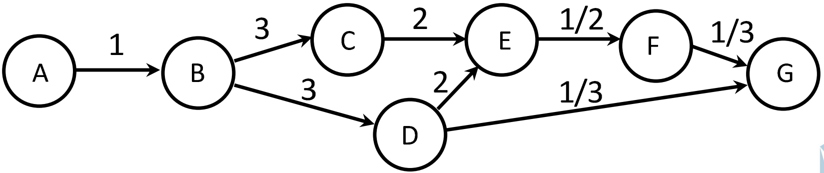
Definition (Well-behaved)

$S = (V, E, d, \alpha)$ is well-behaved if any complete path has balanced-parenthesis signature

Such a graph can be unfolded to a task graph.

Unfolding to Task Graph





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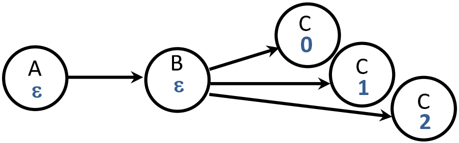

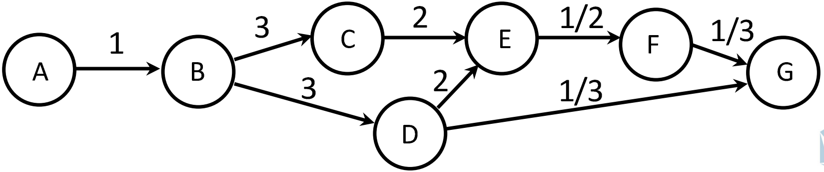

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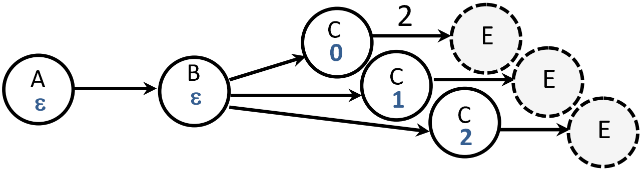

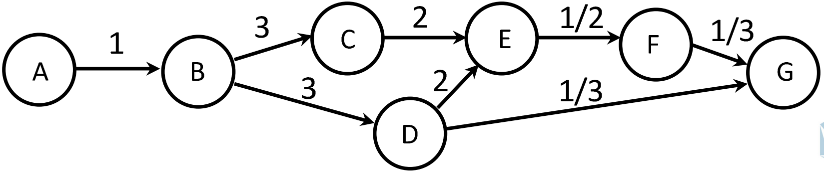

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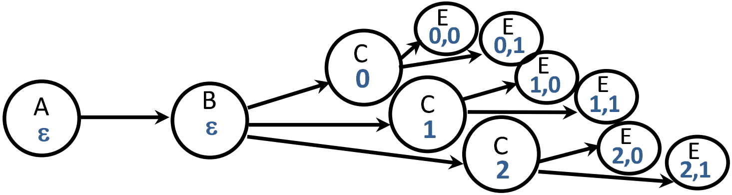

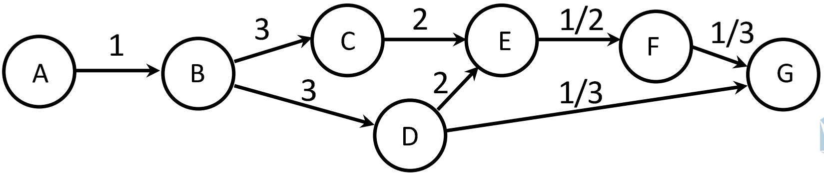

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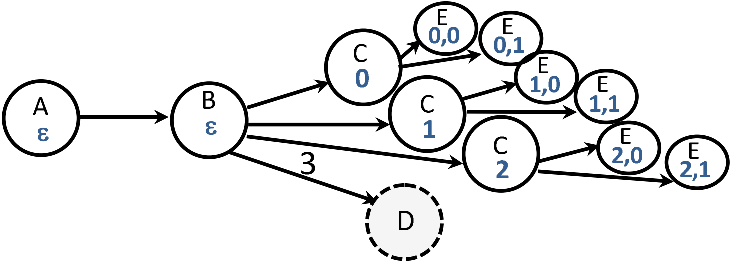

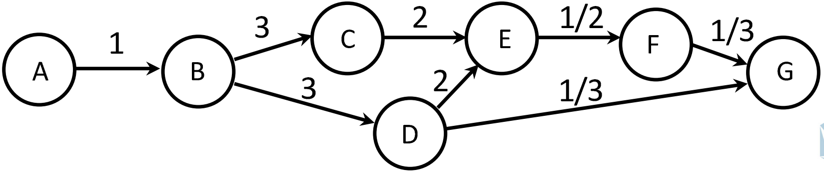

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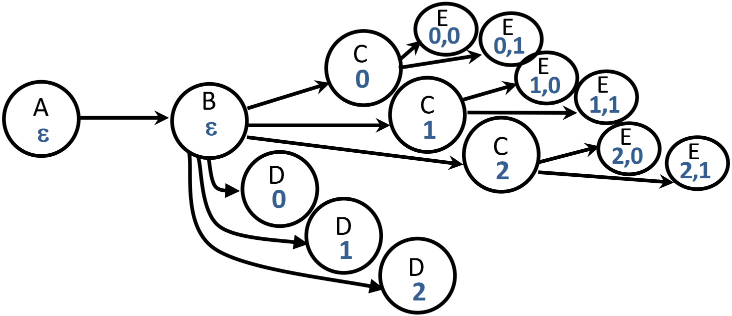

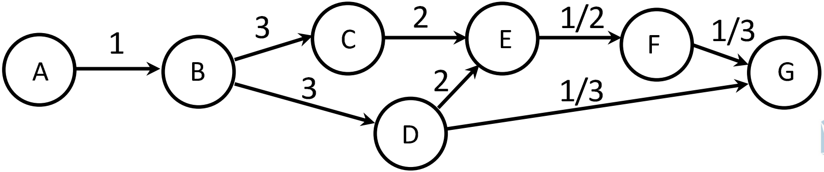

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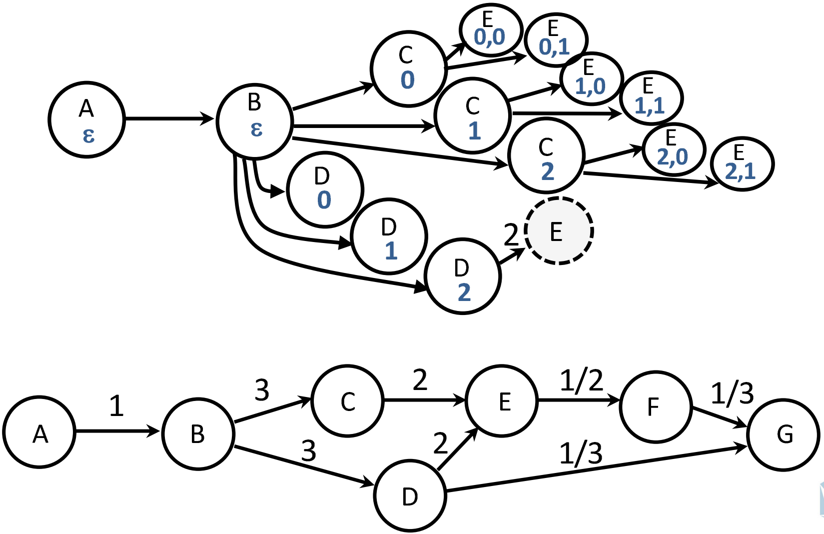

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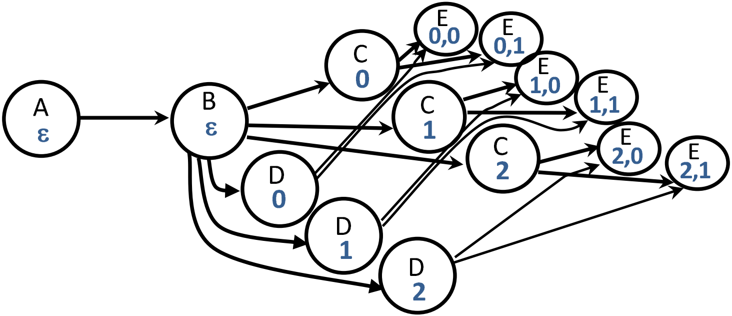

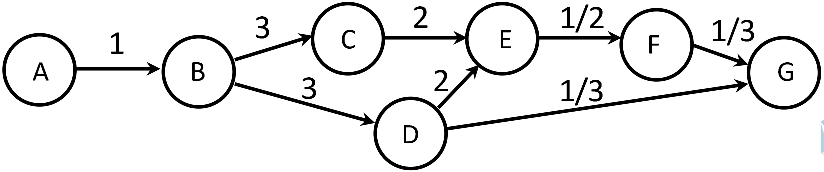

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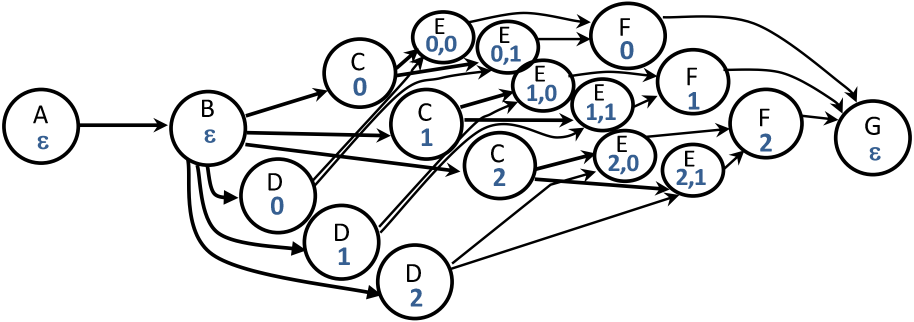

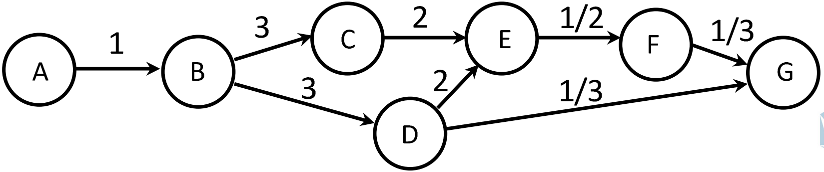

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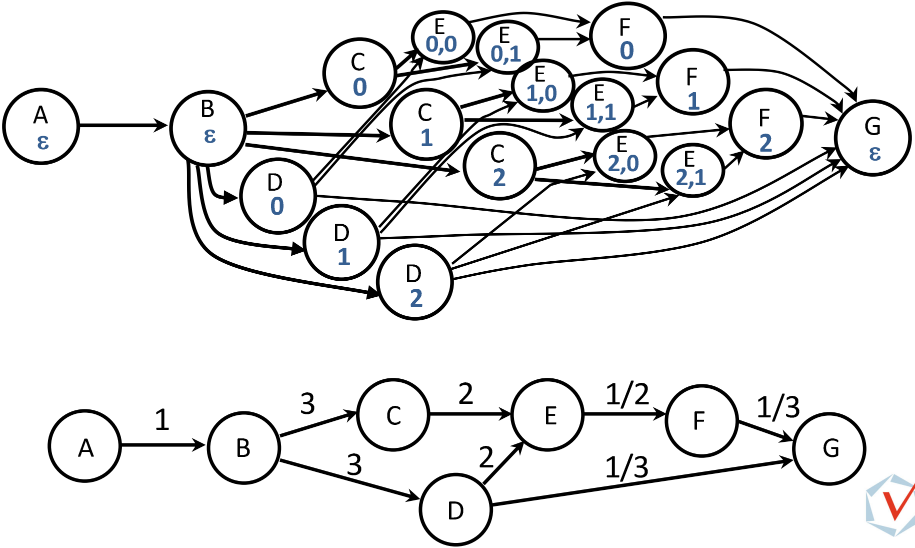
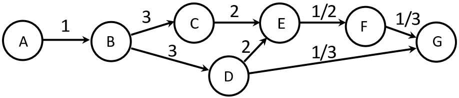


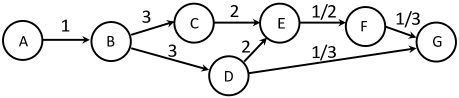
Actors, Tasks, Lexicographic Order

split-join graph: actors *e.g., A, B, C*



Actors, Tasks, Lexicographic Order

notation for actors: v, $v \in V$



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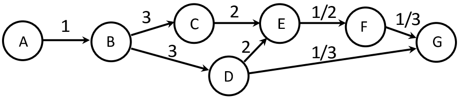
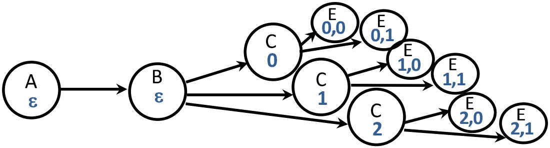


unfolded task graph: tasks *e.g.*, $E_{0,1}$, B, C_2



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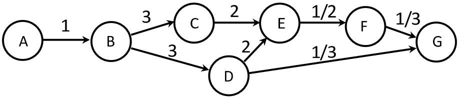
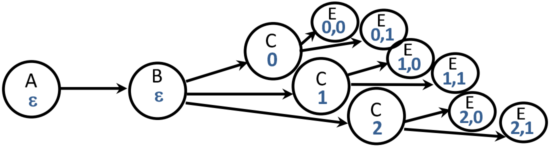


notation for tasks: $u \in U$



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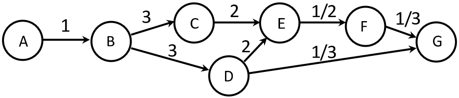

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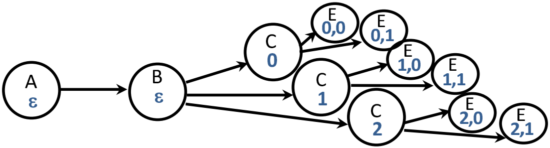
$u = v_h$, $v \in V$ and h - hier. index, *e.g.*, $v_h = E_{0,1}$



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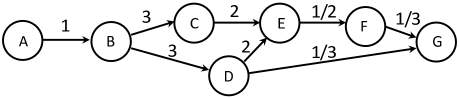
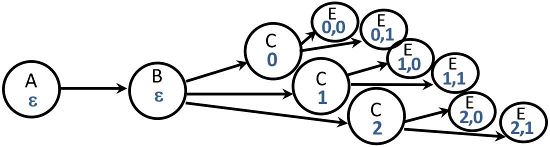


notation for tasks: $u \in U$

$U_v = \{v_h\}$: lexicographically ordered ($\ll$) set of instances of v

$U_E : E_{0,0} \ll E_{0,1} \ll E_{1,0} \ll E_{1,1} \ll E_{2,0} \ll E_{2,1}$



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Decision variables:

- $\mu(u), u \in U$ - the mapping: processor (1,2,...,M) for u
- $s(u)$ - the schedule: start time of u

Constraints

Predicate $\varphi(u, u')$:
task u' starts after the completion of task u

$$\varphi(u, u') : s(u') \geq s(u) + \delta(u)$$

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Mutual exclusion:

$$\bigwedge_{u \neq u' \in U} (\mu(u) = \mu(u')) \Rightarrow \varphi(u, u') \vee \varphi(u', u)$$

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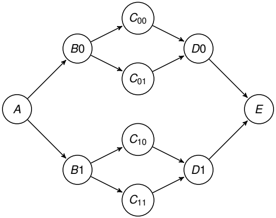
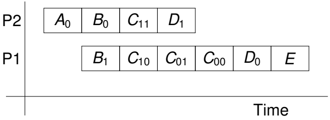
Task Symmetry



task graph



a schedule

- all instances of given actor v are similar (symmetric)

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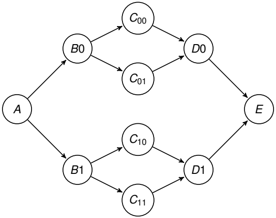

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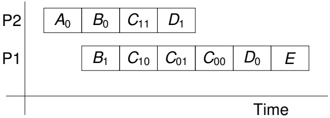

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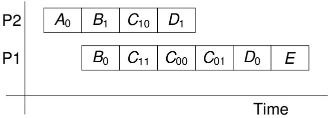

a permuted schedule

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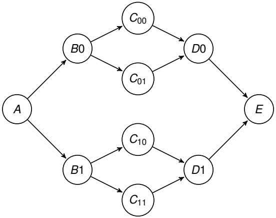

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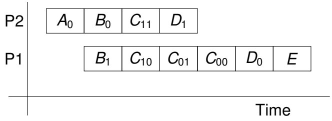

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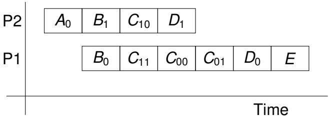
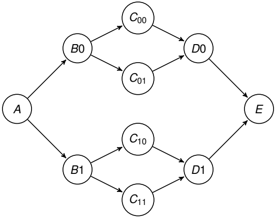


a permuted schedule

- all instances of given actor v are similar (symmetric)
- permutation of symmetric tasks does not change the latency,
- . . . but extends the solution space exponentially

Task Symmetry



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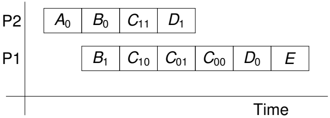
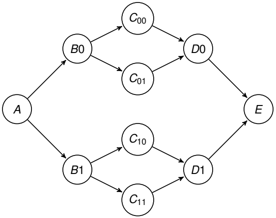


schedule

- enforce the schedule to be compatible with lexicographic order: $s(C_{00}) \leq s(C_{01}) \leq s(C_{10}) \leq s(C_{11})$

Task Symmetry



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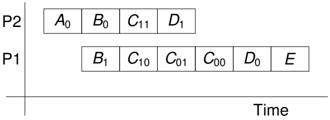

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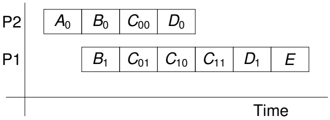

compatible schedule

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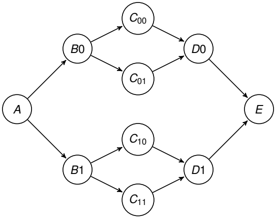

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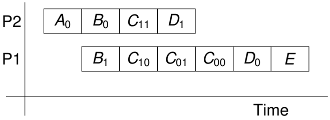
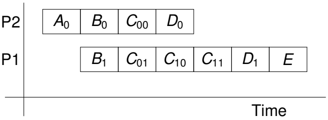


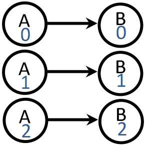
schedule



compatible schedule

- enforce the schedule to be compatible with lexicographic order: $s(C_{00}) \leq s(C_{01}) \leq s(C_{10}) \leq s(C_{11})$
- **Theorem:** adding constraints $s(u) \leq s(u')$ for $u \ll u'$ does not eliminate optimality

Proof Sketch

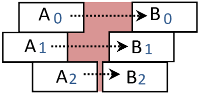


modify a feasible schedule such that:

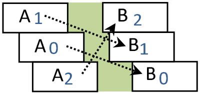
$s(v_0) \leq s(v_1) \leq s(v_2) \leq \ldots$

prove that precedence constraints are satisfied

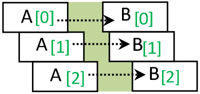
here: for neutral channels ($\alpha = 1$), unfolded to (v_h, v'_h)



lexicographic order

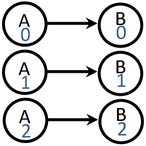


start-time compatible



new hier. index;
new precedence relation

Proof Sketch



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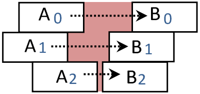

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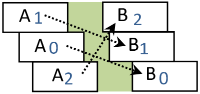

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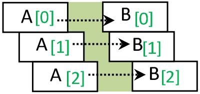

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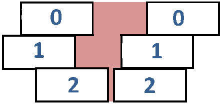

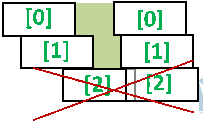

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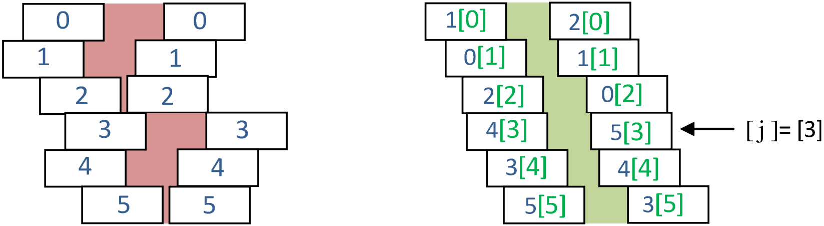
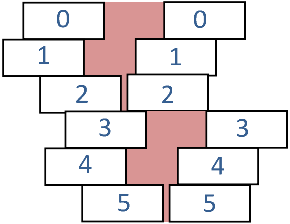
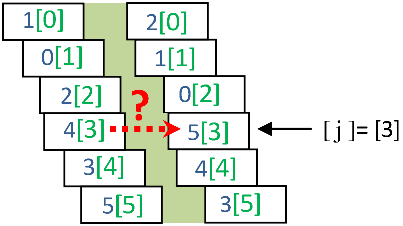


take successor [*j*]

Proof Sketch





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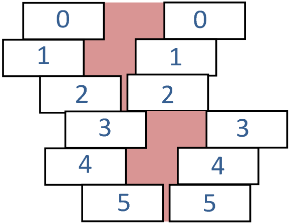

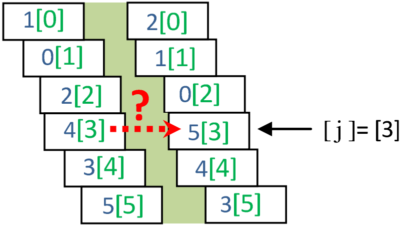

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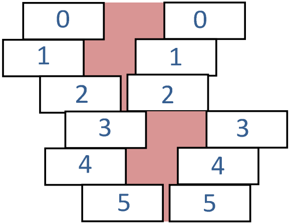
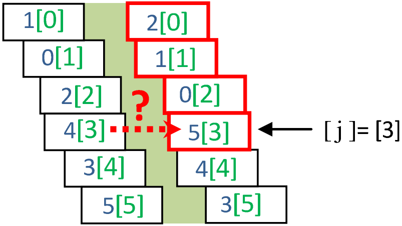
by definition there exist $j + 1$ same or earlier successors

Proof Sketch





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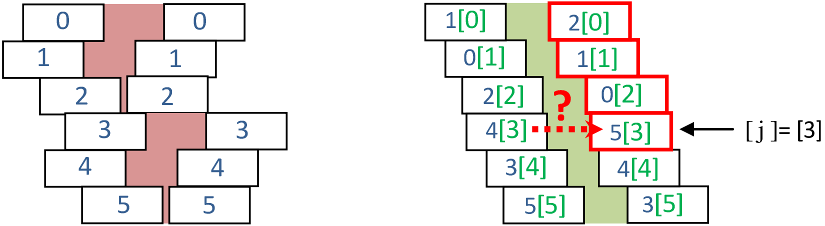

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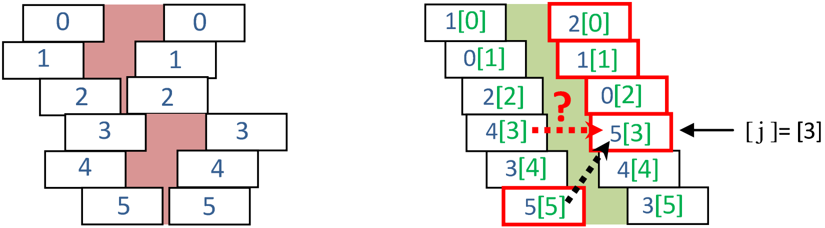
Proof Sketch



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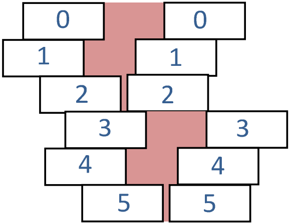

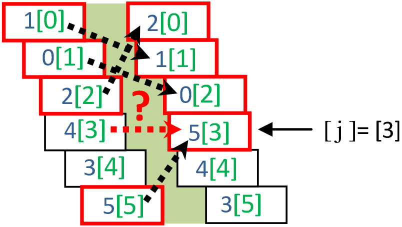

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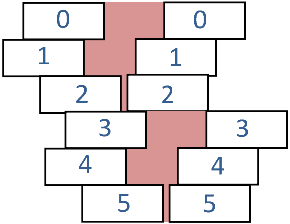

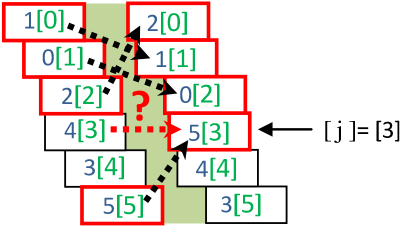

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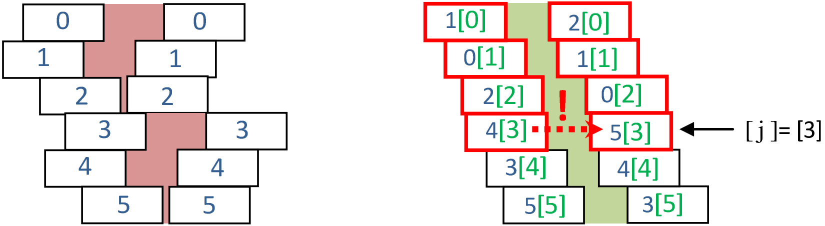
Proof Sketch



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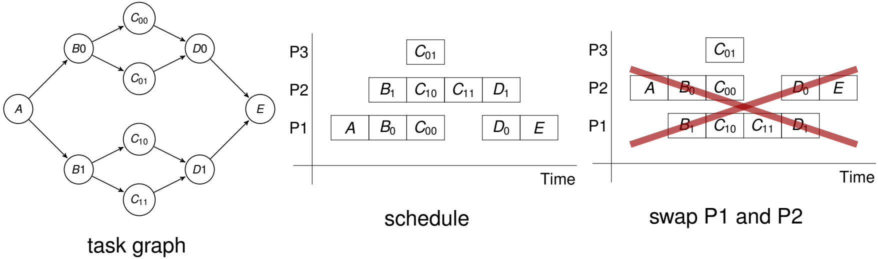
predecessor [j] finishes before successor [j]

Processor Symmetry

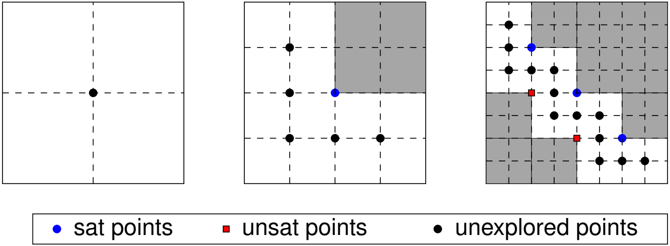


Outline

Exploring the Design Space

One SMT query for a given point (C_L, C_M) in the cost space:

- C_L - latency
- C_M - processor count



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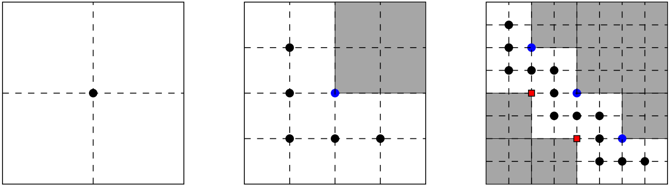


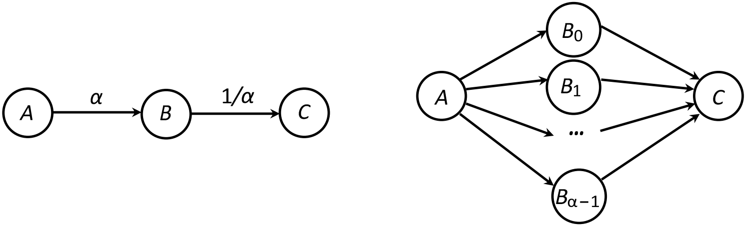
• sat points ▪ unsat points • unexplored points

- Precedence and Mutual Exclusion Constraints
- Cost Constraints

$$\bigwedge_{u \in U} s(u) + \delta(u) \leq C_L \ \wedge \ \bigwedge_{u \in U} \mu(u) \leq C_M$$

Outline

Synthetic-Graph Experiments



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- Increase α and measure increase in computation time
- With(out) breaking of task symmetry and processor symmetry

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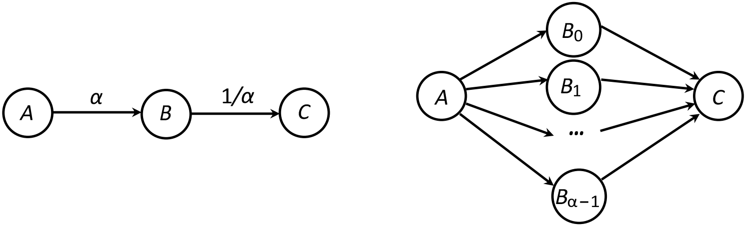

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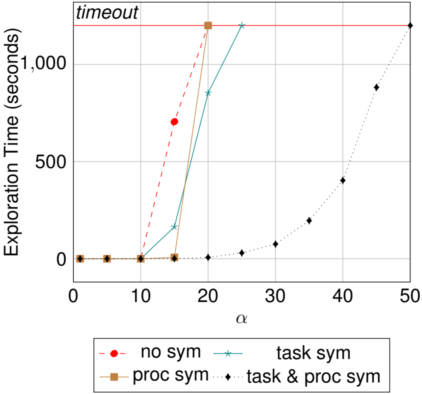
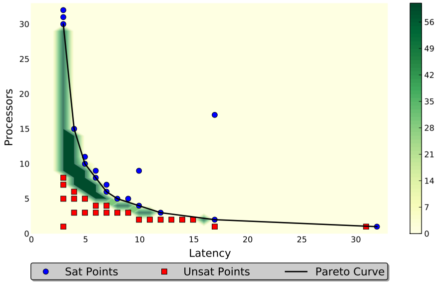


5-processor deployments

Pareto Exploration



without symmetry breaking

cost space (C_L, C_M) exploration for $\alpha = 30$

evaluate task and processor symmetry breaking

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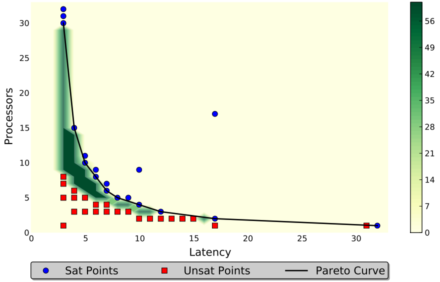

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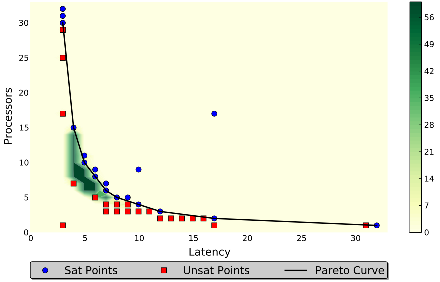

with symmetry breaking

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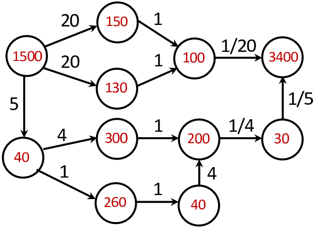
Video Decoder

3D cost space (C_L, C_M, C_B) exploration, C_B - total buffer size

MPEG video decoder:



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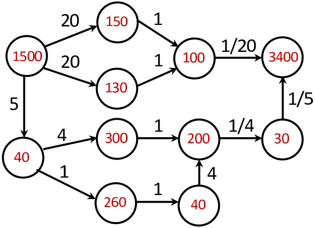

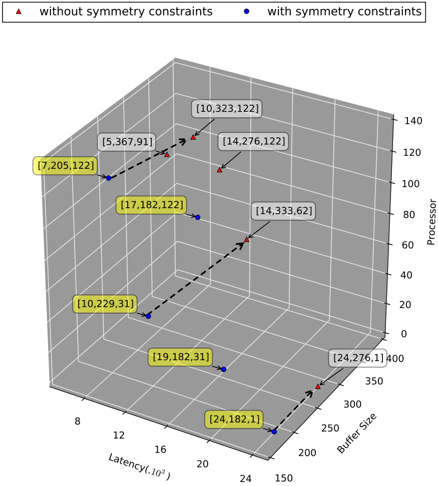

Conclusions

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  - a useful subclass of SDF model
- Framework for multi-criteria optimal deployment
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Conclusions

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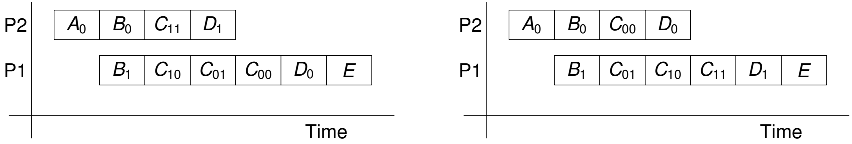
- Future work:
- More symmetry breaking, also approximation and heuristics
- More refined data communication: data transfer delays
- Pipelined scheduling
- Scheduling under uncertainty
- Multistage design flow


P2
A_0 B_0 C_{11} D_1
P1
B_1 C_{10} C_{01} C_{00} D_0 E
Time
P2
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Time

QUESTIONS?