A Gentle Introduction to Program Analysis

Işıl Dillig
University of Texas, Austin

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Programming Languages Mentoring Workshop

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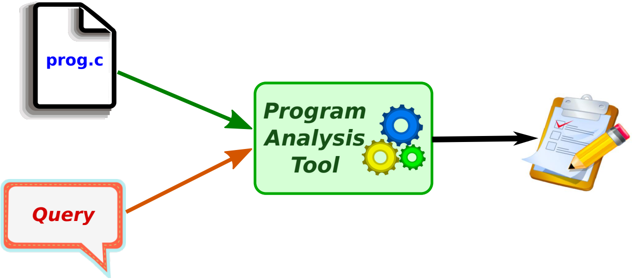

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- **Compiler optimizations.** e.g., which variables should be kept in registers for fastest memory access?
- **Automatic parallelization.** e.g., is it safe to execute different loop iterations on parallel?

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Static

\+ reasons about all executions
\- less precise

Dynamic

\+ more precise
\- results limited to observed executions

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  - **Sound, but incomplete:** May say program is unsafe even though it is safe
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- Many static analysis techniques are sound but incomplete.

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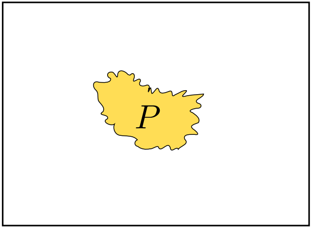

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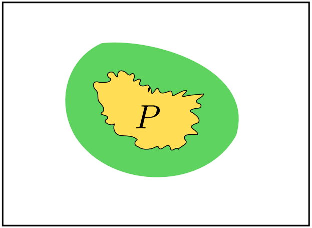

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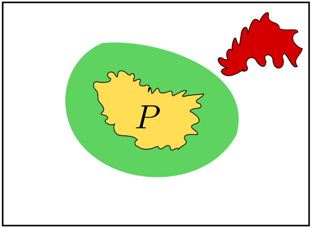

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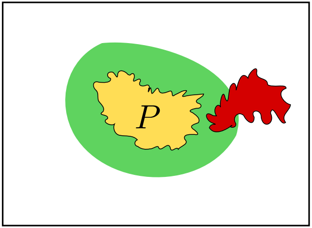

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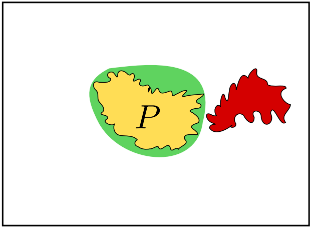

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$\Rightarrow$ **Goal:** Construct abstractions that are precise enough (i.e., few false alarms) and that scale to real programs

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Application	Useful abstraction
No division-by-zero errors	zero vs. non-zero
Data structure verification	list, tree, graph, . . .
No out-of-bounds array accesses	ranges of integer variables

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- Not a specific analysis, but rather a framework for designing sound-by-construction static analyses
- Let's look at an example: A static analysis that tracks the sign of each integer variable (e.g., positive, non-negative, zero etc.)

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  - $\bot$ (bottom): Represents empty-set

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  - $\gamma(\text{pos}) = \{x \mid x \in \mathbb{Z} \wedge x > 0\}$

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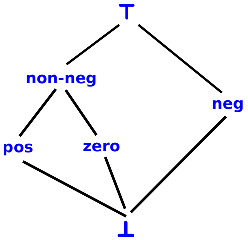

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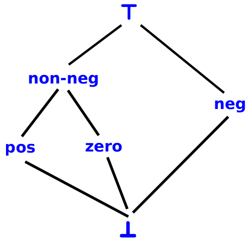

- Least upper bound of two elements is called their join – useful for reasoning about control flow in programs

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$$\alpha(\gamma(A)) = A$$

$$\begin{tikzcd} \mathbf{A} \arrow[r, bend left, "\gamma"] & \mathbf{C} \arrow[l, bend left, "\alpha"] \end{tikzcd}$$

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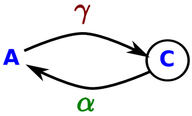

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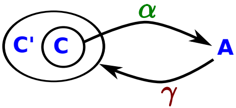

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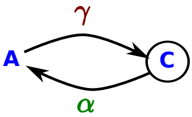

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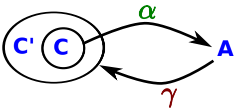

- This is called a **Galois insertion** and captures the soundness of the abstraction

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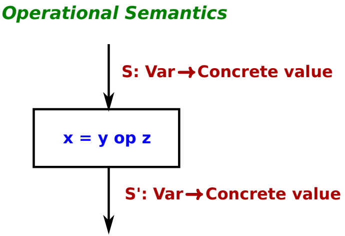

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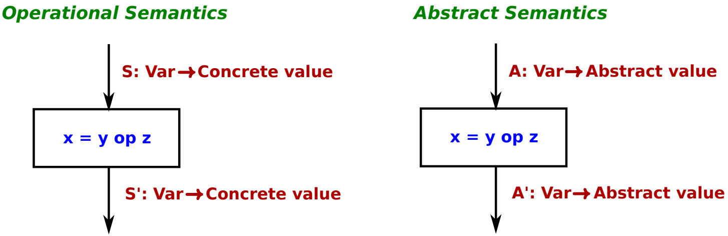

Back to Our Example

- For our sign analysis, we can define abstract transformer for $x = y + z$ as follows:

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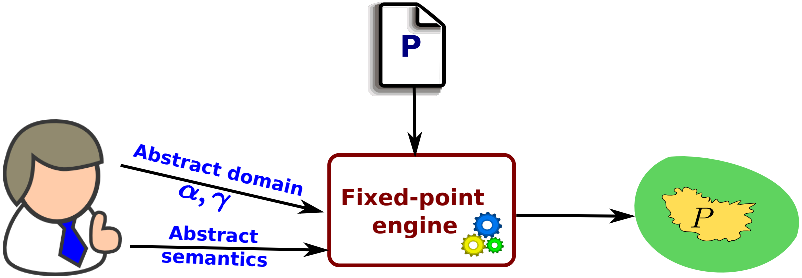
- To ensure soundness of static analysis, must prove that abstract semantics faithfully models concrete semantics

Putting It All Together



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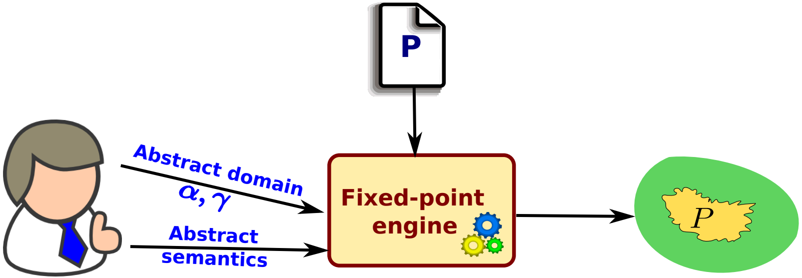

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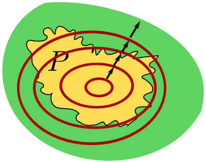

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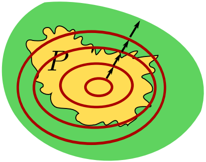
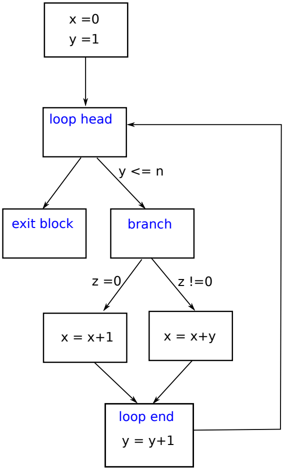


- **Assuming correctness of your abstract semantics, the least fixed point is an overapproximation of the program!**

Performing Least Fixed Point Computation

- Represent program as a **control-flow graph**



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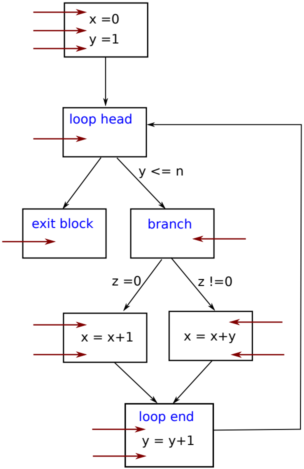

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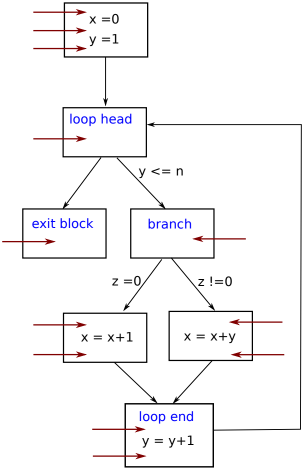

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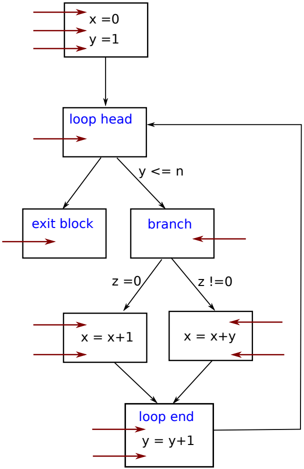

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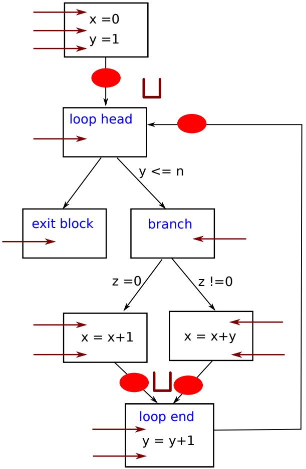

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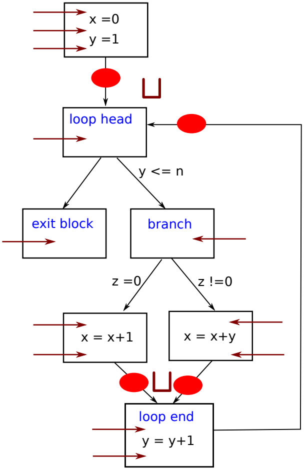
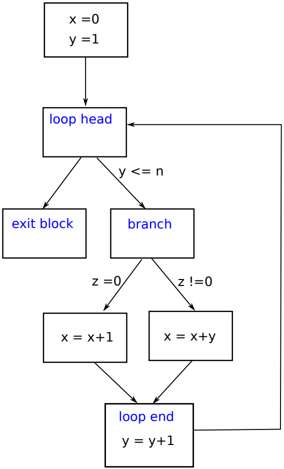
  - Symbolically execute each basic block using abstract semantics



An Example

```
x = 0;
y =0;

while(y <= n)
{
  if (z == 0) {
    x = x+1;
  }
  else {
    x = x + y;
  }
  y = y+1
}
```



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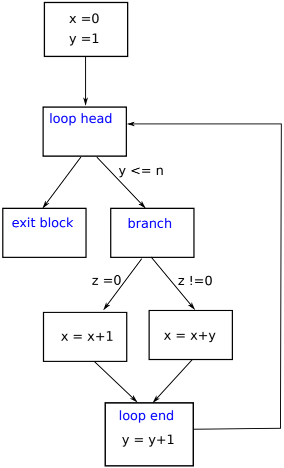
Is x always non-negative inside the loop?



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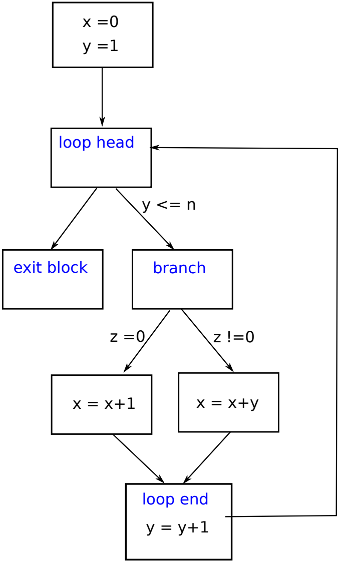

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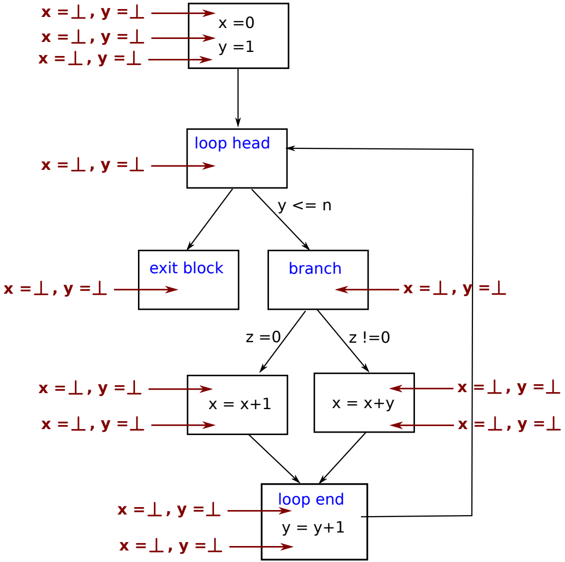

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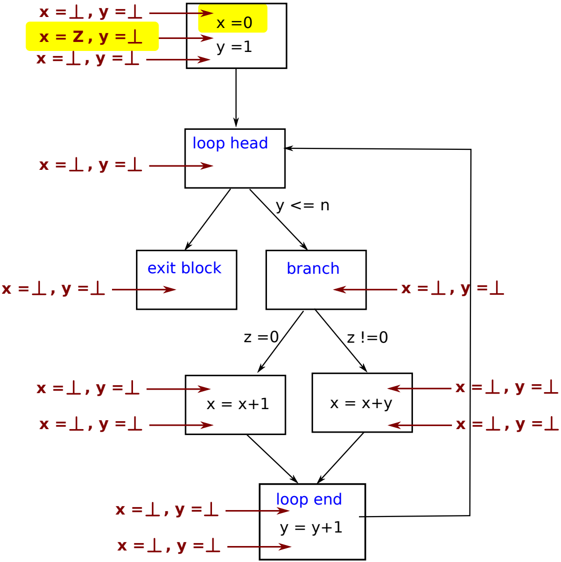

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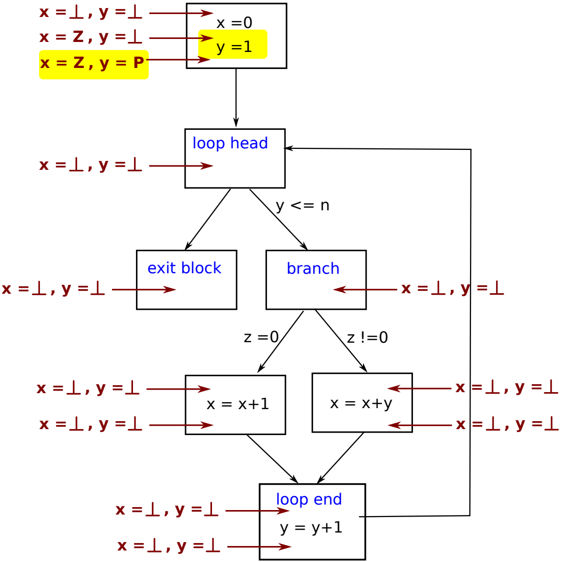

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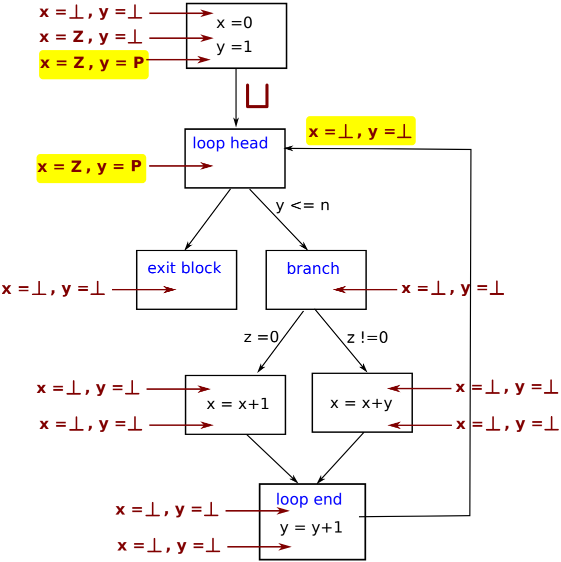

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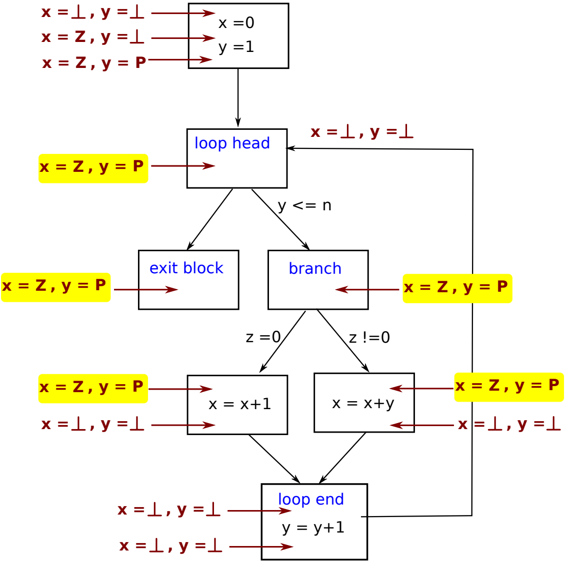

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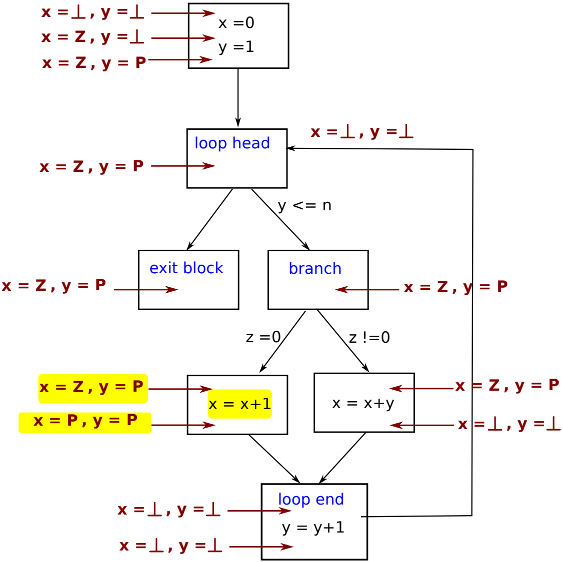

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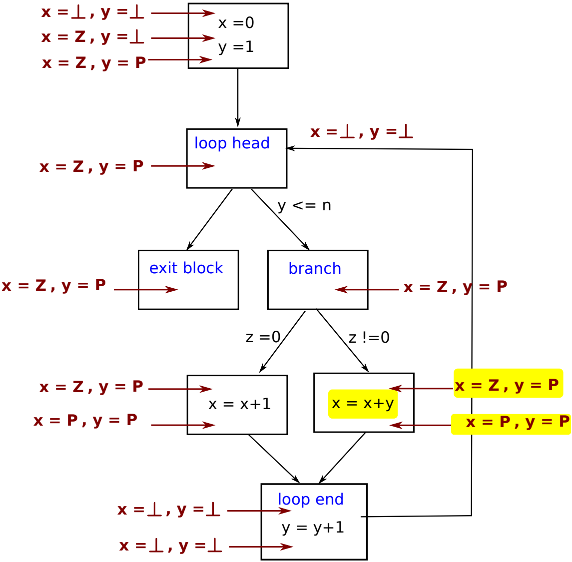

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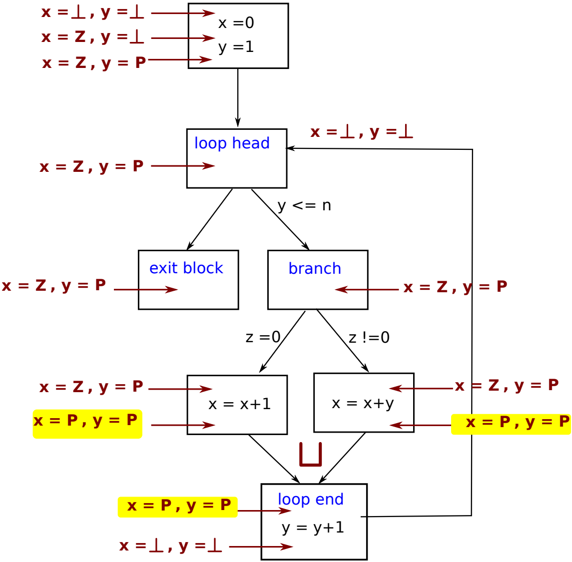

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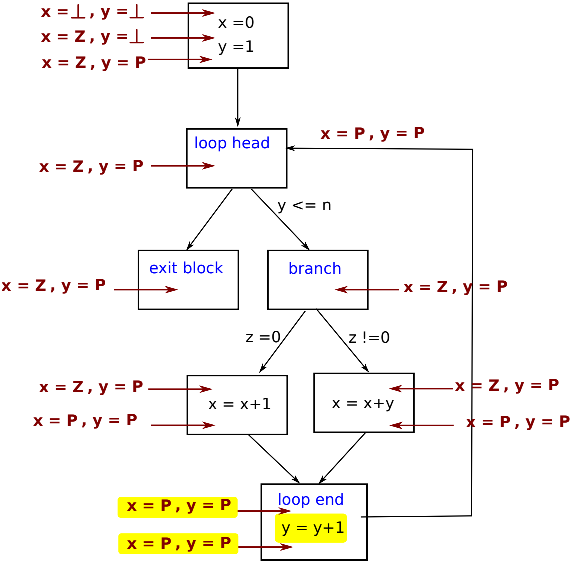

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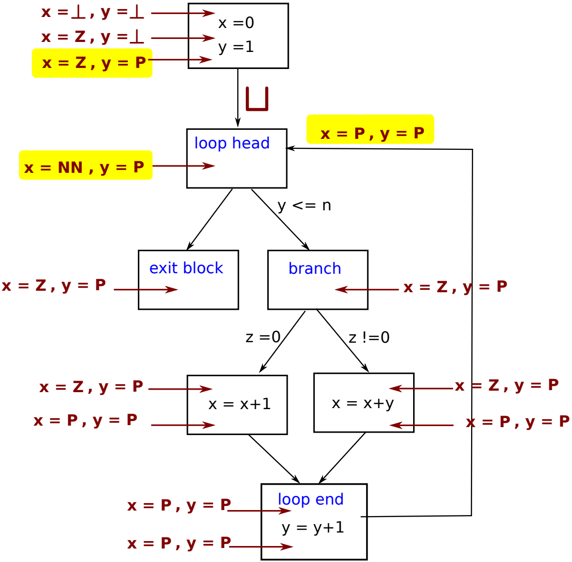

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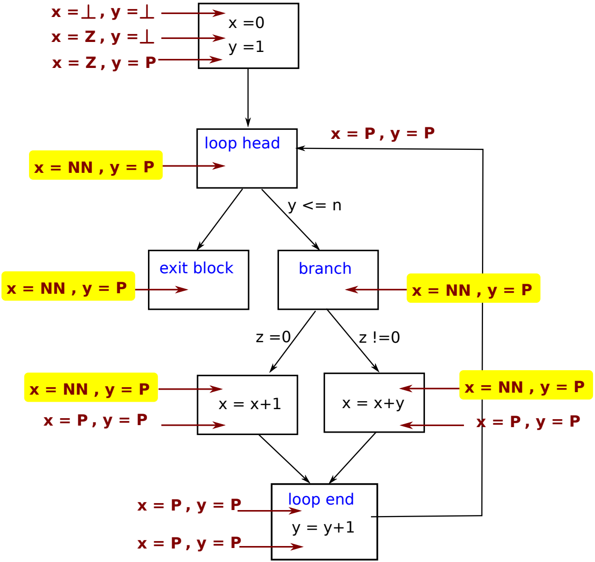

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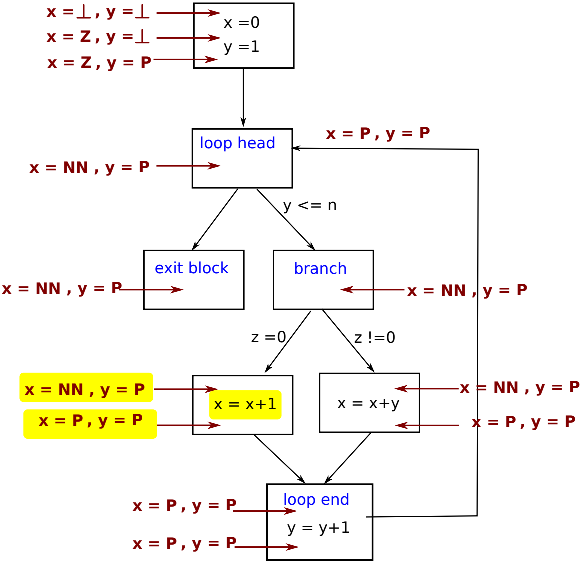

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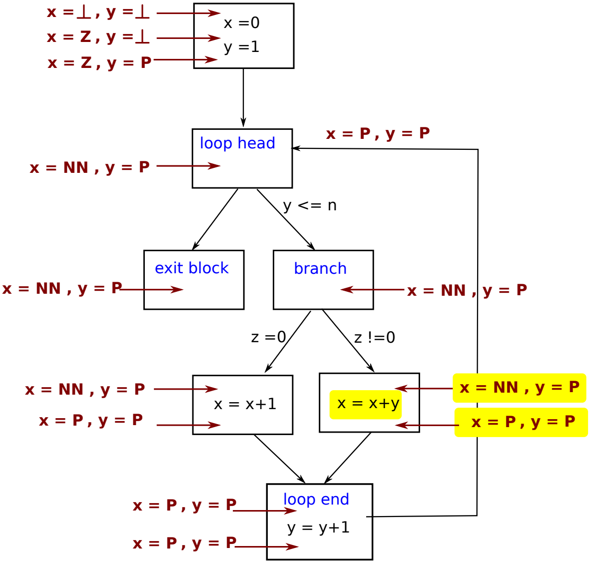

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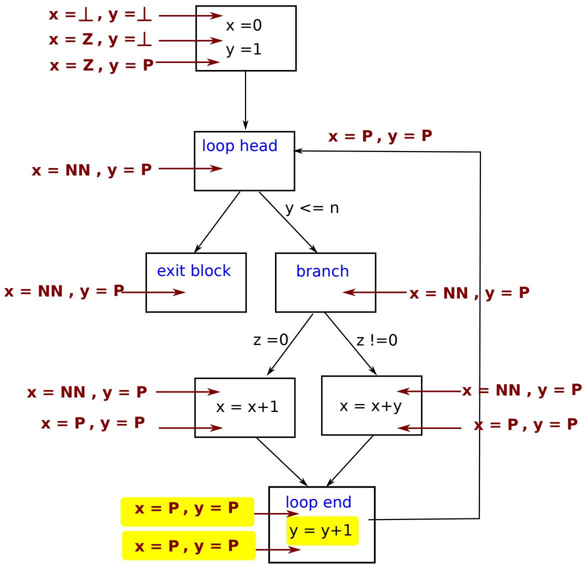

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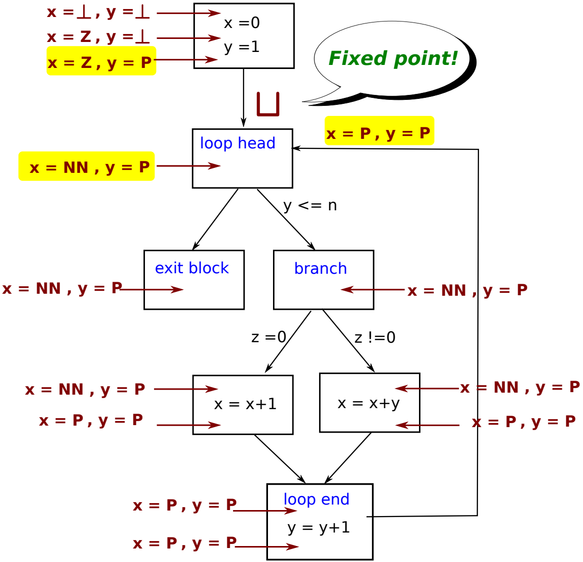

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- Unfortunately, many interesting domains do not have this property, so we need widening operators for convergence.

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Exciting area with lots of interesting topics to work on!

Considering PhD or Postdoc?

If you are interested in program analysis or verification, consider applying to UT Austin!

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