Ultimately Periodic Qualitative Cconstraint Networks for Spatial and Temporal Reasoning

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Abstract

We consider qualitative temporal or spatial constraint networks whose constraints evolve over time in an ultimately periodic fashion: after an initial stretch of time, a fixed pattern of constraints (over an interval) is reproduced indefinitely. We propose a local propagation algorithm which is polynomial, and we show that it decides the consistency problem in some particular cases. We also show that the general problem of consistency for such networks is in PSPACE.

1 Introduction

The need for reasoning about time and space arises in many areas of Artificial Intelligence, including computer vision, natural language understanding, geographic information systems (GIS) and scheduling. Numerous formalisms for representing and reasoning about time and space in a qualitative way have been proposed in the past two decades. The Interval Algebra (IA) introduced by Allen in order to reason about temporal information [1] is a paradigmatic example. Those formalisms consider a finite set of basic relations denoting qualitative relationships between temporal or spatial entities. Intersection, overlapping, containment, precedence are examples of such qualitative relationships.

Typically, qualitative constraint networks are used to express information on a spatial or temporal situation. Each constraint of a constraint network represents a set of acceptable qualitative configurations between some temporal or spatial entities and is defined by a set of basic relations.

Beyond the purely static aspects, many applications also involve studying the evolution of relations with time. Recently, formalisms for reasoning about such evolving relations have been proposed, such as for instance [8, 2]. Roughly speaking, these formalisms are based on logical languages which combine propositional temporal logic with a qualitative constraint-based formalism such as RCC [6] or IA. In these richer languages, constraints implying the positions of objects at different times can be expressed, as well as spatial constraints which have to be satisfied at each point in time. The corresponding networks are often referred to as temporalized constraint networks.

This paper is about temporalized constraint networks of a periodic kind. In the spatial context, the positions of the objects may change in time, and we would like to express how the changes occur over time. The case we consider is the following: after some initial period of arbitrarily constrained change, the same pattern of qualitative constraints will recur indefinitely over time on successive intervals of a fixed length. The analogous situation in the temporal context corresponds to events that, after some initial period, are constrained in a periodic way. In both cases, we refer to this restricted kind of constraints as to ultimately periodic constraints. We consider the consistency problem for these constraint networks and we prove two main results.

2 Basic notions and notations

In the sequel of the paper, we assume we have a qualitative formalism using a finite set B of basic binary relations on a domain D. We assume that the relations in B are jointly exhaustive and pairwise disjoint. Finally, we assume that B contains the identity relation on D (denoted by Id). As a matter of illustration, consider the point algebra (PA) introduced by Vilain *et al.* [7]. PA is based on three basic relations $\mathsf{B} = \{=, <, >\}$ on the set of points of the rational number line, hence $\mathsf{D} = \mathbb{Q}$. The three relations denote the identity relation, the relation of precedence, and the converse of precedence, respectively.

The temporal or spatial information about the configuration of a set of entities can be represented using constraint networks called qualitative networks. Formally, a qualitative constraint network (QCN) $\mathcal{N}$ is a pair (V, C) where V is a finite set of n variables $v_0, \ldots, v_{n-1}$ (n is a positive integer) and C is a map which, to each pair (v_i, v_j) of variables in V associates a subset $C(v_i, v_j)$ of the set of basic relations: $C(v_i, v_j) \subseteq \mathsf{B}$. We will denote by A the set 2^{B} of all subsets of B. For $r \in \mathsf{A}$, two elements $x, y \in D$ satisfy r, which we denote by $x\ r\ y$, iff there is a basic relation $A \in r$ such that $(x, y) \in A$. Hence each element r in A can be considered as the union of all the basic relations it contains. We use the term "relation" to refer to such a union of basic relations. A is equipped with the operations intersection ($\cap$) and union ($\cup$). It is also equipped with the converse operation ($\sim$) and an operation of composition ($\circ$): the converse of a relation r in A is the relation of A corresponding to the transpose of r; the composition $A \circ B$ of $A, B \in \mathsf{B}$ is the relation $r = \{C : \exists x, y, z \in \mathsf{D}, x\ A\ y, y\ B\ z \text{ and } x\ C\ z\}$. $r \circ s$ of $r, s \in \mathsf{A}$ is the relation $t = \bigcup_{A \in r, B \in s}\{A \circ B\}$. We use the following definitions in the sequel:

Definition 1 *Let $\mathcal{N} = (V, C)$ be a* QCN*. A* partial solution *of $\mathcal{N}$ on $V' \subseteq V$ is a map σ of V' to* D *such that $\sigma(v_i)\ \ C(v_i, v_j)\ \ \sigma(v_j)$, for all $v_i, v_j \in V'$. A* solution *of $\mathcal{N}$ is a partial solution on V. $\mathcal{N}$ is* consistent *if and only if it has a solution. $\mathcal{N}$ is* globally consistent *if and only if, for any V', every partial solution σ on $V' \subset V$ can be extended to a partial solution σ' on $V' \cup \{v\} \subseteq V$, for any $v \in V \setminus V'$. $\mathcal{N}$ is* $\circ$-closed *if and only if for all $v_k, v_i, v_j \in V$, $C(v_i, v_j) \subseteq C(v_i, v_k) \circ C(v_k, v_j)$. A* subnetwork *of $\mathcal{N}$ is a network (V, C') where $C'(v_i, v_j) \subseteq C(v_i, v_j)$ for all $v_i, v_j \in V$. $\mathcal{N}' = (V', C')$ is* equivalent *to $\mathcal{N}$ iff $V = V'$ and both networks $\mathcal{N}$ and $\mathcal{N}'$ have the same solutions.*

Given a network $\mathcal{N} = (V, C)$, local constraint propagation algorithms [5, 4] may be used to derive in polynomial time (in $O(|V|^3)$ for some of them) a subnetwork which is $\circ$-closed and equivalent to $\mathcal{N}$. In brief, these algorithms iterate the operation $C(v_i, v_j) \leftarrow C(v_i, v_j) \cap (C(v_i, v_k) \circ C(v_k, v_j))$ for all $v_i, v_j, v_k \in V$ until a fixpoint is obtained. In what follows, we refer to the use of any of these algorithms as to using the $\circ$-closure method.

3 Ultimately periodic qualitative constraint networks

We now define the main notion of this paper, which we call an ultimately periodic qualitative constraint network (UPQCN). This new notion is more readily interpreted in the spatial context, where a UPQCN can be seen as a temporalized QCN. More precisely, consider a set of spatial entities whose spatial location may change over time. At each instant, an object has a given location. We wish to be able to express three types of constraints: constraints between the locations of two objects at a given instant, constraints between the locations of two objects at distinct instants, and constraints between the locations of two objects which have to be satisfied at instants following an initial instant. We assume that each integer $t \geq 0$ corresponds to an instant in time. A UPQCN will allow to represent constraints like these in the following example.

Example 1 *We consider* PA*, interpreted in spatial terms: the objects we consider are points on the rational line. Consider three objects O_0, O_1 and O_2, whose spatial locations are represented by three variables v_0, v_1 and v_2 which stand for rational numbers. These objects change positions over time with the following constraints: At time 0, O_0 is left of O_1 and of O_2; either the location of O_2 at time 0 is left of that at time 1, or both coincide; at time 1, and for all future times, O_0 is right of O_1; starting with time 1, O_0 moves left, and O_1 moves right; starting with time 2, O_2 moves left, and it stays to the left of O_0 and to the right of O_1.*

In the temporal context, a recurrent activity or event can have a finite or infinite number of occurrences over time. These occurrences may have to satisfy a common set of qualitative constraints over time. A UPQCN will allow the specification of such periodic constraints. Now we give a formal definition of a UPQCN:

Definition 2 *A* UPQCN *$\mathcal{R}$ is a structure (V, C, t_{min}, t_{max}), where: $V = \{v_0, \ldots, v_{n-1}\}$ is a finite set of n variables; t_{min}, t_{max} are two integers such that $0 \leq t_{min} \leq t_{max}$; C is a map from $V \times V \times \{0, \ldots, t_{max}\} \times \{0, \ldots, t_{max}\}$ to* A *such that for all $v_i, v_j \in V$ and $t_i, t_j \in \{0, \ldots, t_{max}\}$, $C(v_i, v_i, t_i, t_i) \subseteq \{\mathsf{Id}\}$ and $C(v_i, v_j, t_i, t_j) = C(v_j, v_i, t_j, t_i) \sim$.*

Intuitively, in a spatial context, each variable $v_i \in V$ stands for the spatial component of an object, and $(v_i, t_i) \in V \times \mathbb{N}$ represents the occurrence of this component at time t_i. $C(v_i, v_j, t_i, t_j)$ is a constraint on the relative positions of the occurrence of v_i at time t_i and that of v_j at time t_j. The constraints expressed by C are twofold: firstly, all constraints from 0 up to t_{max} have to be satisfied ; secondly, all constraints from t_{min} to t_{max} have to be satisfied up to t_{max}, but also on all subsequent periods $\{t_{min} + i, \ldots, t_{max} + i\}$ where $i \in \mathbb{N}$. In other words, the data define both initial constraints (up to t_{max}), and a recurrent pattern, or *motif* of constraints (from t_{min} to t_{max}) which repeats itself indefinitely. This is the reason for the choice of the term "ultimately periodic" in our terminology. Hence, the constraints specified in Example 1 can be expressed by a UPQCN $\mathcal{R} = (V, C, t_{min}, t_{max})$, where $V = \{v_0, v_1, v_2\}$, $t_{min} = 1$, $t_{max} = 3$. The constraints defined by C are represented in Figure 1. In a temporal context, each variable $v_i \in V$ stands for a recurrent activity or event, and the pair $(v_i, t_i) \in V \times \mathbb{N}$ represents its $(t_i + 1)^{\text{th}}$ occurrence. $C(v_i, v_j, t_i, t_j)$ constrains the temporal relation between the $(t_i + 1)^{\text{th}}$ occurrence of v_i and the $(t_j + 1)^{\text{th}}$ occurrence of v_j. We formally define a *solution* of a UPQCN as follows:

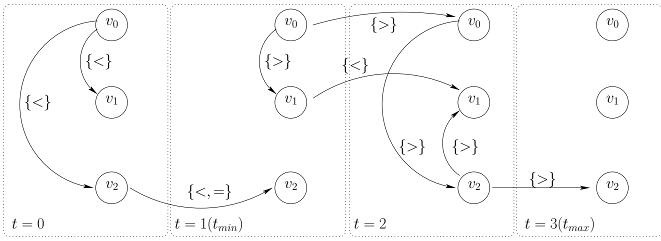

Figure 1. The constraints C of the UPQCN $\mathcal{R}$.

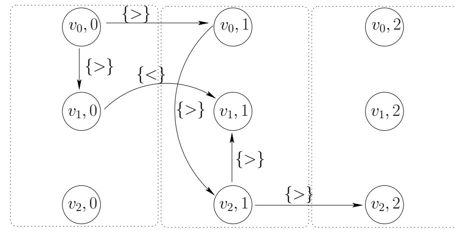

Figure 3. The motif of the UPQCN $\mathcal{R}$.

Definition 3 *A solution σ of a* UPQCN $\mathcal{R} = (V, C, t_{min}, t_{max})$ *is a map from $V \times \mathbb{N}$ to* D *such that, for all $v_i, v_j \in V$ and $t_i, t_j \in \mathbb{N}$ with $t_i \leq t_j$: (1) If $t_j \leq t_{max}$ then $\sigma(v_i, t_i)\ C(v_i, v_j, t_i, t_j)\ \sigma(v_j, t_j)$; (2) if $t_i \geq t_{min}$ and $t_j - t_i \leq t_{max} - t_{min}$ then for all t'_i, t'_j such that $t_{min} \leq t'_i \leq min\{t_{max}, t_i\}$, $t_{min} \leq t'_j \leq min\{t_{max}, t_j\}$ and $t_j - t_i = t'_j - t'_i$ we have $\sigma(v_i, t_i)\ C(v_i, v_j, t'_i, t'_j)\ \sigma(v_j, t_j)$.*

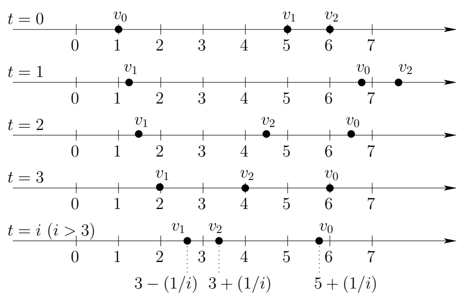

Figure 2. A solution of the UPQCN $\mathcal{R}$.

We extend in an obvious way the notions of consistency and of equivalence to the case of UPQCNs.

4 Making implicit constraints explicit

Our main objective in this paper is to study the consistency of UPQCNs. Our strategy for doing so is as follows: (**1**) Given a UPQCN, which is a potentially infinite network, we associate to it its periodic component, which is a finite network which we call the motif. (**2**) Based on the use of the motif, we define a sequence of finite networks of increasing temporal support, which we call the k-strengthenings. (**3**) We relate the consistency problem for the infinite network to the consistency properties of its successive k-strengthenings. We now proceed to implementing this strategy in detail.

Definition 4 *Let $\mathcal{R} = (V, C, t_{min}, t_{max})$ be a* UPQCN*. The motif of $\mathcal{R}$, denoted by* $\mathsf{motif}(\mathcal{R})$*, is the* QCN $\mathcal{N}_m = (V_m, C_m)$ *where $V_m = V \times \{0, \ldots, \mathsf{lg}\}$ (with $\mathsf{lg} = t_{max} - t_{min}$) and $C_m((v_i, t_i), (v_j, t_j)) = C(v_i, v_j, t_i + t_{min}, t_j + t_{min})$ for all $v_i, v_j \in V$ and for all $t_i, t_j \in \{0, \ldots, \mathsf{lg}\}$.*

In the sequel lg will always denote $t_{max} - t_{min}$. The motif of the UPQCN $\mathcal{R}$ of Figure 1 is shown in Figure 3. We now use the motif to define finite networks which in some sense make explicit the constraints of a UPQCN which must be satisfied during the $(k + 1)$ first instants (where $k \geq t_{max}$). For such a k, we will define the k-strengthening of a UPQCN. Before giving a formal definition, we describe a more intuitive way of describing them. Consider a picture of the original network, whose temporal extension is $\{0, \ldots, t_{max}\}$. Imagine that a picture of the motif is drawn independently on a transparent sheet. Given $k > t_{max}$, we repeatedly superpose the transparent sheet on the original picture, first with a shift of one (at $t_{min} + 1$), then of two (at $t_{min} + 2$), and so on, until k is reached. Each time, we add the constraints of the transparent sheet to the already existing ones (by intersections). The k-strengthening is the network we get when reaching instant k.

Definition 5 *Let $\mathcal{R} = (V, C, t_{min}, t_{max})$ be a* UPQCN *and $\mathcal{N}_m = (V \times \{0, \ldots, \mathsf{lg}\}, C_m)$ its motif. Given $k \geq t_{max}$, the k-strengthening of $\mathcal{R}$, denoted by* $\mathsf{k{-}strengthening}(\mathcal{R})$*, is a* QCN *$\mathcal{N}^k = (V^k, C^k)$ defined inductively by: $V^k = V \times \{0, \ldots, k\}$; $C^{t_{max}}((v_i, t_i), (v_j, t_j)) = C(v_i, v_j, t_i, t_j)$ for all $v_i, v_j \in V$ and $t_i, t_j \in \{0, \ldots, t_{max}\}$. For all $k \geq t_{max}$ and for all $v_i, v_j \in V$ and $t_i, t_j \in \{0, \ldots, k+1\}$ with $t_i \leq t_j$ we have: $C^{k+1}((v_i, t_i), (v_j, t_j)) = C^k((v_i, t_i), (v_j, t_j))$, if $t_i < (k + 1) - \mathsf{lg}$ and $t_j < k + 1$; $C^{k+1}((v_i, t_i), (v_j, t_j)) = C^k((v_i, t_i), (v_j, t_j)) \cap C_m((v_i, t_i - ((k+1) - \mathsf{lg})), (v_j, t_j - ((k + 1) - \mathsf{lg})))$, if $t_i \geq (k + 1) - \mathsf{lg}$ and $t_j < k + 1$; $C^{k+1}((v_i, t_i), (v_j, t_j)) = C_m((v_i, t_i - ((k + 1) - \mathsf{lg})), (v_j, t_j - ((k+1) - \mathsf{lg})))$, if $t_j = k + 1$ and $t_j - t_i \leq \mathsf{lg}$; $C^{k+1}((v_i, t_i), (v_j, t_j)) = \mathsf{B}$, if $t_j = k + 1$ and $t_j - t_i > \mathsf{lg}$; $C^{k+1}((v_j, t_j), (v_i, t_i)) = C^{k+1}((v_i, t_i), (v_j, t_j)) \sim$.*

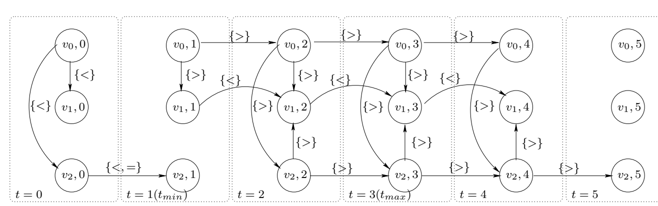

Figure 4. The 5-strengthening of $\mathcal{R}$.

We also need to define the notion of window of a k-strengthening, which is a QCN capturing its constraints over $(\mathsf{lg} + 1)$ consecutive time points:

Definition 6 *Let $\mathcal{N}^k = (V \times \{0,\ldots,k\}, C^k)$ be a k-strengthening of the* UPQCN $\mathcal{R} = (V, C, t_{min}, t_{max})$*, with $k \geq t_{max}$. The t-*window *of $\mathcal{N}^k$, with $t_{min} \leq t \leq k - \mathsf{lg}$, denoted by* $\mathsf{t-window}(\mathcal{N})$*, is the* QCN $\mathcal{N}_t = (V_t, C_t)$ *where $V_t = V \times \{0,\ldots,\mathsf{lg}\}$ and $C_t((v_i,t_i),(v_j,t_j)) = C^k((v_i,t_i+t),(v_j,t_j+t))$, for all $v_i, v_j \in V$ and $t_i, t_j \in \{0,\ldots,\mathsf{lg}\}$.*

The *end* of $\mathcal{N}^k$ corresponds to its last window: its $(k - \mathsf{lg})$-window. We still need a definition before the study of the interactions between the various consistency properties.

Definition 7 *Let $\mathcal{R} = (V, C, t_{min}, t_{max})$ be a* UPQCN *and $\mathcal{N}^k = (V \times \{0,\ldots,k\}, C^k)$ its k-strengthening, for $k \geq t_{max}$. A map σ from $V \times \mathbb{N}$ to* D *is a* solution *of $\mathcal{N}^k$ iff the restriction of σ to $V \times \{0,\ldots,k\}$ is a solution of $\mathcal{N}^k$,* i.e. *iff $\sigma(v_i,t_i)\ C^k((v_i,t_i),(v_j,t_j))\ \sigma(v_j,t_j)$ for all $v_i, v_j \in V$ and $t_i, t_j \in \{0,\ldots,k\}$.*

We can state the following results :

Proposition 1 *Let σ a solution of a* UPQCN $\mathcal{R}$*. Then σ is a solution of* $\mathsf{k-strengthening}(\mathcal{R})$ *for all $k \geq t_{max}$.*

Proposition 2 *Let $\mathcal{R} = (V, C, t_{min}, t_{max})$ be a* UPQCN *and σ a map from $V \times \mathbb{N}$ to* D*. If σ is a solution of* $\mathsf{k-strengthening}(\mathcal{R})$ *for all $k \geq t_{max}$, then σ is a solution of $\mathcal{R}$.*

Proposition 3 *Let $\mathcal{R}$ be a* UPQCN*. For $k \geq t_{max}$,* $\mathsf{end}(\mathsf{k-strengthening}(\mathcal{R}))$ *is a subnetwork of* $\mathsf{motif}(\mathcal{R})$*.*

5 The closure of a UPQCN

Now we have to design methods which, at least in favorable cases, will insure the consistency of a UPQCN. As in the classical cases, the main ingredient consists in constraint propagation methods, resulting in networks with closure properties. This section introduces the necessary tools, in order to get a polynomial algorithm of constraint propagation which transforms a UPQCN into an equivalent UPQCN satisfying a property of closure. This algorithm is based on the $\circ$-closure method. It also uses an operation, called the translation operation. Starting from the motif of a UPQCN, the operation of translation builds a QCN which expresses both the constraints of the motif and the extra constraints which must be satisfied at the next point in time.

Definition 8 *Let $\mathcal{N} = (V', C)$ be a* QCN *with $V' = V \times \{0,\ldots,max\}$ (V being a finite set $\{v_0,\ldots,v_m\}$ and $m, max \geq 0$). The* translation *of $\mathcal{N}$, denoted by* $\mathsf{translation}(\mathcal{N})$*, is the* QCN $\mathcal{N}_{tr} = (V_{tr}, C_{tr})$ *where $V_{tr} = V'$, and for all $v_i, v_j \in V$ and for all $t_i, t_j \in \{0,\ldots,max\}$, $C_{tr}((v_i,t_i),(v_j,t_j)) = C((v_i,t_i),(v_j,t_j)) \cap C((v_i,t_i-1),(v_j,t_j-1))$ if $t_i > 0$ and $t_j > 0$, and $C_{tr}((v_i,t_i),(v_j,t_j)) = C((v_i,t_i),(v_j,t_j))$ otherwise.*

Based on the operation of translation and the $\circ$-closure method, we define the closure of a motif:

Definition 9 *Let $\mathcal{R} = (V, C, t_{min}, t_{max})$ be a* UPQCN *and $\mathcal{N}_m = (V_m, C_m)$ its motif, with $V_m = V \times \{0,\ldots,\mathsf{lg}\}$. The* closure *of $\mathcal{N}_m$, denoted by* $\mathsf{closure}(\mathcal{N}_m)$*, is the* QCN *recursively defined by: $\mathcal{N}^0 = \mathcal{N}_m$ and $\mathcal{N}^{i+1} = \circ-$*$\mathsf{closure}(\mathsf{translation}(\mathcal{N}^i))$*;* $\mathsf{closure}(\mathcal{N}_m) = \mathcal{N}^j$*, where j is the smallest j such that $\mathcal{N}^j = \mathcal{N}^{j-1}$.*

$\mathsf{closure}(\mathcal{N}_m)$ can be computed in $O(u^5)$ time with $u = |V| * (\mathsf{lg}+1)$. A motif of a UPQCN coinciding with its closure will be said *closed*. A closed motif is always a $\circ$-closed QCN. We extend the preceding definitions to UPQCNs:

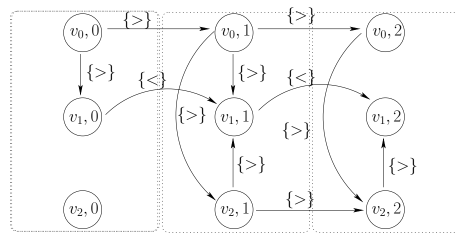

Figure 5. The translation of $\mathsf{motif}(\mathcal{R})$**.**

Definition 10 *Let $\mathcal{R} = (V, C, t_{min}, t_{max})$ be a* UPQCN*. The closure of $\mathcal{R}$ (resp. the $\circ$-closure and the translation of $\mathcal{R}$), denoted by* $\mathsf{closure}(\mathcal{R})$ *(resp.* $\circ - \mathsf{closure}(\mathcal{R})$ *and* $\mathsf{translation}(\mathcal{R})$*), is the* UPQCN $(V, C', t_{min}, t_{max})$ *where C' is the map from $V \times V \times \{0,\ldots,t_{max}\} \times \{0,\ldots,t_{max}\}$ to* A *defined by:*
• *$C'(v_i,v_j,t_i,t_j) = C(v_i,v_j,t_i,t_j)$ for all $v_i, v_j \in V$ and $t_i, t_j \in \{0,\ldots,t_{max}\}$ such that $t_i < t_{min}$ or $t_j < t_{min}$,*
• *$C'(v_i,v_j,t_i,t_j) = C^*_m((v_i,t_i-t_{min}),(v_j,t_j-t_{min}))$ for all $v_i, v_j \in V$ and $t_i, t_j \in \{t_{min},\ldots,t_{max}\}$, where C^*_m denotes the constraints of the closure (resp. the $\circ$-closure and the translation) of the motif of $\mathcal{R}$.*

The closure of a UPQCN can be computed in $O(u^5)$ time. The next proposition states the equivalence of a UPQCN with its transforms.

Proposition 4 *The $\circ$-closure and the translation of a* UPQCN $\mathcal{R}$ *are equivalent to $\mathcal{R}$. The closure of a* UPQCN $\mathcal{R}$ *is equivalent to $\mathcal{R}$.*

6 Complexity results

In this section we give two complexity results for the consistency problem of UPQCNs. We first consider classes of relations $\mathcal{E} \subseteq$ A having the property that all $\circ$-closed QCNs on $\mathcal{E}$ are globally consistent. We say that such a class has the *global consistency property*. We will show that for such a class $\mathcal{E}$ the consistency problem is a polynomial problem. Introducing classes with the global consistency property is

Algorithm 1

Check the consistency of a UPQCN $\mathcal{R} = (V, C, t_{min}, t_{max})$

$\mathcal{R} :=$ closure($\mathcal{R}$)

$\mathcal{N} := \circ -$closure($\mathsf{t_{max}}-$strengthening($\mathcal{R}$))

return not($\mathcal{N}$ contains the empty relation)

a natural move, because such classes actually exist in many qualitative formalisms, which moreover correspond to relations which are relevant for other reasons.

Proposition 5 *Let $\mathcal{E} \subseteq \mathsf{A}$ be a class satisfying the global consistency property. Let $\mathcal{R} = (V, C, t_{min}, t_{max})$ be a closed* UPQCN *on $\mathcal{E}$. Then any solution of the* $\mathsf{k}-$strengthening($\mathcal{R}$)*, with $k \geq t_{max}$, can be extended to a solution of* $(\mathsf{k}+1)-$strengthening($\mathcal{R}$).

The proof is based to the fact that we can extend solutions of $\mathsf{k}-$strengthening($\mathcal{R}$) to solutions of $(\mathsf{k}+1)-$strengthening($\mathcal{R}$). Using propositions 4, 5, 2 and the property of global consistency of $\mathcal{E}$ we get that Algorithm 1 can check the consistency of the UPQCNs on $\mathcal{E}$, hence:

Theorem 1 *Let $\mathcal{E} \subseteq \mathsf{A}$ be a class of relations satisfying the global consistency property. Then the consistency problem for* UPQCN*s on $\mathcal{E}$ is polynomial.*

In many qualitative formalisms, there exists a class $\mathcal{E} \subseteq \mathcal{A}$ satisfying the property of global consistency and containing all basic relations as well as the universal relation. In the sequel, we assume that such is the case for the qualitative formalisms we consider. As a consequence of this assumption, we will now show that the consistency problem for UPQCNs is a PSPACE problem. Firstly, we define particular QCNs.

Definition 11 *Let $\mathcal{R} = (V, C, t_{min}, t_{max})$ be a* UPQCN *and $\mathcal{N}^k = (V \times \{0, \ldots, k\}, C^k)$ the k-strengthening of $\mathcal{R}$ with $k \geq t_{max}$. A k-*scenario *of $\mathcal{R}$ is a* QCN $\mathcal{S}^k = (V \times \{0, \ldots, k\}, C^k_S)$ *such that: for all $v_i, v_j \in V$, $t_i, t_j \in \{0, \ldots, k\}$ with $t_i \leq t_j$, $C^k_S((v_i, t_i), (v_j, t_j)) = \{A\}$ with $A \in C^k((v_i, t_i), (v_j, t_j))$, if $t_j \leq t_{max}$ or, $t_i \geq t_{min}$ and $t_j - t_i \leq \mathsf{lg}$; $C^k_S((v_i, t_i), (v_j, t_j)) = \mathsf{B}$ if $t_i \geq t_{min}$ and $t_j - t_i > \mathsf{lg}$; $C^k_S((v_j, t_j), (v_i, t_i)) = C^k_S((v_i, t_i), (v_j, t_j)) \sim$.*

We extend the notions of window and end to k-scenarios. Properties concerning these particular QCNs (not given here by lack of place) allow to state that the consistency problem for UPQCNs is solved by the non-deterministic algorithm Algorithm 2, which uses a polynomial space (*w.r.t.* $|V| * t_{max}$), hence:

Theorem 2 *Assume that there exists a subclass of relations $\mathcal{E} \subseteq \mathsf{A}$ with the global consistency property and which contain the basic relations and the universal relation. Then the consistency problem for* UPQCN*s on* A *is* PSPACE

Algorithm 2

Input: a UPQCN $\mathcal{R} = (V, C, t_{min}, t_{max})$

1: Guess t_0 and k_0 with $t_{max} < k_0 \leq |B|^{((\mathsf{lg}+1)*|V|)^2} + t_{max}$ and $t_{min} \leq t_0 < k_0 - \mathsf{lg}$.
2: Guess end(S^{k_0}) a consistent atomic subnetwork of motif($\mathcal{R}$)
3: Guess an atomic consistent QCN corresponding to the restriction of S^{k_0} to the variables $V \times \{0, \ldots, t_{max}\}$.
4: For $t \in \{t_{max} + 1 - \mathsf{lg}, \ldots, k_0 - \mathsf{lg} - 1\}$ guess a consistent QCN $\mathsf{t} -$ window($\mathcal{S}^{k_0}$) compatible with $(\mathsf{t}-1) -$ window($\mathcal{S}^{k_0}$) (compatible meaning that the constraints on the same times are equal).
5: Check that end(S^{k_0}) is compatible with $(\mathsf{k_0} - \mathsf{lg} - 1) -$ window($\mathcal{S}^{k_0}$) and that end(S^{k_0}) $= \mathsf{t_0} -$ window($\mathcal{S}^{k_0}$).

7 Conclusions

We have introduced the notion of ultimately periodic qualitative constraint networks. For these networks, we propose a polynomial algorithm to check consistency. In the general case, it is not sufficient for deciding the problem of consistency. However, for classes of relations possessing the global consistency property, it does yield a method which is complete. We have further shown that the consistency problem for the networks we consider is in PSPACE, at least under the assumption that there exists a subclass which satisfies the global consistency property and which contains all basic relations as well as the universal relation. This work also opens new directions for further research. One is concerned with implementing the algorithms we have introduced here. As a matter of fact, we are currently developing such an implementation. Another one consists in trying to characterize in a generic way new cases where the consistency problem is polynomial. A third direction of research would consist in studying similar networks in the context of quantitative constraints, such as for simple temporal problems [3].

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