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Verification of analog and mixed-signal circuits using hybrid systems techniques

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Plan

1. Introduction
2. Verification of analog circuits
3. Verification of mixed-signal circuits
4. Conclusion

Introduction

- Analog and mixed-signal circuits: increasing applications
- Traditional design of analog ICs: mixture of expertise, lots of manual calculations
- Numerical simulation not suitable to prove that the circuit satisfies the specification under all possible inputs and parameter values
- Symbolic analysis methods
  - Most tools are developed for linear or linearized circuits
  - For nonlinear circuits: reduction to linear or weakly nonlinear systems, limited by accuracy trade-off
  - Time-domain symbolic analysis: less well-developed

Our goal and approach

Goal: We want to analyze

- *analog* and *mixed-signal* circuits whose continuous dynamics are described by a system of differential algebraic equations (DAEs)
- *time-domain properties*
- under *all* possible *input signals* and *parameter values*

Approach: use the verification techniques for continuous/hybrid systems

- Extend the reachability analysis techniques for ordinary differential equations (ODEs) to DAEs
- Optimization-based verification methods (to find the worst case behaviors)

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Model of analog circuits

Behavior of an analog circuit can be described by nonlinear differential algebraic equations (DAE):

$$F(x(t), \dot{x}(t), y(t), u(t), p) = 0$$

u: inputs; x: state variables (internal voltages and currents); y: outputs; p: circuit parameters

Analyze time-domain properties

- Characterize solutions under all possible input signals and parameters $\Rightarrow$ *compute reachable sets*
- Reachability analysis techniques for *ordinary differential equations* developed in the context of *hybrid systems* (e.g. tools CheckMate (CMU), d/dt (Verimag), level set methods (Stanford))

$\Rightarrow$ **Extend ODE reachability analysis techniques to DAEs**

Reachability analysis of DAEs

$$F(x, \dot{x}, y, u, p) = 0$$

- Differential index: minimal number of differentiations required to solve for the derivatives of variables
- Practical electronic circuits: often index 1 or 2.
- We focus on DAEs of index 1

Reachability analysis of DAEs (cont'd)

We study the equivalent semi-explicit form of the DAE system:

$$\dot{x} = f(x, y, p)$$
$$0 = g(x, y, p)$$

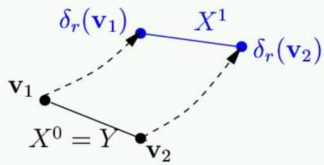
- Transforming into ODEs :
  Differentiating the algebraic eq. once gives $\dot{y} = -g_y^{-1} g_x f$ where $g_y(x, y) = \partial g / \partial y$. (Note that the DAEs are of index 1)
  $\Rightarrow$ Obtain augmented ODEs with variables $z = (x, y)^T$:

$$\dot{z} = (f, -g_y^{-1} g_x f)^T = \tilde{f}$$

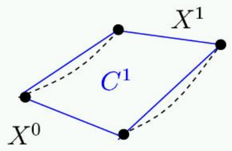
- Retain the algebraic constraint and intepret the original DAEs as the augmented ODEs on a manifold :

$$\dot{z} = \tilde{f}(z, p)$$
$$0 = g(z, p)$$

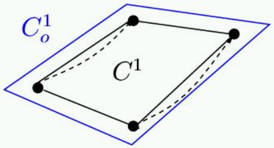
ODE reachability technique of d/dt



$$X^1 = conv\{\delta_r(\mathbf{v}_1), \delta_r(\mathbf{v}_2)\}$$



$$C^1 = conv(X^0 \cup X^1)$$



$$C^1_o = bloat(C^1, \varepsilon)$$

Linear system $\dot{x}(t) = Ax(t) + u(t)$
Second order method (approximation error is of order $\mathcal{O}(r^2)$ where r is time step) [AsarinDangMaler01]

ODE reachability technique of *d/dt* (cont'd)

Extension to nonlinear systems : $\dot{x}(t) = f(x(t))$ [AsarinDangGirard03]

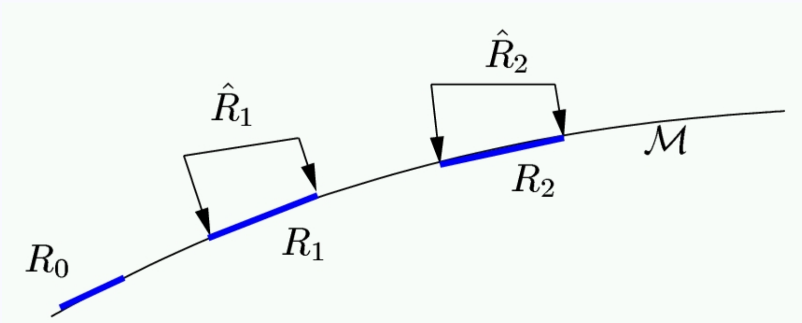
- Approximation of f by a piecewise affine function over a simplicial mesh
- Approximation error μ modeled as uncertain input: $\dot{x}(t) = A_i x(t) + u(t)$ where $||u|| \leq \mu \Rightarrow$ conservativeness
- Second order method (if f is a C^2 function)

Reachability analysis of ODEs on a manifold

We study a system of ODEs on a manifold:

$$\begin{aligned} \dot{z}(t) &= f(z(t)) \\ 0 &= g(z(t)) \quad \Rightarrow \textit{defining a manifold } \mathcal{M} \\ z(0) &\in R_0 \end{aligned}$$

Combining reachability computations techniques for ODEs and ideas from *geometric integration using projection* [Lubich,Hairer,Wanner 2003]



Reachability algorithm for ODEs on a manifold

R_0: initial set
repeat $k = 0, 1, \ldots$
$\hat{R}_{k+1} = \mathit{Reach}_{[0,r]}(R_k)$ /* *computed for the augmented ODEs* */
$R_{k+1} = \Pi_{\mathcal{M}}(\hat{R}_{k+1})$ /* *project on the manifold* $\mathcal{M}$ */
until $R_{k+1} = \bigcup_{i=1}^{k} R_i$

- Projection:

$$\Pi_{\mathcal{M}}(\hat{z}) = \arg\min_{z} |\hat{z} - z| \;\; \text{subject to } g(z) = 0$$

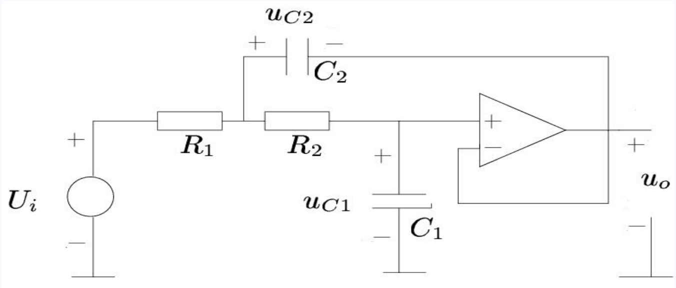
- Convergence : same order as the convergence order of the technique for ODEs (projection does not deteriorate the convergence)
- Second order method

Computational issues

- Nonlinearities in analog circuits often restricted to the device models.
- Not all the nonlinearities are important for the property under study.
- 'Hybridization'
  - approximation of a complex function by a piecewise simpler one
  - can be used for complexity reduction
  - gives rise to a hybrid system
- Hybrid systems: good model of many circuit element characteristics (which are often smoothened to avoid numerical instability)

Example: Biquad low pass filter

[Hartong,Hedrich,Barke 2002]



$$\dot u_{C1} = \frac{u_{C2} + u_o - u_{C1}}{C_1 R_2} \qquad \dot u_{C2} = \frac{U_i - u_{C2} - u_o}{C_2 R_1} - \frac{u_{C2} + u_o - u_{C1}}{C_2 R_2}, \tag{1}$$

$$u_o - V_{max} \tanh\left(\frac{(u_{C2} - u_o)V_e}{V_{max}}\right) + U_{om} = 0, \tag{2}$$

$$U_{om} = \mathcal{V}(i_0), \quad i_o = -C_2\, \dot u_{C2}, \tag{3}$$

$$\mathcal{V}(i_o) = K_1 i_o + 0.5\sqrt{K_1 i_o^2 - 2K_2 i_o I_s + K_1 I_s^2 + K_2} - 0.5\sqrt{K_1 i_o^2 + 2K_2 i_o I_s + K_1 I_s^2 + K_2}. \tag{4}$$

Differentiating (2) $\Rightarrow$ nonlinear ODEs on a manifold with variables $z = (u_{C1}, u_{C2}, u_o)$

Biquad low pass filter: Approximation by hybridization

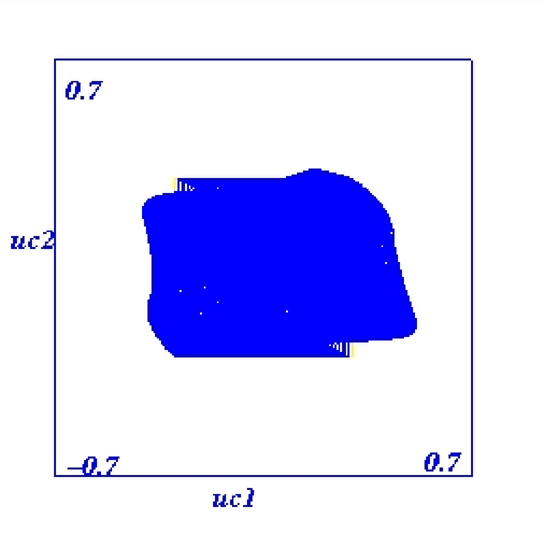
- $tanh$ approximated by polynomial or piecewise affine $\Rightarrow$ loss of important behaviors of the circuit
- the nonlinear characteristics $U_{om} = \mathcal{V}(i_o)$ can be approximated by a piecewise-affine function:

$$U_{om} = \begin{cases} 1e10\, i_o + 0.5e8 & \text{if } i_o \leq -i_s \\ 0 & \text{if } \ -i_s < i_o < i_s \\ 1e10\, i_o - 0.5e8 & \text{if } i_o \geq i_s \end{cases} \quad (5)$$

$\Rightarrow$ Hybrid automaton with 3 continuous modes

Biquad low pass filter: verification results

The property to verify is the *absence of overshoots* .



- $C_1 = 0.5e-8$, $C_2 = 2e-8$, and $R_1 = R_2 = 1e6$ (highly damped case)

- The initial set: $u_{C1} \in [-0.3, 0.3]$, $u_{C2} \in [-0.3, 0.3]$ and $u_o \in [-0.2, 0.2]$

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Verification of mixed-signal circuits

Mixed-signal circuits are circuits mixing analog signals (real-valued) and digital signals (boolean-valued).

Model: We consider circuits that can be modeled by:

$$F(x(k), x(k+1), u(k), \delta(k), p) = 0, \;\; k \in \mathbb{N},$$

- $\delta(k) \in \{0, 1\}^s$ (modelling logical part)
- $u(\cdot) = (u(k))_{k \in \mathbb{N}}$ input sequence
- $x(k)$ state at time k, uniquely defined from $x(0)$ and $u(\cdot)$

Safety verification problem: given a safe set $\mathcal{S}$, verify that the system always remains in $\mathcal{S}$ over a bounded horizon N

Optimisation based verification: Find the *worst-case* behavior and check if it satisfies the property

Optimisation based verification

- Define an objective function J such that:

$$J(x) \text{ is positive } \Leftrightarrow x \text{ is outside } \mathcal{S}$$

- To prove that *the system is safe at time k* : checking whether the maximum value of $J(x(k))$ is not positive by solving:

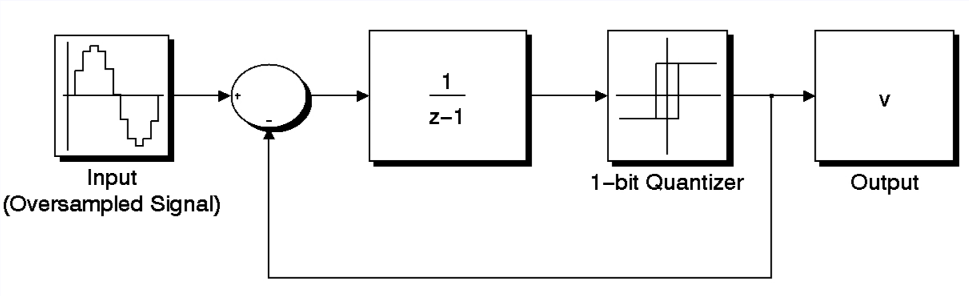
$$\begin{aligned} &\max\ J(x(k)), \\ &s.t.\ \ F(x(t), x(t+1), u(t), \delta(t), p) = 0, \\ &\qquad u(t) \in \mathcal{U},\ \delta(t) \in \{0,1\}^s,\ t \in \{0, 1, \ldots, k-1\}, \\ &\qquad x(0) \in X_0. \end{aligned}$$

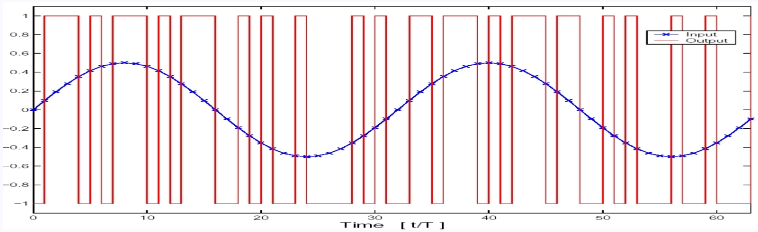
  This constrained optimisation problem is mixed since $\delta(t)$ are integers

- To prove that *the system is safe over horizon N* , we prove that it is *safe at all steps $k = 1, 2, \ldots, N$*

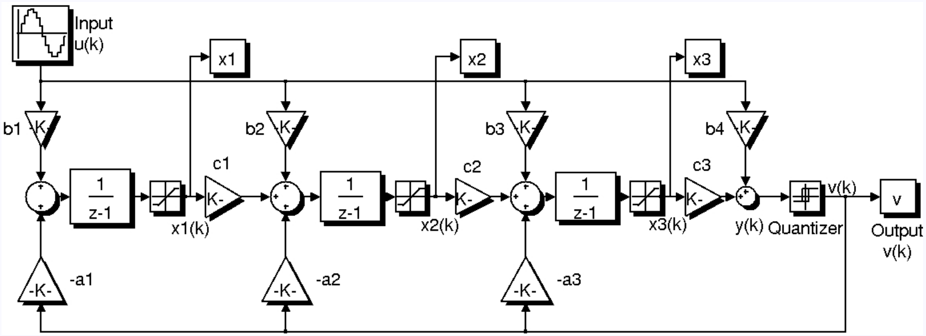
Δ-Σ modulation

- Δ-Σ modulation is a popular technique of analog-to-digital conversion
- 4 steps: anti-aliasing, oversampling, noise shaping, quantization





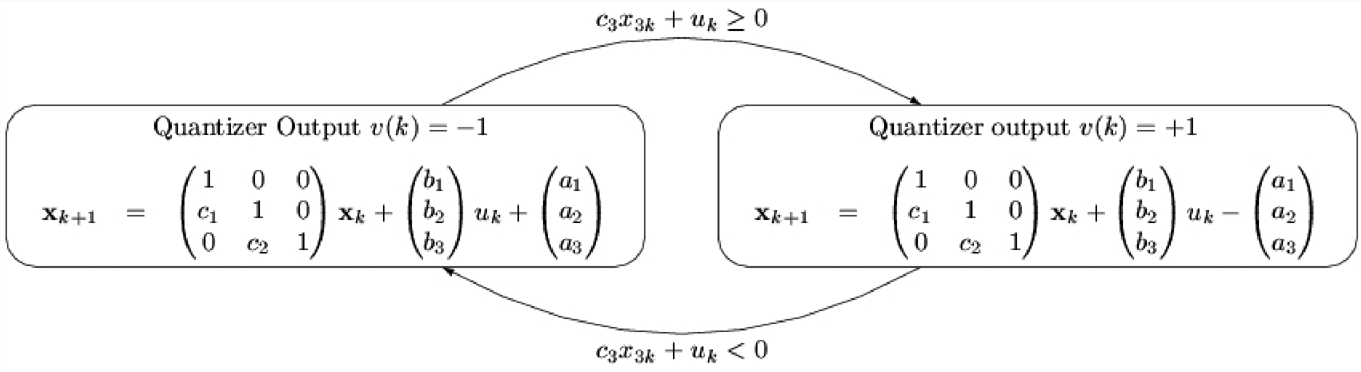
Third-order Δ-Σ modulator



- 3 integrators $\Rightarrow$ third order Δ-Σ modulator
- High order modulators achieve better performance but induce *instability problems*
- Stability: a modulator is stable if the states of all its integrators always remain bounded (under bounded inputs)

Third-order Δ-Σ : hybrid model and verification

Hybrid automaton model



$\mathbf{x}_k = (x_1(k)\ x_2(k)\ x_3(k))^T$: states of the integrators

Verification

To prove that the circuit is stable, we prove that its integrators *never saturate* $\Rightarrow$ a safety property

Safe set $\mathcal{S} = [-x_1^{sat}, x_1^{sat}] \times [-x_2^{sat}, x_2^{sat}] \times [-x_3^{sat}, x_3^{sat}]$

Third-order Δ-Σ verification

- Objective function can be defined as

$$J(x(k)) = \max_{i=1,2,3}(|x_i(k)| - x_i^{sat})$$

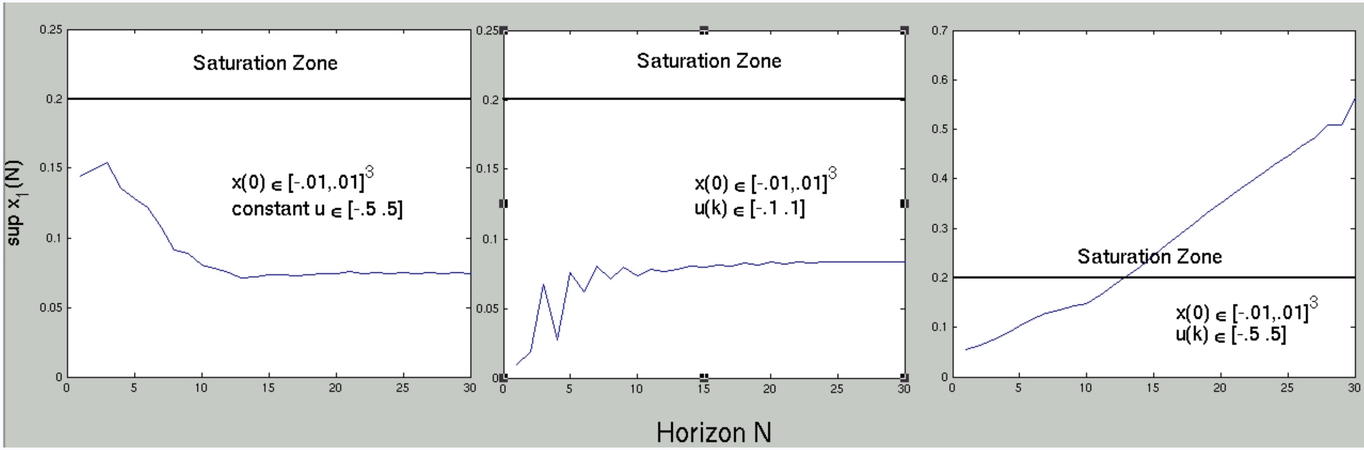
Symmetries of the dynamics and the safe set $\mathcal{S}$ allow to split the optimization problem into three auxiliary problems:

$$\max J(x(k)) = \max_{i=1,2,3}(\max J_i(x(k))$$

where $J_i(x(k)) = x_i(k) - x_i^{sat}$ for $i = 1, 2,$ or 3.

- Objective functions are *linear* and the dynamics of each mode is affine $\Rightarrow$ *Mixed Integer Linear Programs* (for which efficient solvers exist)
- We used MOSEK to solve the above optimisation problem.

Δ-Σ verification: Results



Remark: frequent switching of the quantizer makes the optimisation problem very combinatorial

Conclusion

Results

- A framework for modelling and verification of analog and mixed-signal circuits using hybrid system techniques
- Encouraging experimental results

Future work

To increase scalability:

- Abstraction and model reduction techniques, exploit circuit structure
- Consider refined constraints on the input signals (e.g. bandwidth)

To integrate formal methods in the design process:

- automatic translation from circuit descriptions (such as in VHDL-AMS) to hybrid automata
- formalism for specifying properties of analog signals

End

Thank You For Your Attention