Feedback Control Using The Laplace Transform

Course Feedback Control and Real-time Systems

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Stabilization by Feedback

• Stabilization by feed-back
• Pole Placement
Consideration of Disturbance

- disturbance $w(t)$ represents perturbations or modelling error
- the output $y(t)$ and the setpoint/reference $r(t)$
- the controller $C$ and the system $S$ are supposed to be rational fractions $C(s), S(s)$
We now compute the closed-loop transfer function:

\[
Y(s) = S(s)(C(s)(R(s) - Y(s)) + W(s))
\]

\[
(1 + S(s)C(s))Y(s) = S(s)(C(s)R(s) + W(s))
\]

where \(W(s)\), \(Y(s)\), \(R(s)\) are the Laplace transforms of the disturbance \(w(t)\), the output \(y(t)\) and the reference \(r(t)\).
We obtain the closed-loop transfer function: 

\[ Y(s) = \frac{S(s)C(s)}{1 + S(s)P(s)} R(s) + \frac{S(s)}{1 + S(s)C'(s)} W(s) \]
Control Problem Formulation

\[ Y(s) = S(s)C(s) + R(s) + S(s) + S(s)P(s) \]

given the transfer function \( S(s) \) of the system, find \( C(s) \) such that

1. \( S(s)C(s) + S(s) \) is stable and close to the identity (fidelity)
2. \( S(s) + S(s)P(s) \) is small (robustness or disturbance rejection)
Control Problem Formulation

\[ Y(s) = \frac{S(s)C'(s)}{1 + S(s)C'(s)} R(s) + \frac{S(s)}{1 + S(s)C'(s)} W(s) \]
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We consider the system $S(s) = \frac{1}{s^2}$ (double integrator) and a proportional controller $C(s) = a$.
Exemple: PID controllers

We consider the system \( S(s) = \frac{1}{s^2} \) (double integrator) and a proportional controller \( C(s) = a \).

We calculate the denominator of \( \frac{S(s)C'(s)}{1 + S(s)C'(s)} \) which gives

\[ D = s^2 + a \]

This is a second-order polynomial with purely imaginary roots, unstable!!
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The denominator of \( \frac{S(s)C(s)}{1 + S(s)C(s)} \) is

\[ D = s^2(cs + d) + as + b = cs^3 + ds^2 + as + b \]

This is a third-order polynomial with 3 roots that we can fix as we want. We choose stable roots

\[ (s + 1)(s - e^{\frac{3i\pi}{4}})(s - e^{\frac{5i\pi}{4}}) = (s + 1)(s^2 + \sqrt{2}s + 1) = s^3 + 2.4s^2 + 2.4s + 1 \]
Pole Placement – Example: PID controller

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$$(s + 1)(s - e^{\frac{3i\pi}{4}})(s - e^{\frac{5i\pi}{4}}) = (s + 1)(s^2 + \sqrt{2}s + 1) = s^3 + 2.4s^2 + 2.4s + 1$$

We identify: $c = 1, d = 2.4, a = 2.4, b = 1$
Use Simulink to simulate this PI controller and the system. Add some disturbance (by using the block named “Band-Limited White Noise”). Is the result satisfactory? If not, modify the controller to reject the disturbance.