Discretization of Continuous Controllers

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- One way to design a computer-controlled control system is to make a continuous-time design and then make a discrete-time approximation of this controller $\Rightarrow$ Analog Design Digital Implementation
- The computer-controlled system should now behave as the continuous-time system

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- This is crucially dependent on choosing fairly short sampling periods.

Difference Approximations (1)

- When the continuous-time controller is specified as a transfer function $C(s)$, it is natural to look for methods that will transform the continuous transfer function $C(s)$ to a pulse transfer function $C_d(z)$ so that the corresponding behaviors of the two systems are close to each other.
- z and s are related as $z = exp(sT)$, where T is the sampling period.

Difference Approximations (2)

The difference approximations correspond to the series expansions

- $z = e^{sT} \approx 1 + sT$ (Forward difference or Euler's method)
- $z = e^{sT} \approx \frac{1}{1-sT}$ (Backward difference)
- $z = e^{sT} \approx \frac{1+sT/2}{1-sT/2}$ (Trapezoidal method, or Tustin's approximation)

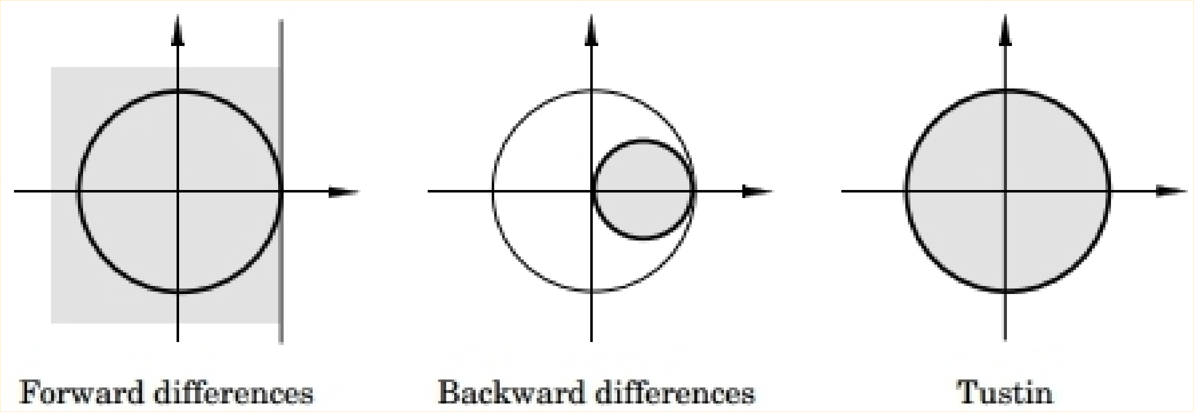
Computing transfer function $C_d(z)$

To calculate $C_d(z)$ we substitute s in $C(s)$ with the following:

- $s \approx \dfrac{z-1}{T}$ (Forward difference or Euler's method)
- $s \approx \dfrac{z-1}{zT}$ (Backward difference)
- $s \approx \dfrac{2}{T}\dfrac{z-1}{z+1}$ (Trapezoidal method, or Tustin's approximation)

Stability

the stability region (corresponding to the left half-plane $Re(s) \leq 0$) in the s-plane is mapped on the z-plane.



Stability - Remarks

- Forward-difference approximation: it is possible that a stable continuous-time system is mapped into an unstable discrete-time system.
- Backward approximation: a stable continuous-time system will always give a stable discrete-time system.
- Tustin's approximation: has the advantage that the left half s-plane is transformed into the unit disc in the z-plane.

Selection of Sampling Period and Anti-aliasing Filters

Choice of sampling rates and anti-aliasing flters are important

- Preserve stability
- Preserve performance

Exercise: Application to LEGO robots

1. Compute the continuous-time transfer function of the open loop $F_{BO}(s)$

2. Estimate the crossover frequency ω_c of $F_{BO}(s)$ using the function "margin" in matlab.

3. Choose sampling period T_e according to the rule: $\omega_c T_e$ **is between** 0.05 **and** 0.14

4. Discretize the controllers using one of the approximation methods and the chosen sampling period. (Note that instead of replacing the whole continuous-time controller with its discretized version, we can replace only its components containing continuous-time blocs, such as the integrators $\frac{1}{s}$)

5 Add the digital anti-aliasing filter