Security Models: Proofs, Protocols and Certification

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Outlines

1. **Introduction : attack models and security properties**
2. Security definitions and proofs - Perfect secrecy
3. Elementary notions in probability theory
4. Shannon' theorem on perfect secrecy

Security : what cryptography should provide

CAIN

- Confidentiality
- Authentication
- Integrity
- Non-repudiation

Kerckhoffs' principle [1883]

A cryptosystem should be secure even if everything about the system, except the key, is public knowledge.

Attack models : COA / KPA / CPA / CCA (1/2)

COA : Ciphertext-Only Attack

the attacker is assumed to have access only to a set of ciphertexts

Eg :*vulnerabilities to COA :*

- *WEP : bad design ;*
- *DES : too small key space*

KOA : Known-Plaintext Attack

the attacker has samples of both the plaintext and its encrypted version ; he uses them to get the secret key.

Eg : *vulnerabilities to KOA : encrypted ZIP archive : knowing only one unencrypted file from the archive is enough to calculate the key*

Attack models : COA / KPA / CPA / CCA (2/2)

CPA : Chosen-Plaintext Attack

the attacker chooses a plaintext and can crypt it to obtain the corresponding ciphertexts ; i.e.
he has access to an encryption machine.

Eg : *vulnerabilities to COA : dictionary attack on Unix* `passwd` *file.*
`Crack`, `John the Ripper`, `L0phtCrack`, `Cain&Abel`, ...

CCA : Chosen-Ciphertext Attack

the attacker chooses a ciphertext and can decrypt without knowing the key.
He has access to a decryption machine (oracle).
↪ Important for smart cards designers, since the attacker has full control on the device !

Eg : *vulnerabilities to CCA : ElGamal, early versions of RSA in SSL, ...*

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Security definitions. A cryptosystem is

Computationally secure :

if any successful attack requires at least N operations, with N large eg $10^{120} \simeq 2^{400}$.

Provable secure :

if any attack exists, a known hard problem could be efficiently solved.
[Proof : reduction, complexity, P, NP]

Semantic secure – for *asymmetric cryptosystem* – :

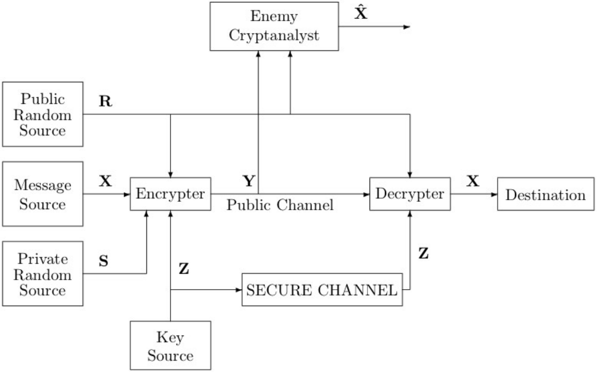
knowing the public key and a ciphertext (COA), it must be infeasible for a computationally-bounded adversary to derive significant information about the plaintext

NB equivalent to the property of *ciphertext indistinguishability*[Blum,Micali]

Unconditionally secure : (or *perfect secrecy*)

cannot be broken, even by a computationally-**un**bounded attack
$\hookrightarrow$ "Information Theory" [Shannon]

Model of a symmetric cryptosystem



Shannon model

- perfect secrecy : i.e., informally,
  the knowledge of Y gives no information on X
- $\hookrightarrow$ definition : "Information Theory"

Outlines

Discrete Random Variable (1/2)

- **Sample space** S : finite set whose elements are called "elementary events"
  Eg : can be viewed as a possible outcome of an experiment
- an **event** is a subset of S.
  $\emptyset$= the *null* event S= the *certain* event
- events A and B are **mutually exclusive** iff $A \cap B = \emptyset$

- **Probability distribution**
  a function $\Pr : X \subset S \mapsto [0,1]$ satisfying probability axioms :
  1. $\forall$ event A : $\Pr(A) \geq 0$;
  2. if A and B mutually exclusive : $\Pr(A \cup B) = \Pr(A) + \Pr(B)$
  3. $\Pr(S) = 1$

Discrete Random Variable (2/2)

Definition : Discrete Random Variable

a function X from a finite space S to the real numbers.

$\hookrightarrow$ *quantity whose values are random*

For a real number x, the event $X = x$ is $\{s \in S : X(s) = x\}$. Thus

$$\Pr(X = x) = \sum_{s \in S : X(s) = x} \Pr(s)$$

Experiment = rolling a pair of fair 6-sided dice 

- Random variable X : the maximum of the two values
- $\Pr(X = 3) = \frac{5}{36}$

Conditional probability and independence

Def : Conditional property of an event A given another event B :

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Def : Two events A and B are independent iff

$$\Pr(A \cap B) = \Pr(A).\Pr(B)$$

So, if $\Pr(B) \neq 0$, A and B independent $\iff \Pr(A|B) = \Pr(A)$

Bayes's theorem

From definition,
$\Pr(A \cap B) = \Pr(B \cap A) = \Pr(B)\Pr(A|B) = \Pr(A)\Pr(B|A)$.
This, if $\Pr(B) \neq 0$, we have :

$$\Pr(A|B) = \frac{\Pr(A)\Pr(B|A)}{\Pr(B)}$$

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1. Introduction : attack models and security properties
2. Model of a symmetric cryptosystem
3. Elementary notions in probability theory
4. **Information theory - Shannon' theorem on perfect secrecy**

Information and entropy

Shannon's measure of information

- Hartley's measure of information : $I(X) = \log_2 \frac{1}{p_i}$ *bit* (*logon*)
- Def : entropy $H(X)$ (or uncertainty) of a disc. rand. var. X :

$$H(X) = -\sum_{x \in \mathrm{Supp}(X)} \Pr(X = x).\log_2 \Pr(X = x)$$

i.e. the "average Hartley information".

Basic properties of entropy

- Let $n =$Card(Sample space) ; then $H(X) \leq \log_2 n$
  The entropy is maximum for the uniform probability distribution Gibbs'lemma
- $H(X|Y) \leq H(X) + H(Y)$
- $H(XY|Z) = H(Y|Z) + H(X|YZ)$
- $H(X|Y) \leq H(XZ|Y)$

Unconditional security / Perfect secrecy

Characterization of perfect secrecy

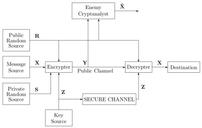
The knowledge of the ciphertext Y brings no additional information on the plaintext X, i.e.

$$H(X|Y) = H(X)$$

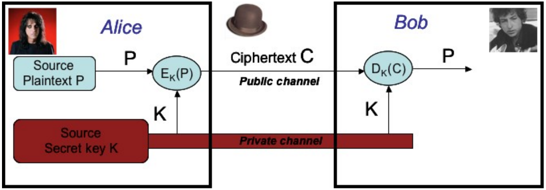
Outlines

- Lecture 1 : attacks ; security defs ; unconditional security and entropy.
  Unconditional secure symmetric cipher
  - Proof of Shannon's theorem : lower bound on the key size.
  - Vernam's cipher – binary and generalization to arbitrary group.
  - Generalized Vernam's cipher is unconditionally secure.
- Lecture 2 : Asymmetric protocols and provable security
  - Asymmetric cryptography is not unconditionally secure
  - Provable security : arithmetic complexity and reduction
  - Complexity and lower bounds : exponentiation
  - P, NP classes

Model of a symmetric cryptosystem



General model



Simplified model

Definition : *Unconditional security* or *Perfect secrecy*

The symmetric cipher is unconditionally secure iff $H(P|C) = H(P)$

i.e. the cryptanalyst's a-posteriori probability distribution of the plaintext, after having seen the ciphertext, is identical to its a-priori distribution.

Shannon's theorem : necessary condition, lower bound on K

In any unconditionally secure cryptosystem : $H(K) \geq H(P)$.

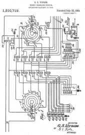
Proof : $H(P) = H(P|C) \leq H((P,K)|C) = H(K|C) + H(P|(K,C)) = H(K|C) \leq H(K)$

Vernam's cipher : unconditionally secure cryptosystem

Shannon's Theorem part 2 : existence

It exists unconditionally secure cipher.

Example : OTP (One-Time Pad) / Vernam's cipher



Vernam's patent, 1917

Symmetric cipher of a bit stream :

- let $\oplus$ = boolean *xor*; let $n = |P|$.
- for $i = 1, \ldots, n$: $\quad C_i = P_i \oplus K_i$

OTP : One-Time Pad [AT&T Bell labs]

NB : size of the (boolean) key K = size of the (boolean) plaintext P.

OTP applications

- Unbreakable if used properly. A one-time pad must be **truly random data** and must be **kept secure** in order to be unbreakable.
- intensively used for diplomatic communications security in the 20th century. E.g. telex line Moscow–Washington : keys were generated by hardware random bit stream generators and distributed via trusted couriers.
- In the 1940s, the (Soviet Union) KGB used recycled one-time pads, leading to the success of the NSA code-breakers of the project VENONA [http ://www.nsa.gov/venona/]

Generalized Vernam's cipher

Generalization to a group $(G, \otimes)$ with $m = |G|$ elements

- For $1 \leq i \leq n$, let K_i be uniformly randomly chosen in G.
- Ciphertext $C = E_K(P)$ is computed by : $C_i = P_i \otimes K_i$
- What is the deciphering $P = D_K(P)$?

Theorem : Generalized Vernam's cipher is unconditionally secure

Proof : We have $H(P) \leq \log_2 m^n = n \log_2 m$.
Besides,

$$\Pr(P = p | C = c) = \Pr(C \otimes K^{-1} = p | C = c) = \Pr(K = p^{-1} \otimes c) = \frac{1}{m^n}$$

then $H(P|C) \geq H(P)$. Since $H(P|C) \leq H(P)$, we have $H(P|C) = H(P)$.

Summary

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Training exercises (tutoring and home)

- probability ; entropy and secret.

Next lecture

1. Provable security of Asymmetric protocols