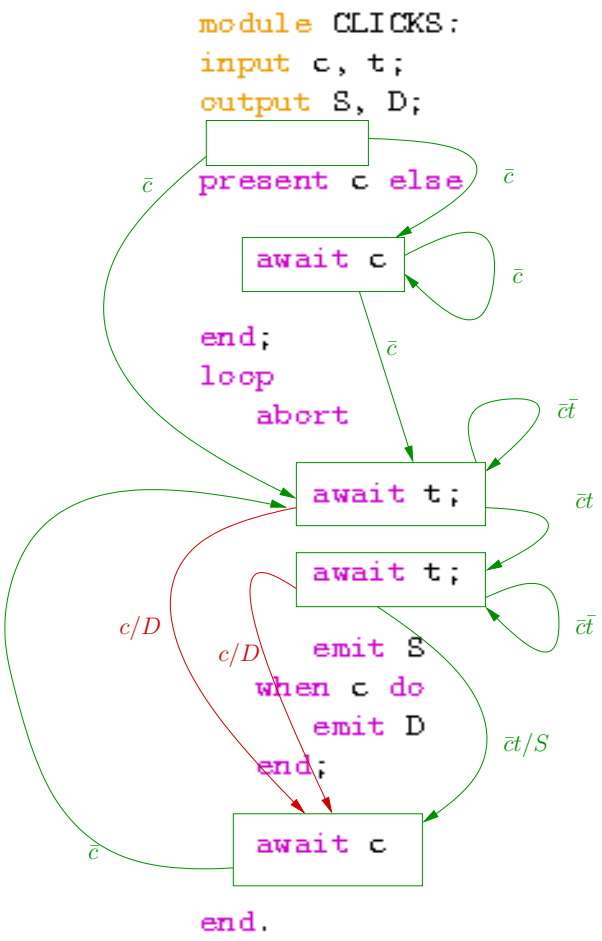
Question 1

Follow the method viewed in course (cf. slides) : 1 init state at the beginning, one state per `await` statement.



Question 2

There are many solutions. This one uses a *delayed switch* : conversely to the classical one (cf. slides) that correspond to a time interval `[on,off[` (on included, off excluded), `dswitch` corresponds to an interval `]on,off]` (on excluded, off excluded).

```
node dswitch(on,off:bool) returns (state: bool);
let
   state = false -> pre (if state then not off else on);
tel

node clics(c,t: bool) returns (S,D: bool);
var w1,w2: bool;
let
   assert #(c,t);
   w1 = dswitch(c, S or D);
   w2 = dswitch(w1 and t and not c, S or D);
   D = w1 and c;
   S = w2 and t;
tel
```

Question 3
Structural constraints :

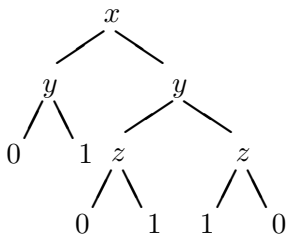
```
a = 1;
k = 1;
a + j = b + k;
b = c + d;
c + d = f + e;
e + f = g + h;
g + h = j;
```

Objective :

```
max: 15 c + 10 d + 8 e + 15 f + 20 g + 12 h;
```

Question 4
The loop is executed at most 10 times (in fact exactly 10, but it does not matter for WCET). New constraint : `b <= 10` or equivalently `j <= 10`.
First estimation : 500, corresponding to take 10 times the path c.f.g
Question 5
During each iteration, it is impossible to take the sequence : c.f.j, because : the iteration starts with `y = x[i]` (b), then (c) `z = 0`, then `y += z` (f) and since $z = 0$ y does not change (`y = x[i]`), then `y > x[i]` is false and (g) is impossible.
For each iteration t, we have $c_t + f_t + g_t \leq 2$, since there are 10 iterations : $c + f + g \leq 20$.
With this constraint, the worst path at each iteration is d.f.g (45) thus the WCET is $10 * 45 = 450$

Question 6



Question 7

The formula has no implicant of length 1 (i.e. none of x, $\bar{x}$, y, $\bar{y}$, z, $\bar{z}$ are implicant).
It has 2 implicants of length 2, that are thus prime :
(A) $\bar{x}.y$
(B) $y.\bar{z}$
And finally 5 implicants of length 3 :
$\bar{x}.y.z$, not prime since it implies (A)
$\bar{x}.y.\bar{z}$ not prime since it implies (A)
$x.y.\bar{z}$ not prime, not prime since it implies (B)
$x.\bar{y}.z$ prime

Question 8

Whenever the right branch is 1, we can deduce that the root variable x implies the whole formula, thus x is a (minimal) implicant :

$$\mathcal{I}\left(\begin{array}{ccc} & x & \\ \diagup & & \diagdown \\ 0 & & 1 \end{array}\right) = \mathcal{I}\left(\begin{array}{ccc} & x & \\ \diagup & & \diagdown \\ f_0 & & 1 \end{array}\right) = x$$

Same reasoning when the left branch is 1 : $\bar{x}$ is a (minimal) implicant :

$$\mathcal{I}\left(\begin{array}{ccc} & x & \\ \diagup & & \diagdown \\ 1 & & 0 \end{array}\right) = \mathcal{I}\left(\begin{array}{ccc} & x & \\ \diagup & & \diagdown \\ 1 & & f_1 \end{array}\right) = \neg x$$

When the left branch is 0, it means that x MUST TRUE to satisfy the formula, i.e. x necessarily appears in any implicant. To obtain a "minimal" implicant then : compute a "minimal" implicant for the right branch (f_1), and concatenates x to the result :

$$\mathcal{I}\left(\begin{array}{ccc} & x & \\ \diagup & & \diagdown \\ 0 & & f_1 \end{array}\right) = x \cdot \mathcal{I}(f_1)$$

Similarly, when the left branch is 0, $\bar{x}$ is necessary to satisfy the formula, and thus it must appear in any implicant :

$$\mathcal{I}\left(\begin{array}{ccc} & x & \\ \diagup & & \diagdown \\ f_0 & & 0 \end{array}\right) = \bar{x} \cdot \mathcal{I}(f_0)$$

Finally, when none of the branches is a constant, we have to find a "strategy" that hopefully gives a "small" implicant. A reasonable strategy consist in searching, if it exists, an implicant where the variable x does not appear (directly or negated). For that, we first build the *consensus* formula :

$$g = f_0 \cdot f_1$$

(Note : cf. course, $g = \forall x \ \ f$)

If g is NOT IDENTICALLY 0, it means that any implicant of g is also an implicant of f, and, since it does not contain the variable x, we can expect it to be small (with no guaranty !) :

$$\left(\begin{array}{c} x \\ / \quad \backslash \\ f_0 \quad f_1 \end{array} \right) = \mathcal{I}(g) \text{ if } g = f_0 \cdot f_1 \neq 0$$

If g is IDENTICALLY 0, it means that x or $\bar{x}$ is necessary in any implicant ; in this case one can chose *arbitrarily* to compute an implicant with x or with $\bar{x}$, for instance :

$$\left(\begin{array}{c} x \\ / \quad \backslash \\ f_0 \quad f_1 \end{array} \right) = x \cdot \mathcal{I}(f_1) \text{ if } g = f_0 \cdot f_1 = 0$$

or :

$$\left(\begin{array}{c} x \\ / \quad \backslash \\ f_0 \quad f_1 \end{array} \right) = \bar{x} \cdot \mathcal{I}(f_0) \text{ if } g = f_0 \cdot f_1 = 0$$

NOTE : with the previous definition, $\mathcal{I}$ is strictly linear w.r.t. the size of the operand, but has no guaranty on the quality (the minimality) of the result. We can have a better result, but with a much costly algorithm, (not only non-linear, but non polynomial !). The idea is simple : when there are several choices, computes all the cases, and keep the "best one". Suppose that we have a procedure shortest that takes a list of monomial and returns one of them with a minimal length :

$$\left(\begin{array}{c} x \\ / \quad \backslash \\ f_0 \quad f_1 \end{array} \right) = \text{shortest}\left(\mathcal{I}(g), x \cdot \mathcal{I}(f_1), \bar{x} \cdot \mathcal{I}(f_0)\right) \text{ if } g = f_0 \cdot f_1 \neq 0$$

$$\left(\begin{array}{c} x \\ / \quad \backslash \\ f_0 \quad f_1 \end{array} \right) = \text{shortest}\left(x \cdot \mathcal{I}(f_1), \bar{x} \cdot \mathcal{I}(f_0)\right) \text{ if } g = f_0 \cdot f_1 = 0$$