Time-Triggered Mixed-Critical Scheduler on Single- and Multi-processor Platforms

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Verimag - Université Joseph Fourier - Grenoble

HPCC 2015

1The research leading to these results has received funding from the European Community’s Seventh Framework Programme [FP7/2007-2013] under grant agreement no. 288175 [Certainty] and by ESA Project MoSaTT-CMP, Contract No. 4000111814/14/NL/MH
Multi-cores and Many-cores Systems

The introduction of multi-cores and many-cores is leading to an increasing trend towards integrating multiple subsystems upon a single chip.

Effects of integration:
- Increase efficiency
- Reduce device count
- Reduced Size, Weight and Power consumption
- Increase design cost
- Increase certification complexity
Mixed Critical Systems

The certification problem

- In some domain (avionics, automotive) systems must pass certification test issued by Certification Authorities
- Applications at different levels of criticality interact and co-exist

Avionic **DO-178B**

defines 5 levels of criticality, based on the effect of a failure:
A (Catastrophic), B (Hazardous), C (Major), D (Minor) and E (No Effect).

Unmanned Air Vehicles (**UAVs**)  

defines two levels: flight-critical and mission-critical
  - both levels are critical and hard real-time
  - Only flight-critical needs to be certified by a certification authority!
Vestal Model (S. Vestal, 2007) in Dual Critical Systems

Scheduling in Mixed Criticality System

very pessimistic estimation of WCET for jobs that undergo certification

- risk of very low processor usage
- solution: interleaving the execution of jobs of different criticality levels
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Vestal Model for finite set of jobs in dual critical systems

- every job is classified as hi-critical (HI) or lo-critical (LO)
- every job is labeled with two Worst Case Execution Times:
  - \(C(LO)\) computed with industrial standard tools
    (realistic estimation)
  - \(C(HI)\) computed with tools compliant to certification authority standards
    (very pessimistic estimation)
Schedulability conditions of Mixed Criticality System in Vestal Model

a set of jobs is correctly scheduled if:

Condition 1
If all jobs respect their LO WCET, then both HI and LO jobs must meet their deadline.

Condition 2
If at least one job's execution time exceeds its LO WCET, then all HI jobs must complete before their deadline, whereas LO jobs may be even dropped.
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the scheduling problem is NP-complete
Example of scheduling problem

Let us apply different scheduling to the instance:

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Criticality Monotonic

priority order = 1-3-2
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![EDF Scheduling Diagram]

**Criticality Monotonic**

![Criticality Monotonic Scheduling Diagram]

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D. Socci et al. (UJF / Verimag)
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Time Triggered scheduler approach

Time Triggered (TT) Schedulers

- the most used in real safety-critical system
- the scheduler intervals are statically precomputed
  - interference analysis is easier
  - certification is easier
Single Time Table per Mode (STTM)

- extends TT to mixed-critical Systems (*Baruah and Fohler, 2011*)
- two tables: LO table and HI table

![Diagram of STTM](chart.png)

- normally execute LO table, jump to HI table when $C(LO)$ is violated
- one must ensure that each HI jobs gets at least $C(HI)$ time to execute
- scheduling on 2 tables is not trivial!
  In general, for $n$ jobs $O(n)$ tables are needed! (*Baruah et al., 2012*)
Our solution

- numerous solutions have been proposed in literature for the MCS scheduling problem
  - mainly priority based

- solutions considering STTM approach are a few and make restrictive assumptions
Our solution

- Numerous solutions have been proposed in literature for the MCS scheduling problem
  - Mainly priority based

- Solutions considering STTM approach are few and make restrictive assumptions

- We propose an algorithm that can map memoryless mixed critical scheduling solutions to a STTM one

Proposed Algorithm

**INPUT:** MCS scheduling solution  
**OUTPUT:** STTM scheduling: TT table \( \text{LO} \) and \( \text{HI} \)
The algorithm

- generate **LO** by simulating the execution when all jobs runs for $C(LO)$
- generate **HI** by simulation of **HI** jobs under the following conditions:
  - each job executes for $C(HI)$ time units
  - a job $J$ runs at time $t$ if it is enabled by the input scheduling and one of the following *rules* is true:
    1. $J$ is terminated in **LO**
    2. $J$ has executed in **LO** more then it has in **HI**
    3. $J$ has executed in **LO** as much as it has in **HI**, and it is running in **LO**

- Rules 2 and 3 assure that $J$ will not run in **HI** for more time than in **LO** before it terminates in **LO**
- once $J$ is terminated in **LO**, Rule 1 permanently enables it
Single processor correctness

on single processor systems the following holds:

Theorem

*If the input policy is correct (i.e., meets all deadlines), then the output STTM policy is a correct as well.*
Multiprocessor Experiments

on multi processors experiment were performed:

Experiments

1. generate random instances for different values of 
   \( s = \text{Stress}(\text{LO}) = \text{Stress}(\text{HI}) \)

2. find solution using priority based algorithm MCEDF

3. translate the solutions using our methodology

we use \text{Stress} (utilization metric) to measure the “hardness” of an instance

\[
\text{Stress}(\text{LO}) = \max_{0 \leq t_1 < t_2} \left\{ \frac{m}{\min\{1, |J'|\}} \cdot \sum_{J'=J_i \mid t_1 \leq A_i \wedge D_i \leq t_2} \frac{C_i(\text{LO})}{t_2 - t_1} \right\}
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\[
\text{Stress}(\text{HI}) = \max_{0 \leq t_1 < t_2} \left\{ \frac{m}{\min\{1, |J'|\}} \cdot \sum_{J'=J_i \mid \chi_i = \text{HI} \wedge t_1 \leq A_i \wedge D_i \leq t_2} \frac{C_i(\text{HI})}{t_2 - t_1} \right\}
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Experimental Results

- on horizontal axes, $s = Stress(LO) = Stress(HI)$
Conclusions

- we proposed an algorithm to translate MCS scheduling solutions into STTM ones
- the algorithm is optimal for the single processor case
- on multicores, we have a very low failure rate
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Future work

- support dependencies
- implement bus and cache interference analysis
- more criticality levels
Thanks for the attention!
Example

\[ \text{PT}_{HI} = J_2 \succ J_3 \succ J_1 \]

\[ T_{j}^{LO}(t) = C_{j}(LO) \quad (1) \]

\[ T_{j}^{HI*}(t) < T_{j}^{LO}(t) \quad (2) \]

\[ T_{j}^{HI*}(t) = T_{j}^{LO}(t) \land E^{LO}(t) = J_j \quad (3) \]
Conclusions

Example

\[ PT_{HI} = J_2 \succ J_3 \succ J_1 \]

\[ T_{j}^{LO}(t) = C_{j}(LO) \quad (1) \]
\[ T_{j}^{HI}(t) < T_{j}^{LO}(t) \quad (2) \]
\[ T_{j}^{HI}(t) = T_{j}^{LO}(t) \land E^{LO}(t) = J_{j} \quad (3) \]

\[ t = 0 \quad E^{LO}(t) = J_{1} \]

<table>
<thead>
<tr>
<th>Job</th>
<th>STATUS</th>
<th>( T^{HI}(t) )</th>
<th>( T^{LO}(t) )</th>
<th>( C(LO) )</th>
<th>( C(HI) )</th>
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</table>
Example

\[ PT_{HI} = J_2 \succ J_3 \succ J_1 \]

\[ T_{j}^{LO}(t) = C_{j}(LO) \quad (1) \]
\[ T_{j}^{HI*}(t) < T_{j}^{LO}(t) \quad (2) \]
\[ T_{j}^{HI*}(t) = T_{j}^{LO}(t) \wedge E^{LO}(t) = J_{j} \quad (3) \]

<table>
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<th>Job</th>
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</tbody>
</table>
**Example**

\[ PT_{HI} = J_2 \succ J_3 \succ J_1 \]

\[ T_j^{LO}(t) = C_j(LO) \quad (1) \]
\[ T_j^{HI^*}(t) < T_j^{LO}(t) \quad (2) \]
\[ T_j^{HI^*}(t) = T_j^{LO}(t) \land E^{LO}(t) = J_j \quad (3) \]

<table>
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<th>Job</th>
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<th>( T_j^{LO}(t) )</th>
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</tbody>
</table>

\( t = 2 \)
\( E^{LO}(t) = J_1 \)
Example

\[ PT_{HI} = J_2 \succ J_3 \succ J_1 \]

\[ T_{j}^{LO}(t) = C_{j}(LO) \quad (1) \]

\[ T_{j}^{HI*}(t) < T_{j}^{LO}(t) \quad (2) \]

\[ T_{j}^{HI*}(t) = T_{j}^{LO}(t) \land E^{LO}(t) = J_{j} \quad (3) \]

<table>
<thead>
<tr>
<th>Job</th>
<th>STATUS</th>
<th>( T^{HI*}(t) )</th>
<th>( T^{LO}(t) )</th>
<th>C(LO)</th>
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</tbody>
</table>

\( t = 3 \quad E^{LO}(t) = J_1 \)
Example

\[ PT_{HI} = J_2 > J_3 > J_1 \]

\[
T_j^{LO}(t) = C_j(LO) \quad (1) \\
T_j^{HI*}(t) < T_j^{LO}(t) \quad (2) \\
T_j^{HI*}(t) = T_j^{LO}(t) \land E^{LO}(t) = J_j \quad (3)
\]

<table>
<thead>
<tr>
<th>Job</th>
<th>STATUS</th>
<th>( T_{HI*}(t) )</th>
<th>( T_{LO}(t) )</th>
<th>( C(LO) )</th>
<th>( C(HI) )</th>
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\( t = 5 \) \quad \( E^{LO}(t) = \bot \)
Example

\[ PT_{HI} = J_2 \succ J_3 \succ J_1 \]

\begin{align*}
T_j^{LO}(t) &= C_j(LO) \quad (1) \\
T_j^{HI*}(t) &< T_j^{LO}(t) \quad (2) \\
T_j^{HI*}(t) &= T_j^{LO}(t) \land E^{LO}(t) = J_j \quad (3)
\end{align*}

\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
Job & STATUS & \( T^{HI*}(t) \) & \( T^{LO}(t) \) & \( C(LO) \) & \( C(HI) \) & \( (1) \) \( (2) \) \( (3) \) \\
\hline
1 & Enabled & 4 & 3 & 3 & 5 & ✓ \\
2 & Term. & 2 & 1 & 1 & 2 & & \\
3 & Enabled & 0 & 0 & 2 & 4 & ✓ \\
\hline
\end{tabular}

\( t = 6 \)

\( E^{LO}(t) = J_3 \)
Example

\[ PT_{HI} = J_2 \succ J_3 \succ J_1 \]

\[ T_{jLO}(t) = C_j(LO) \quad (1) \]
\[ T_{jHI*}(t) < T_{jLO}(t) \quad (2) \]
\[ T_{jHI*}(t) = T_{jLO}(t) \land E^{LO}(t) = J_j \quad (3) \]

\[ t = 7 \quad E^{LO}(t) = J_4 \]

<table>
<thead>
<tr>
<th>Job</th>
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<th>( T_{HI*}^{HI}(t) )</th>
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</table>
Example

\[ PT_{HI} = J_2 \succ J_3 \succ J_1 \]

\[ T_j^{LO}(t) = C_j(LO) \quad (1) \]

\[ T_j^{HI*}(t) < T_j^{LO}(t) \quad (2) \]

\[ T_j^{HI*}(t) = T_j^{LO}(t) \land E^{LO}(t) = J_j \quad (3) \]
Example

\[ PT_{HI} = J_2 \succ J_3 \succ J_1 \]

\[ T_{j_{LO}}(t) = C_j(LO) \quad (1) \]
\[ T_{j_{HI}^*}(t) < T_{j_{LO}}(t) \quad (2) \]
\[ T_{j_{HI}^*}(t) = T_{j_{LO}}(t) \land E_{LO}(t) = J_j \quad (3) \]

<table>
<thead>
<tr>
<th>Job</th>
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<th>( T_{LO}(t) )</th>
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</table>
Example

\[ PT_{HI} = J_2 \succ J_3 \succ J_1 \]

\[ T_{HI}^L(t) = C_{j}(LO) \quad (1) \]
\[ T_{HI}^H(t) < T_{LO}^L(t) \quad (2) \]
\[ T_{HI}^H(t) = T_{LO}^L(t) \wedge E_{LO}^L(t) = J_j \quad (3) \]

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{Job} & \text{STATUS} & T_{HI}^H(t) & T_{LO}^L(t) & C(LO) & C(HI) & (1) & (2) & (3) \\
\hline
1 & \text{Term.} & 5 & 3 & 3 & 5 & & & \\
2 & \text{Term.} & 2 & 1 & 1 & 2 & & & \\
3 & \text{Term.} & 4 & 1 & 2 & 4 & & & \\
\hline
\end{array}
\]

\[ t = 11 \quad E_{LO}^L(t) = \bot \]