Exercise 1
Let $f$ and $g$ be two negligible functions, then

1. $f \cdot g$ is negligible.

2. For any $k > 0$, $f^k$ is negligible.

3. For any $\lambda, \mu$ in $\mathbb{R}$, $\lambda f + \mu g$ is negligible.

Exercise 2
Prove that $X$ and $Y$ are independent if and only if for all values $x$ taken by $X$ with non-zero probability, the conditional distribution of $Y$ given the event $X = x$ is the same as the distribution of $Y$.

Exercise 3
Consider the algorithm $D2$ that outputs 1 iff the input string contains more zeros than ones. If $D2$ can be implemented in polynomial time, then prove that $X$ and $Y$ are polynomial-time-indistinguishable.

Exercise 4
Let $X := \{X_n\}_{n \in \mathbb{N}}$, $Y := \{Y_n\}_{n \in \mathbb{N}}$ and $Z := \{Z_n\}_{n \in \mathbb{N}}$ three ensembles. If $X$ and $Y$ are indistinguishable in polynomial time, $Y$ and $Z$ are indistinguishable in polynomial time then $X$ and $Z$ are indistinguishable in polynomial time.

Exercise 5
Recall that the distributions $D_0, D_1$ are said to be indistinguishable ($0 \leq \epsilon \leq 1$) if

$$|Pr[A(x_0) = 1] - Pr[A(x_1) = 1]| \leq \epsilon$$

holds for all adversaries $A$ running in time at most $t$, where the random variable $x_0$ is distributed according to $D_0$ and $x_1$ is distributed like $D_1$.

Now, let’s call the distributions $D_0, D_1$ inseparable just if

$$\frac{1}{2} - \frac{\epsilon}{2} \leq Pr[A(x) = b] \leq \frac{1}{2} + \frac{\epsilon}{2}$$
holds for all adversaries $A$ running in time at most $t$, where the random variable $b$ is a uniformly random bit and where the random variable $x$ is distributed according to $D_b$. This is a very natural notion, because it talks about our chances of guessing correctly which distribution $x$ came from, and whether we can do much better than simply flipping a coin.

Prove: $D_0, D_1$ are indistinguishable if and only if they are inseparable. (Hence the notion of inseparability is redundant.)