Programming Languages and Compiler Design

Programming Language Semantics
Compiler Design Techniques

Yassine Lakhnech & Laurent Mounier

{lakhnech,mounier}@imag.fr

http://www-verimag.imag.fr/~lakhnech

http://www-verimag.imag.fr/~mounier.

Master of Sciences in Informatics at Grenoble (MoSIG)

Grenoble Universités

(Université Joseph Fourier, Grenoble INP)

Code Optimization

Objective (of this chapter)

- give some indications on general optimization techniques:
  - data-flow analysis
  - register allocation
  - software pipelining
  - etc.
- describe the main data structures used:
  - control flow graph
  - intermediate code (e.g., 3-address code)
  - Static Single Assignment form (SSA)
  - etc.
- see some concrete examples

But not a complete panorama of the whole optimization process

(e.g.: a real compiler, for a modern processor)

Objective of the optimization phase

Improve the ***efficiency*** of the target code, while preserving the source semantics.
efficiency $\rightarrow$ several (antagonist) criteria

- execution time
- size
- memory used
- energy consumption
- etc.

$\Rightarrow$ no optimal solution, no general algorithm
$\Rightarrow$ a bunch of optimization techniques:

- inter-dependant each others
- sometimes heuristic based

Two kinds of optimizations

Independant from the target machine

"source level" or "assembly level" pgm transformations:

- dead code elimination
- constant propagation, constant folding
- code motion
- common subexpressions elimination
- etc.

Dependant from the target machine

optimize the use of the hardware resources:

- machine instruction
- memory hierarchy (registers, cache, pipeline, etc.)
- etc.

Overview

1. Introduction
2. Some optimizations independant from the target machine
3. Some optimizations dependant from the target machine

Some optimizations independant from the target machine

Main principle

Input: initial intermediate code

Output: optimized intermediate code

Several steps:

1. generation of a control flow graph (CFG)
2. analysis of the CFG
3. transformation of the CFG
4. generation of the output code

Intraprocedural 3-address code (TAC)

"high-level" assembly code:

- binary logic and arithmetic operators
- use of temporary memory location `ti`
- assignments to variables, temporary locations
- a label is assigned to each instruction
- conditional jumps `goto`

Examples:

- `l:  x := y op x`
- `l:  x := op y`
- `l:  x := y`
- `l:  goto l'`
- `l:  if x oprel y goto l'`

Basic block (BB)

A maximal instruction sequence $S = i_1.\cdots i_n$ such that:

- S execution is never "broken" by a jump
  $\Rightarrow$ no `goto` instruction in $i_1.\cdots i_{n-1}$
- S execution cannot start somewhere in the middle
  $\Rightarrow$ no `label` in $i_2.\cdots i_n$

$\Rightarrow$ execution of a basic bloc is atomic

Partition of a 3-address code BBs:

1. computation of Basic Block heads:
   1st inst., inst. target of a jump, inst. following a jump
2. computation of Basic Block tails:
   last inst, inst. before a Basic Block head

$\Rightarrow$ a single traversal of the TAC

Control Flow Graph (CFG)

A representation of how the execution may progress inside the TAC

$\rightarrow$ a graph (V, E) such that:

$$V = \{B_i \mid B_i \text{ is a basic block}\}$$

$$E = \{(B_i, B_j) \mid$$
"last inst. of B_i is a jump to 1st inst of B_j" $\vee$
"1st inst of B_j follows last inst of B_i in the TAC"$\}$

Example

Give the Basic Blocks and CFG associated to the following TAC sequence:

```
0. x := 1
1. y := 2
2. if c goto 6
3. x := x+1
4. z := 4
5. goto 8
```

```
6. z := 5
7. if d goto 0
8. z := z+2
9. r := 1
10 y := y-1
```

Optimizations performed on the CFG

Two levels:

Local optimizations:

- computed inside each BB
- BBs are transformed independent each others

Global optimizations:

- computed on the CFG
- transformation of the CFG:
  - code motion between BBs
  - transformation of BBs
  - modification of the CFG edges

Local optimizations

- algebraic simplification, strength reduction
  $\rightarrow$ replace costly computations by less expensive ones
- copy propagation
  $\rightarrow$ suppress useless variables
  (i.e., equal to another one, or equal to a constant)
- constant folding
  $\rightarrow$ perform operations between constants
- common subexpressions
  $\rightarrow$ suppress duplicate computations
  (already computed before)
- dead code elimination $\rightarrow$ suppress useless instructions
  (which do not influence pgm execution)

Example of local optimizations

Initial code:

```
a := x ** 2
b := 3
c := x
d := c * c
e := b * 2
f := a + d
g := e * f
```

Example of local optimizations

Algebraic simplification:

```
a := x ** 2          a := x * x
b := 3               b := 3
c := x               c := x
d := c * c           d := c * c
e := b * 2           e := b << 1
f := a + d           f := a + d
g := e * f           g := e * f
```

Example of local optimizations

Copies propagation:

```
a := x * x          a := x * x
b := 3              b := 3
c := x              c := x
d := c * c          d := x * x
e := b << 1         e := 3 << 1
f := a + d          f := a + d
g := e * f          g := e * f
```

Example of local optimizations

Constant folding:

```
a := x * x          a := x * x
b := 3              b := 3
c := x              c := x
d := x * x          d := x * x
e := 3 << 1         e := 6
f := a + d          f := a + d
g := e * f          g := e * f
```

Example of local optimizations

Elimination of common subexpressions:

```
a := x * x          a := x * x
b := 3              b := 3
c := x              c := x
d := x * x          d := a
e := 6              e := 6
f := a + d          f := a + d
g := e * f          g := e * f
```

Example of local optimizations

Copies propagation:

```
a := x * x          a := x * x
b := 3              b := 3
c := x              c := x
d := a              d := a
e := 6              e := 6
f := a + d          f := a + a
g := e * f          g := 6 * f
```

Example of local optimizations

Dead code elimination (+ strength reduction):

```
a := x * x          a := x * x          a:= x * x
b := 3
c := x
d := a
e := 6
f := a + a          f := a + a          f := a << 1
g := 6 * f          g := 6 * f          g := 6 * f
```

Local optimization: a more concrete example

Inital source program: addition of matrices

```
for (i=0 ; i < 10 ; i ++)
     for (j=0 ; j < 10 ; j++)
        S[i,j] := A[i,j] + B[i,j]
```

Basic blocks:

B1: i := 0

B2: if i $>$ 10 goto B7

B3: j := 0

B4: if j $>$ 10 goto B6

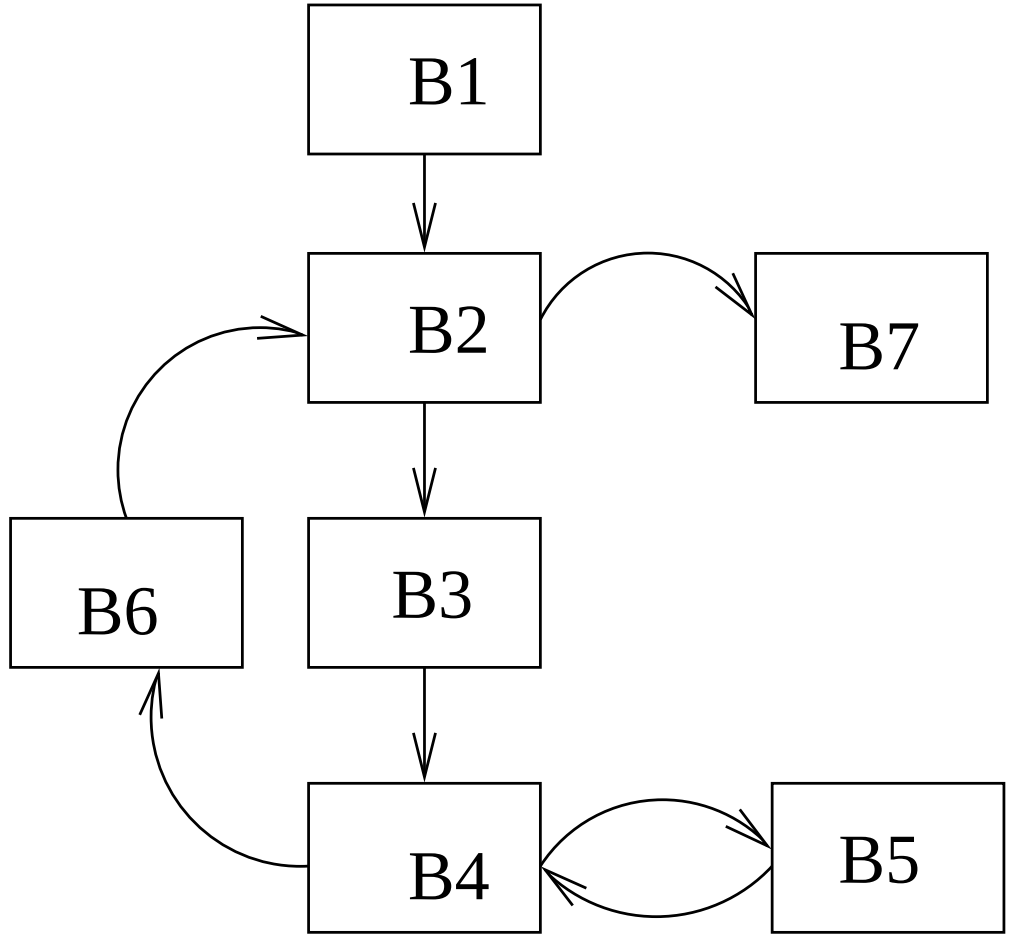
B5

B6: i := i + 1

goto B2

B7: end

Control Flow Graph



Inital Block B5

```
B5:   t1 := 4 * i
      t2 := 40 * j
      t3 := t1 + t2
      t4 := A[t3]
      t5 := 4 * i
      t6 := 40 * j
      t7 := t5 + t6
      t8 := B[t7]
      t9 := t4 + t8
      t10:= 4 * i
      t11:= 40 * j
      t12:= t10 + t11
      S[t12] := t9
      j := j + 1
      goto B4
```

Optimization of B5 (1/4)

B5:

```
t1 := 4 * i
t2 := 40 * j
t3 := t1 + t2
t4 := A[t3]
t5 := 4 * i
t6 := 40 * j
t7 := t5 + t6
t8 := B[t7]
t9 := t4 + t8
t10:= 4 * i
t11:= 40 * j
t12:= t10 + t11
S[t12] := t9
j := j + 1
goto B4
```

A same value is assigned to temporary locations t1, t5, t10

Optimization of B5 (2/4)

B5:
t1 := 4 * i
t2 := 40 * j
t3 := t1 + t2
t4 := A[t3]
t6 := 40 * j
t7 := t1 + t6
t8 := B[t7]
t9 := t4 + t8
t11:= 40 * j
t12:= t1 + t11
S[t12] := t9
j := j + 1
goto B4

A same value is assigned to temporary locations t2, t6, t11

Optimization of B5 (3/4)

B5:
t1 := 4 * i
t2 := 40 * j
t3 := t1 + t2
t4 := A[t3]
t7 := t1 + t2
t8 := B[t7]
t9 := t4 + t8
t12:= t1 + t2
S[t12] := t9
j := j + 1
goto B4

A same value is assigned to temporary locations t3, t7, t12

Optimization of B5 (4/4): the final code obtained

```
B5:   t1 := 4 * i
      t2 := 40 * j
      t3 := t1 + t2
      t4 := A[t3]
      t8 := B[t3]
      t9 := t4 + t8
      S[t3] := t9
      j := j + 1
      goto B4
```

Global optimizations

Global optimization: the principle

Typical examples of global optimizations:

- constant propagation trough several basic blocks
- elimination of global redundancies
- code motion: move invariant computations outside loops
- dead code elimination

How to “extrapolate” local optimizations to the whole CFG ?

1. associate (local) properties to entry/exit points of BBs
   (set of active variables, set of available expressions, etc.)
2. propagate them along CFG paths
   $\rightarrow$ enforce consistency w.r.t. the CFG structure
3. update each BB (and CFG edges) according to these global properties

$\Rightarrow$ a possible technique: data-flow analysis

Data-flow analysis

Static computation of data related properties of programs

- (local) properties φ_i associated to some pgm locations i
- set of data-flow equations:
  $\longrightarrow$ how φ_i are transformed along pgm execution
  Rks:
  - forward vs backward propagation (depending on φ_i)
  - cycles inside the control flow $\Rightarrow$ fix-point equations !
- a solution of this equation system:
  $\longrightarrow$ assigns "globaly consistent" values to each φ_i
  Rk: such a solution may not exist . . .
- decidability may require abstractions and/or approximations

Example: elimination of redundant computations

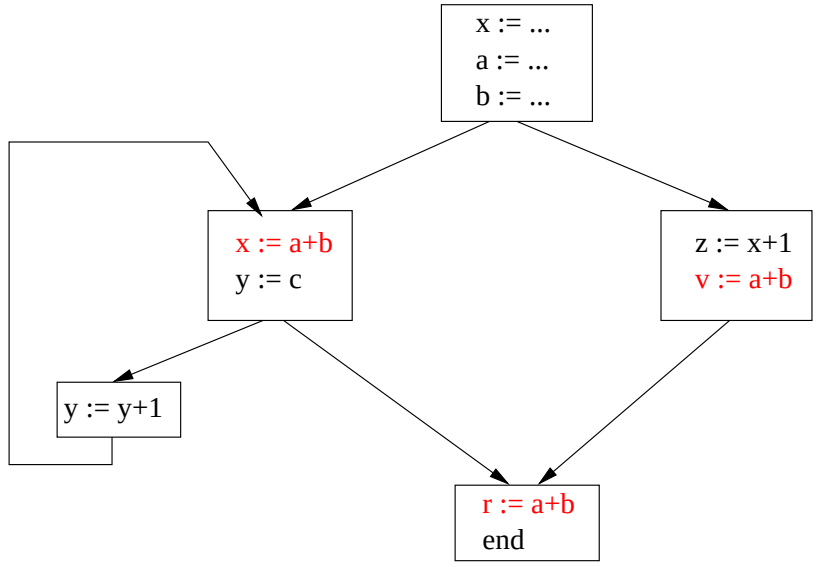
An expression e is redundant at location i iff

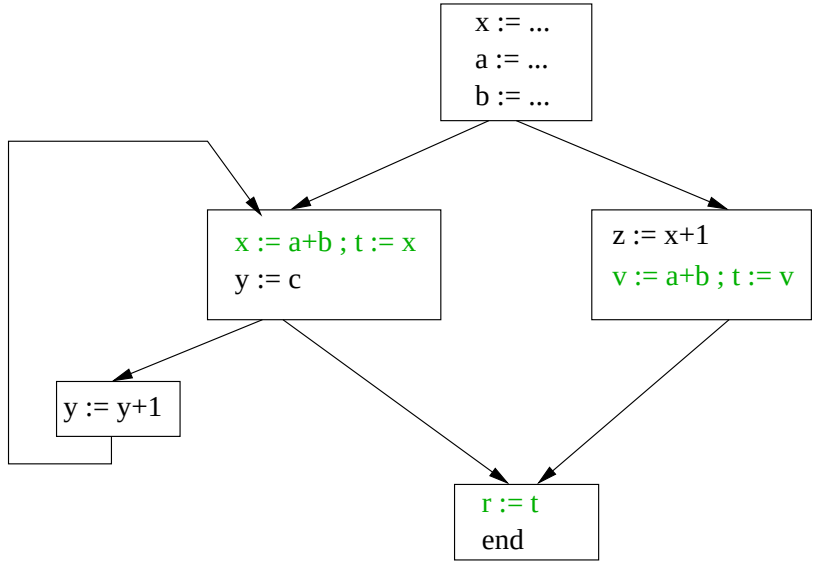
- it is computed at location i
- this expression is computed on every path going from the initial location to location i
  Rk: we consider here syntactic equality
- on each of these paths: operands of e are not modified between the last computation of e and location i

Optimization is performed as follows:

1. computation of available expressions (data-flow analysis)
2. $x := e$ is redundant at loc i if e is available at i
3. $x := e$ is replaced by $x := t$
   (where t is a temp. memory containing the value of e)

Elimination of redundant computation: an example





Data-flow equations for available expressions (1/2)

For a basic block b, we note:

- $In(b)$: available expressions when entering b
- $Kill(b)$: expressions made non available by b (because an operand of e is modified by b)
- $Gen(b)$: expressions made available by block b (computed in b, operands not modified afterwards)
- $Out(b)$: available expressions when exiting b

$$Out(b) = (In(b) \setminus Kill(b)) \cup Gen(b) = F_b(In(b))$$

F_b = transfer function of block b

Data-flow equations for available expressions (2/2)

How to compute $In(b)$?

- if b is the initial block:

$$In(b) = \emptyset$$

- if b is not the initial block:
  An expression e is available at its entry point iff it is available at the exit point of each predecessor of b in the CFG

$$In(b) = \bigcap_{b' \in Pre(b)} Out(b')$$

$\Rightarrow$ forward data-flow analysis along the CFG paths

Q: cycles inside the CFG $\Rightarrow$ fix-points computations
greatest vd least solutions ?

Solving the data-flow equations (1/2)

Let $(E, \leq)$ a partial order.

- For $X \subseteq E, a \in E$:
  - a is an upper bound of X if $\forall x \in X.\ x \leq a$
  - a is a lower bound of X if $\forall x \in X.\ a \leq x$
- The least upper bound (*lub*, $\sqcup$) is the smallest upper bound
- The great lower bound (*glb*, $\sqcap$) is the largest lower bound
- $(E, \leq)$ is a lattice if every subset of E admits a *lub* and a *glb*.
- A function $f : 2^E \rightarrow 2^E$ is monotonic if:

$$\forall X, Y \subseteq E \quad X \leq Y \ \Rightarrow\ f(X) \leq f(Y)$$

- $X = \{x_0, x_1, \ldots x_n, \ldots\} \subseteq E$ is an (increasing) chain if $x_0 \leq x_1 \leq \ldots x_n \leq \ldots$
- A function $f : 2^E \rightarrow 2^E$ is ($\sqcup$-)continuous if $\forall$ increasing chain X, $f(\sqcup X) = \sqcup f(X)$

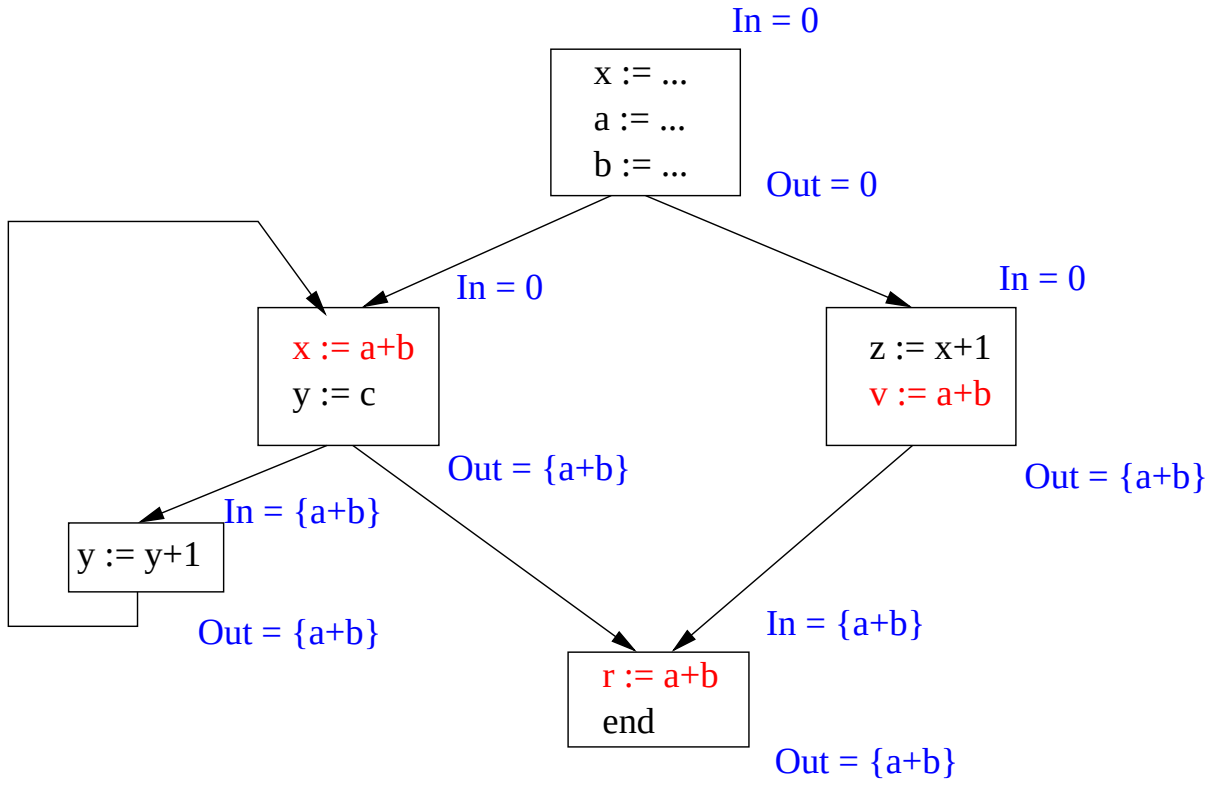
Solving the data-flow equations (2/2)

Fix-point equation: solution ?

- properties are finite sets of expressions $\mathcal{E}$
- $(2^{\mathcal{E}}, \subseteq)$ is a complete lattice
  $\bot$: least element, $\top$: greatest element
  $\sqcap$: greatest lower bound, $\sqcup$: least upper bound
- data-flow equations are defined on monotonic and continuous operators ($\cup$, $\cap$) on $(2^{\mathcal{E}}, \subseteq)$
- Kleene and Tarski theorems:
  - the set of solution is a complete lattice
  - the greatest (resp. least) solution can be obtained by successive iterations w.r.t. the greatest (resp. least) element of $2^{\mathcal{E}}$

$$\text{lfp}(f) = \sqcup\{f^i(\bot) | i \in \mathbb{N}\} \qquad \text{gfp}(f) = \sqcap\{f^i(\top) | i \in \mathbb{N}\}$$

Back to the example



Generalization

- Data-flow properties are expressed as finite sets associated to entry/exit points of basic blocs: `In`(b), `Out`(b)

- For a forward analysis:
  - property is "false" ($\bot$) at entry of initial block
  - $\texttt{Out}(b) = F_b(\texttt{In}(b))$
  - `In`(b) depends on `Out`(b'), where $b' \in Pred(b)$ ($\sqcap$ for "$\forall$ paths", $\sqcup$ for "$\exists$ path")

- For a backward analysis:
  - property is "false" ($\bot$) at exit of final block
  - $\texttt{In}(b) = F_b(\texttt{Out}(b))$
  - `Out`(b) depends on `In`(b'), where $b' \in Succ(b)$

Data-flow equations: forward analysis

Forward analysis, least fix-point	$\texttt{In}(b) = \begin{cases} \bot & \text{if b is initial} \\ \bigsqcup_{b' \in Pre(b)} \texttt{Out}(b') & \text{otherwise.} \end{cases}$ $\texttt{Out}(b) = F_b(\texttt{In}(b))$
Forward analysis, greatest fix-point	$\texttt{In}(b) = \begin{cases} \bot & \text{if } b \text{ is initial} \\ \bigsqcap_{b' \in Pre(b)} \texttt{Out}(b') & otherwise. \end{cases}$ $\texttt{Out}(b) = F_b(\texttt{In}(b))$

Data-flow equations: backward analysis

Backward analysis, least fix-point	$\texttt{Out}(b) = \begin{cases} \bot & \text{if } b \text{ is final} \\ \bigsqcup_{b' \in Succ(b')} \texttt{In}(b') & otherwise. \end{cases}$ $\texttt{In}(b) = F_b(\texttt{Out}(b))$
Backward analysis, greatest fix-point	$\texttt{Out}(b) = \begin{cases} \bot & \text{if } b \text{ is final} \\ \bigsqcap_{b' \in Succ(b)} \texttt{In}(b') & otherwise. \end{cases}$ $\texttt{In}(b) = F_b(\texttt{Out}(b))$

Active Variable

- A variable `x` is inactive at location `i` if it is not used in every CFG-path going from `i` to `j`, where `j` is:
  - either a final instruction
  - or an assignement to `x`.

- An instruction `x := e` at location `i` is useless if `x` is inactive at location `i`.

$\Rightarrow$ useless instuctions can be removed . . .

Rk: used means

“in a right-hand side assignment or in a branch condition”.

Data-flow analysis for inactive variables

We compute the set of active variables . . .

Local analysis

$\texttt{Gen}(b)$ is the set of variables x s.t. x is used in block b, and, in this block, any assignement to x happens after the (first) use of x.

$\texttt{Kill}(i)$ is the set of variables x assigned in block b.

Global analysis : backward analysis, $\exists$ a CFG-path (least solution)

- $$\begin{aligned} \texttt{Out}(b) &= \bigcup_{b' \in Succ(b)} \texttt{In}(b') \\ \texttt{In}(b) &= (\texttt{Out}(b) \setminus \texttt{Kill}(b)) \cup \texttt{Gen}(b) \end{aligned}$$
- $\texttt{Out}(b) = \emptyset$ if b is final.

Computation of functions Gen and $Kill$

Recursively defined on the syntax of a basic bloc B:

$\texttt{B} ::= \varepsilon \mid \texttt{B ; x := a} \mid \texttt{B ; if b goto l} \mid \texttt{B ; goto l}$

$Gen(B)$	$=$	$Gen_l(\texttt{B}, \emptyset)$
$Kill(B)$	$=$	$Kill_l(\texttt{B}, \emptyset)$
$Gen_l(\texttt{B ; x := a}, X)$	$=$	$Gen_l(\texttt{B}, X \setminus \{\texttt{x}\} \cup \mathsf{Used}(\texttt{a}))$
$Gen_l(\texttt{B ; if b goto l}, X)$	$=$	$Gen_l(\texttt{B}, X \cup \mathsf{Used}(\texttt{b}))$
$Gen_l(\texttt{B ; goto l}, X)$	$=$	$Gen_l(\texttt{B}, X)$
$Gen_l(\varepsilon, X)$	$=$	X
$Kill_l(\texttt{B ; x := a}, X)$	$=$	$Kill_l(\texttt{B}, X \cup \{\texttt{x}\})$
$Kill_l(\texttt{B ; if b goto l}, X)$	$=$	$Kill_l(\texttt{B}, X)$
$Kill_l(\texttt{B ; goto l}, X)$	$=$	$Kill_l(\texttt{B}, X)$
$Kill_l(\varepsilon, X)$	$=$	X

Used(e): set of variables appearing in expression e

Removal of useless instructions

1. Compute the sets $In(B)$ and $Out(B)$ of active variables at entry and exit points of each blocks.
2. Let $F : Code \times 2^{Var} \rightarrow Code$
   $F(b, X)$ is the code obtained when removing useless assignments inside b, assuming that variables of X are active at the end of b execution.

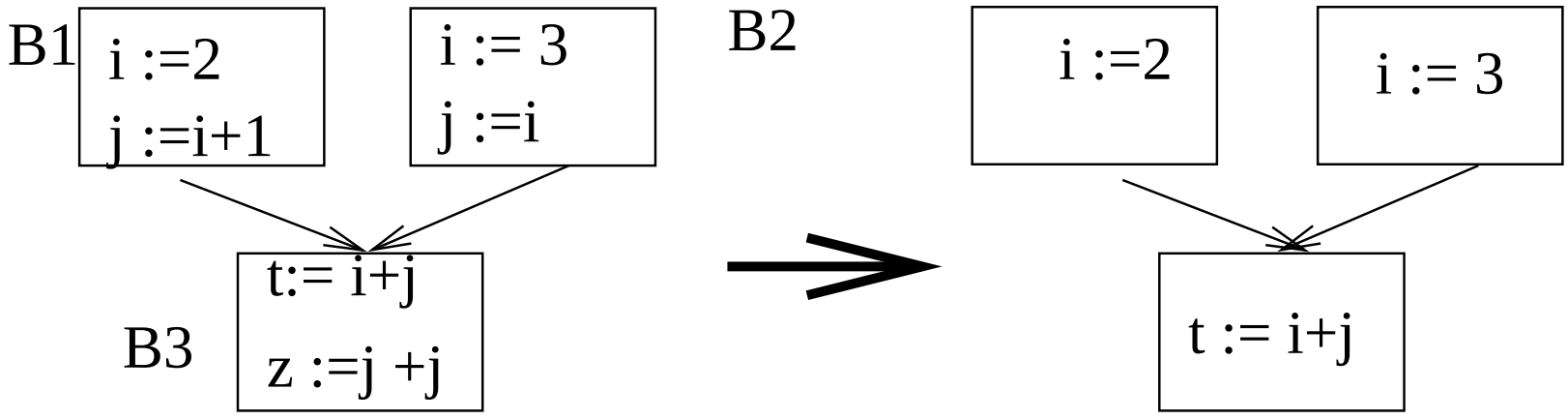
$$
\begin{array}{lcl}
F(\texttt{B ; x := a}, X) & = & \begin{cases} F(B, X) & \text{if } x \notin X \\ F(B, (X \setminus \{x\}) \cup \texttt{Used}(a)); x := a & \text{if } x \in X \end{cases} \\
F(\texttt{ B ; if b goto l}, X) & = & F(B, X \cup \texttt{Used}(b)); \texttt{if b goto l} \\
F(\texttt{B ; goto l}, X) & = & F(B, X); \texttt{goto l} \\
F(\epsilon, X) & = & \epsilon
\end{array}
$$

3. Replace each block B by $F(B, Out(B))$.

Rk: this transformation may produce new inactive variables . . .

Constant propagation

Example:



- A variable is constant at location `l` if its value at this location can be computed at compilation time.
- At exit point of B1 and B2, `i` and `j` are constants
- At entry point of B3, `i` is not constant, `j` is constant.

Constant propagation: the lattice

- Each variable takes its value in $D = \mathbb{N} \cup \{\top, \bot\}$, where:
  - $\top$ means “non constant value”
  - $\bot$ means “no information”

- Partial order relation $\leq$:
  if $v \in D$ then $\bot \leq v$ and $v \leq \top$.

- The least upper bound $\sqcup$:
  for $x \in D$ and $v_1, v_2 \in \mathbb{N}$

$x \sqcup \top = \top$	$x \sqcup \bot = x$	$v_1 \sqcup v_2 = \top$ if $v_1 \neq v_2$	$v_1 \sqcup v_1 = v_1$

Rk: relations $\leq$ is extended to functions $Var \rightarrow D$

$f1 \leq f2$ iff $\forall x. f1(x) \leq f2(x)$

Constant propagation: data-flow equations

- property at location `l` is a function $Var \to D$.
- Forward analysis:

$$
\begin{array}{lcl}
In(b) & = & \begin{cases} \lambda x.\bot & \text{if } b \text{ is initial,} \\ \bigsqcup_{b' \in Pred(b)} Out(b') & \text{otherwise} \end{cases} \\
Out(b) & = & F_b(In(b))
\end{array}
$$

Transfer function F_b ?

a basic block = sequence of assignements

b ::= ϵ | x:=e ; b

F_b defined by syntactic induction:

$$
\begin{array}{lcll}
F_{\mathsf{x:=e\ ;\ b}}(f) & = & F_{\mathsf{b}}(f[x \mapsto f(e)]) & \text{(assuming variable initialization)} \\
F_\epsilon(f) & = & f &
\end{array}
$$

Pgm transformation:

$\forall$ block b, $f \in In(b), f(\mathsf{e}) = v \Rightarrow$ x:=e replaced by x:=v

Exercise

Constant propagation can be viewed as abstraction of the standard semantics where expressions values are interpreted other domain D

1. Write this abstract semantics for the `while` language in an operational style (relation $\longrightarrow_{\#}$)
2. Define a program transformation which removes useless computations (i.e., computations between constant operands)
3. Give the equations which express the correctness of this transformation

Another example of data-flow analysis

A computation of an expression e can be anticipated at loc. p iff:

- all paths from p contains a location p_i s.t. e is computed at p_i
- e operands are not modified between p and p_i

Example:

```
if (x>0)
      x = i + j;
else
      repeat y = (i + j) * 2; x := x+1 ; until x>10
```

can be changed to

```
tmp = i + j;
if (x>0)
      x = tmp;
else
      repeat y = tmp * 2; x := x+ 1 ; until x>10
```

Application: moving invariants outside loops

Interprocedural analysis

```
main()
{
  int i,j ;
  void f(){
    int x,y ;
    y = i+j ; x = y ;
  }
  i = 0 ;
  f() ;
  j = 1;
}
```

- a dedicated basic block B_{call} for the `call` instruction
- $In(B_{call}) = In(B_{f_{in}}), Out(B_{call}) = Out(B_{f_{out}})$

Rks:

- static binding is be assumed
- parameters ?

Exercice: Computation of active variables

Control-flow analysis

$\longrightarrow$ retrieve program control structures from the CFG ?
Application: loop identification

$\Rightarrow$ use of graph-theoretic notions:

- dominator, dominance relation
- strongly connected components

Rk1: most loops are easier to identify at syntactic level, but:

- use of `goto` instruction still allowed in high-level languages
- optimization performed on intermediate representations (e.g., CFG)

Rk2: other approaches can be used to identify loops . . .

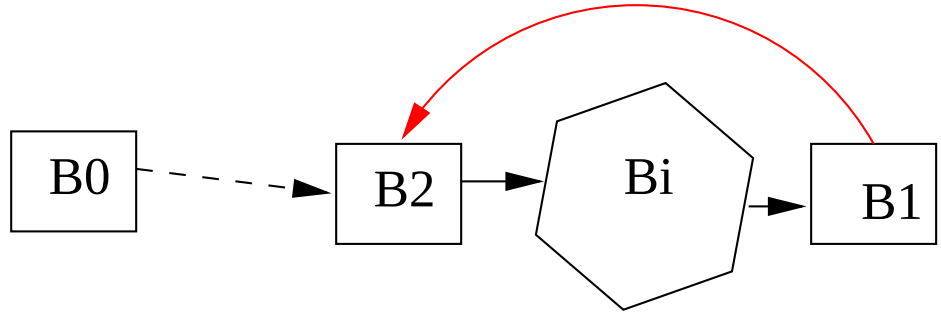
Loop identification

Node B_1 is a dominator of B_2 ($B_2 \leq B_1$) iff every path from the entry block to B_2 goes through B_1. $Dom(B) = \{B_i | B_i \leq B\}$.

An edge (B_1, B_2) is a loop back edge iff $B_2 \leq B_1$

To find “natural loops”:

1. find a back edge (B_1, B_2)
2. find $Dom(B_2)$
3. find blocks $B_i \in Dom(B_2)$ s.t. there is a path from B_i to B_2 not containing B_1.



Some machine level optimization techniques

Register Allocation

Pb:

- expression operands are much efficiently accessed when liying in registers (instead of RAM)
- the “real” number of registers is finite (and usually small)

$\Rightarrow$ register allocation techniques:

- assigns a register to each operand (variable, temporary location)
- performs the memory exchange (`LD`, `ST`) when necessary
- optimality ?

Several existing techniques:

- optimal code generation for arithmetic expressions
- graph-coloring techniques (more general case)
- etc.

Code generation for arithmetic expressions: example

code generation for `(a+b) - (c - (d+e))`
with 2 registers, and instruction format = `OP Ri, Ri, X` (where `X=Ri` or `X=M[x]`)

Solution 1: one register needs to be saved

```
LD R0, M[a]
ADD R0, R0, M[b]
LD R1, M[d]
ADD R1, R1, M[e]
ST R1, M[t1]          ! register R1 needs to be saved ...
LD R1, M[c]
SUB R1, R1, M[t1]
SUB R0, R0, R1
```

Solution 2: no register to save

```
LD R0, M[c]
LD R1, M[d]
ADD R1, R1, M[e]
SUB R0, R0, R1
LD R1, M[a]
ADD, R1, R1, M[b]
SUB, R1, R1, R0
```

Code generation for arithmetic expressions: principle

Evaluation of `e1 op e2` , assuming:

- r registers are available, evaluation of `ei` requires r_i registers
- intsruction format is “op reg, reg, ad” where “ad” is a register or a memory location

Several cases:

- $\boxed{r_1 > r_2:}$
  - after evaluation of `e1`, $r_1 - 1$ registers available
  - $r_1 - 1 \geq r_2 \Rightarrow r_1 - 1$ registers are enough for `e2`
  - $\Rightarrow \boxed{r_1 - r}$ register allocations are required
- $\boxed{r_1 = r_2:}$
  - after evaluation of `e1`, $r_1 - 1$ registers available
  - $r_1 - 1 < r_2, \Rightarrow r_2$ (=r_1) registers required for `e2`
  - $\Rightarrow \boxed{r_1 + 1 - r}$ register allocations are required
- $\boxed{r_1 < r_2:}$
  - after evaluation of `e1`, $r_1 - 1$ registers available
  - $r_1 - 1 < r_2, \Rightarrow r_2$ ($> r_1$) registers required for `e2`
  - $\Rightarrow r_2 + 1 - r$ register allocations are required
  - $\boxed{r_2 - r \text{ allocations are enough if e2 is evaluated first !}}$

A two-phase algorithm

Step 1: each AST node is labeled with the number of registers required for its evaluation

rNb : $Aexp \rightarrow \mathbb{N}$ (rNb(`e`) is the number of registers required to evaluate e)

$$\text{rNb}(\texttt{e}) = \begin{cases} 1 & \text{if } e \quad \text{is a left leaf} \\ 0 & \text{if } e \quad \text{is a right leaf} \end{cases}$$

$$\text{rNb}(\texttt{e1 op e2}) = \begin{cases} max(\text{rNb}(\texttt{e}_1), \text{rNb}(\texttt{e}_2)) & \text{if rNb}(\texttt{e}_1) \neq \text{rNb}(\texttt{e}_2) \\ \text{rNb}(\texttt{e}_1) + 1 & \text{if rNb}(\texttt{e}_1) = \text{rNb}(\texttt{e}_2) \end{cases}$$

Step 2: "optimal" code generation using these labels (exercice)

$\rightarrow$ for a binary node `e1 op e2`:

- evaluate the more register demanding sub-expression first
- write the result in a register Ri (save one if necessary)
- evaluate the other sub-expression, write the result in a register Rj
- generate `OP, Ri, Ri, Rj`

A more general technique

1. Intermediate code is generated assuming ∞ numbers of "symbolic" registers S_i
2. Assign a real register R_i to each symbolic register s.t.
   - if R_i is assigned to S_i, R_j is assigned to S_j
   - then Lifetime$(S_i) \cap$ lifetime$(S_j) \neq \emptyset \Rightarrow R_i \neq R_j$

   where Lifetime(S_i): sequences of pgm location where S_i is active

How to ensure this condition ?

Collision graph G_C:

- Nodes denote lifetime symbolic registers: $N_i = (S_i, \text{Lifetime}(S_i))$
- Edges are the set $\{((S_1, L_1), (S_2, L_2) \mid L_1$ and L_2 overlap$\}$

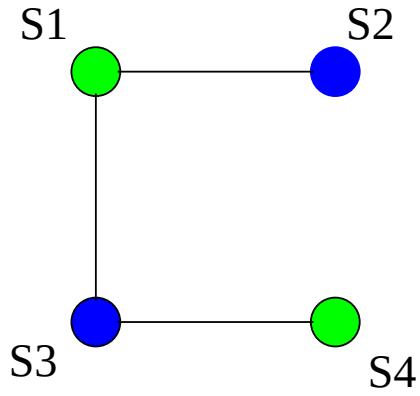
$\Rightarrow$ register allocation with k real register = k-coloring problem of G_C

(i.e., assign a distinct *colour* to each pair of adjacent nodes)

Example 1

```
S1 := e1
S2 := e2
...
... S2 ...                  S2 used
S3 := S1+S2                 S1 and S2 used
...
S4 := S1*5                  S1 used
... S4 ...                  S4 used
... S3 ...                  S3 used
```

Collision Graph:



Can be colored with 2 colors $\Rightarrow$ 2 real registers are enough

k-coloring in practice ? (1)

When $k > 2$, this problem is NP-complete ...

An efficient heuristic:

Repeat:

if exists a node N of G_C such that degree$(N) < k$

(N can receive a distinct colour from all its neighbours)

remove N (and corresponding edges) from G_C and push it on a stack S

else (G_C is assumed to be non k-colourable)

choose a node N **(1)**

remove N from G_C **(2)**

until G_C is empty

While S is not empty

pop a node from S

add it to G, give it a colour not used by one of its neighbours

Rk: this algo may sometimes miss k-colorable graphs ...

k-coloring in practice ? (2)

What happens when there is no node of degree < k ?

(1) choose a node N to remove:
$\rightarrow$ high degree in G_C, not corresponding to an inner loop, etc.
(2) remove node N:
$\rightarrow$ save a register into memory before (register spilling)

Several attempts to improve this algorithm:

node coalescing:

`S1 := S2`, Lifetime$(S1) \cap$ Lifetime$(S2) = \emptyset$
$\Rightarrow$ nodes associated to `S1` and `S2` could be merged
pb: it increases the graph degree ...

lifetime splitting:

long lifetime increases the graph degree
$\Rightarrow$ split it into several parts ...
pb: where to split ?

Instruction scheduling

Motivation: exploit the instruction parallelism provided in many target architectures (e.g., VLIW processors, instruction pipeline, etc.)

Pbs:

- possible data dependancies between consecutive instructions (e.g., `x := 3 ; y := x+1`)
- possible resource conflicts between consecutive instructions (ALU, co-processors, bus, etc.)
- consecutive instructions may require various execution cycles
- etc.

$\Rightarrow$ Main technique: change the initial instruction sequence (instruction scheduling)

- preserve the initial pgm semantics
- better exploit the hardware resources

Rks: “loop unrolling” and “expression tree reduction” may help . . .

Dependency Graph

Data dependencies:

$\longrightarrow$ execution order of 2 instructions should be preserved in the following situation:

Read After Write (RAW) : inst. 2 read a data written by inst. 1

Write After Read (WAR) : inst. 2 write a data read by inst. 1

Write After Write (WAW) : inst. 2 write a data written by inst. 1

Dependency graph G_D

- nodes = { instructions }
- edges = $\{(i_1, d, i_2) \mid$ there is a dependency d from i_1 to $i_2\}$

Rk: if we consider a basic block, G_D is a directed acyclic graph.

Any topological sort of G_D leads to a valid result (w.r.t. pgm semantics).
This sort can be influenced by several factors:

- the resources used by the instruction ($\exists$ a static reservation table)
- the number of cycles it requires (latency)
- etc.

Example

1. Draw the dependency graph G_D associated to the following program
2. Give a topological sort of G_D
3. Rewrite this program with a “maximal” parallelism

```
1. a := x+1
2. x := 2+y
3. y := z+1
4. t := a*b
5. v := a*c
6. v := 3+t
```

Software pipelining (overview . . .)

Idea: exploit the parallelism between instrutions of distinct loop iterations

```
for k in 1 .. N loop
   r := T[k] ;          - inst. A
   x := x + r ;         - inst. B
   T[k] := x ;          - inst. C
end loop
```

Assumptions: 3 cycles per instruction, 1 cycle delay when no dependencies

- Initial exec. sequence: A(1), B(1), C(1), A(2), B(2), C(2), . . . A(k), B(k), C(k)
  $\Rightarrow$ 7 cycles / iteration
- "Pipelined exec. sequence": A(1), A(2), A(3), B(1), B(2), B(3), C(1), C(2), C(3), . . .
  $\Rightarrow$ 3 cycles / iteration !

(real life) pbs:

- `N` not always divisible by the number of instruction in the loop body
  `for k in 1 to N-2 step 3 loop A(k) ; A(k+1) ; A(k+2) ...`
- high latency instruction in the loop body
- possible overhead when k is not "large enough"
- `...`