How to obtain and prove invariants

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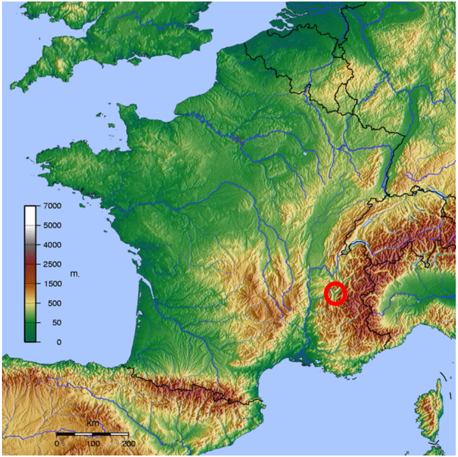
Project STATOR, VERIMAG, Grenoble

http://stator.imag.fr/



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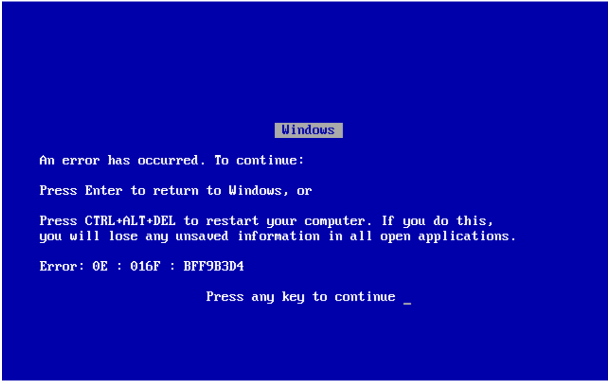
In short

Prove that a **program** does not end in the **wrong place**.



Systems we consider

Software considered as a discrete-time transition system.

Nondeterminism:

External components inputs, operating system...

Due to modeling libraries that we do not want to consider in detail (e.g. $\sin(x) \in [-1, 1]$)...

Safety properties

In this talk: properties of the form
"$\forall$ execution, $\forall$ state along execution, property P holds"

Otherwise said: "the system cannot reach $\neg P$"

Obvious use: $\neg P$ is the **error states**

- runtime errors (division by zero, null pointer, array access out of bounds, invalid cast...)
- assertion violations

Safety properties on states for input/output specifications

Often, specifications "$\forall x \in I\ f(x) = f_{\text{spec}}(x)$" (deterministic function)
Or "$\forall x \in I\ r(x, f(x))$" (nondeterministic function)

Solution Keep a copy of input state x in the current state (x, l, y) where l program location and y current program variables
$P(x, l, y)$ is $l = \text{final} \Rightarrow r(x, y)$

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Safety properties for termination

1. Choose an integer ranking function ρ
2. Prove that it decreases along all executions
3. Prove that it remains nonnegative

3 is directly a safety property

2 also is (record previous state along with current state)

First idea: direct proof by induction

Prove

1. for all initial state σ, $P(\sigma)$
2. **(induction step)** for all consecutive states $\sigma \rightarrow \sigma'$, $P(\sigma) \Rightarrow P(\sigma')$

Weakness?

A mathematical example

- initialization $x := 0$
- step $x \to x^2$

Prove $x < 100$.
Not inductive.
Yet $x = 0$ always! Property obviously always true!

Note: $x = 0 \Rightarrow x < 100$.
Strenghtened property is inductive.

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A loopy example

```
for(int i=0; i!=100; i++) { }
```

Loop body:

- initialization $i := 0$
- step $i \to i + 1$ if $i \neq 100$

Prove $i < 200$. Inductive?

Not inductive. Yet **stronger** $i \leq 100$ inductive!

A loopy example

```
for(int i=0; i!=100; i++) { }
```

Loop body:

- initialization $i := 0$
- step $i \to i + 1$ if $i \neq 100$

Prove $i < 200$. Inductive?

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Vocabulary: invariants

For some authors:

- **Invariant:** property that always holds on all traces
- **Inductive invariant:** invariant that can be proved correct by a proof of induction

For other authors or e.g. Floyd-Hoare proofs: "invariant" means "inductive invariant".

Strenghtening: executive summary

The property to prove is almost never inductive.

Replace it by a **stronger, inductive property**.
Invariant strengthening

(Basis of Floyd-Hoare rule for loops.)

Relative completeness

If a property is always true, does there necessarily exist a stronger, inductive property?

Yes, always.

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The set of reachable states

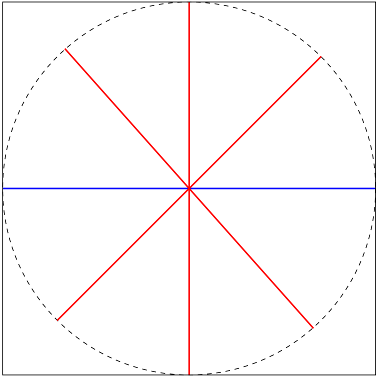
The set R of reachable states is the **least inductive invariant**:

- contains the initial states I
- stable by $\rightarrow$

Equal to $\{y \mid x \rightarrow^* y, x \in I\}$

P always holds on all traces iff $R \Rightarrow P$
(identify sets and properties)

Example: (non)inductive invariants



Initialize: $x \in [-1, 1]$, $y = 0$.
Step: rotate by 45°

Reachable states = star
Non inductive invariant: $[-1, 1] \times [-1, 1]$
Inductive invariants: octagon or disc

As a least fixed point

Initialize $S_0 = \emptyset$

Iterate $S_{k+1} \mapsto \text{step}(S_k) \cup \text{initial states}$

Ascending sequence, its limit is the set of reachable states

If finite state space (size n), converges in n
If infinite state space... quite useless!

Decidability

Assume initial state + transition relation in linear arithmetic (e.g. counter machine)

Is membership in the set of reachable states decidable?

No, since it allows deciding the **halting problem** over deterministic counter machines (test reachability of final state).

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Halting

In Peano arithmetic

(Cook's completeness theorem)

If

- memory state is in $\mathbb{Z}^n$
- initial states: first-order formula over n variables
- transition relation: first-order formula over $n + n$ variables

Then the set of reachable states is defined in Peano arithmetic (Gödel encoding).

(But Peano arithmetic is undecidable! Can't even show that the invariant implies the property!)

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Undecidability of arithmetic

Reversing the arrows

So far: compute inductive invariant, show no bad states inside (**forward**)

Alternative: compute inductive invariant on **reverse** transition system, starting from bad states; show no initial state in inside (**backward**)

Combinations

Our weapons

Abstraction Search for invariant in a restricted class

Extrapolation Find inductive invariant in a finite number of steps

Refinement If invariant too coarse, refine it

Tricks Cheap and not-so cheap improvements

Our goals

Depending on usage:

Guided by a property Prove a given property P

Unguided Provide "strong" invariants (best-effort)

A simple idea

Replace arbitrary sets of states (or traces etc.) by **symbolically** represented sets and **compute** on them.

Intervals

State space $= \mathbb{Z}^d$ (d integer variables)
Compute one interval per variable per program point

```
int x, y, z; //x ∈ [−7, 5]
if (x >= 0) { //x ∈ [0, 5]
  y = x; //y ∈ [0, 5]
} else { //x ∈ [−7, −1]
  y = -x; //y ∈ [1, 7]
} //y ∈ [0, 7]
z = 2*y + x; //z ∈ [−7, 19]
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Lack of relationality

```
int x, y, z; //x ∈ [0,1]
y = x; //y ∈ [0,1]
z = x - y; //z ∈ [−1,1]
```

No relation tracked between x and y thus z **over-approximated**.

Recovering relationality

E.g. SSA form and symbolic expressions.

```
int x, y, z; //x ∈ [0,1]
y = x; //y ∈ [0,1]
z = x - x;
```

Recovering relationality

Simplification step (see Miné)

```
int x, y, z; //x ∈ [0,1]
y = x; //y ∈ [0,1]
z = 0; //z ∈ [0,0]
```

Code motion

```
int x, y, z; //x ∈ [−7, 5]
if (x >= 0) { //x ∈ [0, 5]
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} else { //x ∈ [−7, −1]
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(Recall: without motion, $z \in [-7, 19]$)

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Code motion and symbolic expressions

```
int x, y, z;
if (x >= 0)
  y = x;
  z = 2*x + x;
} else {
  y = -x;
  z = 2*(-x) + x;
}
```

Code motion and symbolic expressions

```
int x, y, z; //x ∈ [−7, 5]
if (x >= 0) { //x ∈ [0, 5]
  y = x; //y ∈ [0, 5]
  z = 3*x; //z ∈ [0, 15]
} else { //x ∈ [−7, −1]
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Implementation note: exploit sparsity

```
/* 10000 variables initialized */
if (x > 0) {
  y = 1;
}
```

The merge operation at the end of if-then-else should **not** perform 9999 useless interval merges.

Intervals: summary

- $+$ Easy to implement
- $+$ Low complexity
- $+$ Handles $+$, $-$, $\times$, $/$, $\sqrt{}$...
- \- No relations
- \- Enforces convexity
- $=$ Some relations may be recovered by symbolic rewriting

Forward analysis can be insufficient

```
double x, z;
if (abs(x) > 0.1) {
  assert(z != 0);
  z = 1.0 / x;
  ...
}
```

```
double x, y, z;
if (x >= 0) { //x ≥ 0
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Forward, then backward

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}
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double x, y, z; //x = 0
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  y = x; //y ≥ 0 ∧ x = 0
} else { //
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double x, y, z; //x = 0
if (x >= 0) { //x = 0
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} else { //⊥
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if (y > 0.1) { //y > 0.1 ∧ x = 0
  if (x == 0) {
    fail(); //   x = 0
  }
  z = 1.0 / x;
}
```

```
double x, y, z; //x = 0
if (x >= 0) { //x = 0
  y = x; // y = 0
} else { // ⊥
  y = -x; // ⊥
} // y = 0
if (y > 0.1) {  // ⊥
  if (x == 0) {  // ⊥
    fail();  // ⊥
  }
  z = 1.0 / x;
}
```

Forward, then backward, then forward

```
double x, y, z; //x = 0
if (x >= 0) { //x ≥ 0
  y = x; //y ≥ 0 ∧ x = 0
} else { //x < 0
  y = -x; //y > 0 ∧ x = 0
}
if (y > 0.1) { //y > 0.1 ∧ x = 0
  if (x == 0) {
    fail(); //  x = 0
  }
  z = 1.0 / x;
}
```

```
double x, y, z; //x = 0
if (x >= 0) { //x = 0
  y = x; // y = 0
} else { // ⊥
  y = -x; // ⊥
} // y = 0
if (y > 0.1) {  // ⊥
  if (x == 0) {  // ⊥
    fail();  // ⊥
  }
  z = 1.0 / x;
}
```

Forward, then backward, then forward

```
double x, y, z; //x = 0
if (x >= 0) { //x = 0
  y = x; //y ≥ 0 ∧ x = 0
} else { //⊥
  y = -x; //y > 0 ∧ x = 0
}
if (y > 0.1) { //y > 0.1 ∧ x = 0
  if (x == 0) {
    fail(); //  x = 0
  }
  z = 1.0 / x;
}
```

```
double x, y, z; //x = 0
if (x >= 0) { //x = 0
  y = x; // y = 0
} else { // ⊥
  y = -x; // ⊥
} // y = 0
if (y > 0.1) {  // ⊥
  if (x == 0) {  // ⊥
    fail();  // ⊥
  }
  z = 1.0 / x;
}
```

Forward, then backward, then forward

```
double x, y, z; //x = 0
if (x >= 0) { //x = 0
  y = x; //y ≥ 0 ∧ x = 0
} else { //⊥
  y = -x; //y > 0 ∧ x = 0
}
if (y > 0.1) { //y > 0.1 ∧ x = 0
  if (x == 0) {
    fail(); //  x = 0
  }
  z = 1.0 / x;
}
```

```
double x, y, z; //x = 0
if (x >= 0) { //x = 0
  y = x; // y = 0
} else { // ⊥
  y = -x; // ⊥
} // y = 0
if (y > 0.1) {  // ⊥
  if (x == 0) {  // ⊥
    fail();  // ⊥
  }
  z = 1.0 / x;
}
```

Forward, backward, forward with Interproc

http://pop-art.inrialpes.fr/interproc/concurinterprocweb.cgi

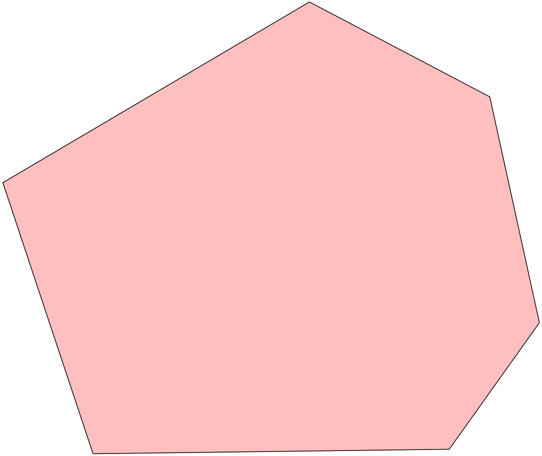
```
var x,y,z: real;

begin
  if x >= 0 then
    y = x;
  else
    y = -x;
  endif;
  if 10 * y >= 1 then
    if x == 0 then
      fail;
    endif;
  endif;
end
```

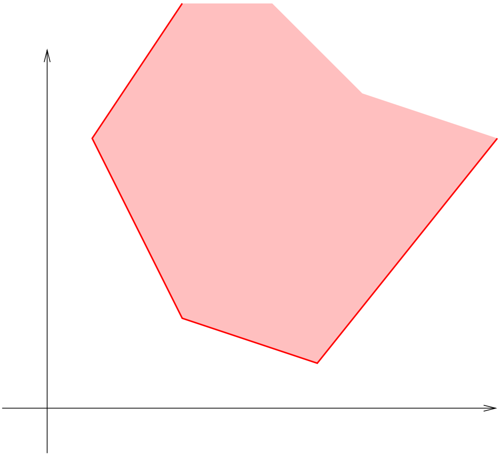
Forward: gets to “fail”

Forward — backward — forward: “fail” unreachable

Convex polyhedra



Unbounded convex polyhedra



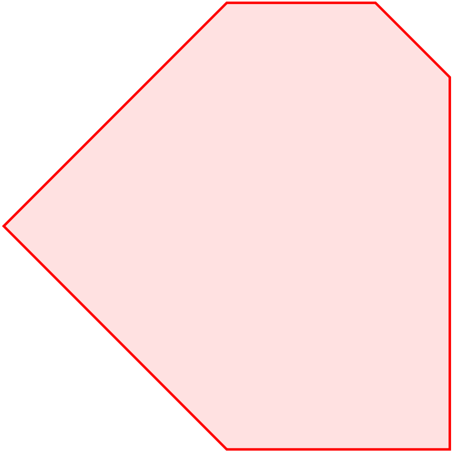
Example of an inductive polyhedron

```
int i, n;
assume(n >= 0);
i = 0;
while(i < n) {
  i = i+1;
}
```

```
int i, n;
assume(n >= 0);
i = 0;
LOOP: // 0 ≤ i ≤ n
  if (i >= n) goto EXIT;
  i = i+1;
  goto LOOP;
EXIT:
```

Octagons

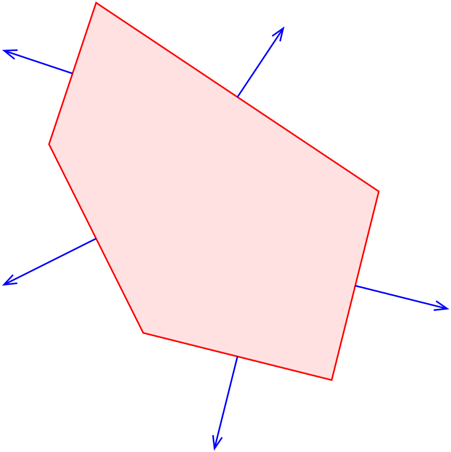
$\pm x \pm y \leq C$ for all variables x and y



(Algorithmics: variants of Floyd-Warshall)

Linear templates

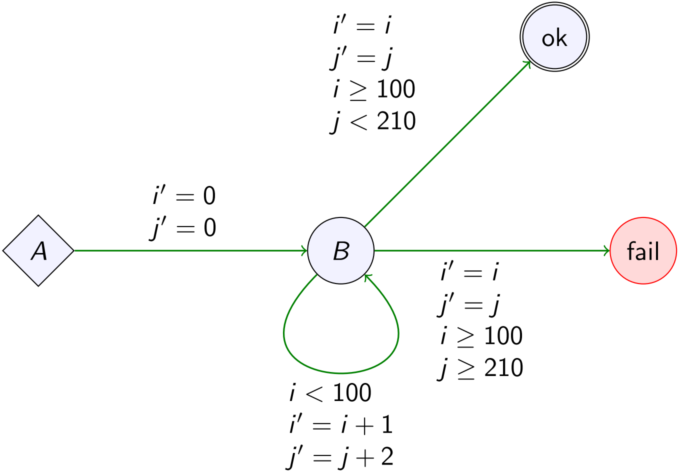
$A\vec{x} \leq B$, A set matrix, $\vec{x}$ program variables



(Algorithmics: linear programming)

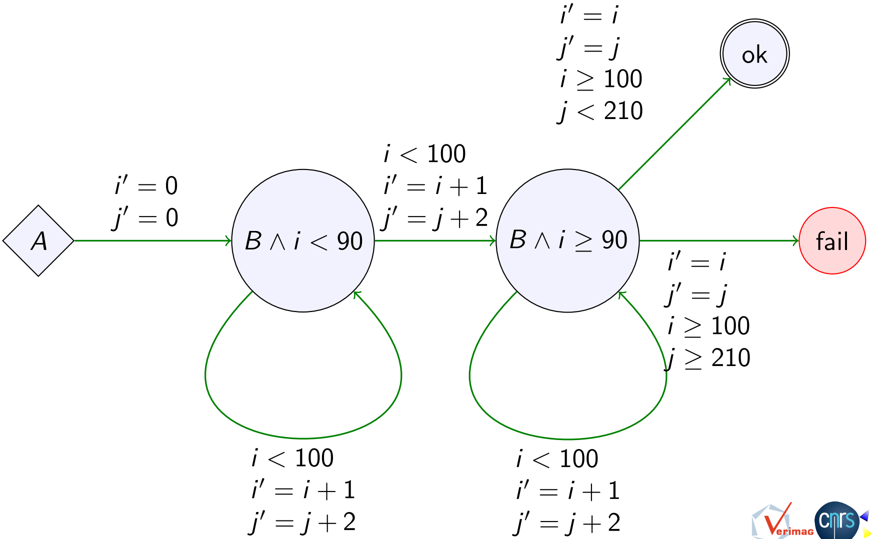
A simple example

```
for(int i=0; i<100; i++) {
  j = j+2;
}
assert(j < 210);
```



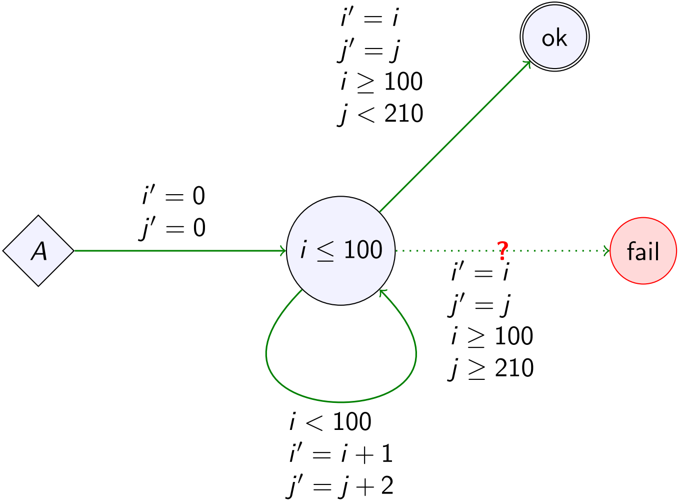
Unsuccessful predicate abstraction

Split control node B according to $i < 90$



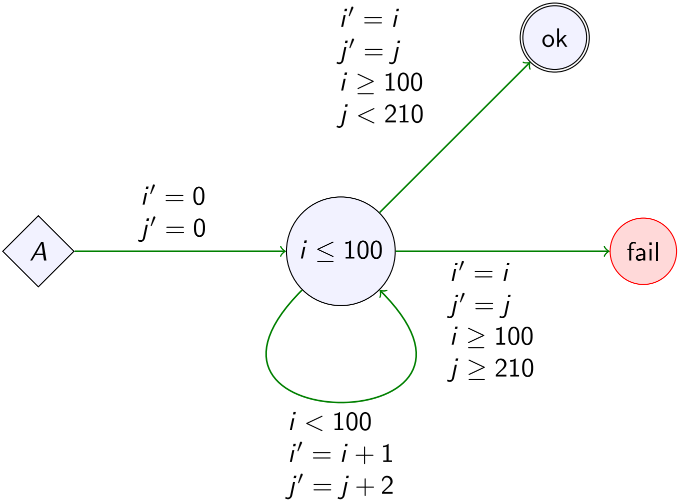
Unsuccessful predicate abstraction

Split control node B according to $i \leq 100$



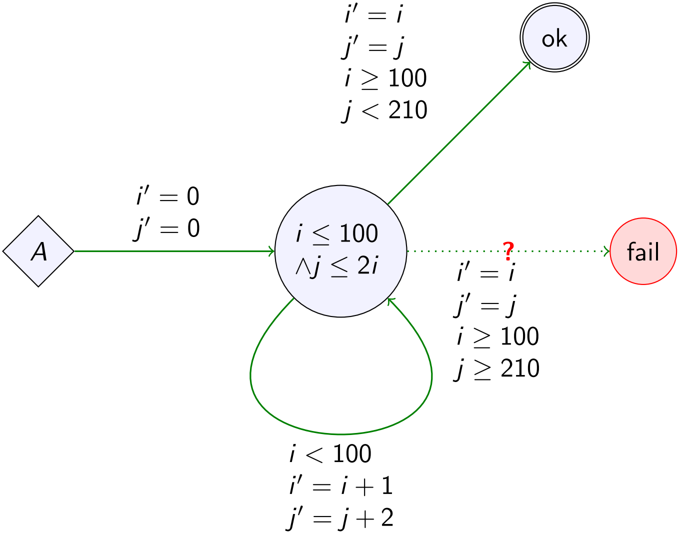
Unsuccessful predicate abstraction

Split control node B according to $i \leq 100$



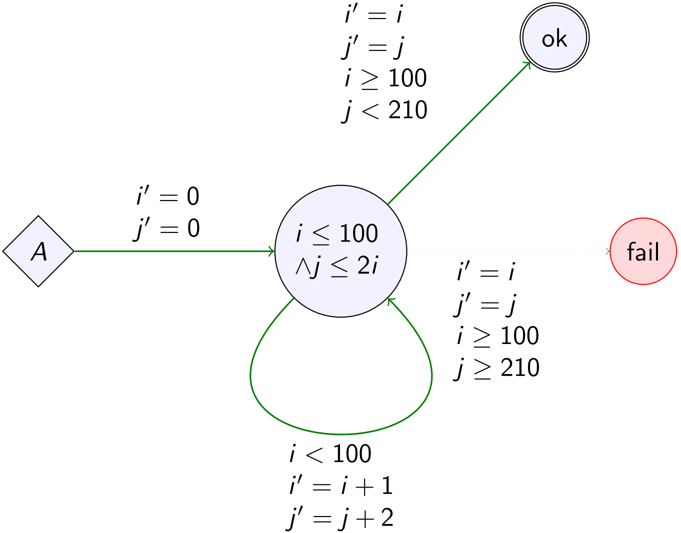
Successful predicate abstraction

Split control node B according to $i \leq 100$ and $j \leq 2i$



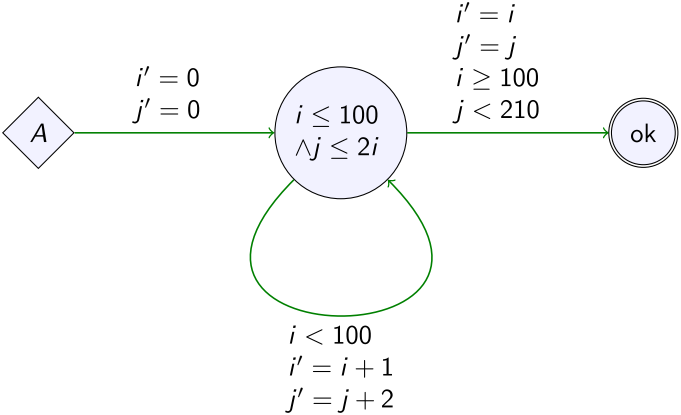
Successful predicate abstraction

Split control node B according to $i \leq 100$ and $j \leq 2i$



Pruning by construction

Do not generate unreachable states



Summary

Select a finite set of predicates (possibly depending on control location)

Split each control state according to predicates (use subsumption)

Draw feasible transitions

Procedure in finite time: only finitely many abstract states

Some decision problem

Given

- initial state
- transition relation (in fixed class)
- final state

answer whether there exist inductive invariant in fixed class.

Vary class of relations and class of invariants.

Proved on template polyhedra:

- ???? for intervals for linear transitions (related to mean-payoff?)
- Σ_2^P-completeness (even single $x \leq B$ interval bound) for transitions with $\vee, \wedge, \exists$-first-order linear arithmetic
- EXPTIME-hardness, in NEXPTIME, for implicit CFG (in the Papadimitriou-Yannakakis sense)

Concrete acceleration

Given $\rightarrow$, compute transitive closure $\rightarrow^{+}$

example: $x \rightarrow x' \iff x < n \wedge x' = x + 1$
transitive closure: $y \rightarrow^{+} y' \iff y < y' \leq n$

Possible for certain subclasses of formulas.
e.g. conjunction of "octagonal" inequalities $\pm x \pm y \leq C$ (Bozga, Iosif, Konečný)

Disjunctions

Let a, b... be accelerable relations.

```
if (*) {
  a
} else {
  b
}
```

$(a|b)^+ = a^+|b^+|a^+b^+|b^+a^+|\ldots$

Replace normal Kleene iterations by partial acceleration.

(Bozga, Iosif, Konečný) FLATA tool

Disjunctions

Let a, b... be accelerable relations.

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```

$(a|b)^+ = a^+|b^+|a^+b^+|b^+a^+|\ldots$
Replace normal Kleene iterations by partial acceleration.

(Bozga, Iosif, Konečný) FLATA tool

Abstract acceleration

Given τ, compute $\alpha(\tau^+)$ where $\alpha(X)$ is "strongest abstraction" in a certain domain.

(Gonnord & Halbwachs; Jeannet, Schrammel & Sankaranarayanan)

Finite automata

Saturation in finite time:

- Finite concrete state (e.g. n Boolean variables): compute exact reachable states
- Predicate abstraction with fixed set of predicates: compute reachable abstract states

Terminates (can be slow)

Intervals

```
int i=0;
while(i < 1000000) { //
  i++;
}
```

Follow edges, merge etc. until stabilization.

Intervals

```
int i=0;
while(i < 1000000) { // i ∈ [0,0]
  i++;
}
```

Follow edges, merge etc. until stabilization.

Intervals

```
int i=0;
while(i < 1000000) { // i ∈ [0,1]
  i++;
}
```

Follow edges, merge etc. until stabilization.

Intervals

```
int i=0;
while(i < 1000000) { // i ∈ [0,2]
  i++;
}
```

Follow edges, merge etc. until stabilization.

Intervals

```
int i=0;
while(i < 1000000) { // i ∈ [0,3]
  i++;
}
```

Follow edges, merge etc. until stabilization.

Intervals

```
int i=0;
while(i < 1000000) { // i ∈ [0,4]
  i++;
}
```

Follow edges, merge etc. until stabilization.

Intervals

```
int i=0;
while(i < 1000000) { // i ∈ [0,5]
  i++;
}
```

Follow edges, merge etc. until stabilization.

Intervals

```
int i=0;
while(i < 1000000) { // i ∈ [0,6]
  i++;
}
```

Follow edges, merge etc. until stabilization.

Intervals

```
int i=0;
while(i < 1000000) { // i ∈ [0,7]
  i++;
}
```

Follow edges, merge etc. until stabilization.

Intervals

```
int i=0;
while(i < 1000000) { // i ∈ [0,8]
  i++;
}
```

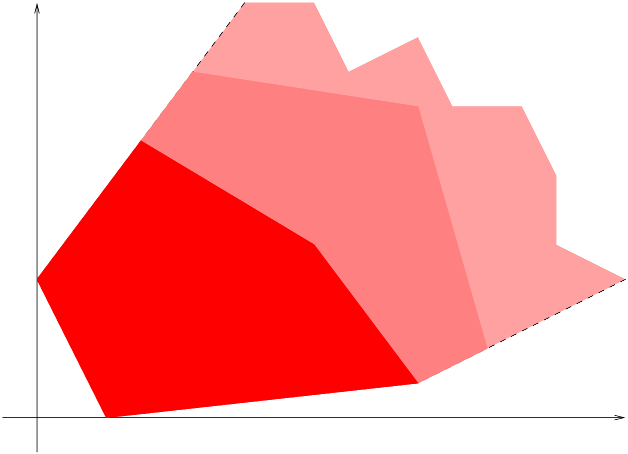
Follow edges, merge etc. until stabilization.

Interval widening

```
int i=0;
while(i < 1000000) { // i ∈ [0, +∞)
  i++;
}
```

Idea: unstable bounds go to $+\infty$

Widening on polyhedra



(Halbwachs)

Interval widening "up to"

Syntactic tracking/extraction of bounds in the program; here $i \to 1000000$

```
int i=0;
while(i < 1000000) { // i ∈ [0, 1000000]
  i++;
}
```

Resembles predicate abstraction!
Can be generalized to polyhedra (see Jeannet).

Upward "Kleene" iterations

- Iterate (upward, using merges) until a fixed point is reached.
- If possibility of infinite ascending sequence (e.g. intervals, polyhedra), apply **widening** instead of normal merge.

Where do we need to apply widening?

Need to apply widening only at "cut set" of control states.

Upward “Kleene” iterations

- Iterate (upward, using merges) until a fixed point is reached.
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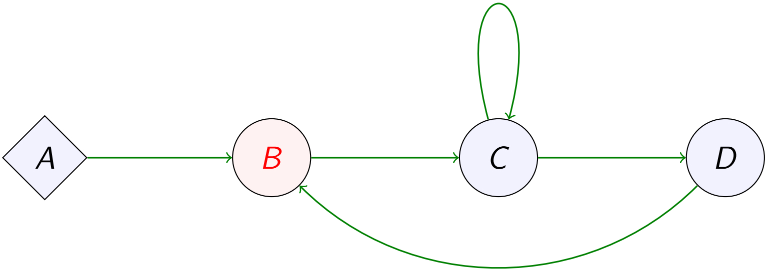
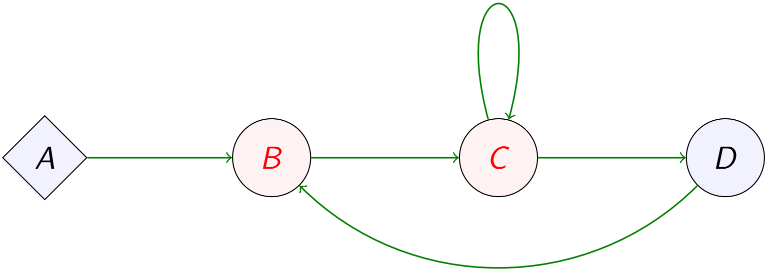
Need to apply widening only at “cut set” of control states.

Cut sets



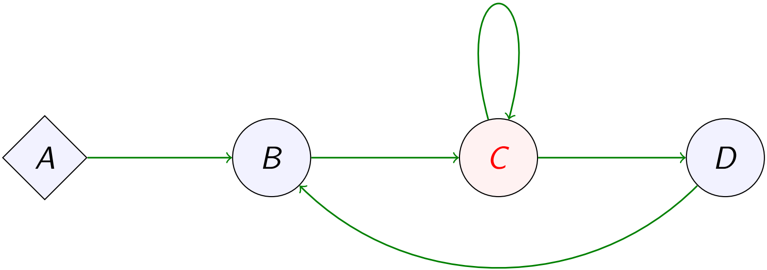
Insufficient: uncut loop at C

Cut sets



Loop heads

Cut sets



Minimal

How to compute a cut set

- Simple but not optimal: target of back edges in depth-first search
- Least cardinality: NP-complete in general
- . . . but linear time for structured programs (reducible cfg) [Shamir '79]

How to compute a cut set

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- Least cardinality: NP-complete in general
- ...but linear time for structured programs (reducible cfg) [Shamir '79]

Non monotonicity

```
int i=0;
while(i != 1000000) { //
  i++;
}
```

Non monotonicity

```
int i=0;
while(i != 1000000) { // i ∈ [0,0]
  i++;
}
```

Non monotonicity

```
int i=0;
while(i != 1000000) { // i ∈ [0,1]
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}
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Non monotonicity

```
int i=0;
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Non monotonicity

```
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while(i != 1000000) { // i ∈ [0,3]
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}
```

Non monotonicity

```
int i=0;
while(i != 1000000) { // i ∈ [0,4]
  i++;
}
```

Non monotonicity

```
int i=0;
while(i != 1000000) { // i ∈ [0,5]
  i++;
}
```

Non monotonicity

```
int i=0;
while(i != 1000000) { // i ∈ [0,6]
  i++;
}
```

Non monotonicity

```
int i=0;
while(i != 1000000) { // i ∈ [0, +∞)
  i++;
}
```

Non monotonicity

```
int i=choose(0, 1000000); // i ∈ [0,1000000]
while(i != 1000000) { //
  i++;
}
```

Non monotonicity

```
int i=choose(0, 1000000); // i ∈ [0,1000000]
while(i != 1000000) { // i ∈ [0,1000000]
  i++;
}
```

A very simple loop

```
i=0;
while (i < 100) {
  i=i+1;
}
```

Find an inductive loop invariant as an interval $[-l, h]$:

- $[-l, h]$ must contain the initial state: $l \geq 0,\ h \geq 0$
- $[-l, h]$ must be stable by "pushing the interval through the loop"
  - test maps $[-l, h]$ to $[-l, \min(h, 99)]$
  - then $i = i + 1$ maps $[-l, \min(h, 99)]$ to $[-(l-1), \min(h, 99) + 1]$

  Thus inclusion: $l \geq l - 1$ and $h \geq \min(h, 99) + 1$

Thus the least solution satisfies

- $l = \max(0, l - 1)$
- $h = \max(0, \min(h, 99) + 1)$

How to solve min-max equations

We end with equations with “min”, “max”, and monotone affine-linear expressions

$$h = \max(0, \min(h, 99) + 1)$$

How to solve them?

Naive approach:

- Enumerate all argument choices for “min” and “max”
- For each choice, compute solution of linear equation system
- Discard if not a solution of the original problem (wrong choices of arguments of “min” and “max”)
- Take the least one

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- Enumerate all argument choices for “min” and “max”
- For each choice, compute solution of linear equation system
- Discard if not a solution of the original problem (wrong choices of arguments of “min” and “max”)
- Take the least one

Solving the naive way

$$h = \max(0, \min(h, 99) + 1) \quad (1)$$

Turned into 3 different equations:

- $h = \max(\underline{0}, \min(h, 99) + 1) \rightsquigarrow h = 0$ (left-arg to "max"), solution $h = 0$, but not solution of (1): $\max(0, \min(0, 99) + 1)$, the right argument of "max" is greater $\Rightarrow$ discarded
- $h = \max(0, \underline{\min(\underline{h}, 99) + 1}) \rightsquigarrow h = h + 1$ (right-arg to "max", left-arg to "min"), solution $h = +\infty$, but not solution of (1): $\min(+\infty, 99)$, the argument of "min" is smaller $\Rightarrow$ discarded
- $h = \max(0, \underline{\min(h, \underline{99}) + 1}) \rightsquigarrow h = 99 + 1 = 100$ (right-arg to "max", right-arg to "min"), **solution** of the original problem.

But **exponential blowup**.

Max-policy iteration

(Developed by H. Seidl, T. Gawlitza)

$$h = \max(-\infty, 0, \min(h, 99) + 1)$$

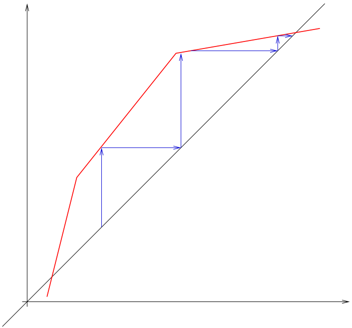
Pick an argument for "max":

- Initial value for $h = -\infty$
- $h = \max(\underline{-\infty}, 0, \min(h, 99) + 1)$; $h = -\infty$; replace: $\max(-\infty, \underline{0}, -\infty)$, found higher argument $h = 0$
- $h = \max(-\infty, \underline{0}, \min(h, 99) + 1)$; $h = 0$; replace: $\max(-\infty, 0, \underline{1})$, found higher argument $h = 1$
- $h = \max(-\infty, 0, \underline{\min(h, 99) + 1})$; solve $h = \min(h, 99) + 1$ for solution $h \geq 1$:
  Solve $h \leq h + 1 \wedge h \leq 99 + 1$ for maximal finite h: $h = 100$.

High level view

Transforms the original problem (with "max") into a sequence of problems (without "max") with increasing "value".

Intuition: solution is maximum of "order-concave" functions



It's like solving $h = F(h)$ by infinite ascending sequence
$-\infty, F(-\infty), F \circ F(-\infty), F \circ F \circ F(-\infty)\dots$
but taking "big strides"!

Max policy iteration: executive summary

- Works for linear templates (and some nonlinear ones, but much more difficult)
- For linear transitions (and...)
- "Policy": for each control node, and each bound in the template, pick an incoming edge ($-\infty$ = unreachable, $+\infty$ = unbounded)
- Solve resulting system for greatest finite fixed point
- Change policy if unstable

Replaces blind widening and widening "up to" by a kind of acceleration of ω Kleene iteration steps.

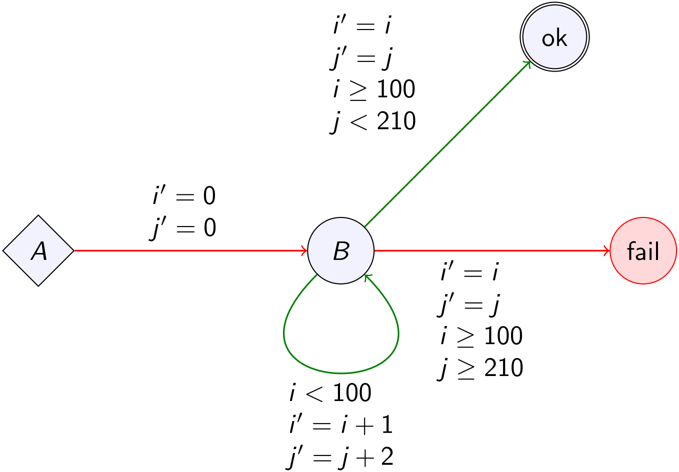
Refinement

From a property or invariant, get a stronger one.

- **Unguided** Get a stronger one.
- **Guided** Get a stronger one that "kills" some counterexample.

In predicate abstraction

```
for(int i=0; i<100; i++) {
  j = j+2;
}
assert(j < 210);
```



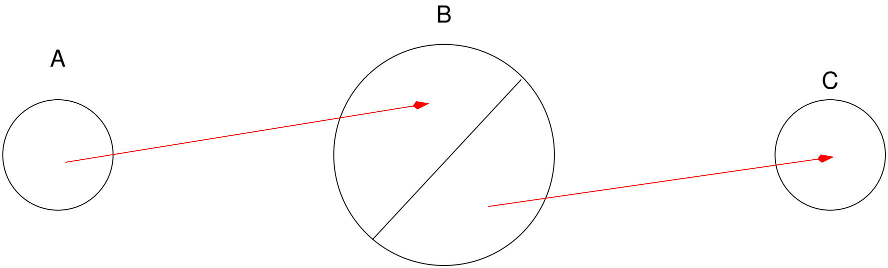
A bad counterexample

Try to find values for the red path:

$i_1 = 0 \wedge j_1 = 0 \wedge i_2 = i_1 \wedge j_2 = j_1 \wedge i \geq 100 \wedge j \geq 210$

UNSAT

Why wrong? Can move from one state in A to one state in B, from one state in B to one in "fail". But states in B not the same.



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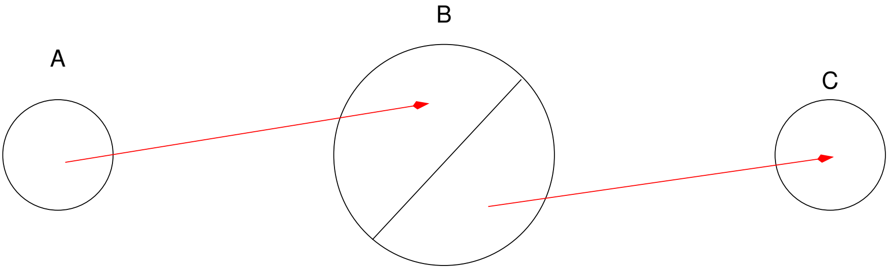
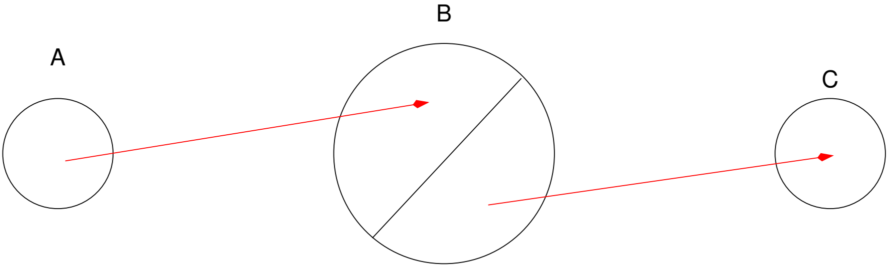


A refinement

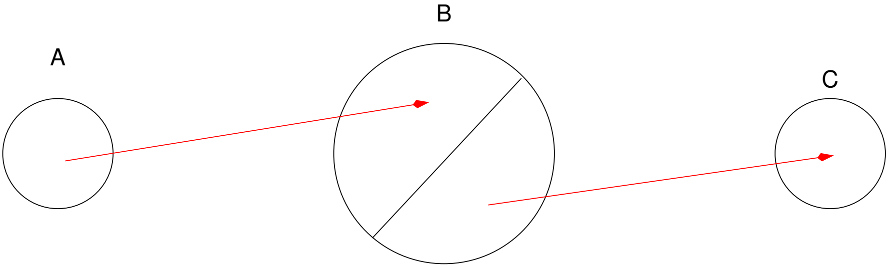


Two step explanation for infeasible path:

- $i_1 = 0 \wedge j_1 = 0 \wedge i_2 = i_1 \wedge j_2 = j_1 \Rightarrow j_2 = 2i_2 \wedge i \geq 100$
- $j_2 = 2i_2 \wedge i_2 \geq 100 \Rightarrow j_2 < 210$

This is a **Craig interpolant**.

A refinement



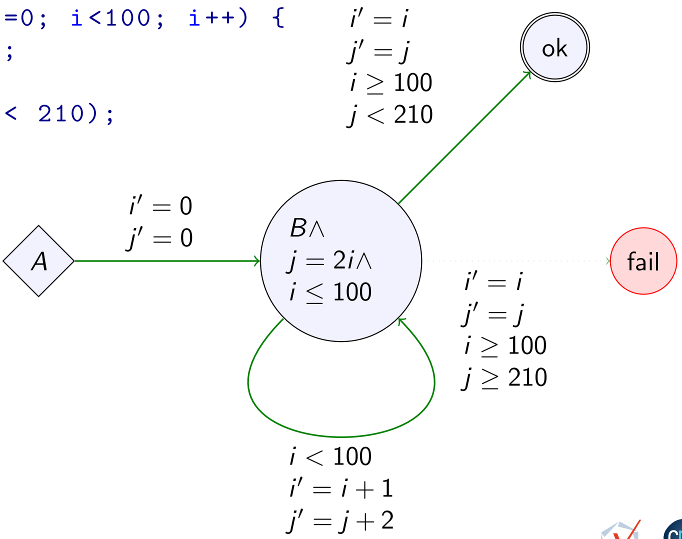
Two step explanation for infeasible path:

- $i_1 = 0 \wedge j_1 = 0 \wedge i_2 = i_1 \wedge j_2 = j_1 \Rightarrow j_2 = 2i_2 \wedge i \geq 100$
- $j_2 = 2i_2 \wedge i_2 \geq 100 \Rightarrow j_2 < 210$

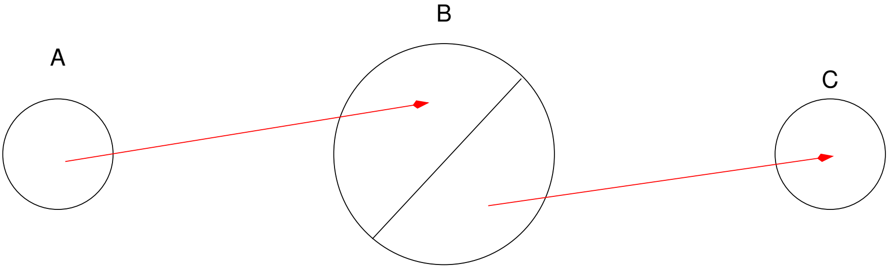
This is a **Craig interpolant**.

A good refinement

```
for(int i=0; i<100; i++) {
  j = j+2;
}
assert(j < 210);
```



Another refinement



Two step explanation for infeasible path:

- $i_1 = 0 \wedge j_1 = 0 \wedge i_2 = i_1 \wedge j_2 = j_1 \Rightarrow i_2 = 0 \wedge j_2 = 0$
- $i_2 = 0 \wedge j_2 = 0 \Rightarrow j_2 < 210$

This is another **Craig interpolant**.

Another refinement

```
for(int i=0; i<100; i++) {
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assert(j < 210);
```
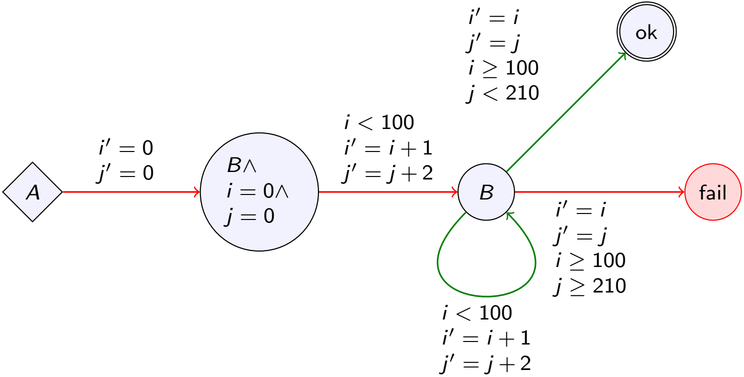



Further refinement

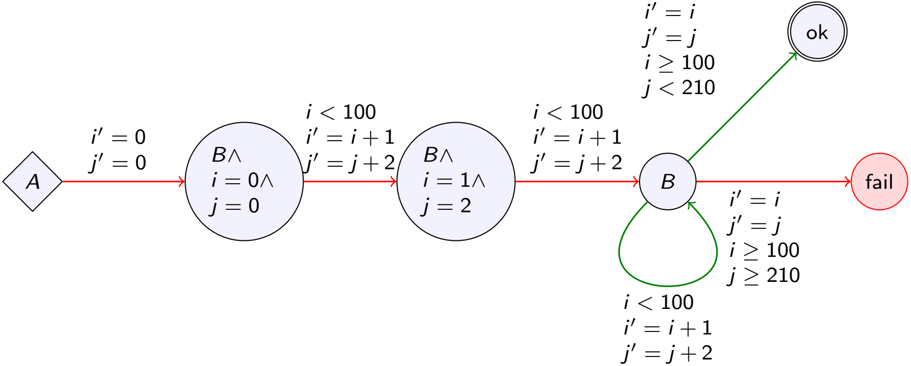
Two step explanation for infeasible path:

- $i_1 = 0 \land j_1 = 0 \land i_2 = i_1 \land j_2 = j_1 \Rightarrow i_2 = 0 \land j_2 = 0$
- $i_2 = 0 \land j_2 = 0 \land i_3 = i_2 + 1 \land j_3 = j_2 + 2 \Rightarrow i_3 = 1 \land j_3 = 2$
- $i_3 = 1 \land j_3 = 2 \Rightarrow j_2 < 210$

This is another **Craig interpolant**.

Further refinement

```
for(int i=0; i<100; i++) {
  j = j+2;
}
assert(j < 210);
```



Overfitting and convergence

- Interpolant $j = 2i \wedge i \leq 100$ (polyhedral inductive invariant) proves the property.
- Interpolants $i = 0 \wedge j = 0$, $i = 1 \wedge j = 2$, $i = 2 \wedge j = 4$ ... (exact post-conditions) lead to non-termination.

Challenge: find "good" interpolants "likely" to become inductive
Problem similar to widening
McMillan: find "short" interpolants using few "magic" constants?

When simple induction is insufficient

```
int index = 0, tab[8];
while(true) {
  out = tab[index];
  tab[index] = input(-10, 10);
  index++;
  if (index==8) index=0;
}
```

Cannot prove $out \in [-10, 10]$ by simple induction.

Otherwise said

```
while(true) {
  int t0, t1, t2, t3,
      t4, t5, t6, t7;
  out = t7;
  t7=t6; t6=t5;
  t5=t4; t4=t3;
  t3=t2; t2=t1;
  t1=t0;
  t0 = input(-10, 10);
}
```

Or in Lustre

```
node top
  (in : int)
returns
  (ok : bool);

var
  t0 : int;
  t1 : int;
  t2 : int;
  t3 : int;
  t4 : int;
  t5 : int;
  t6 : int;
  t7 : int;
  out : int;

let
```

```
  ok = out >= -10
   and out <= 10;
  out = 0 -> pre t7;
  t7 = 0 -> pre t6;
  t6 = 0 -> pre t5;
  t5 = 0 -> pre t4;
  t4 = 0 -> pre t3;
  t3 = 0 -> pre t2;
  t2 = 0 -> pre t1;
  t1 = 0 -> pre t0;
  t0 = if in >= -10
       and in <= 10
       then in else 0;

--%PROPERTY ok;

tel
```

Kind

`http://clc.cs.uiowa.edu/Kind/`
Proves the above program correct by 9-induction.

Descending sequence in abstract interpretation

```
int i=0, n;
assume(n >= 0);
while(i < n) {
  i = i+1;
}
```

Invariant inferred after widening: $i \geq 0$

Reasoning: "in order to be at loop head, either coming from initialization, either from preceding iteration"
Preceding iteration:

```
assume(i >= 0);
assume(i < n);
i = i+1;
```

Thus: $i = 0 \vee 1 \leq i \leq n$, thus $0 \leq i \leq n$ better invariant.

Executive summary: k-induction and descending sequence

k-induction Given P to prove:

- initialization Prove that P holds on k first steps of execution
- induction If P holds on k steps $x_0 \to x_1 \to \cdots \to x_{k-1}$ and $x_{k-1} \to x_k$ then P holds on x_k

Descending sequence Given any invariant I:

- initialization Keep states in k first steps of execution in I'
- induction Assume I holds on k steps $x_0 \to x_1 \to \cdots \to x_{k-1}$ and $x_{k-1} \to x_k$, add x_k states to I'.

Intuition

(E. Goubault's group)

If precondition $\rightarrow$ program $\rightarrow$ $x \leq 10$ provable in, say, octagons

a proof of $x \leq 10$ can be written using only a subset

- of the guards (tests) and other constraints (operations)
- of possible reasoning steps (e.g. "if $y - x \leq 3$ and $z - y \leq 5$ then $z - x \leq 8$")

Iterative refinement: if another subset is "obviously" better, choose it.

A very simple loop

```
i=0;
while (i < 100) {
  i=i+1;
}
```

Find an inductive loop invariant as an interval $[-l, h]$:

- $[-l, h]$ must contain the initial state: $l \geq 0,\ h \geq 0$
- $[-l, h]$ must be stable by "pushing the interval through the loop"
  - test maps $[-l, h]$ to $[-l, \min(h, 99)]$
  - then $i = i + 1$ maps $[-l, \min(h, 99)]$ to $[-(l-1), \min(h, 99) + 1]$

  Thus inclusion: $l \geq l - 1$ and $h \geq \min(h, 99) + 1$

Thus the least solution satisfies

- $l = \max(0, l - 1)$
- $h = \max(0, \min(h, 99) + 1)$

Min-policy iteration

Only choose for "min":

- $h = \max(0, \min(\underline{h}, 99) + 1) \rightsquigarrow h = \max(0, h + 1) \rightsquigarrow$ find least solution of $h \geq 0 \wedge h \geq h + 1$ (linear programming) $\rightsquigarrow h = +\infty$ $\min(+\infty, 99) = 99$, so flip to right argument of "min"
- $h = \max(0, \min(h, \underline{99}) + 1) \rightsquigarrow h = \max(0, 100) \rightsquigarrow$ find least solution of $h \geq 0 \wedge h \geq 100$ (linear programming) $\rightsquigarrow h = 100$

Solution: $h = 100$

Always the least one?

In general, the min-policy iteration process may stop on a solution of the system of min-max equation that is not the least one.

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Solution: $h = 100$
Always the least one?

In general, the min-policy iteration process may stop on a solution of the system of min-max equation that is not the least one.

Min-policy iteration, executive summary

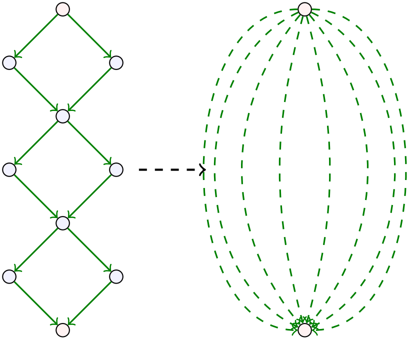
- Replaces original system by an over-approximation: leave out some guards and other constraints on the reachable states
- Solve the resulting, simple, system
- If the system solves the property, terminate
- Check whether some "obvious" improvement exists, loop

Code motion or trace partitioning

```
double x, y, z;
if (x >= 0) {
  y = x;
  if (y > 0.1) {
    assert(x != 0);
    z = 1.0 / x;
  }
} else {
  y = -x;
  if (y > 0.1) {
    assert(x != 0);
    z = 1.0 / x;
  }
}
```

In general: exponential cost

Considering individual paths



Use of SMT-solving

(Satisfiability modulo theory: given a first-order formula in a theory, say "unsat" or give a solution)

Instead of explicitly considering all 2^n paths
consider them **implicitly**, as needed

Solve successive SMT problems "is my current candidate invariant inductive"?

Applied to

- Kleene iterations + widening [Monniaux & Gonnord, C. Tinelli]
- max-policy iteration [Gawlitza & Monniaux, Monniaux & Schrammel]

Floating-point

Most abstract domains: ideal mathematics ($\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$)
Intervals: handle floating-point by directed rounding
But relational domains?

$$x \oplus y = x + y + \epsilon,\ |\epsilon| \leq \epsilon_r |x + y|$$

$$x \otimes y = x \times y + \epsilon,\ |\epsilon| \leq \min(\epsilon_a, \epsilon_r |x + y|)$$

ϵ_r "error at last bit of precision"
ϵ_a least positive floating-point value

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Data structures

- Point-to graph (may/must)
- Abstract into a single variable
  - all data allocated at single location? (but beware of malloc-like functions)
  - all fields with same field identifier (e.g. in Java)
- Recursive decomposition of memory
- Separation logic?

Warning

List of tools **certainly not exhaustive**.

General framework

(not all tools exactly this way)

compiler front-end (Java bytecode, LLVM, CIL, Eclipse CDT...)
$\downarrow$
iterator
$\downarrow$
data structure abstractions
$\downarrow$
numeric abstractions

Polyspace

(commercial) now Mathworks

Astrée

Cousot et al. (commercial)
`http://www.astree.ens.fr/`
`http://www.absint.com/astree/index.htm`

- home-made front-end
- only abstract interpretation
- intervals and octagons
- specialized abstractions for numerical filters
- limited memory abstractions
- now support for parallel programs

Astrée applications

Large plane, large fly-by-wire code

CPAChecker

`http://cpachecker.sosy-lab.org/`

- Eclipse CDT front-end
- mostly predicate interpretation
- intervals
- limited support for octagons and polyhedra

Verasco

Proved correct in Coq

- Compcert front-end
- (floating point) intervals
- polyhedra
- memory
- in the future: filters

No public release yet

`http://verasco.imag.fr/`

APRON & Interproc

Jeannet & Miné

APRON: abstract domains
(Concur)Interproc: demonstrator analyzer

`http://apron.cri.ensmp.fr/library/`
`http://pop-art.inrialpes.fr/interproc/concurinterprocweb.cgi`

PAGAI

http://pagai.forge.imag.fr/ (Julien Henry)

- LLVM front-end
- uses APRON abstract domains
- path-focusing for SMT-solving
- extra applications to worst case execution time (WCET) analysis

Demo

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Demo

Questions?

http://stator.imag.fr/

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