Some motivations for building an approximation theory of hybrid signals and systems

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Introduction

- Model-based development widely recognised as a method of choice for efficient and safely design
- More effectively used in embedded control
  - e.g., automatic code generation in Airbus fly-by-wire (1984)
  - Simulink-CAN, Simulink-TTA,...
- But rather empirical, lack of foundations

Some issues:

1. Model-based development in control and in computer science
2. Sampling theory for discrete event and hybrid systems
3. Fault-tolerance in GALS

Model-based design in computer science and control

Model-based design in computer science

- Starts from a non deterministic specification
- Based on successive property-preserving refinements
- Until an implementation is reached

Remarks:

- This is an idealised scheme, seldom fulfilled
- Yet has a paradigmatic value
- Some real-world impressive achievements in control!!
  - B method (Abrial): Paris, Barcelona, New York subways

Model-based design in computer science

$MACHINE$	$initial$
$SETS$	$persons, buildings$
$ABSTRACT\_CONSTANTS$	$state, authorisation$
$PROPERTIES$	$persons \neq \emptyset \ \wedge \ buildings \neq \emptyset$
	$\wedge \quad state \in persons \rightarrow buildings$
	$\wedge \quad authorisation \in persons \leftrightarrow buildings$
$INVARIANT$	$state \subseteq authorisation$
$OPERATION$	$move \mathrel{\hat{=}} \ ANY \quad (p, b)$
	$WHERE \quad (p, b) \in authorisation$
	$\wedge \ state(p) \neq b$
	$THEN \quad state(p) := b$
	END
END	

Model-based design in computer science

Further steps:

- Add implementation details:
  - paths, doors, badge controls,...
- Separate controllers from environment !!!
- Generate control programs

Model-based design in computer science and control

Model-based design in control science

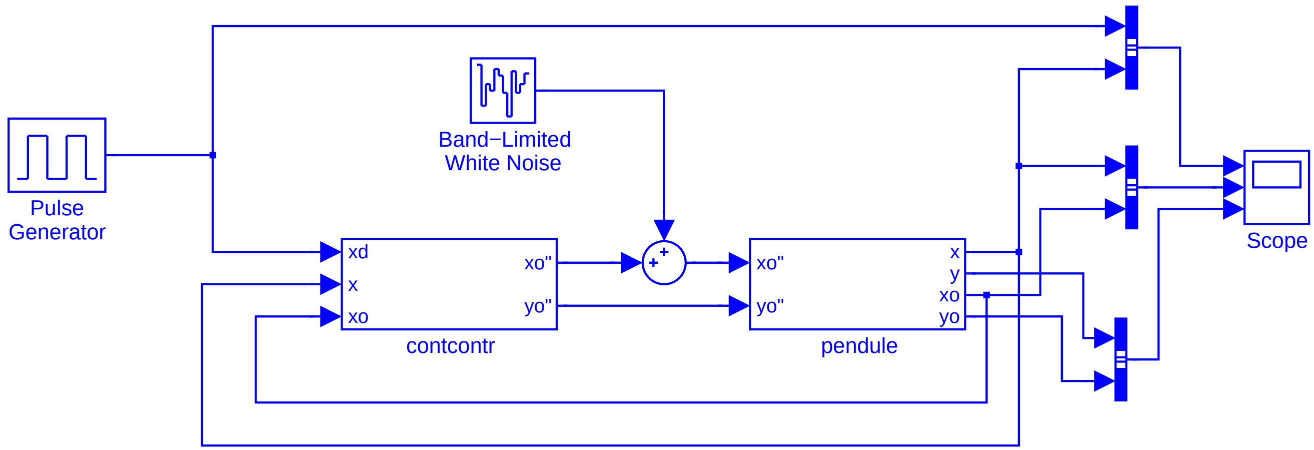
- Start from a perfect model
- Design a robust controller
- Add perturbations and implementation details and checks for robustness

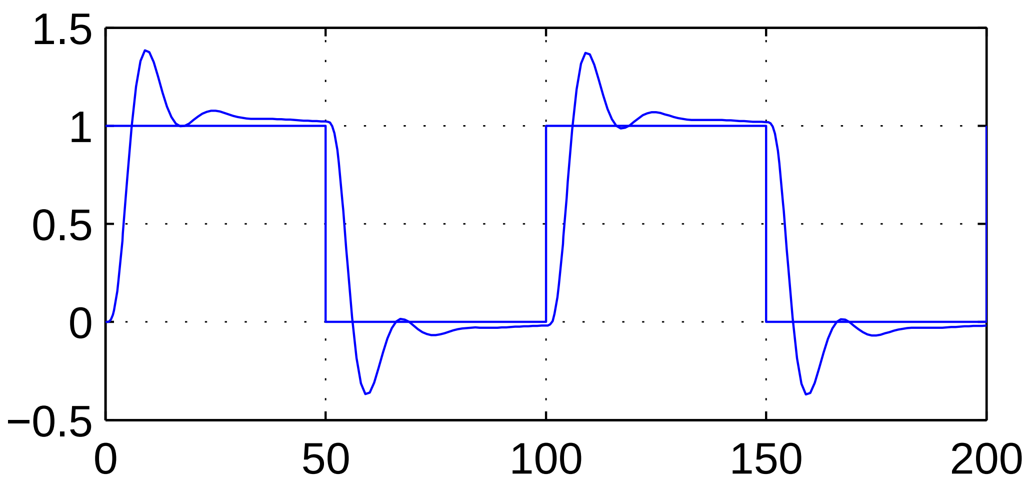
Remarks:

- This is also an idealised scheme

Model-based design in control

Perfect model





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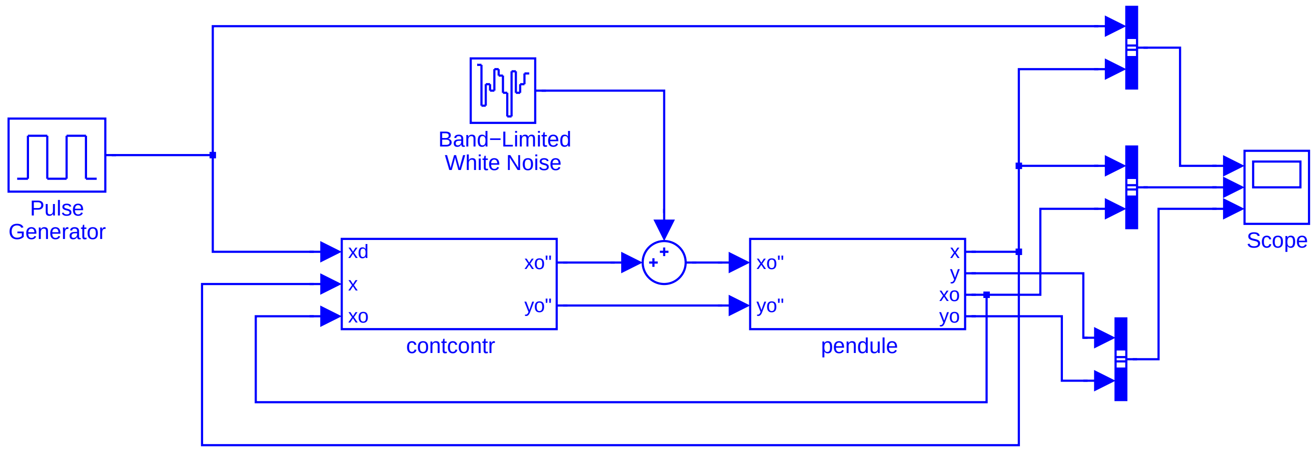
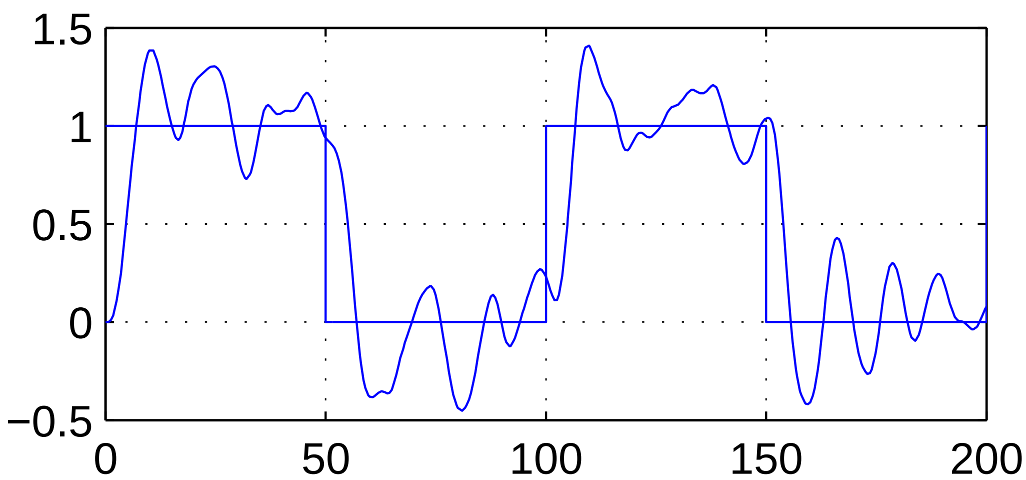
Model-based design in control

Perfect model with noise





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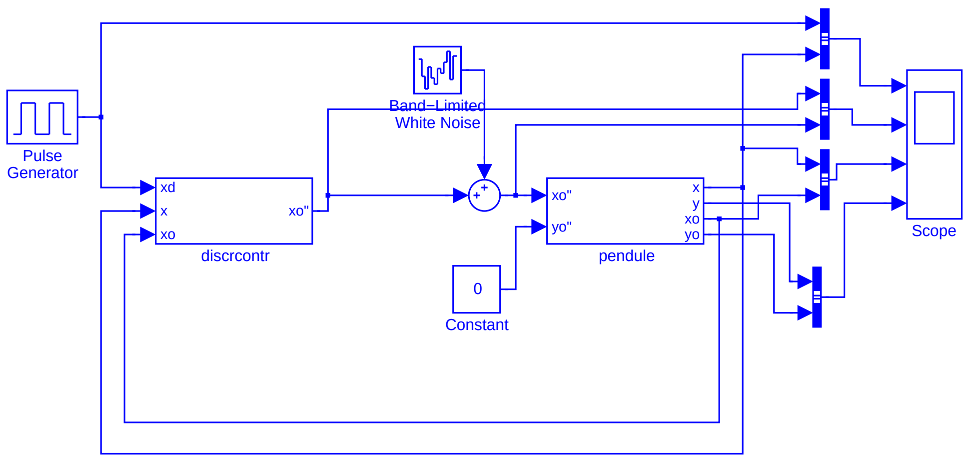
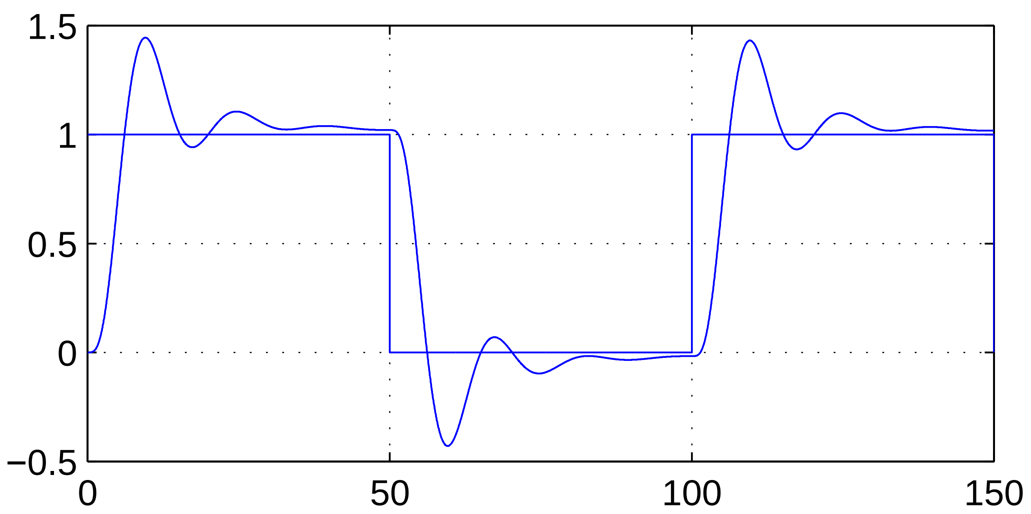
Model-based design in control

Discrete-time controller





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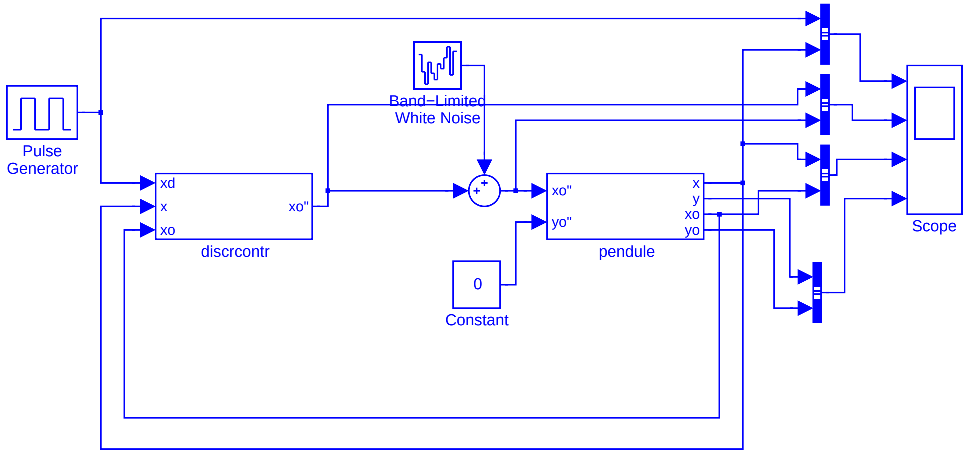
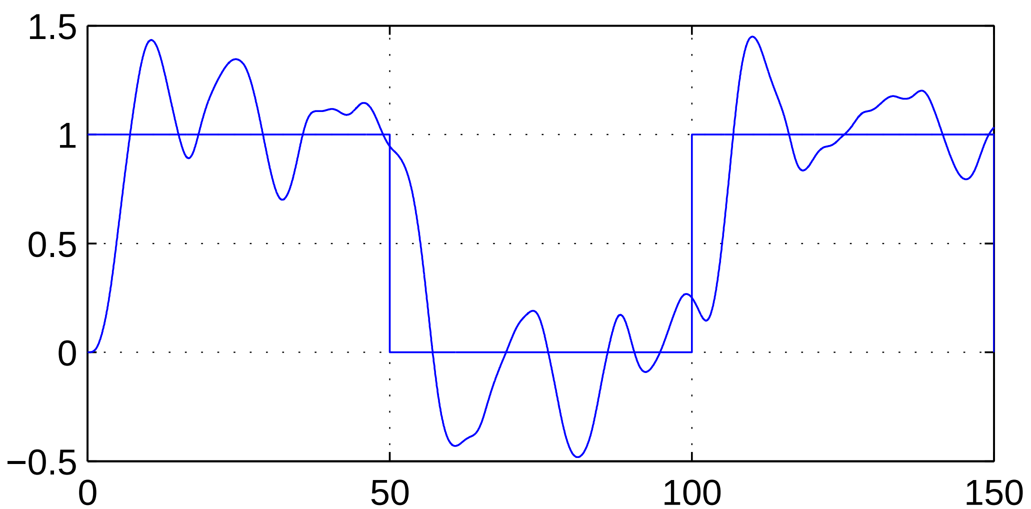
Model-based design in control

Discrete-time controller with noise





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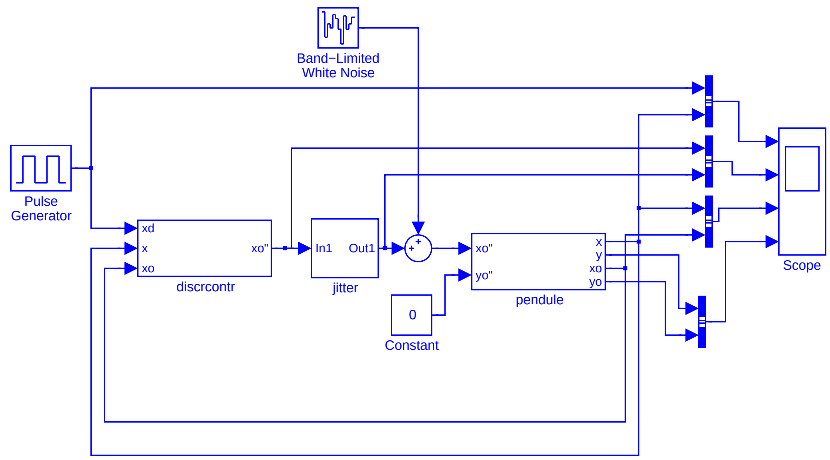
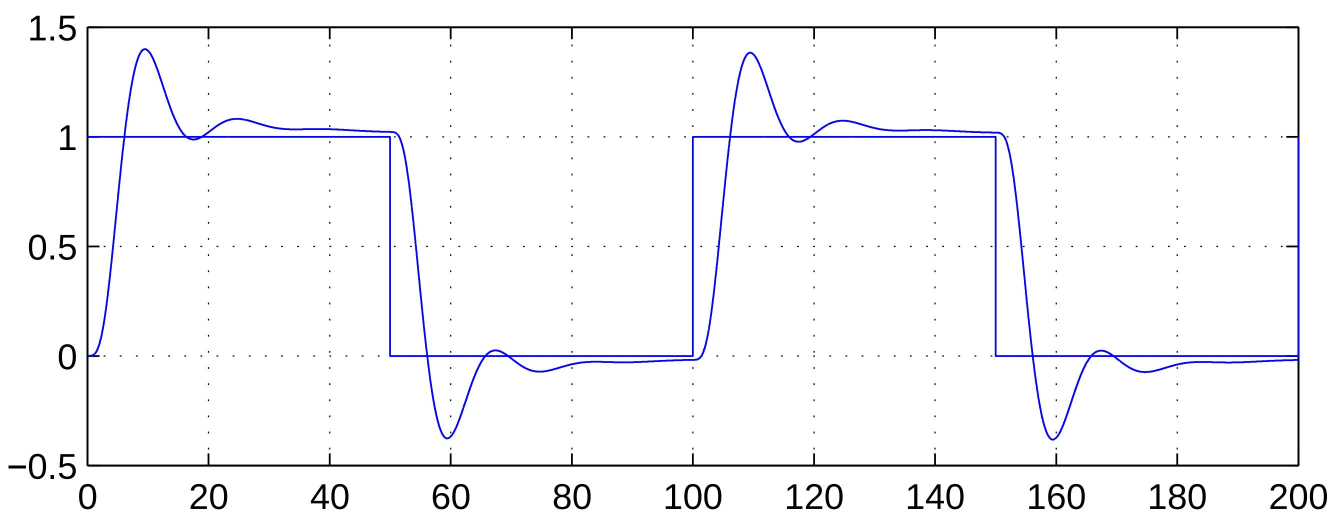
Model-based design in control

Discrete-time controller with jitter





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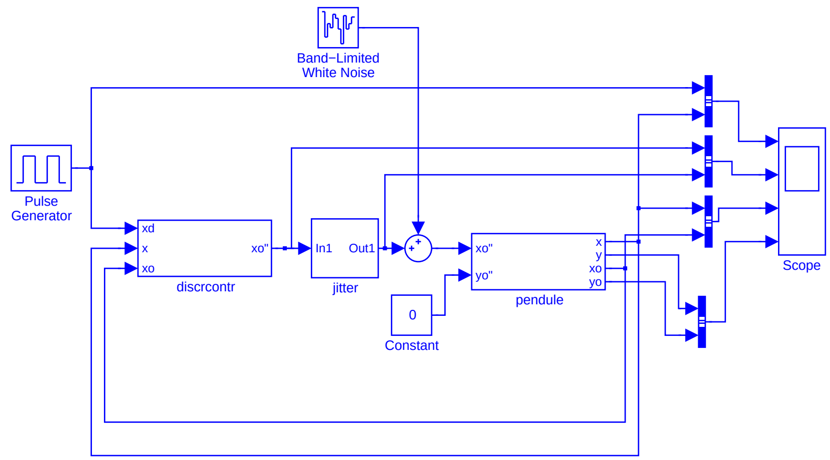
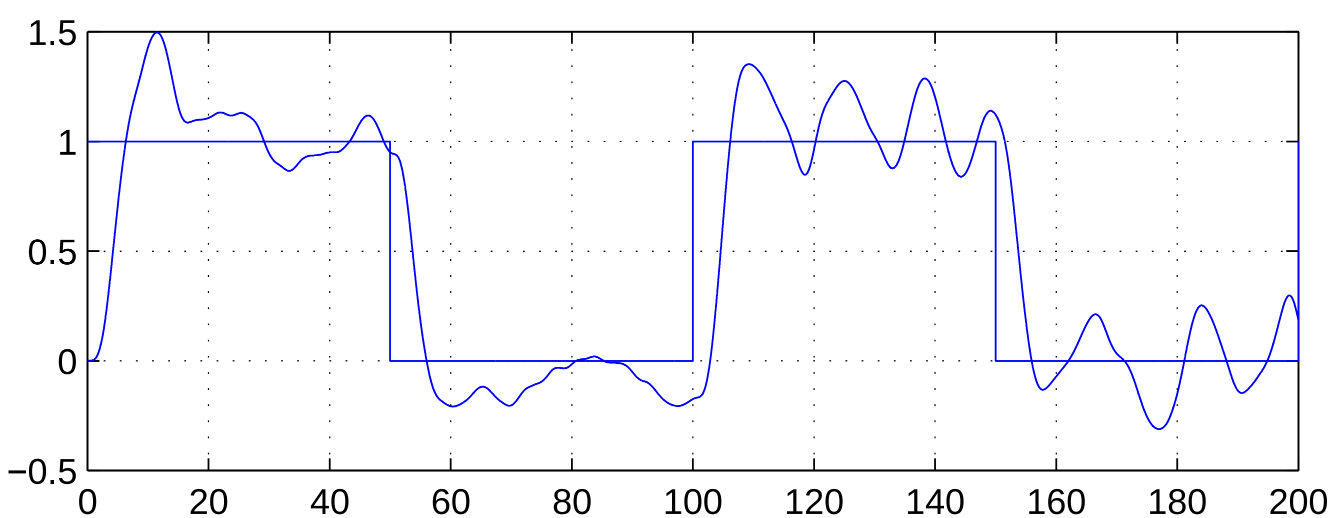
Model-based design in control

Discrete-time controller with jitter and noise





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Model-based design in computer science and control

How can we make them converge ??

A suggestion :

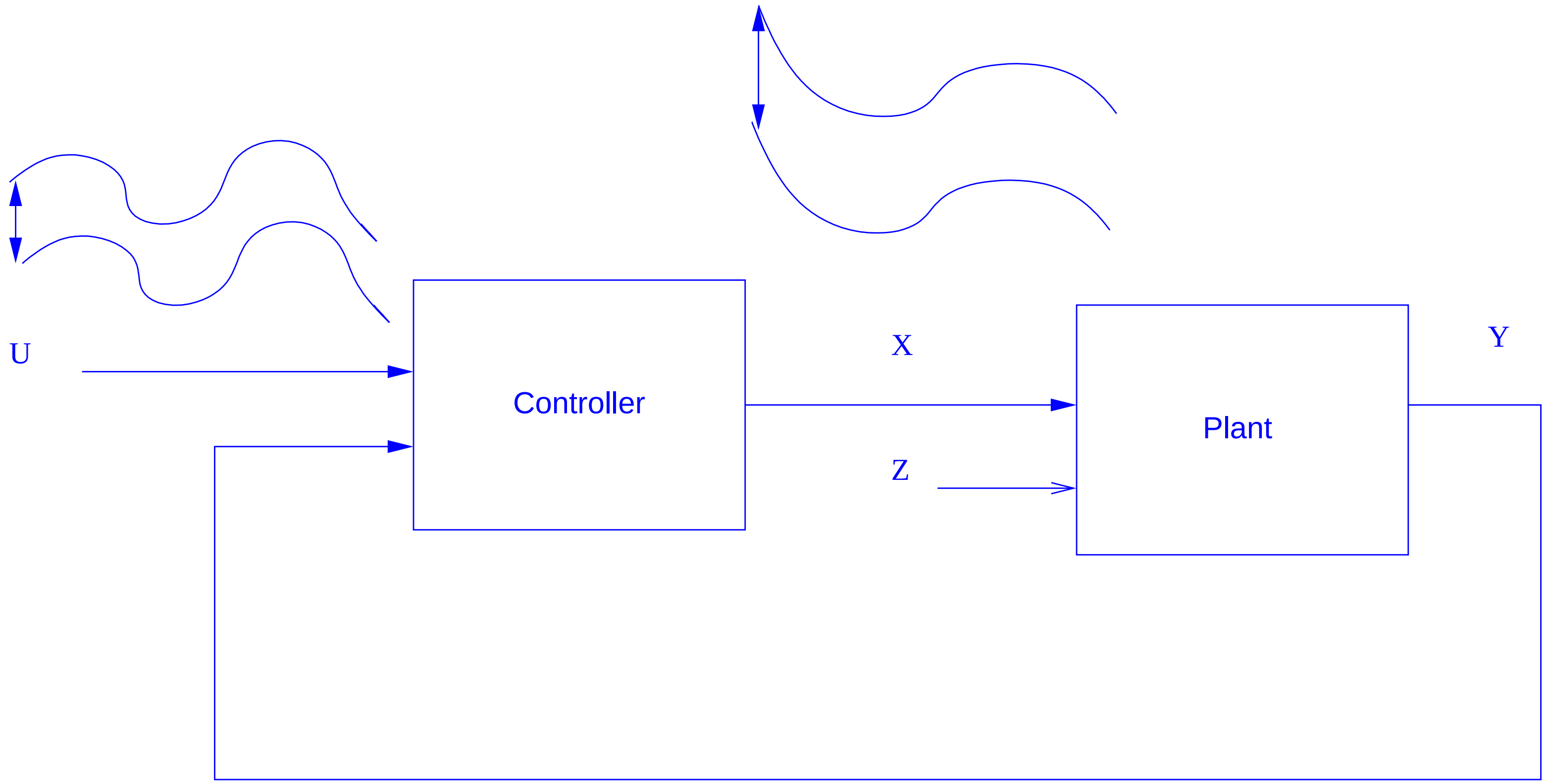
Consider the perfect control model as specifying a set of behaviours, those behaviours which are within some “distance” of the perfect model behaviour.

This requires some notion of “distance”, able to account for

- perturbations
- modelling errors
- discretisation
- jitter and communication delays
- ...

Model-based design in computer science and control

How can we make them converge ??



Sampling discrete event and hybrid systems

Continuous control is implemented by periodic sampling (time-triggered)

- sampled-data control theory
- numerical analysis

Discrete event control is implemented by event triggered systems

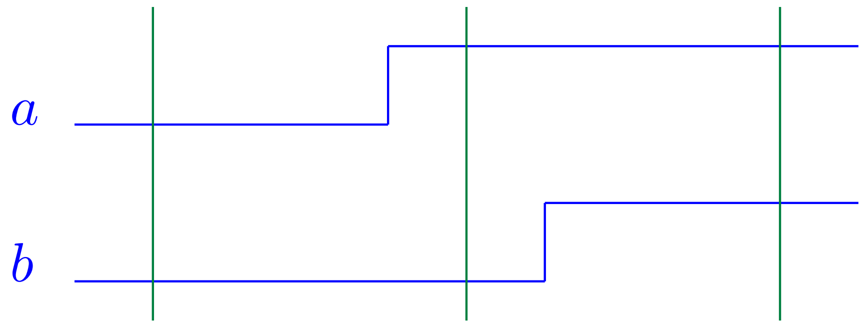
What about mixed (hybrid) systems ??

Experience shows that periodic sampling is a popular solution

but an empirical one

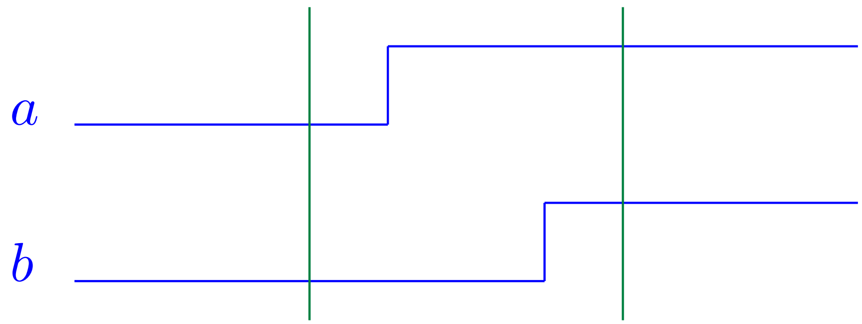
Sampling discrete event systems

A possible sampling



Sampling discrete event systems

Another one

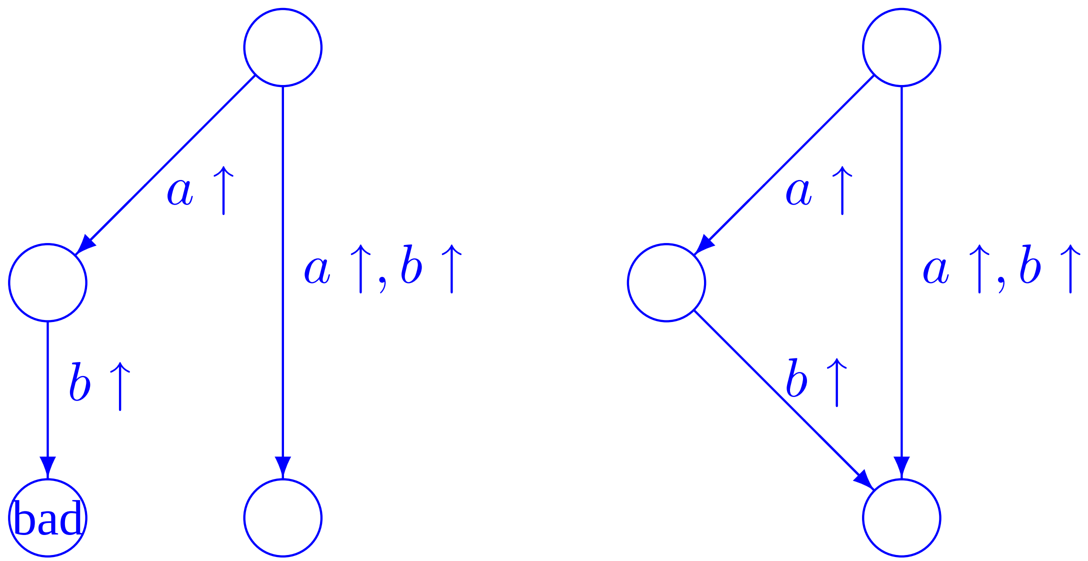


Races

A race takes place when two variables can change in distinct orders

A race is critical if different states can be reached according to which variable changes first

A critical race　　　　A non-critical race



Sampling discrete event and hybrid systems

Which kind of “distance” can account for

- perturbations
- modelling errors
- discretisation
- jitter and communication delays
- sampling discrete events
- races
- ...

Fault tolerance in GALS

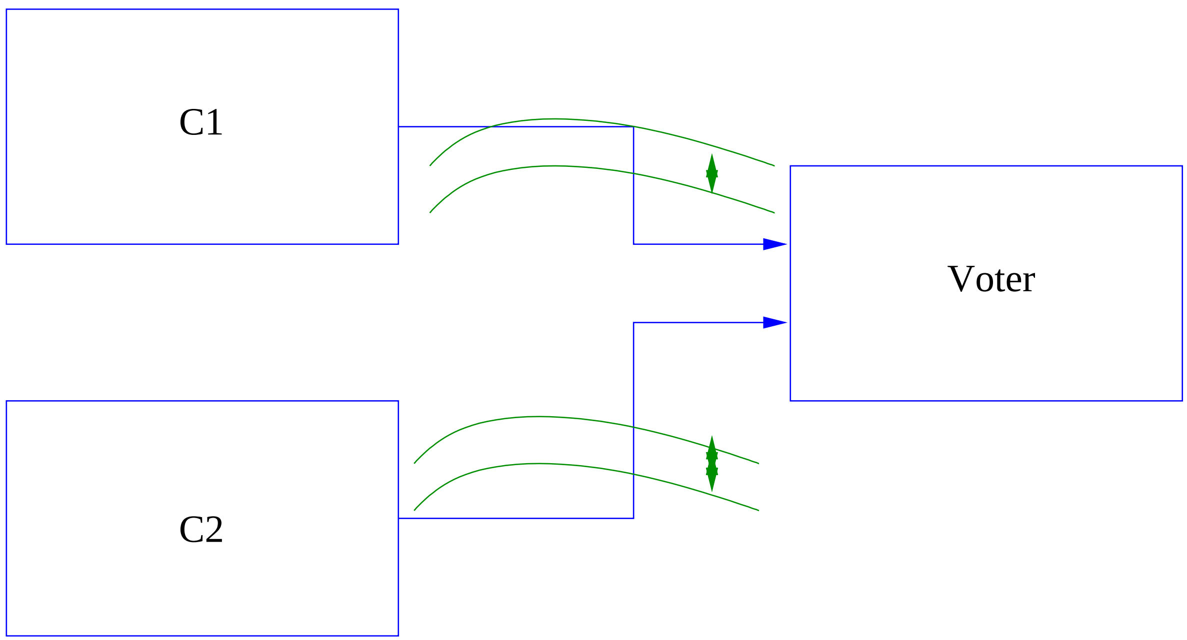
Communication "by periodic sampling" is a popular technique in distributed control

- autonomy, non blocking
- akin to shared memory

Example: Airbus "fly-by-wire" architecture

Entails a need for distance voting in fault-tolerant systems based on this communication principle

Distance voting



When do two signals differ for more than their “normal deviation” ?

Conclusion

This seems to show some need for

An approximation, sampling and voting theory for hybrid signals and systems

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Thank you for your attention