Control of Hybrid Systems

Theory, Computation & Applications

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Outline

1. Basics
   - Hybrid Modeling
   - Controller Computation
2. Low Complexity Controllers
   - The Three Levers of Complexity
   - Minimum-Time Controller
   - *N*-Step Controller
3. Industrial Applications
   - Control of a DC-DC Converter
   - Vibration Damping
4. Conclusions

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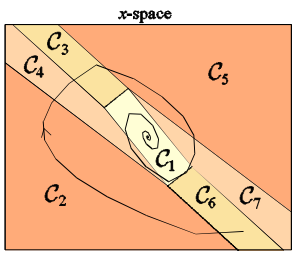
PWA Hybrid Models

- Piecewise affine (PWA) systems.
- Polyhedral partition of the state space.
- Affine dynamics in each region.



$$\mathcal{C}_r \triangleq \{[x \quad u]^T \in \mathbb{R}^n | \; H_r[x \quad u]^T \leq K_r\}$$

$$x(t+1) = A_r x(t) + B_r u(t) + f_r \qquad \text{if } [x(t) \quad u(t)] \in \mathcal{C}_r$$

Equivalence of Hybrid Models

Equivalent classes of discrete-time hybrid models:

- Mixed Logical Dynamical (MLD)
- Piecewise affine (PWA)
- Linear Complementarity (LC)
- Extended Linear Complementarity (ELC)
- Max-Min-Plus-Scaling (MMPS)

The above discrete-time hybrid models can be proven to be equivalent (under some mild assumptions). Equivalence implies that for all feasible initial states and for all feasible input trajectories, the models yield the same state and output trajectories.

[Heemels, De Schutter, Bemporad, 2001]

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Optimal Control for Constrained PWA Systems

System

- Discrete ***PWA*** Dynamics $x(k+1) = f_{\text{PWA}}(x(k), u(k))$
- Constraints on the state $x(k) \in \mathcal{X}$
- Constraints on the input $u(k) \in \mathcal{U}$

$$\left.\begin{array}{l} x(k) \in \mathcal{X} \\ u(k) \in \mathcal{U} \end{array}\right\} C^x x(k) + C^u u(k) \leq C^0$$

Objectives

- Stability (feedback is stabilizing)
- Feasibility (feedback exists for all time)
- Optimal Performance

Constrained Finite Time Optimal Control of PWA Systems

Linear Performance Index (p=1,∞)

$$\boxed{J^*(x) := \min_U \|Px_T\|_p + \sum_{k=0}^{T-1} \|Qx_k\|_p + \|Ru_k\|_p}$$

Constraints

$$\begin{cases} x_0 = x, \\ x_{k+1} = f_{\text{PWA}}(x_k, u_k), \\ C^x x_k + C^u u_k \leq C^0 \end{cases}$$

Algebraic manipulation ➡ Mixed Integer Linear Program (MILP)

$$U^*(x) = \{u_0^*, u_1^*, \ldots, u_{T-1}^*\}$$

Constrained Finite Time Optimal Control of PWA Systems

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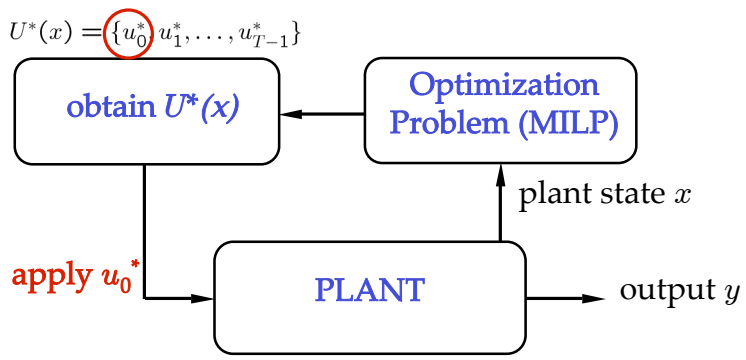
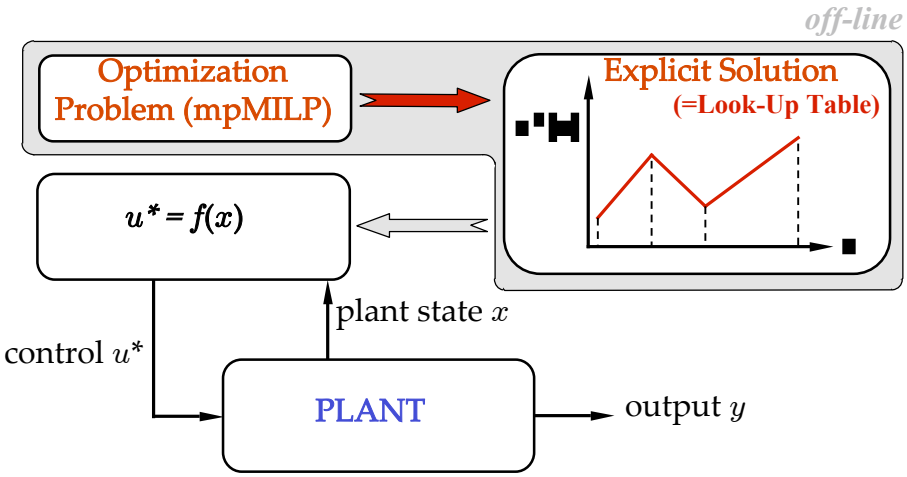
Receding Horizon Control

Receding Horizon Control *On-Line* Optimization



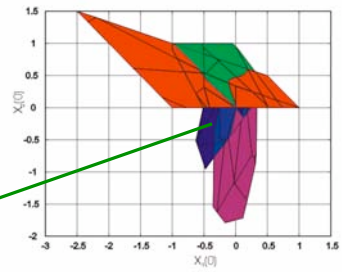
Receding Horizon Policy: *Off-Line* Opt. Multi-Parametric Programming



Characteristics of mp-MILP Solution

- The optimizer $z^*(x)$ is piecewise affine
- The feasible set X^* is partitioned into critical regions (CR).
- The value function $J^*(x)$ is piecewise affine.

$\| Qx \|_{\{1,\infty\}}$ CR is polyhedral



Receding Horizon Control
Stability Guarantee

Linear Performance Index (p=1,∞)

$$J^*(x) := \min_U \|Px_T\|_p + \sum_{k=0}^{T-1} \|Qx_k\|_p + \|Ru_k\|_p$$

Stability Guarantee

- Choose horizon $T=\infty$
  => Constrained Infinite Time Optimal Control

or

- Choose $||Px_T||_p$ to be a (Control) Lyapunov Function

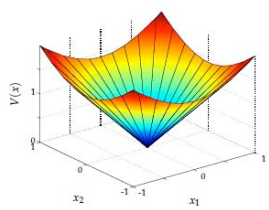
Receding Horizon Control
Stability Guarantee

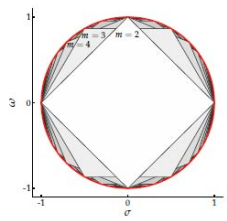
Linear Performance Index (p=1,∞)

$$J^*(x) := \min_U \|Px_T\|_p + \sum_{k=0}^{T-1} \|Qx_k\|_p + \|Ru_k\|_p$$

Stability Guarantee

Choose $||Px_T||_p$ to be a (Control) Lyapunov Function
=> Simple LP or algebraic test *Christophersen, 2006*





Multi-parametric controllers

Algorithms have been developed for over 5 years:

…Minimization of linear and quadratic objectives

(Baotic, Baric, Bemporad, Borrelli, De Dona, Dua, Goodwin, Grieder, Johansen, Mayne, Morari, Pistikopoulos, Rakovic, Seron, Toendel)

…Minimum-Time controller computation

(Baotic, Grieder, Kvasnica, Mayne, Morari, Schroeder)

…Infinite horizon controller computation

(Baotic, Borrelli, Christophersen, Grieder, Morari, Torrisi)

…Computation of robust controllers

(Borrelli, Bemporad, Kerrigan, Grieder, Maciejowski, Mayne, Morari, Parrilo, Sakizlis)

⇒ Advanced computation schemes are available !

Multi-parametric controllers

PROs:

- Easy to implement
- Fast on-line evaluation
- Analysis of closed-loop system possible

CONs:

- Number of controller regions can be large
- Off-line computation time may be prohibitive
- Computation scales badly.

⇒ controller complexity is the crucial issue

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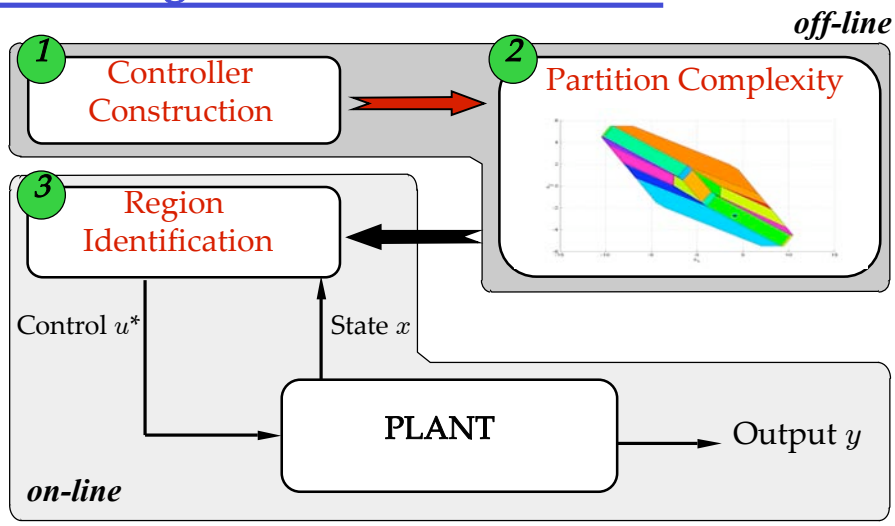
3 Complexity Levers of Receding Horizon Control



1st Lever for Complexity Reduction

Objective

Compute controller partition as quickly as possible.

Why is computation time an issue?

- For LTI systems, controller computation time correlates to controller complexity
- For PWA systems, controller computation time may not correlate to controller complexity

However…

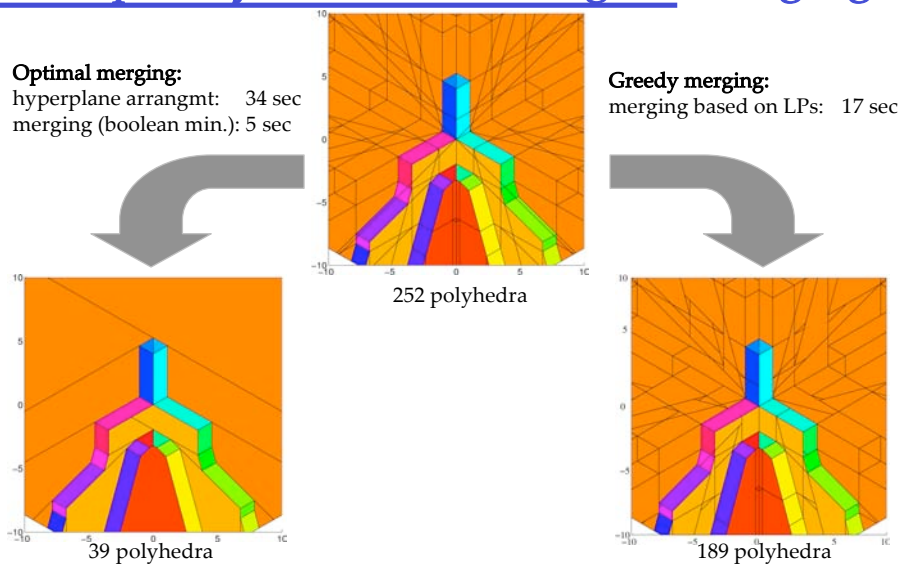
- Controller complexity and runtime grows exponentially with problem size

⇒ Controller computation is not a bottleneck

2nd Lever for Complexity Reduction - Region Merging

Optimal merging:
hyperplane arrangmt: 34 sec
merging (boolean min.): 5 sec

Greedy merging:
merging based on LPs: 17 sec



3rd Lever for Complexity Reduction

Objective

For a given partition, identify controller region as quickly as possible.

Algorithms

Identification of region in time logarithmic in the number of regions is possible *(Bemporad, Grieder, Johansen, Jones, Rakovic, Toendel)*

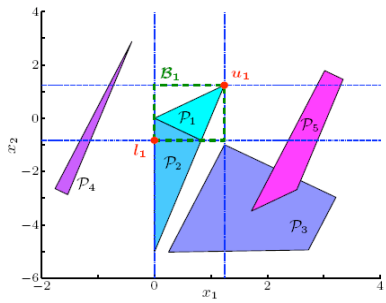
However…

- Pre-processing for most schemes is very expensive
- Usually not applicable to general partitions (e.g. overlapping regions, holes, non-convex partitions, etc.)

Region Identification using Bounding Boxes

Idea

Construct an *interval tree* based on bounding boxes. The search then reduces to answering stabbing queries in O(log(N)) time.



Advantage

- Very cheap pre-processing.
- Applicable to any type of partitions (not nec. polyhedral).

On-line strategy

- The tree selects possible candidate regions.
- Locally search the list of candidates.

(Christophersen et. al, submitted to CDC 2006)

Control Objective vs. Partition Complexity

Objective

– Compute controller partition with as few regions as possible
– Guarantee stability and constraint satisfaction

Observation

Complex objectives yield complex controllers

Approach

Use simpler objectives to obtain simpler controllers

⇒ Controller complexity is a bottleneck

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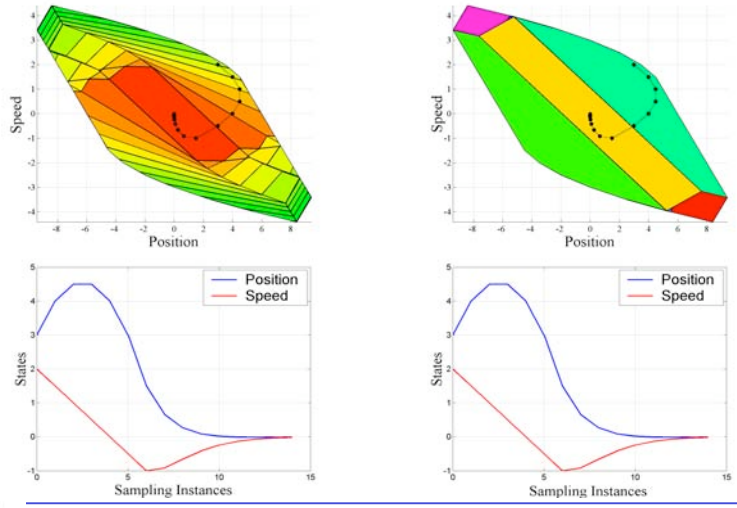
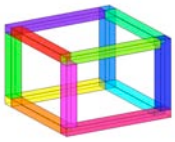
Numerical Example

Optimal Control | *N*-Step Control



Multi Parametric Toolbox

- All results and plots were obtained with the MPT toolbox



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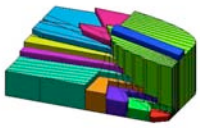
- MPT is a MATLAB toolbox that provides efficient code for
  - (Non)-Convex Polytope Manipulation
  - Multi-Parametric Programming
  - Control of PWA and LTI systems



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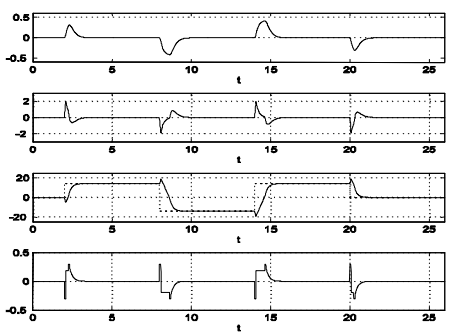
Furuta Pendulum

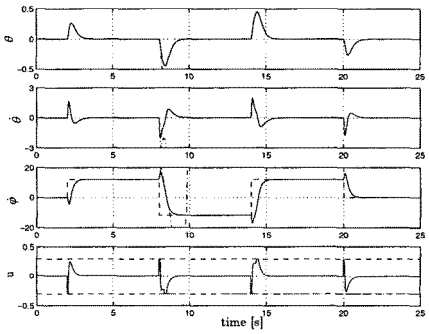
- Rotating inverted pendulum
- Explicit controller with 5 regions
- Results equivalent to *[Åkesson and Åström, 2001]*

$$d/dt \begin{bmatrix} \theta \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \alpha & 0 & 0 \\ \beta & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \theta \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ \beta \\ \delta \end{bmatrix} u(t)$$

$$|u(t)| < 0.3$$

[Akesson and Aström, 2001]

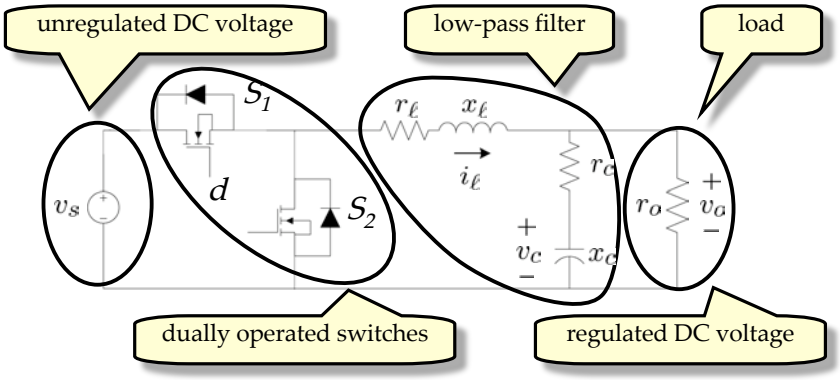




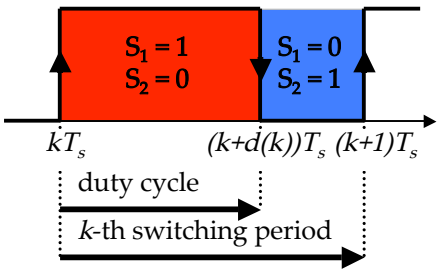
Switch-mode DC-DC Converter

Switched circuit: supplies power to load with constant DC voltage

Illustrating example: **synchronous step-down** DC-DC converter

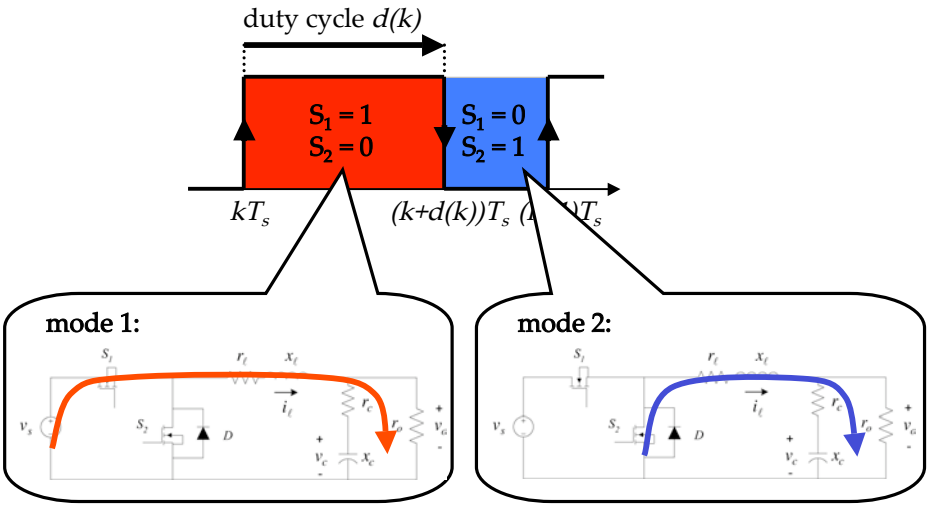


Operation Principle



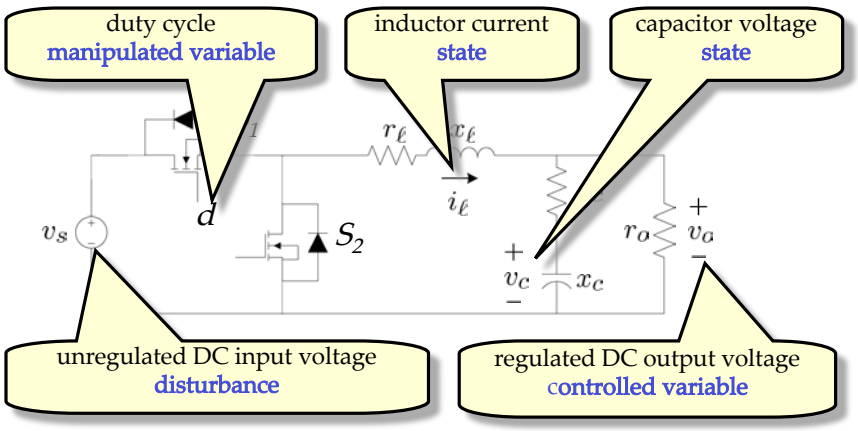
- Length of **switching period** T_s constant *(**fixed switching frequency**)*
- **Switch-on** transition at kT_s, $k \in N$
- **Switch-off** transition at $(k+d(k))T_s$ *(**variable pulse width**)*
- **Duty cycle** $d(k)$ is real variable bounded by 0 and 1

Mode 1 and 2



Control Objective

Regulate DC output voltage by appropriate choice of duty cycle

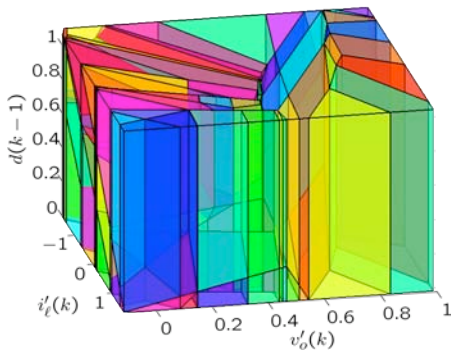


State-feedback Controller: Polyhedral Partition

PWA state-feedback control law:

computed in 100s using the MPT toolbox

121 polyhedra after simplification with optimal merging



Colors correspond to the 121 polyhedra

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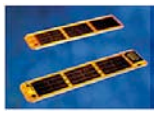
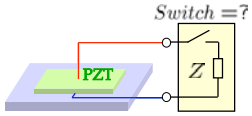
Smart Damping Materials

Niederberger, Moheimani






- Demands
  - Device *suppresses vibration*
  - *External power* source for operation is not required
  - *Weight* and *size* of the device have to be kept to a minimum
- Idea
  - *Switched Piezoelectric (PZT) Patches*



- Problem
  - *What is the optimal switching law for optimal vibration suppression?*

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Application: Brake Squeal Reduction

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Neubauer, Popp

- Friction induced vibration in brakes
  - Strong vibration radiates unwanted noise
  - One frequency, small bandwidth
  - Frequency can vary



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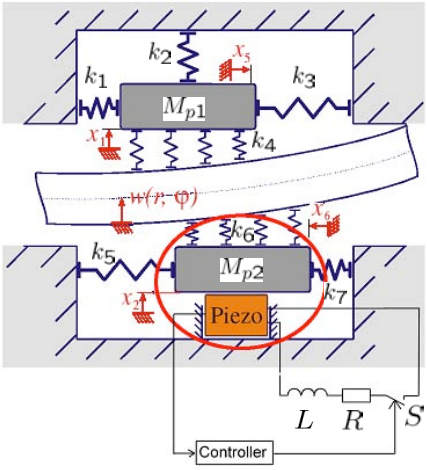
Brake Squeal Reduction using Shunt Control

INSTITUT FÜR MECHANIK
der Universität Hannover

- Vibration reduction
  - Piezoelectric actuator between brake pad and calliper
  - Switching shunt control
- Advantages
  - Tracks resonance frequency
  - Cheap solution
  - No electrical power required



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Other Applications of Hybrid Systems Control Tools

- Control / Scheduling of Cogeneration Power Plants
- Control / Scheduling of Cement Kilns and Mills
- Electronic Throttle Control / Traction Control
- Control of Thermal Print Heads
- Control of Voltage Stability for Electric Power Networks
- Direct Torque Control of 3 phase Induction Motors
- Control of Anaesthesia (Inselspital Bern)
- Adaptive Cruise Control

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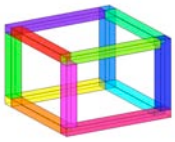

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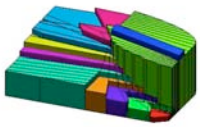

Multi Parametric Toolbox

4000+ downloads

Rated 4.5 / 5 on `mathworks.com`

Conclusions

- Foundations of a theoretical framework for for practical controller design for PWA system have been established
- Complexity reduction and robustness are main research issues
- Applications in industry are beginning