Relay Systems - The Simplest Hybrid Systems?

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1. Introduction
2. Examples of Relay Systems
3. Relay Auto-Tuning
4. Limit Cycles
5. Summary

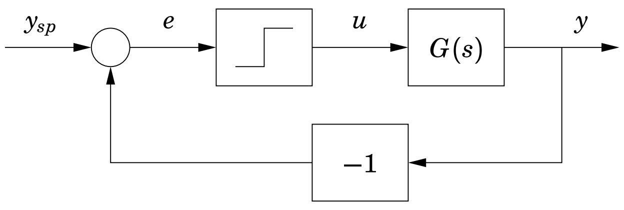
Thanks to K. H. Johansson, A Rantzer and S. E. Mattson

Introduction

- The Simplest hybrid system?
  - Two discrete states
  - Linear continuous behavior
- Relay systems are common
- Relay systems have been studied for a long time
- Relays systems are still widely used
  - Delta-Sigma modulators
  - Relay auto-tuning
- Relay systems have rich behavior
- Challenges
- Can theory for hybrid systems help?

A Challenge

Understand the behavior of the system



Some things are well known but important problems remain

Find all transfer functions such that there is a stable limit cycle

I hope to tell you why this is interesting

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Relay Systems are Common

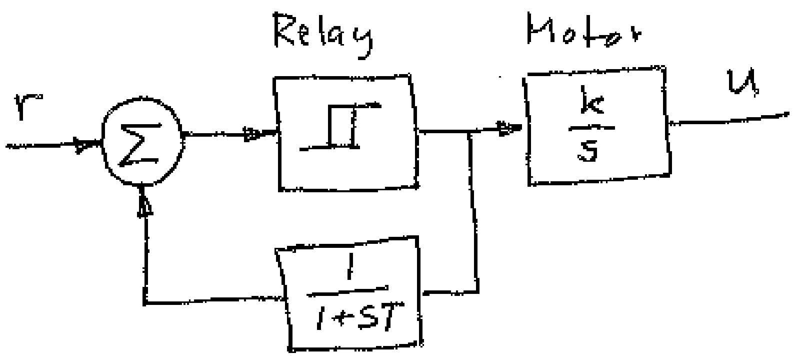
Applications of relay feedback include

- On–off control, temperature control (Hawkins,1887)
- Amplifiers
- Self-oscillating adaptive systems
- Relay auto-tuning
- DC/DC converters
- Δ-Σ modulators
- Coulomb friction
- Variable structure systems

A Classic Field

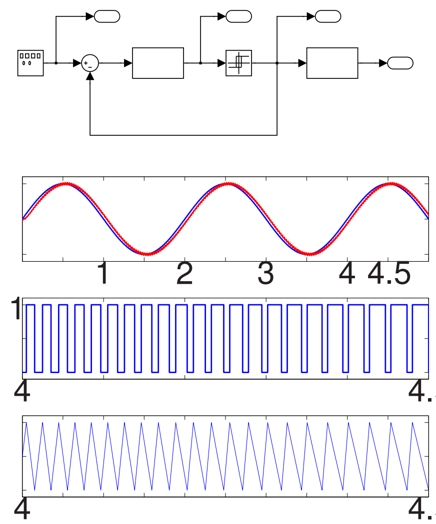
- J. T. Hawkins Automatic regulators for heating apparatus. Trans ASME 1887.
- I. Flügge-Lotz Discontinuous Automatic Control. 1953
- Andronov, Vitt and Khaikin, Theory of Oscillators. 1965
- Ahterton Nonlinear Control Engineering. 1975
- Tsypkin Relay Control Systems. 1984
- Filippov Differential equations with Discontinuous Right Hand Side. 1988.
- Utkin. Sliding Mode Control. 1992
- Aizerman 1974

PI Temperature Controller



$$G(s) = \frac{k}{s}\frac{k_r}{1+\frac{k_r}{1+sT}} = k\frac{1+sT}{s} = kT + \frac{k}{s}$$

The Delta-Sigma Modulator



Cousins of the Delta-Sigma Modulator

- One-bit AD converter
  - Sampled versions
  - Telecom systems
- One bit stereo amplifier

Challenging problems

- How to avoid ringing
- System design
  - Trial and error - patent coefficients
  - Model predictive control
  - Daniel Quevedo: Quantized Moving Horizon Optimization PhD thesis University of Newcastle

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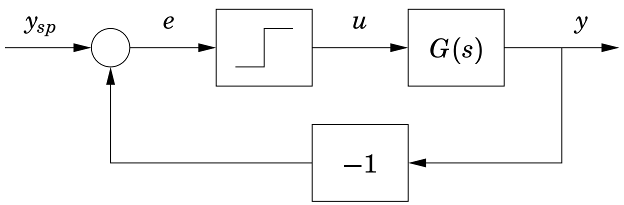
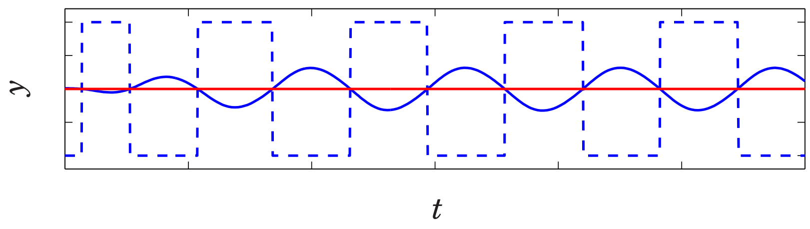
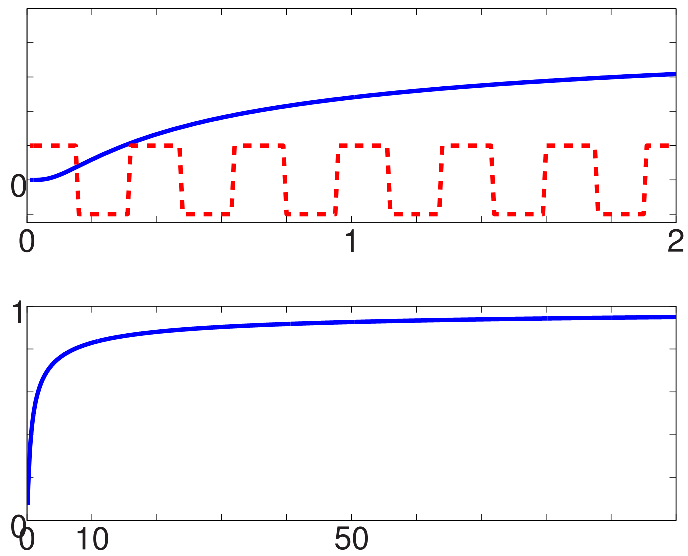
Relay Auto-Tuning

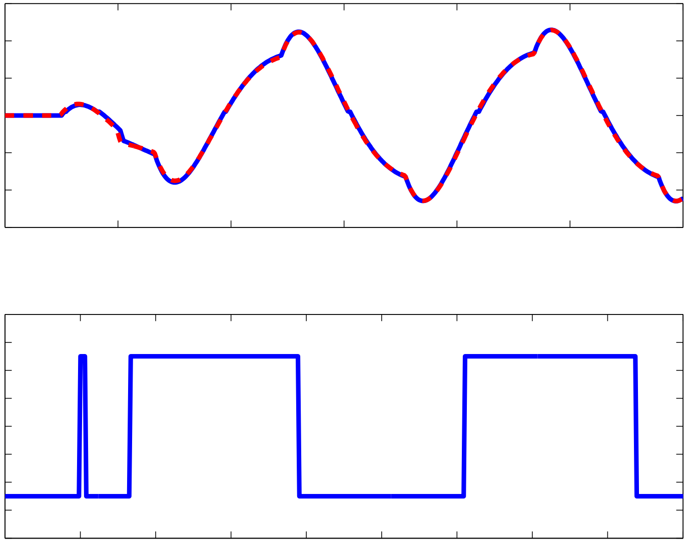




Short Experiment Time $G(s) = exp(-\sqrt{s})$

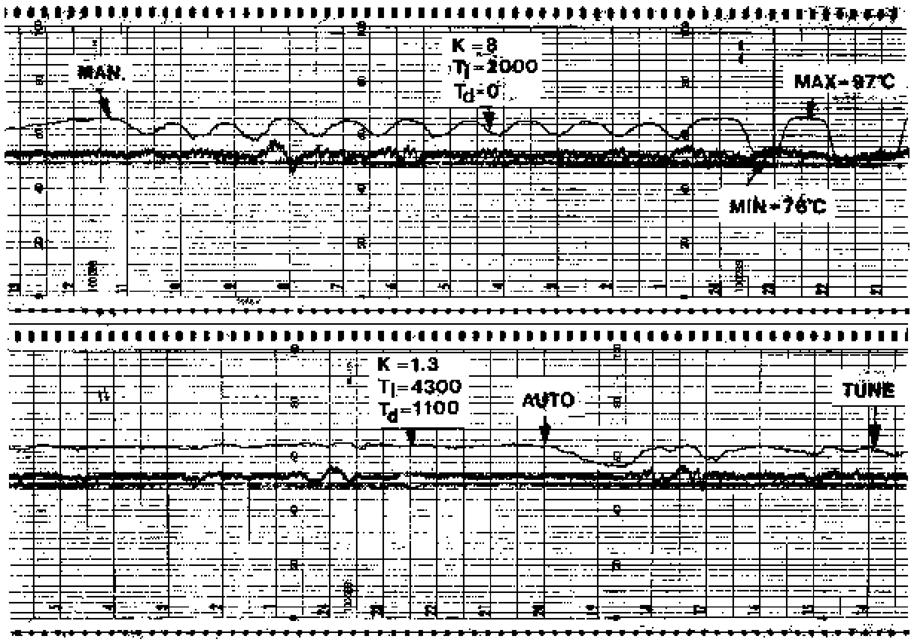


Good Excitation



Commercial Auto-Tuners

- Easy to use
  - one-button tuning
- Robust
- Many versions
  - Stand alone
  - DCS systems
  - Estimation methods
  - Control design
- Large numbers
- Good industrial experience



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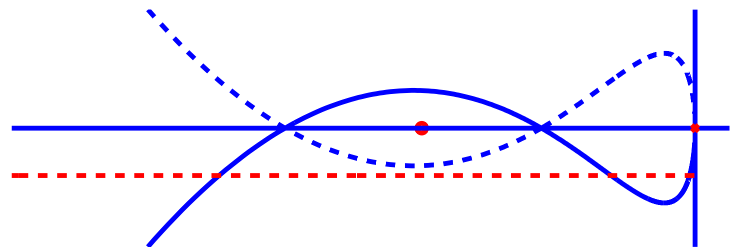
Limit Cycles

- Describing functions

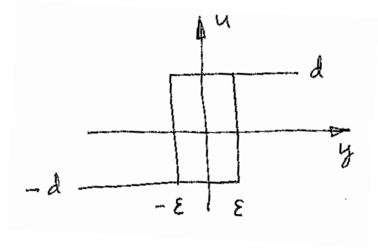


- Tsypkins frequency domain conditions
- Necessary conditions
- Local stability
- Limit cycles with chattering

Necessary Conditions

Assume $CB = 0$, relay with hysteresis. Assume that the system is in state -a on a symmetrical periodic orbit when control switches from $-d$ to d. Integrate over a half period h

$$
\begin{aligned}
\frac{dx}{dt} &= Ax + Bu, \quad y = Cx \\
x(h) &= a = -e^{Ah}a + \int_0^h e^{As} ds B d \\
&= -\Phi(h)a + \Gamma(h)d \\
Cx(h) &= \varepsilon
\end{aligned}
$$



$$
f(h) = C\left(I + e^{Ah}\right)^{-1} \int_0^h e^{As\tau} d\tau = \frac{\varepsilon}{d}
$$

Necessary Conditions for a Limit Cycle

Consider a linear system with hysteretic relay feedback. Assume that there exists a symmetric periodic solution with period $T = 2h$. Then the following conditions hold.

$$
(i) \qquad f(h) = C(I + e^{Ah})^{-1} \int_0^h e^{A\tau} B d\tau = \frac{\varepsilon}{d}
$$

$$
(ii) \qquad y(t) = C\left(e^{At}a - \int_0^t e^{A\tau} d\tau B d\right) < \varepsilon \text{ for } 0 \leq t < h
$$

It follows from (ii) that

$$
y'(0) = -Ca
$$

The periodic solution is obtained with the initial condition

$$
x(0) = a = (I + e^{Ah})^{-1} \int_0^h e^{A\tau} B d\tau d
$$

Local Stability

Assume that there is a symmetric periodic solution with period $2h$. Let a be the initial state that generates the periodic motion. The Jacobian of the Poincaré map is given by

$$
W = \left(I - \frac{vC}{Cv}\right) e^{Ah},
$$

where $v = Aa + Bd$. The limit cycle is locally stable if and only if W has all its eigenvalues inside the unit disk.

The eigenvalues give the convergence rate.

Global stabiliy. Conditions given by Goncalves using Lyapunov functions and LMIs.

Analysis of Limit Cycles for a Particular System

Straight forward computations

- Solve
$$f(h) = C(I + e^{Ah})^{-1} \int_0^h e^{A\tau} B d\tau = \frac{\varepsilon}{d}$$
- Compute initial state
$$a = (I + e^{Ah})^{-1} \int_0^h e^{A\tau} B d\tau d$$
- Check y
- Check gloabal stability using LMIs

Frequency Domain Interpretation

Let $H(z)$ be the pulse transfer function corresponding to zero-order hold sampling of the system with sampling period h

$$\frac{dx}{dt} = Ax + Bu, \qquad y = Cx$$

i.e.

$$H(z) = C\Big(zI - \Phi(h)\Big)^{-1}\Gamma(h)$$

The the condition

$$f(h) = C\Big(I + \Phi(h)\Big)^{-1}\Gamma(h) = \frac{\varepsilon}{d}$$

can be written as

$$H(-1) = -\frac{\varepsilon}{d}$$

Physical interpretation! Infinite dimensional systems.

Frequency Domain Interpretation - Tsypkin Locus

Condition for oscillation

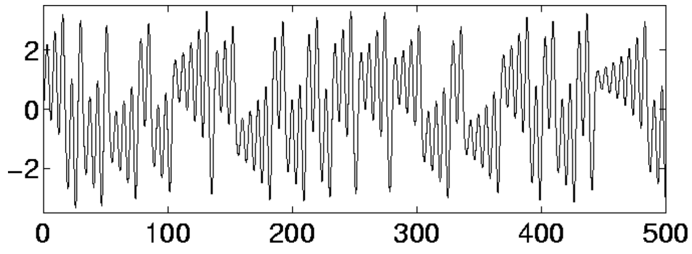
$$H(z) = \sum_{n=-\infty}^{\infty} \frac{1 - z^{-1}}{\log z + 2\pi i n} G\left(\frac{1}{h}\log z + \frac{2\pi i n}{h}\right) = -\frac{\varepsilon}{d}$$

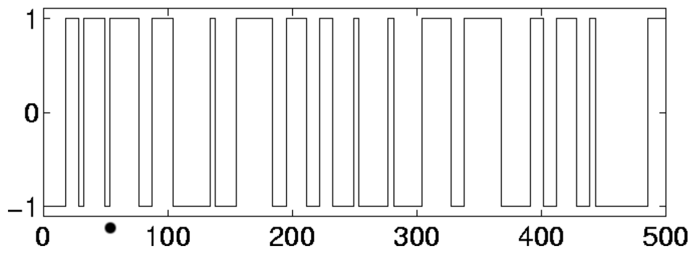
Introducing $\omega = \pi/h$ gives

$$H(-1) = \frac{2}{i\pi} \sum_{n=-\infty}^{\infty} \frac{G(i\omega(1+2n))}{1+2n} \approx \frac{4}{i\pi} \operatorname{Im} G(i\omega)$$

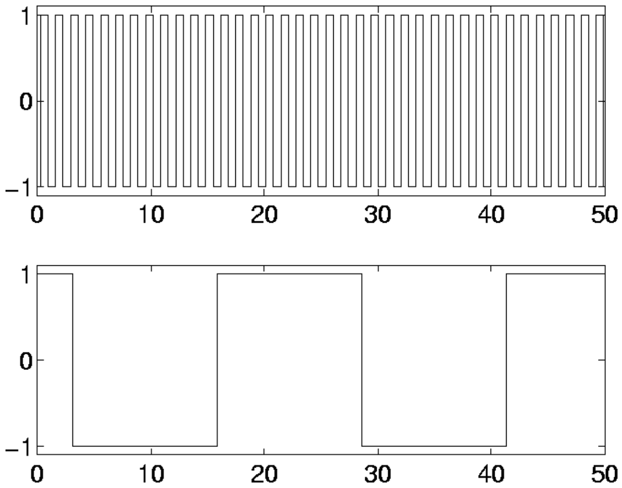
and the condition for oscillation is that the Nyquist plot of $G(s)$ intersects the half line $s = -i\varepsilon\pi/4d$. (Describing function condition.) The Tsypkin locus (hodograph) gives exact condition.

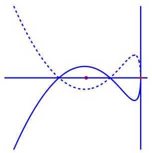
Asymmetrical Oscillations $G(s) = \dfrac{1}{s^2 - 0.1s + 1}$





Many Limit Cycles $\dfrac{(s+1)^2}{(s+0.1)^3(s+7)^2}$





Limit Cycles with Sliding Modes $\dfrac{(s-1)^2}{(s+1)^3}$

$$\dot{x}(t) = \begin{pmatrix} -3 & 1 & 0 \\ -3 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} x(t)$$

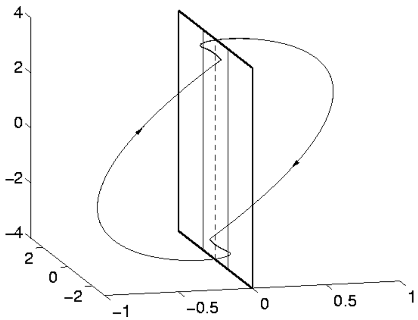
$$u(t) = -\operatorname{sgn} y(t) = -\operatorname{sgn}\Big(x_1 + u\Big)$$

Sliding mode $|x_1| < 1$

$$x_1 = 0$$

$$\frac{dx_2}{dt} = x_3$$

$$\frac{dx_2}{dt} = 0$$



Limit Cycles with Sliding Modes $G(s) = \dfrac{(s-0.2)^2}{(s+1)^4}$

$$\dot{x}(t) = \begin{pmatrix} -4 & 1 & 0 & 0 \\ -6 & 0 & 1 & 0 \\ -4 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 1 \\ -0.4 \\ 0.04 \end{pmatrix} u(t)$$

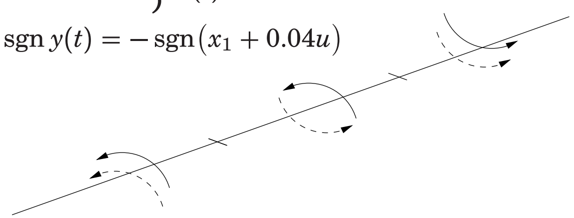
$$y(t) = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} x(t)$$

$$u(t) = -\operatorname{sgn} y(t) = -\operatorname{sgn}(x_1 + 0.04u)$$

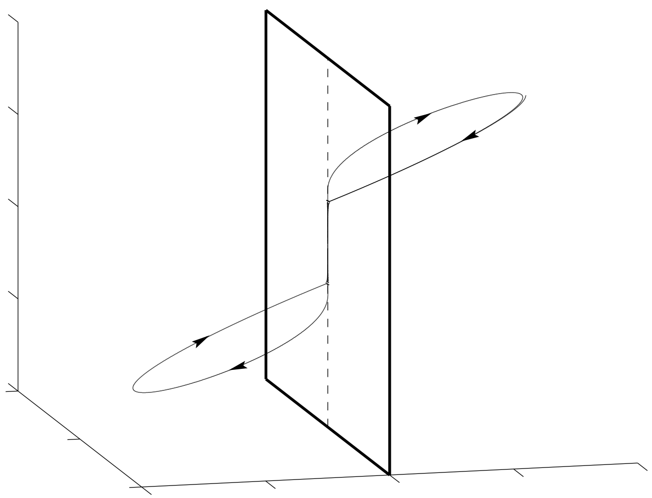
$CB = 0$, $CAB = 1$

2nd order sliding

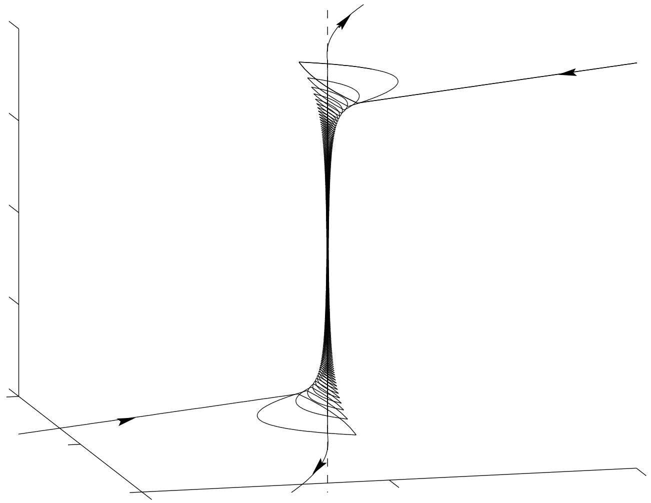
$|x_1| < 0.04$



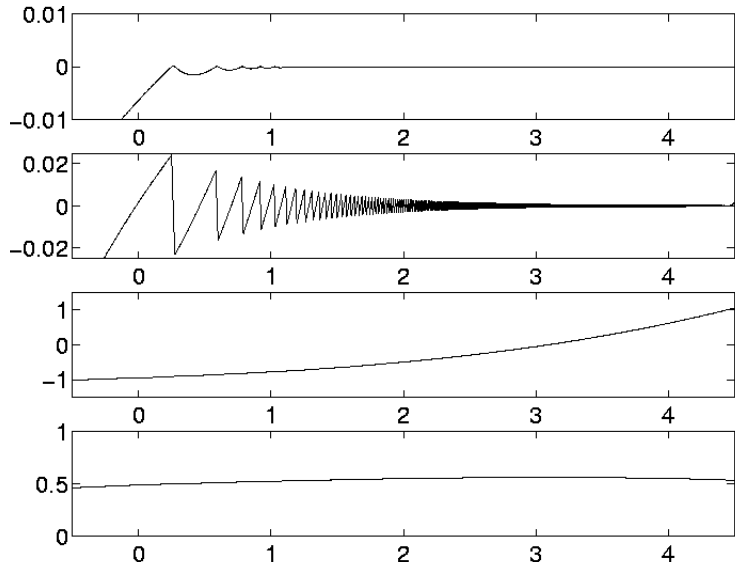
Projection of State Space on R^3



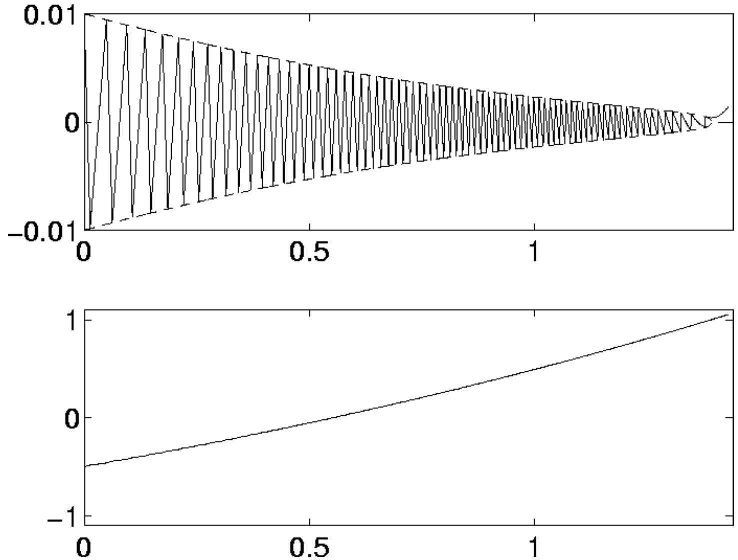
Projection of State Space on R^3 - Details



Time Functions



Accurate Estimates of Amplitudes



Simulation Difficult when Chattering Occurs

- Hybrid systems very useful
  - Detect that chattering occurs
  - Switch to Fillipov solution
  - Switch to regular solver when chattering stops $x_1 = 1$!
- Detection straight forward analytically but difficult to do automatically
- Omola-Omsim (Mattson et al)
- Modelica `http://modelica.org`

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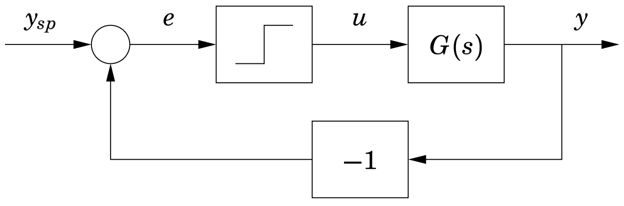
Summary

- Relay system
  - Simple discrete behavior - only two states
  - Continuous behavior - linear
- In spite of this they have rich behavior
- Many useful applications
  - Simple feedback loops
  - Delta-sigma modulators
  - Relay auto tuners
- Challenging problems
  - Sampling theorems for amplitude quantized signals
  - Design theory for delta-sigma modulators
  - Classify systems where relay tuning works

Classify Systems where Relay Feedback Works



Find all transfer functions such that there is a stable limit cycle!

Conjectures

- Monotone frequency responses
- Almost monotone step responses

References

Andronov, A. A Khaikin, S. E. and Vitt,A. A. Theory of Oscillators, Dover 1987.

Åström, K. J. Oscillations in systems with relay feedback. In Åström, et al (Editors) Adaptive Control, filtering and signal processing. IMA Volumes in Mathematics and Its Applications Vol 75, pp 1–25 Springer 1995.

Åström, K. J. and Hägglund, T. Advanced PID Control. ISA 2006.

Atherton, D. P. Nonlinear control engineering: Describing functin analysis and design. Van Nostrand 1975.

Filippov,A. F. Differential Equations with Discontinuous Right Hand Sides.Kluwer 1988.

Flügge-Lotz, I. Discontinuous automatic control. Princeton 1953.

Flügge-Lotz, I. Discontinuous and optimal control. Princeton 1968.

References

Johansson, K. H. Rantzer, A. and K.J. Åström. Fast switches in relay feedback systems. Automatica 35 (1999) 539–552.

Johansson, K. H. Barabanov and Åström, K. J. Limit cycles with chattering in in relay feedback systems. IEEE TAC-47 (2002)1414-1423

Misawa E. A. and L. M. Fridman. (Eds)Fundamentals of Sliding Mode Control Wiley-IEEE 2006

Tsypkin, Ya. Z. Relay Control Systems, Cambridge 1984. (Original Russian book 1955)

Utkin, V. I. Sliding Modes in Control and Optimization. Springer 1992.