On Synthesizing Controllers from Bounded-Response Properties

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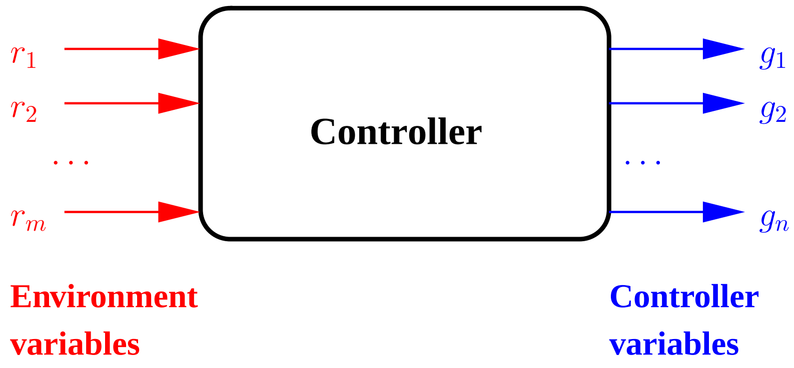
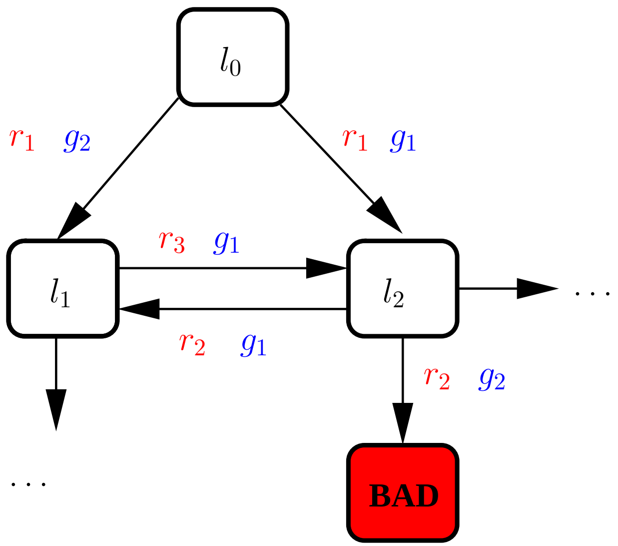
Overview

- Introduction
- Property-based Synthesis
  - Bounded-response Properties
- MTL-B
  - Syntax and Semantics
  - Non-Determinism
- From MTL-B to Deterministic Temporal Testers
  - Pastification of MTL-B formulae
  - Bounded-variability assumption
- Application to Synthesis: Arbiter Example
  - Specification in MTL-B
  - Experimental Results
- Conclusion





- Automatic controller synthesis from high-level specifications
  - Problem posed in [Chu63]
  - Theoretically solved in [BL69,TB73]

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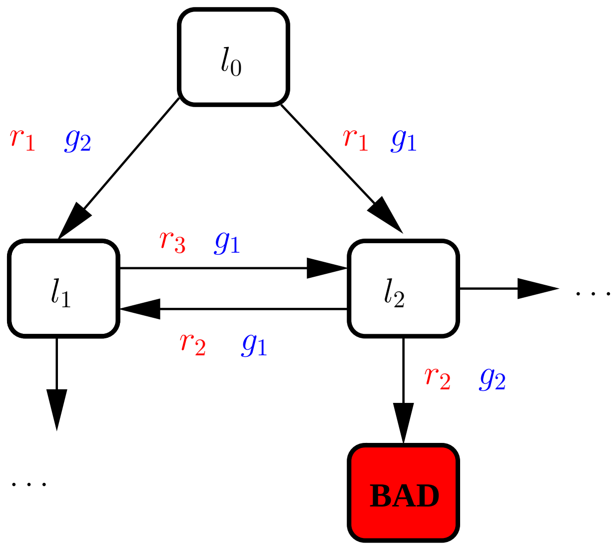

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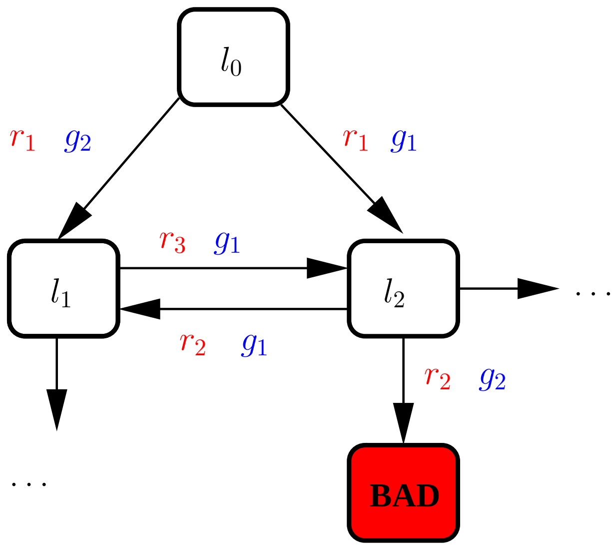

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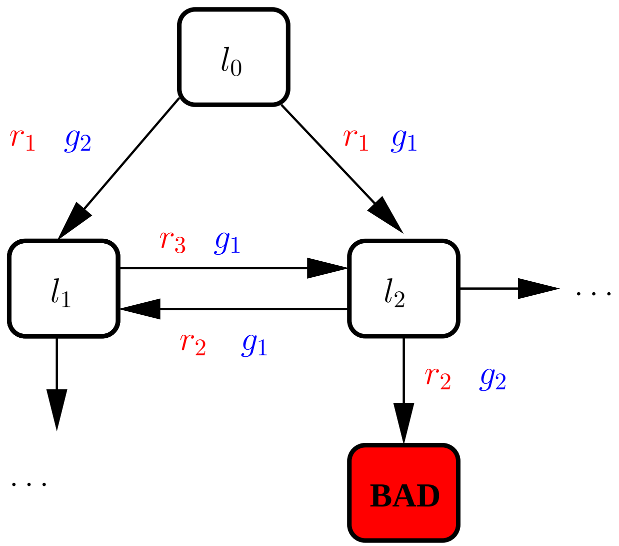

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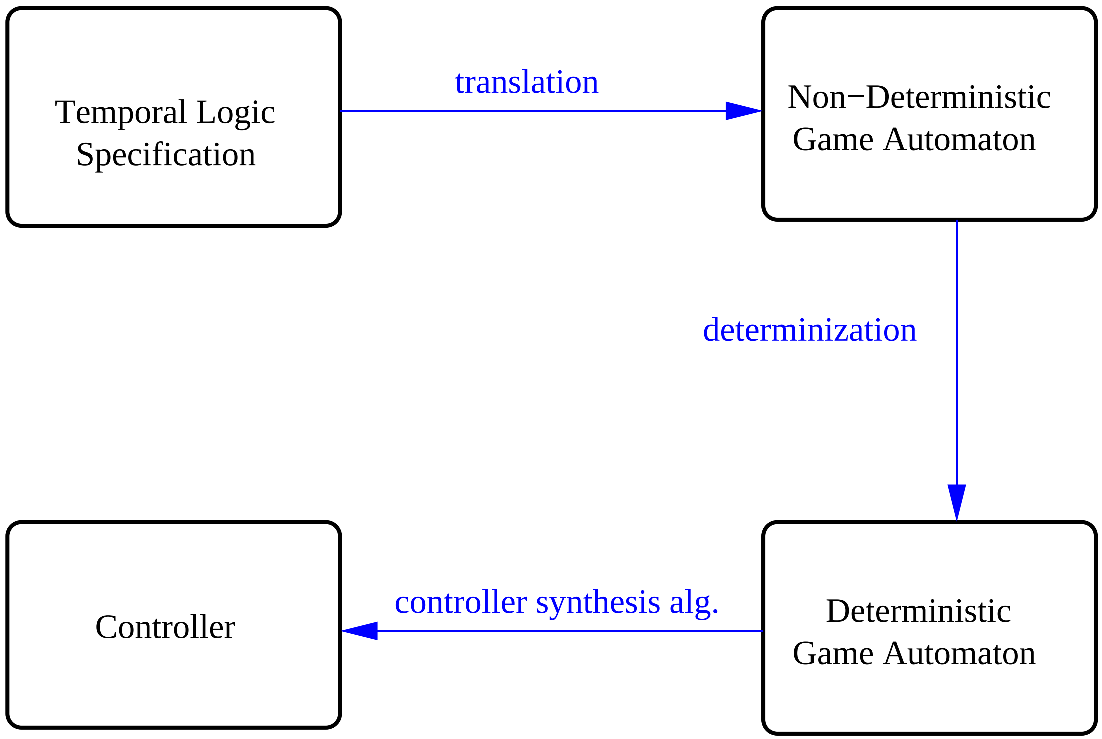

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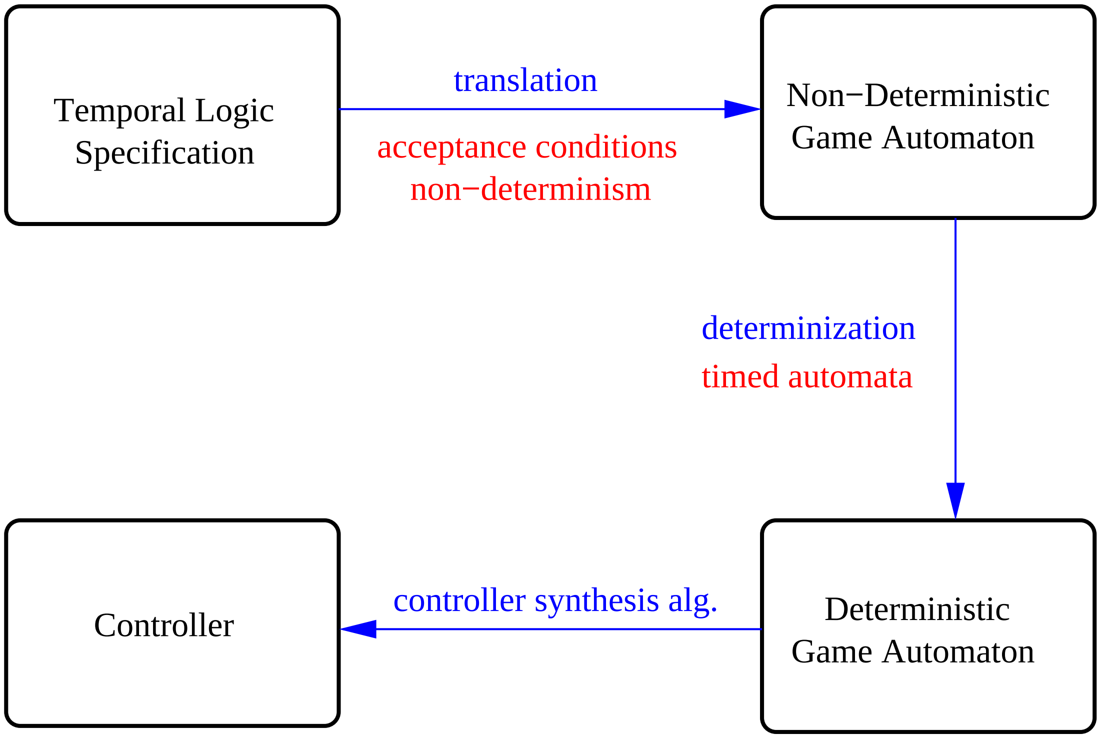

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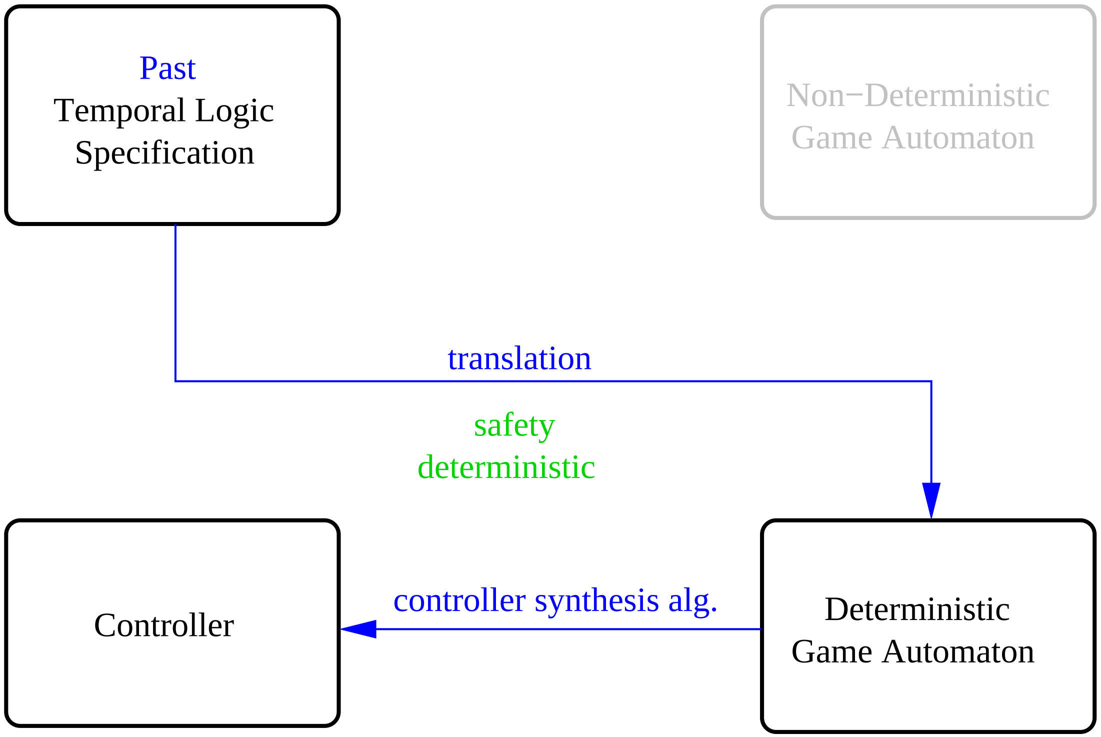

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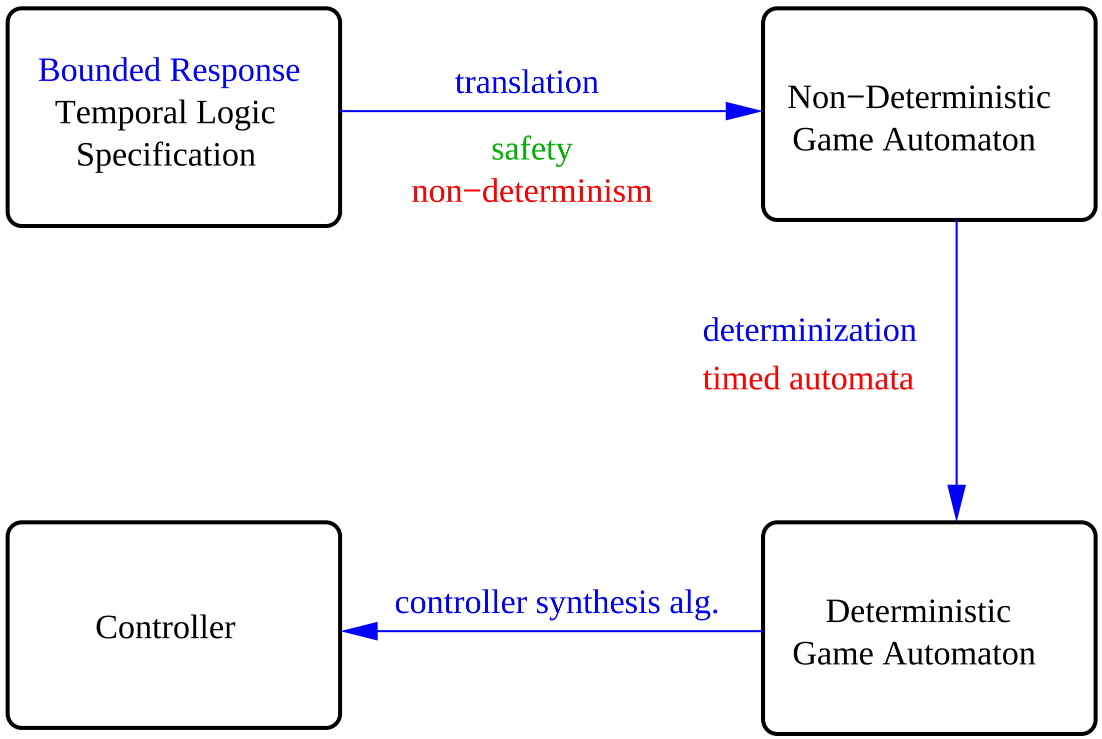

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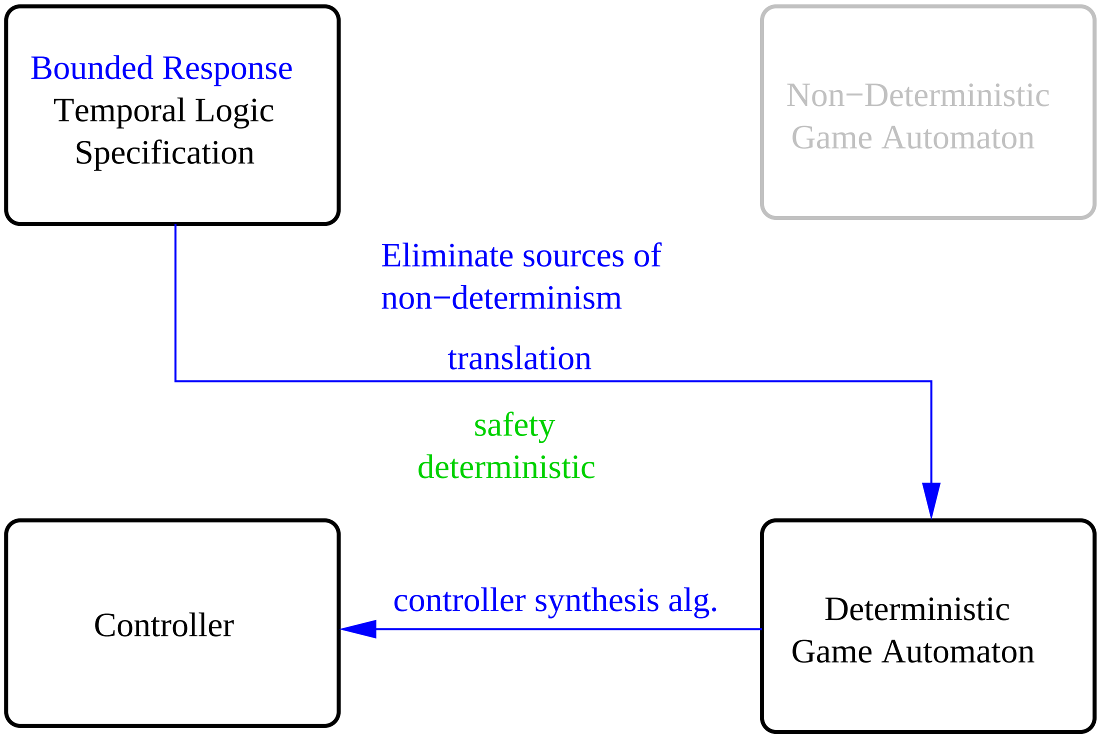

Motivation for Bounded-Response Properties

- **Bounded-response** correspond to **safety** properties
  - $\longrightarrow$ Limited scope wrt more general **liveness** properties
- Liveness properties abstract away the upper bound requirement of occurrence of events
  - But many applications require specifying explicitly such upper bound:
    - Hard real-time systems
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  - MTL [Koy90] without **unbounded until**
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MTL-B: *Syntax and Semantics*

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$$\varphi := p \mid \neg\varphi \mid \varphi_1 \vee \varphi_2 \mid \varphi_1 \mathcal{U}_{[a,b]} \varphi_2 \mid \varphi_1 \mathcal{S}_{[a,b]} \varphi_2 \mid \varphi_1 \mathcal{S} \varphi_2 \mid \varphi_1 \mathcal{P}_{[a,b]} \varphi_2$$

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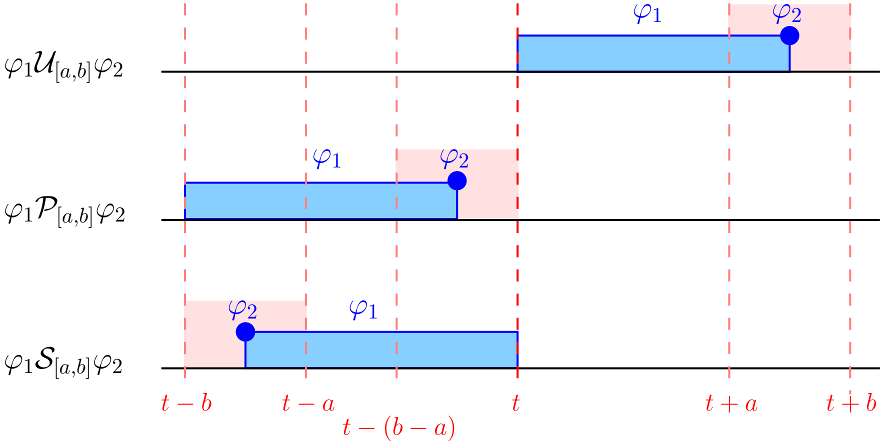

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- **Notes:**
  - “Handshake” semantics of bounded until
  - **Precedes** operator $\sim$ past equivalent of bounded until
- **Derived operators:** $\diamondminus_{[a,b]}, \boxminus_{[a,b]}, \Diamond_{[a,b]}, \Box_{[a,b]}$

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MTL-B *and Non-Determinism*

- Two sources of non-determinism
- Acausality
  - Semantics of **future** temporal logics **acausal**
    - Satisfiability of φ at time t depends on the input signal value at time $t' \geq t$
  - **Past** fragments of temporal logics have **causal** semantics
- Unbounded Variability
  - **No bound** on the **variability** of input signals
  - $\rightarrow$ remember **unbounded number of events**
    - **Example:** $\diamondminus_1 p$ - perfect shift register for p

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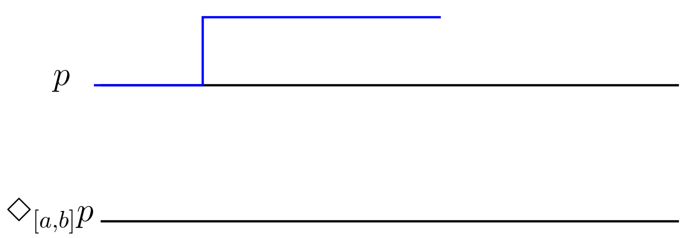

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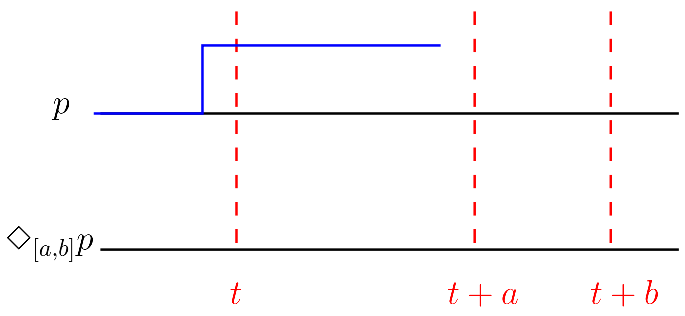

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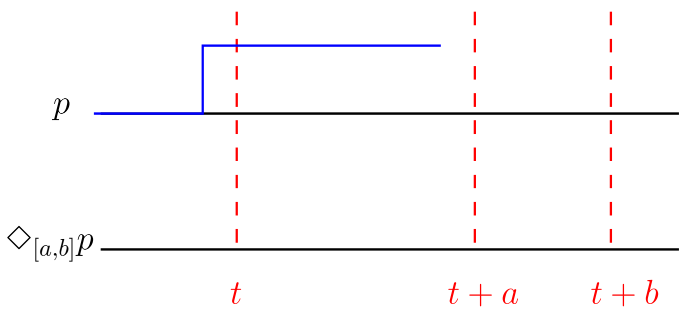

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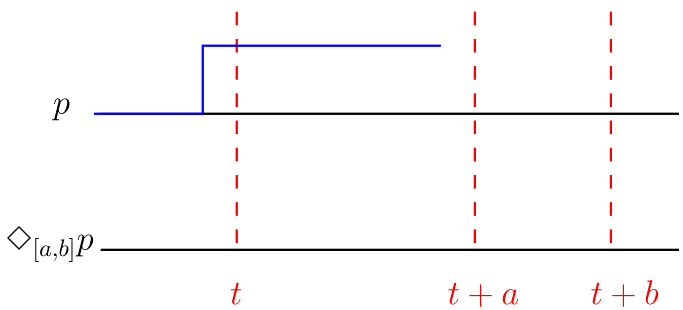

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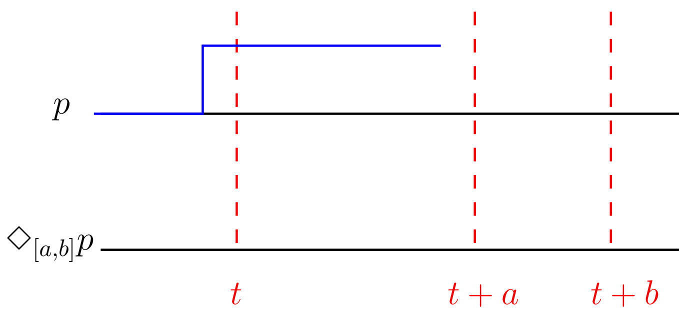

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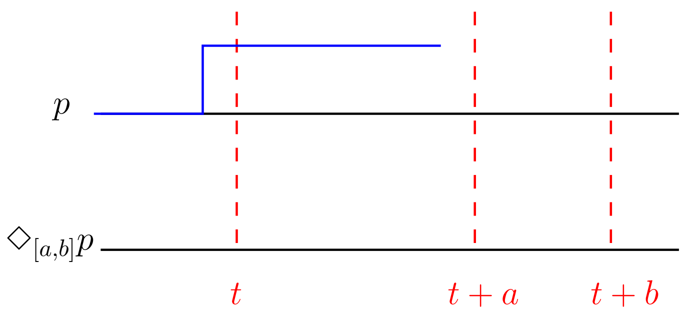

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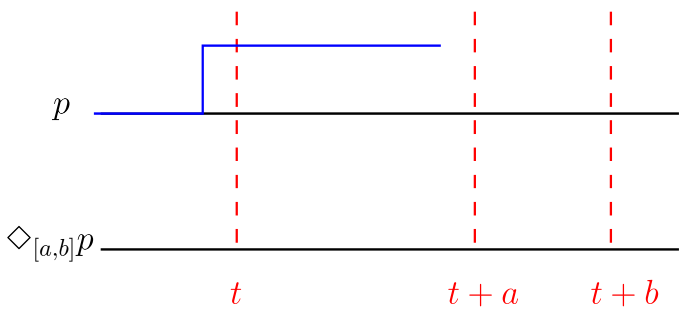

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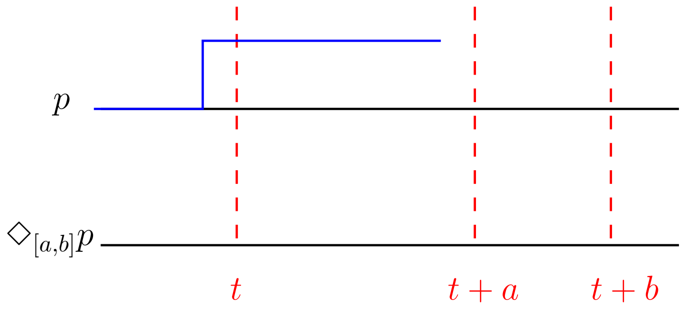

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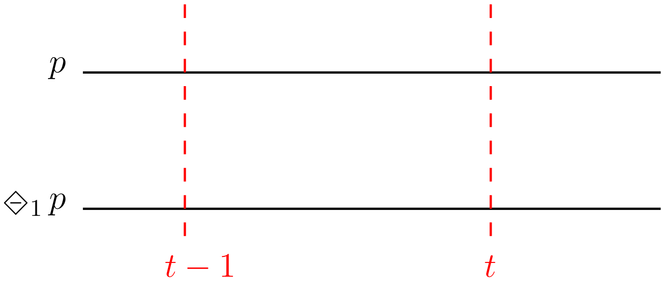

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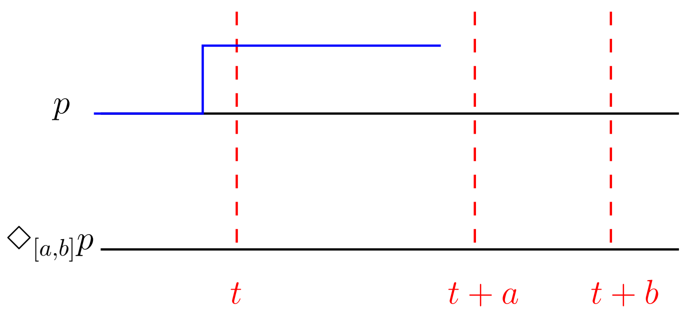

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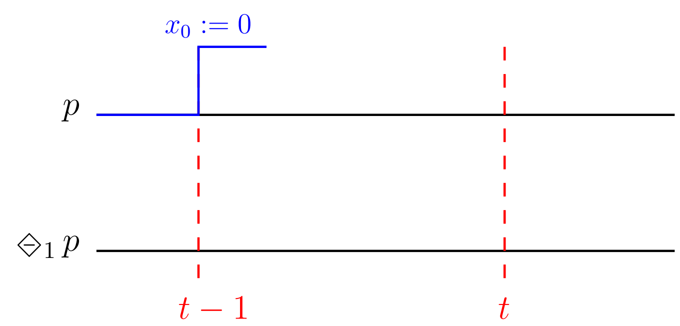

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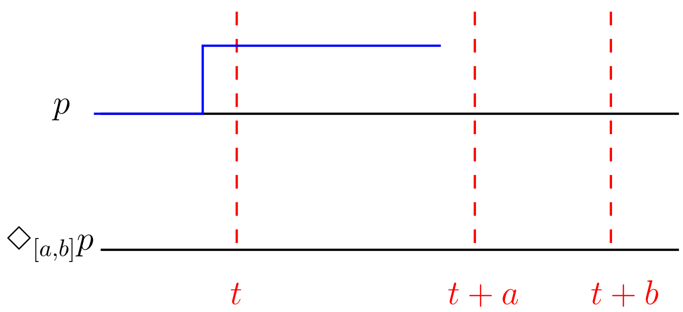

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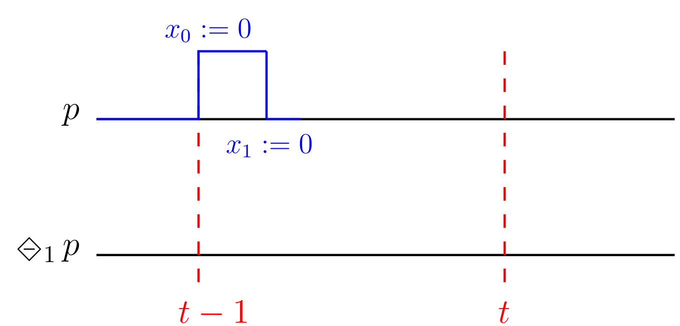

- Two sources of non-determinism
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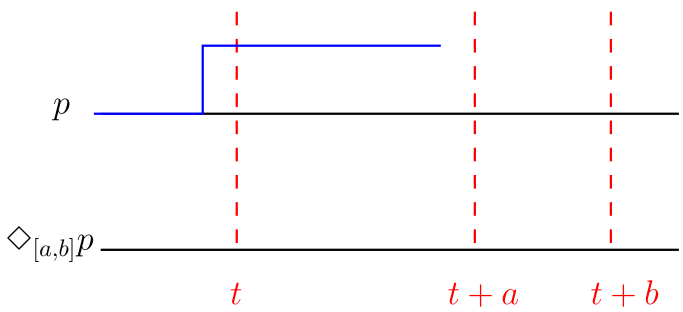

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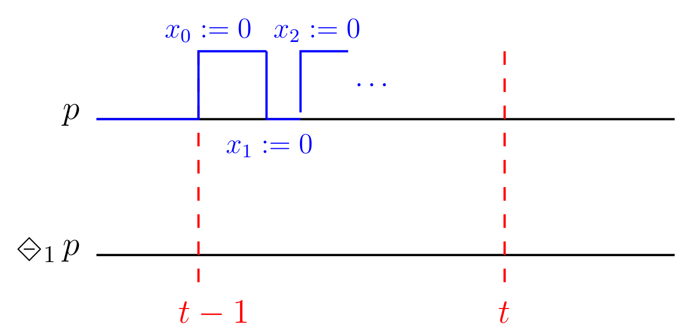
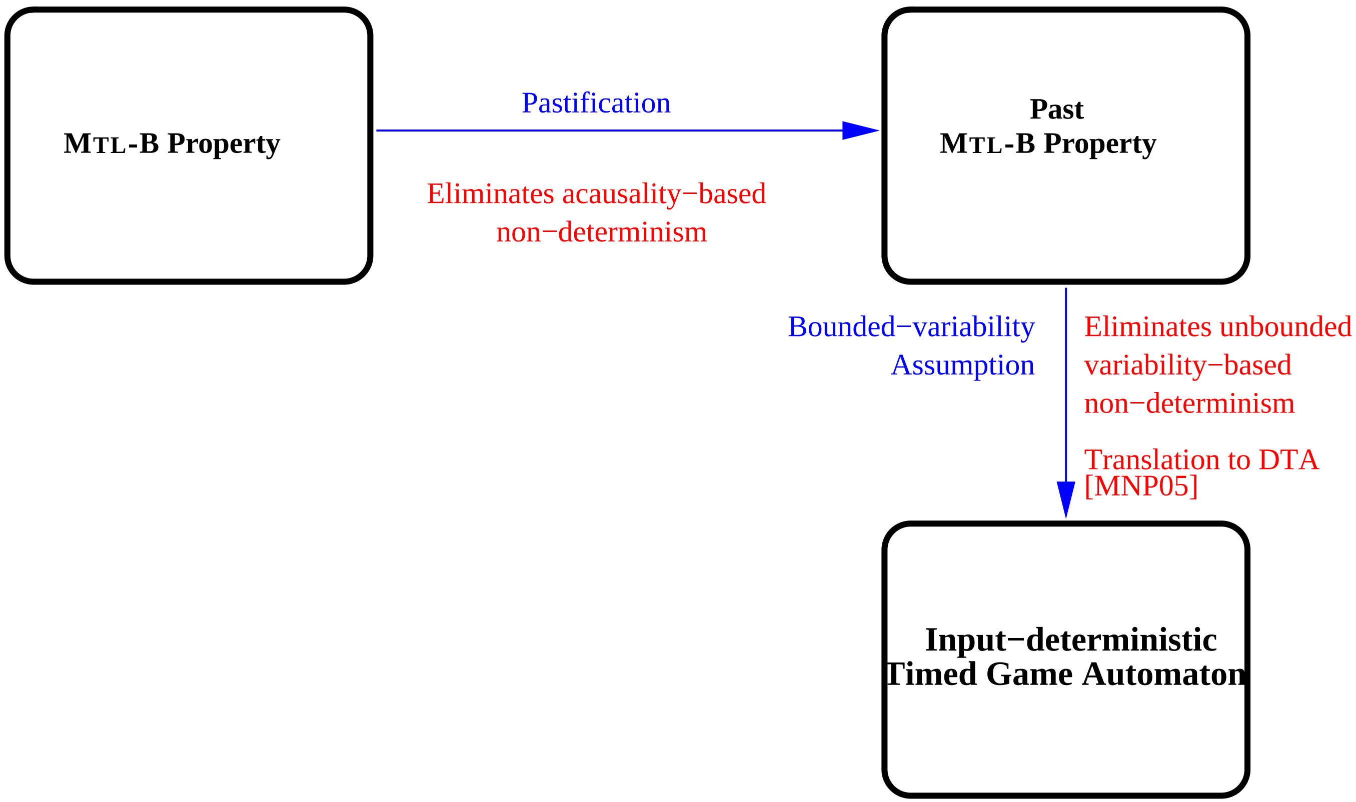


From MTL-B *to Deterministic Timed Automata: Overview*

MTL-B Property

From MTL-B to Deterministic Timed Automata: Overview


MTL-B Property
Pastification
Eliminates acausality−based non−determinism
Past MTL-B Property
Bounded−variability Assumption
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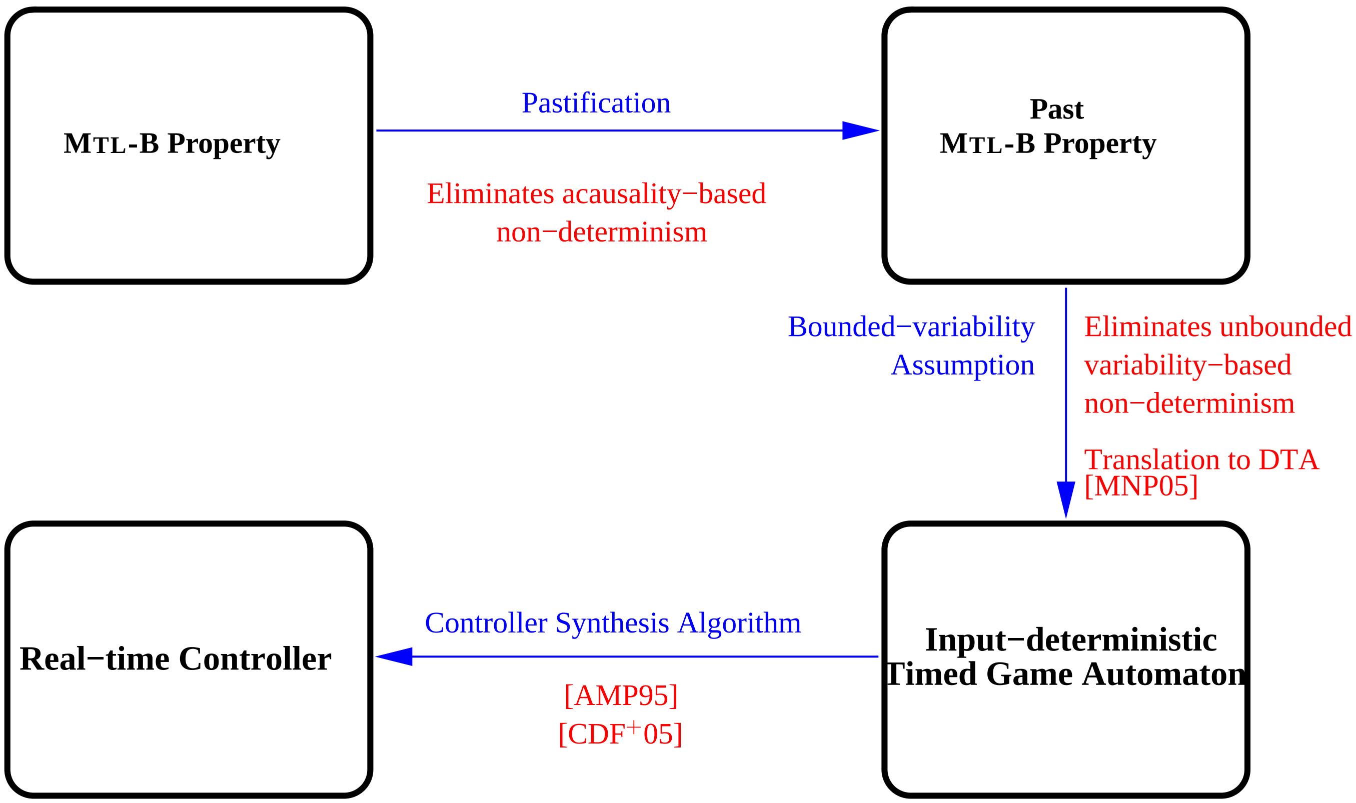

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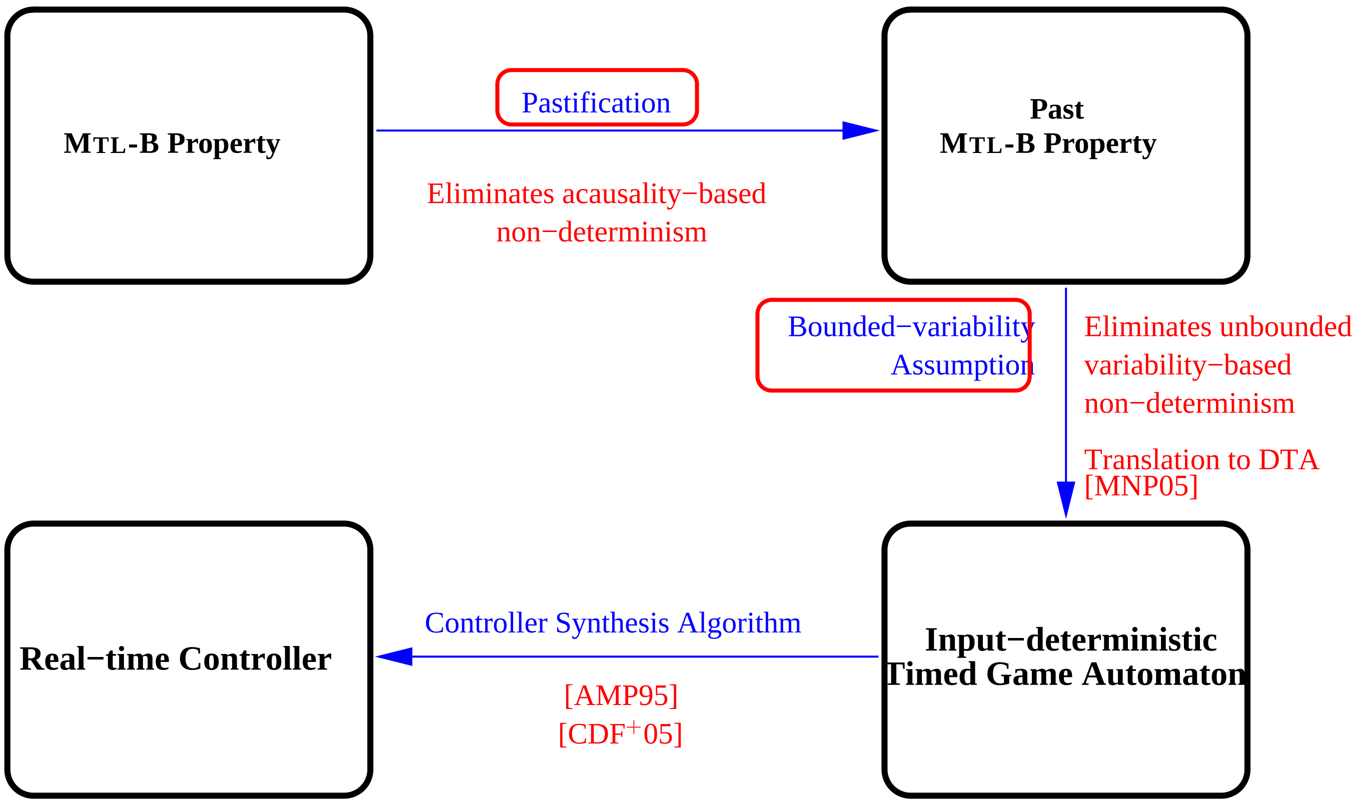

Pastification of MTL-B *formulae*

- **Key idea:** Change the time direction from **future** to **past**
  - MTL-B formula fully determined withing a bounded horizon
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- **Example:** $\varphi = p \rightarrow \Diamond_{[1,2]} \Box_{[0,2]} q$
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$\ldots$	$p*$	$*q$	$*q$	$*q$	$**$	$\ldots$
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D(p) & = & 0 \\
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- **Relation between φ and $\psi = \Pi(\varphi, d)$:**

$$(\xi, t) \models \varphi \quad \leftrightarrow \quad (\xi, t+d) \models \psi$$

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Bounded Variability of Input Signals

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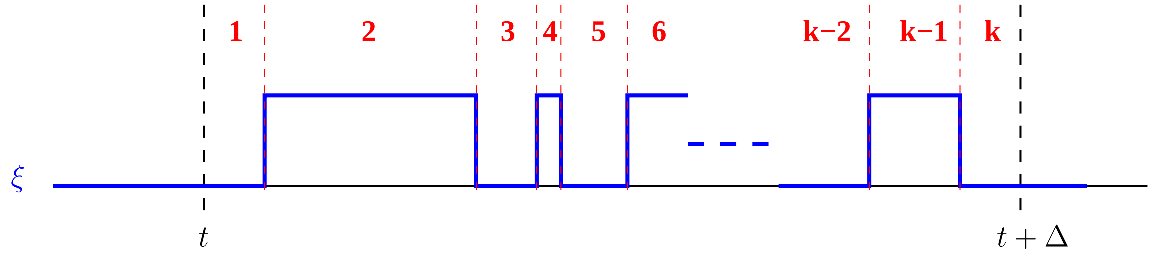

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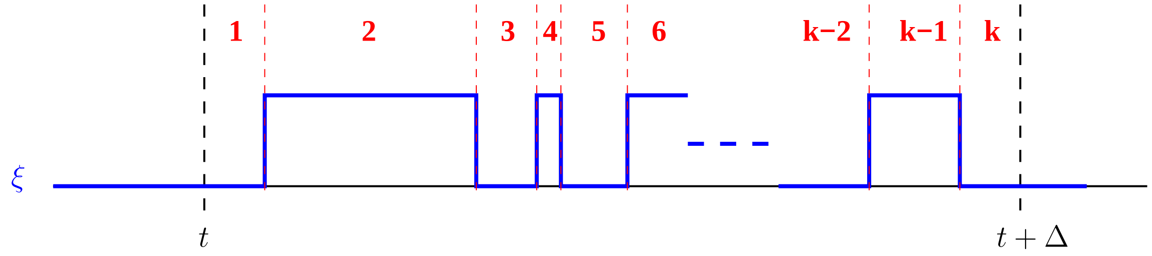

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Temporal testers for MTL-B *formulae*

- Temporal testers for LTL proposed in [KP05]
  - Compositional basis for automata construction corresponding to LTL formulae
  - Extension to real-time temporal logics
    - Past-MITL [MNP05]
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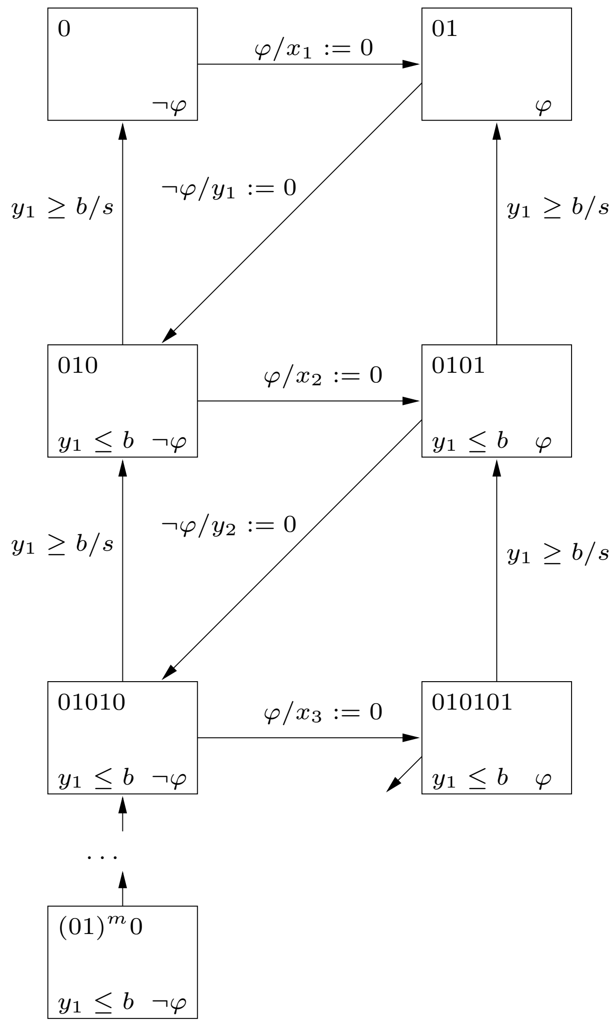

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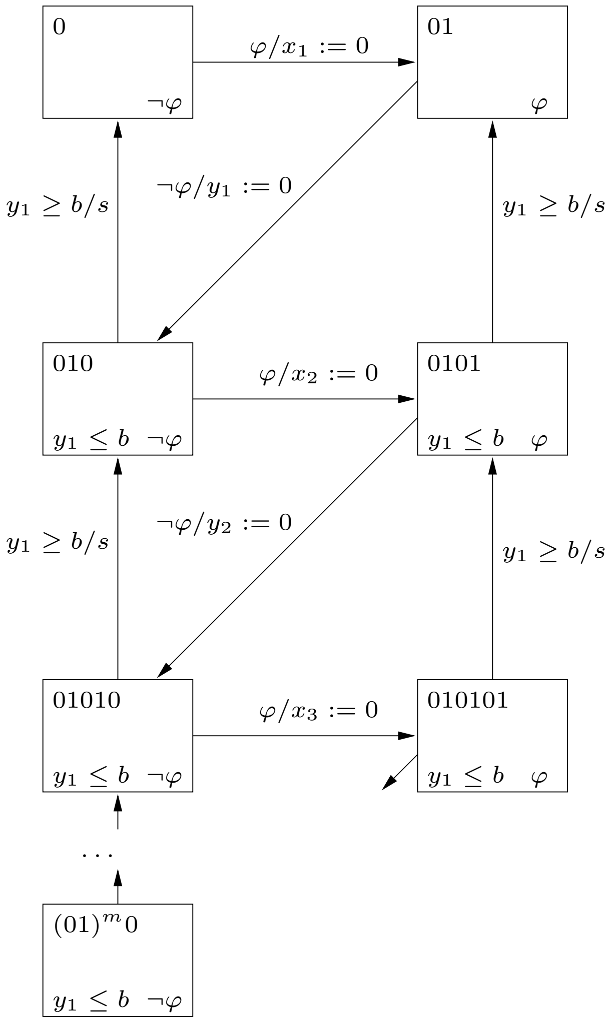

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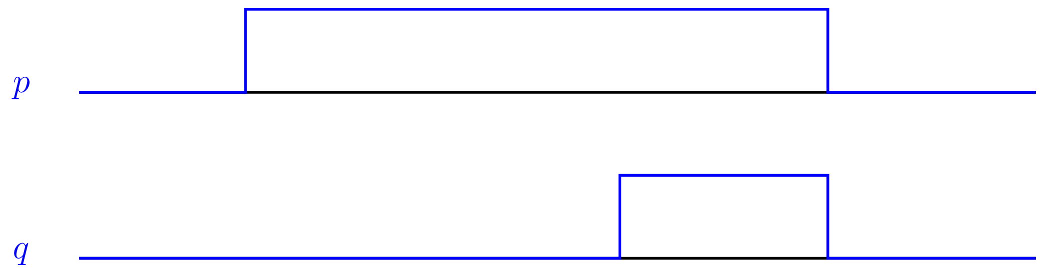

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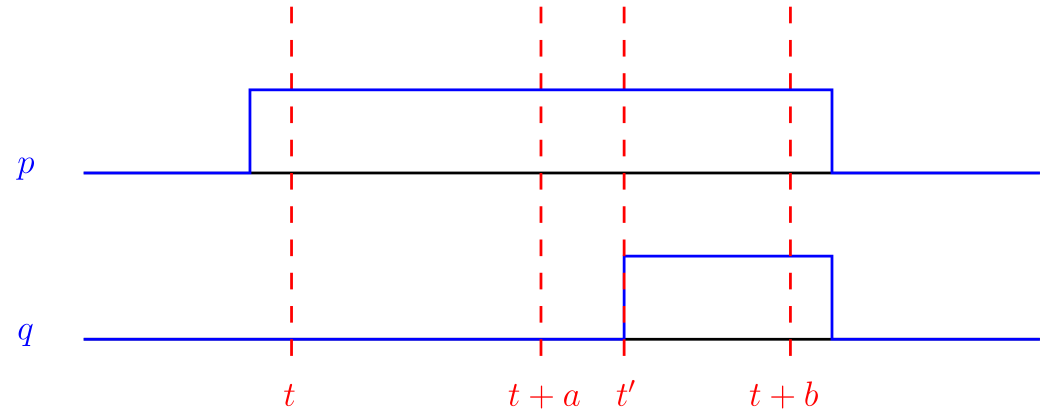

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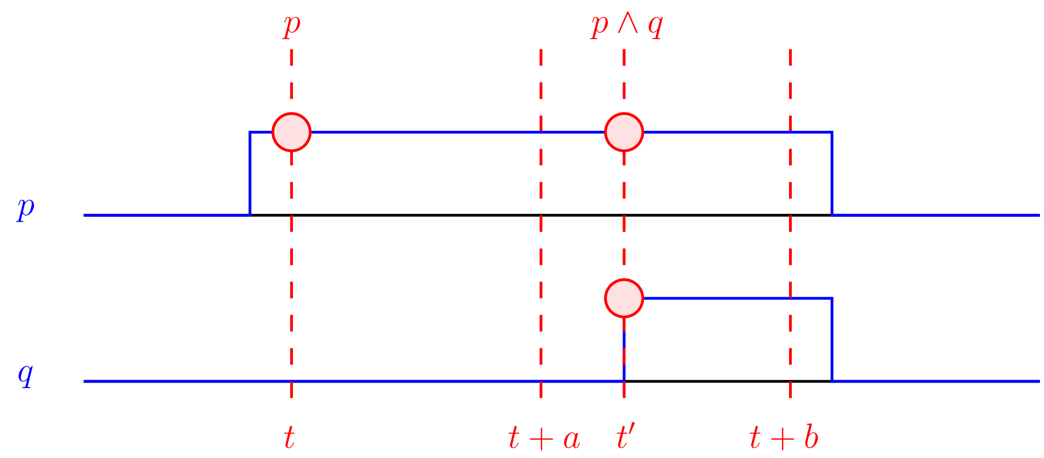

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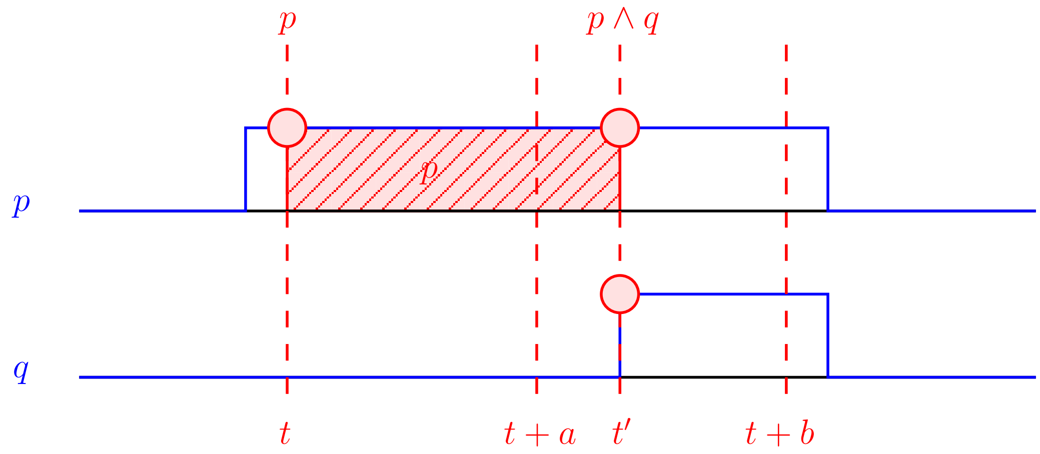

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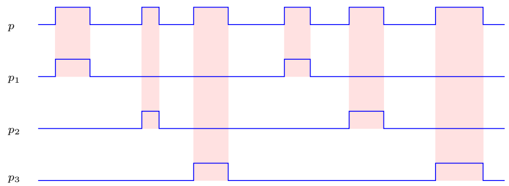

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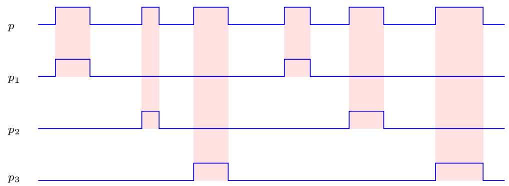

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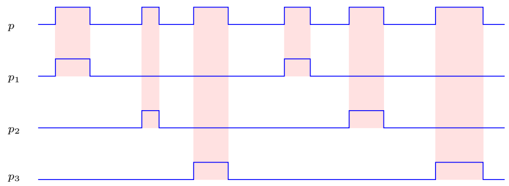

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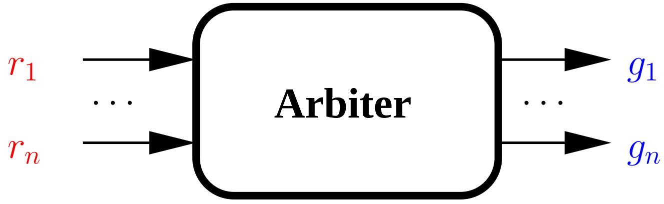
Synthesis of an Arbiter

- Architecture of an arbiter



- Typical timed interaction between the arbiter an a client i

- Communication protocol between the arbiter an a client i

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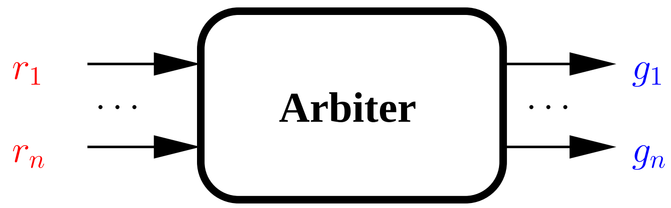

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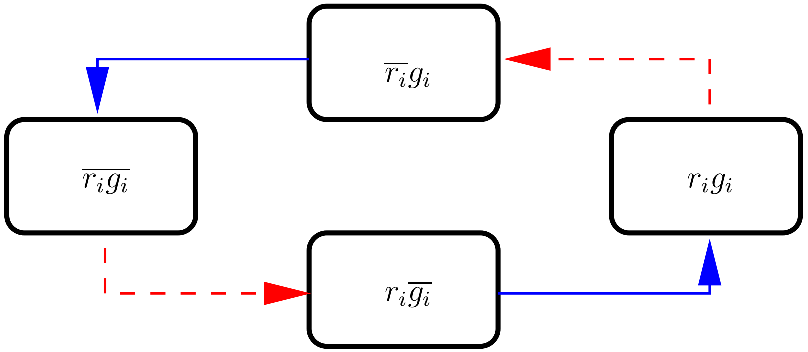

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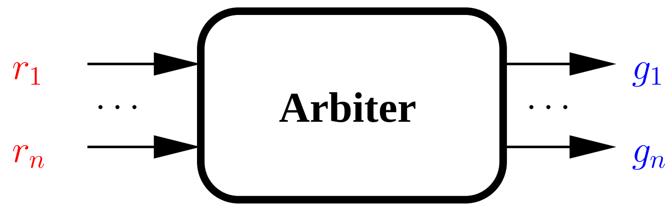

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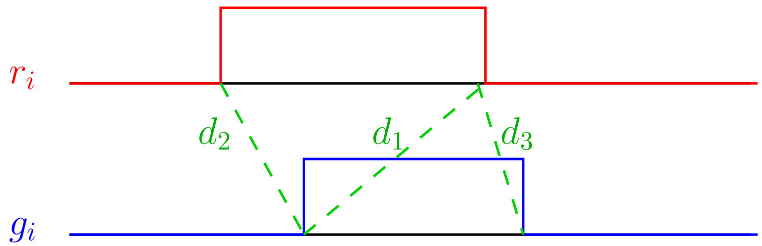

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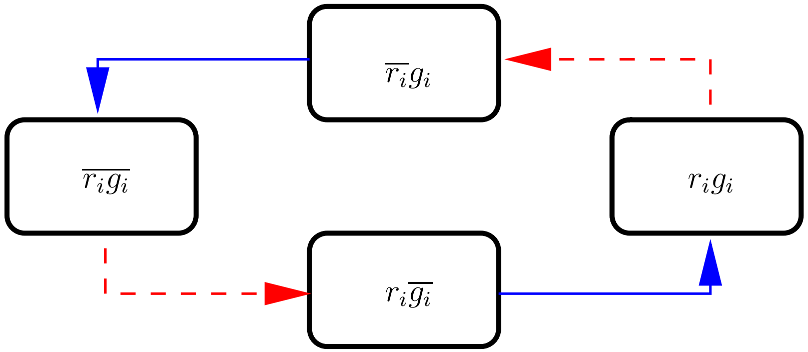

Synthesis of an Arbiter: MTL-B *Specification*

- Initial conditions
  - $I^E : \bigwedge_i \overline{r_i}$
  - $I^C : \bigwedge_i \overline{g_i}$
- Safety requirements
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Synthesis of an Arbiter: MTL-B *Specification*

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Synthesis of an Arbiter: Experimental Results

- Discrete time synthesis
- $d_3 = 1$

N	d_1	d_2	Size	Time	d_1	d_2	Size	Time	d_1	d_2	Size	Time
2	2	4	466	0.00	3	5	654	0.01	4	6	946	0.02
3	2	8	1382	0.14	3	10	2432	0.34	4	12	4166	0.51
4	2	12	4323	0.63	3	15	7402	1.12	4	18	16469	2.33
5	2	16	13505	1.93	3	20	26801	4.77	4	24	50674	10.50
6	2	20	43366	8.16	3	25	84027	22.55	4	30	168944	64.38
7	2	24	138937	44.38	3	30	297524	204.56	4	36	700126	1897.56

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- Complete chain that allows to synthesize controllers automatically from real-time bounded-response temporal specifications
  - Bounded-response temporal property $\longrightarrow$ deterministic timed automaton
    - Pastification of MTL-B formulae
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References

[AFH96] R. Alur, T. Feder, and T.A. Henzinger, The Benefits of Relaxing Punctuality, *Journal of the ACM* **43**, 116–146, 1996 (first published in *PODC'91*).

[AMP95] E. Asarin, O. Maler and A. Pnueli, Symbolic Controller Synthesis for Discrete and Timed Systems, *Hybrid Systems II*, 1–20, LNCS 999, 1995.

[BL69] J.R. Büchi and L.H. Landweber, Solving Sequential Conditions by Finite-state Operators, *Trans. of the AMS* **138**, 295–311, 1969.

[CDF$^+$05] F. Cassez, A. David, E. Fleury, K.G. Larsen and D. Lime, Efficient On-the-Fly Algorithms for the Analysis of Timed Games, *CONCUR'05*, 66–80, 2005.

[Chu63] A. Church, Logic, Arithmetic and Automata, in *Proc. of the Int. Cong. of Mathematicians 1962*, 23–35, 1963.

[KP05] Y. Kesten and A. Pnueli, A Compositional Approach to CTL* Verification, *Theoretical Computer Science* **331**, 397–428, 2005.

[Koy90] R. Koymans, Specifying Real-time Properties with Metric Temporal Logic, *Real-time Systems* **2**, 255–299, 1990.

References

[MN04] O. Maler and D. Nickovic, Monitoring Temporal Properties of Continuous Signals, *FORMATS/FTRTFT'04*, 152–166, LNCS 3253, 2004.

[MNP05] O. Maler, D. Nickovic and A. Pnueli, Real Time Temporal Logic: Past, Present, Future, *FORMATS'05*, 2–16, LNCS 3829, 2005.

[MNP06] O. Maler, D. Nickovic and A. Pnueli, From MITL to Timed Automata, *FORMATS'06*, 274–289, LNCS 4202, 2006.

[MPS95] O. Maler, A. Pnueli and J. Sifakis, On the Synthesis of Discrete Controllers for Timed Systems, *STACS'95*, 229–242, LNCS 900, 1995.

[PPS06] N. Piterman, A. Pnueli and Y. Sa'ar, Synthesis of Reactive(1) Designs, *VMCAI'06*, 364–380, 2006.

[PP06] N. Piterman and A. Pnueli, Faster Solutions of Rabin and Streett Games, *LICS'06*, 275–284, 2006.

[RW89] P.J. Ramadge and W.M. Wonham, The Control of Discrete Event Systems, *Proc. of the IEEE* **77**, 81–98, 1989.