Specifying Timed Patterns using Temporal Logic

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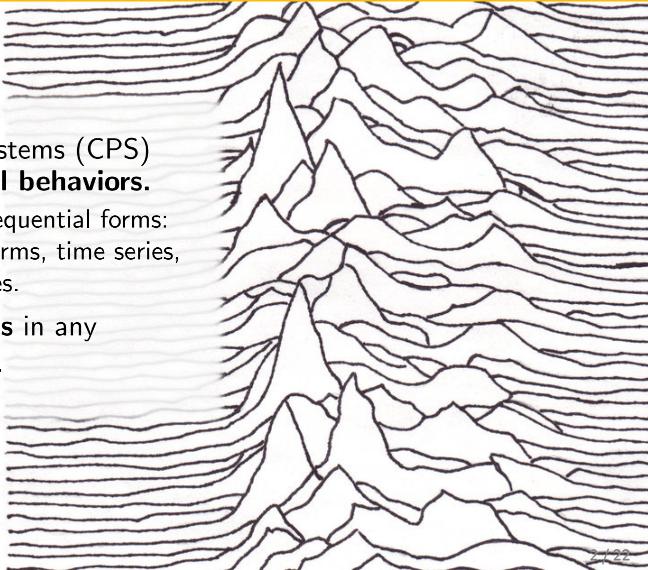
CPSWEEK::HSCC 2018

Porto, Portugal
April 13

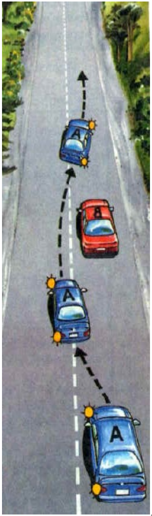
Temporal Behaviors



- Cyber-Physical Systems (CPS) generate **temporal behaviors.**
  - Expressed in sequential forms: signals, waveforms, time series, event sequences.
- There are **patterns** in any temporal behavior.

Detecting Patterns in Temporal Behaviors

- Specific shapes on waveforms:
    - Rise and falls, various pulses, decays, ...
- Specific arrangements of physical observations.
    - High speed period after high acceleration, ...
- Sequences of actions, simultaneous occurrences.
    - Overtaking a car.
    - Speeding-up while overtaken. (illegal pattern)



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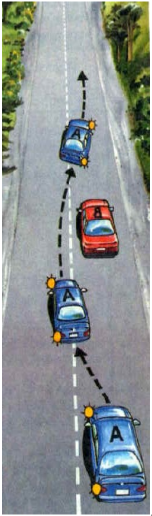
- **Find such pre-defined temporal patterns.**



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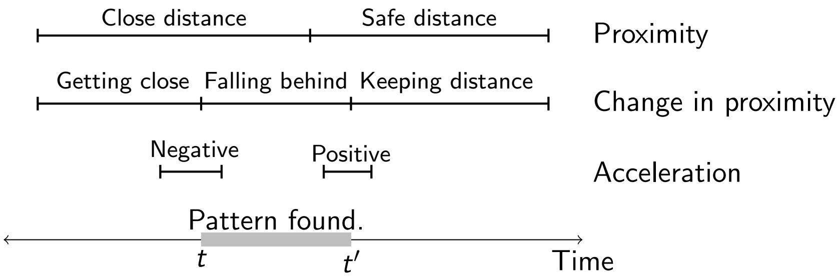
- Applications: Text search, lexers, parsers, NLP.
- A temporal behavior is different than a text — one-dimensional discrete sequence of single chars.
  - Time is dense (continuous).
  - Temporal behaviors are multi-dimensional (multi-variate/multi-channel behaviors).
  - Many patterns in time talk about different dimensions.
  - Durations (and timings) are important.

An Example



— **Find all** *falling behind periods* **begun by** *a deceleration period* **and followed by** *a period of safe* **and** *keeping distance* **at least 30 seconds**.

Related Work

- Timed Pattern Matching (2014):
  - Inspired by textual pattern matching.
  - Defined to be a computation to find all instances of a timed pattern over temporal behaviors.
  - Solved for timed regular expressions by an offline algorithm over dense-time Boolean behaviors.

Related Work

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- Later extended by online algorithms, measurements, timed automata patterns, skipping, quantitative semantics, and tools.

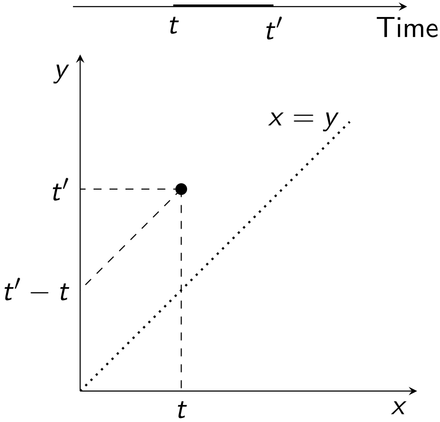
Contribution

- Explore **temporal logic patterns** for TPM.
- Propose period-based TL for the specs.
- Introduce Metric Compass Logic (MCL)
  - Period-based Temporal Logic + Timing Constraints.
- Present an offline pattern matching algorithm for MCL.

Logic of Time Periods

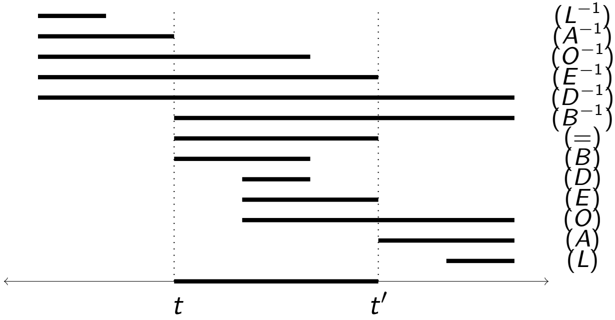
Time Periods

- A time period (t, t') is a pair such that $t < t'$.
- It begins at t, ends at t', and has a duration of $t' - t$.
- Illustrated on the xy-plane.



Relations between Time Periods

- Known as Allen's interval relations.



- Relations: Adjacent (A), Begins (B), Ends (E), Overlaps (O), Later (L), During (D), and their inverses.

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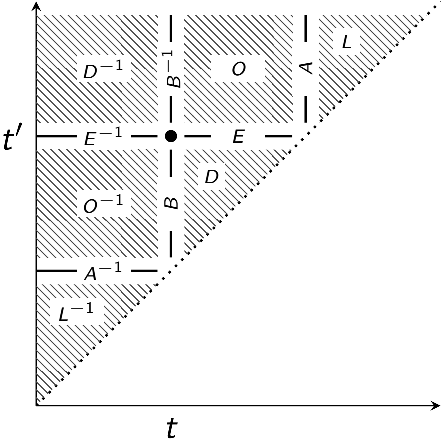

- Relations: Adjacent (A), Begins (B), Ends (E), Overlaps (O), Later (L), During (D), and their inverses.
- You can ask questions if there exists or for all ... related periods.

A Temporal Logic of Time Periods

- We have more relations (Allen's) for time periods and consequently more temporal operators. (cf. time periods)
- It is shown that six of them is enough.
- Known as Halpern-Shoham (HS) logic[1].
- Intractable for satisfiability, validity, model checking.

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- Known as Halpern-Shoham (HS) logic[1].
- Intractable for satisfiability, validity, model checking.
- No problem for pattern matching. :)

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Temporal Operators (Diamonds and Boxes)

There exists a time period

- $\Diamond$ — **Begins** (Begin at the same time, End earlier)
- $\Diamond$ — **Begun-by** (Begin at the same time, End later)
- $\Diamond$ — **Ends** (Begin earlier, End at the same time)
- $\Diamond$ — **Ended-by** (Begin later, End at the same time)
- $\Diamond$ — **Adjacent in the past** (Ends where it begins)
- $\Diamond$ — **Adjacent in the future** (Begins where it ends)

the current time period.

- Boxes: $\Box \equiv \neg \Diamond \neg$
- Also called compass logic due to the decoration.

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- Defined inductively over a set P of atomic propositions:
  - An atomic proposition $p \in P$ is a MCL formula.
  - If φ_1 and φ_2 are formulas, then $\varphi_1 \cup \varphi_2$, $\varphi_1 \cap \varphi_2$, and $\overline{\varphi_1}$ are formulas.
  - If φ is a formula, then $\Diamond_I\, \varphi$, $\Diamond_I\, \varphi$, $\Diamond_I\, \varphi$, $\Diamond_I\, \varphi$, $\Diamond_I\, \varphi$, and $\Diamond_I\, \varphi$ are formulas.

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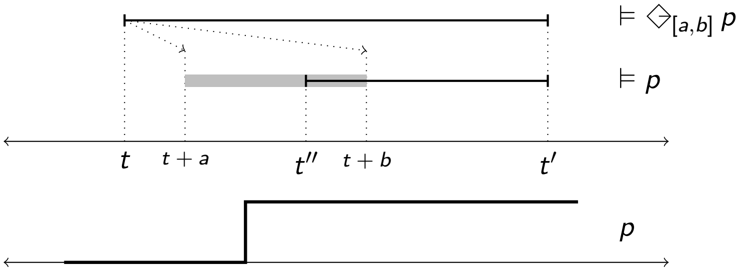
- One diamond for relations A^{-1}, A, B^{-1}, B, E^{-1}, E.
- The rest of operators/relations is derivable.

One Diamond Explained

- $\diamond$ — Ended-by (Begin later, End at the same time)



An Example

— **Find all** *falling behind periods* **begun by** *a deceleration period* **and followed by** *a period of safe* **and** *keeping distance* **at least 30 seconds**.

$$\varphi : \texttt{fall-behind} \ \wedge \ \diamondminus \texttt{decel} \ \wedge \ \diamondplus_{[30,\infty)}(\texttt{safe} \wedge \texttt{keep-dist})$$

Computing Match Sets

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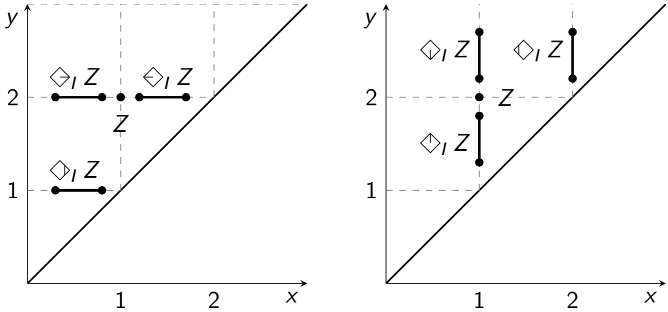
Compute the match set $\mathcal{M}_w(\varphi)$ in the following.

Skeleton Algorithm

$Z = \text{EVAL}_W(\varphi)$ is the match set of the pattern φ over W.

select (φ)
case p**:**
$Z := V(p)$
case $\overline{\psi}$**:**
$Z := \text{COMPLEMENT}(\text{EVAL}_W(\psi))$
case $\psi_1 \cup \psi_2$**:**
$Z := \text{UNION}(\text{EVAL}_W(\psi_2),\ \text{EVAL}_W(\psi_2))$
case $\psi_1 \cap \psi_2$**:**
$Z := \text{INTERSECT}(\text{EVAL}_W(\psi_2),\ \text{EVAL}_W(\psi_2))$
case $\Diamond_I\, \psi$**:**
$Z := \Diamond\text{-SHIFT}(\text{EVAL}_W(\psi), I)$
end select
return Z

Shifting Time Periods



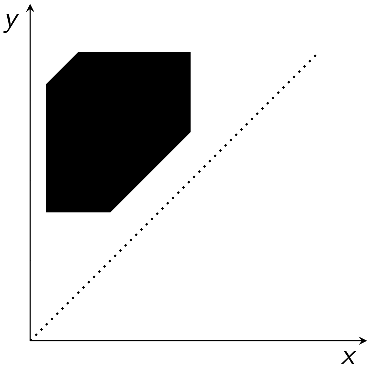
- Look at the effect of $\Diamond_I$ on a single period.
- Need to represent and manipulate sets of time periods.

Representing Match Sets

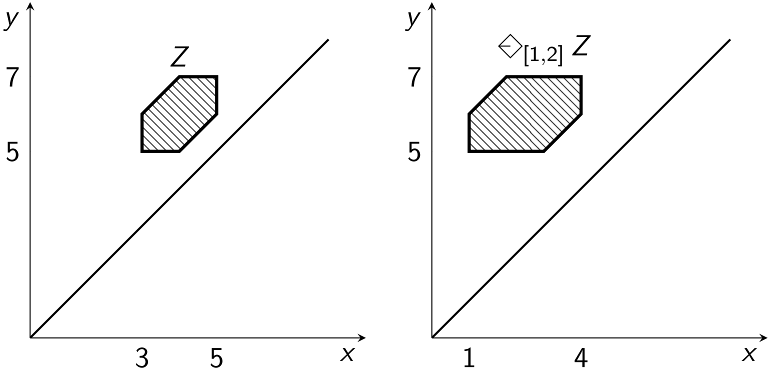
- A **zone** is a convex set of time periods, formed by constraints on begins, ends, and durations.

$$\begin{array}{ccccc} c_1 & \prec & x & \prec & c_2 \\ c_3 & \prec & y & \prec & c_4 \\ c_5 & \prec & y - x & \prec & c_6 \end{array}$$

- We represent a match set Z by a finite union R_Z of zones.



Shifting Zones



- Theorem: Zones are closed under $\diamondsuit_I$ operators.

Experiments

Test Patterns	Offline Algorithm Input Size 100K	500K	1M
$\overline{p}$	0.18/12	0.95/45	1.88/92
$\diamondminus_I\, p$	0.07/16	0.29/65	0.66/163
$\boxdashv_I\, p$	0.49/23	1.98/100	3.92/163
$\diamondminus_I \diamondbar_J\, p$	0.08/20	0.32/37	0.96/60
$\diamondminus \diamondbar(\boxdashv p \cdot q)$	0.40/31	1.98/143	3.93/268
$\diamondminus \diamondbar(\boxdashv p \cdot q) \cap \diamondminus_I\, q$	0.43/38	2.17/179	4.30/304

- Diamonds are cheap, complementation is expensive.
- Linear execution time for typical inputs.

Conclusions

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- We should consider more concise, expressive, and elegant formalisms for monitoring and pattern matching even though they are not good for other tasks.
- Expressiveness? (FAQ):
  - $\Diamond$, $\Diamond$ is not expressible in regular expressions.
  - Concatenation is not expressible in compass logic.
- Online algorithm? (A theoretical and practical challenge)

Thank you!