Specifying Timed Patterns using Temporal Logic

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Temporal Behaviors

- Cyber-Physical Systems (CPS) generate **temporal behaviors**.
  - Expressed in sequential forms: signals, waveforms, time series, event sequences.
- There are **patterns** in any temporal behavior.
Detecting Patterns in Temporal Behaviors

- Specific shapes on waveforms:
  - Rise and falls, various pulses, decays, ...
- Specific arrangements of physical observations.
  - High speed period after high acceleration, ...
- Sequences of actions, simultaneous occurrences.
  - Overtaking a car.
  - Speeding-up while overtaken. (illegal pattern)
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- Find such pre-defined temporal patterns.
Timed Patterns

- We are inspired from textual pattern matching.
- Applications: Text search, lexers, parsers, NLP.
Timed Patterns

- We are inspired from textual pattern matching.
- Applications: Text search, lexers, parsers, NLP.
- A temporal behavior is different than a text — one-dimensional discrete sequence of single chars.
  - Time is dense (continuous).
  - Temporal behaviors are multi-dimensional (multi-variate/multi-channel behaviors).
  - Many patterns in time talk about different dimensions.
  - Durations (and timings) are important.
An Example

Find all falling behind periods begun by a deceleration period and followed by a period of safe and keeping distance at least 30 seconds.
Related Work

- Timed Pattern Matching (2014):
  - Inspired by textual pattern matching.
  - Defined to be a computation to find all instances of a timed pattern over temporal behaviors.
  - Solved for timed regular expressions by an offline algorithm over dense-time Boolean behaviors.
Related Work

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- Later extended by online algorithms, measurements, timed automata patterns, skipping, quantitative semantics, and tools.
Contribution

- Explore **temporal logic patterns** for TPM.
- Propose period-based TL for the specs.

- Introduce Metric Compass Logic (MCL)
  - Period-based Temporal Logic + Timing Constraints.

- Present an offline pattern matching algorithm for MCL.
A time period \((t, t')\) is a pair such that \(t < t'\).

It begins at \(t\), ends at \(t'\), and has a duration of \(t' - t\).

Illustrated on the \(xy\)-plane.
Relations between Time Periods

- Known as Allen’s interval relations.

- Relations: Adjacent (A), Begins (B), Ends (E), Overlaps (O), Later (L), During (D), and their inverses.
Relations between Time Periods

- Relations: Adjacent (A), Begins (B), Ends (E), Overlaps (O), Later (L), During (D), and their inverses.
- You can ask questions if there exists or for all ... related periods.
A Temporal Logic of Time Periods

- We have more relations (Allen’s) for time periods and consequently more temporal operators. (cf. time periods)
- It is shown that six of them is enough.
- Known as Halpern-Shoham (HS) logic\(^1\).
- Intractable for satisfiability, validity, model checking.

\(^1\)Halpern and Shoham. A propositional modal logic of time intervals. 1986.
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- Intractable for satisfiability, validity, model checking.
- No problem for pattern matching. :)

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Temporal Operators (Diamonds and Boxes)

There exists a time period

- $\Diamond \quad \text{Begins}$ (Begin at the same time, End earlier)
- $\Box \quad \text{Begun-by}$ (Begin at the same time, End later)
- $\bigtriangleup \quad \text{Ends}$ (Begin earlier, End at the same time)
- $\bigtriangledown \quad \text{Ended-by}$ (Begin later, End at the same time)
- $\Diamond \quad \text{Adjacent in the past}$ (Ends where it begins)
- $\Diamond \quad \text{Adjacent in the future}$ (Begins where it ends)

the current time period.

- Boxes: $\square \equiv \neg \Diamond \neg$
- Also called compass logic due to the decoration.
Metric Compass Logic (MCL)

- We add timing constraints to HS logic.
- Use as a timed pattern specification language.
Metric Compass Logic (MCL)

- We add **timing constraints** to HS logic.
- Use as a timed pattern specification language.
- Defined inductively over a set $P$ of atomic propositions:
  - An atomic proposition $p \in P$ is a MCL formula.
  - If $\varphi_1$ and $\varphi_2$ are formulas, then $\varphi_1 \cup \varphi_2$, $\varphi_1 \cap \varphi_2$, and $\overline{\varphi_1}$ are formulas.
  - If $\varphi$ is a formula, then $\Diamond_{I} \varphi$, $\Diamond_{I} \varphi$, $\Diamond_{I} \varphi$, $\Diamond_{I} \varphi$, $\Diamond_{I} \varphi$, and $\Diamond_{I} \varphi$ are formulas.
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• Use as a timed pattern specification language.
• Defined inductively over a set $P$ of atomic propositions:
  • An atomic proposition $p \in P$ is a MCL formula.
  • If $\varphi_1$ and $\varphi_2$ are formulas, then $\varphi_1 \cup \varphi_2$, $\varphi_1 \cap \varphi_2$, and $\neg \varphi_1$ are formulas.
  • If $\varphi$ is a formula, then $\diamond_I \varphi$, $\square_I \varphi$, $\diamond_I \varphi$, $\square_I \varphi$, $\diamond_I \varphi$, and $\diamond_I \varphi$ are formulas.
• One diamond for relations $A^{-1}$, $A$, $B^{-1}$, $B$, $E^{-1}$, $E$.
• The rest of operators/relations is derivable.
One Diamond Explained

- $\diamondsuit$ — Ended-by (Begin later, End at the same time)

$\vdash [a, b] \ p$

$\vdash p$
An Example

— Find all falling behind periods begun by a deceleration period and followed by a period of safe and keeping distance at least 30 seconds.

$$\varphi: \text{fall-behind} \land \lozenge \text{decel} \land \lozenge [30,\infty)(\text{safe} \land \text{keep-dist})$$
Computing Match Sets
Timed Pattern Matching

- A computation for identifying all time periods of a temporal behavior that satisfy a timed pattern.
Timed Pattern Matching

- A computation for identifying **all time periods** of a temporal behavior that satisfy a timed pattern.
- Patterns specified in **Metric Compass Logic**. (This Paper)
Timed Pattern Matching

- A computation for identifying all time periods of a temporal behavior that satisfy a timed pattern.
- Patterns specified in Metric Compass Logic. (This Paper)
- The set of all satisfying segments is called the match set of the pattern $\varphi$ over a temporal behavior $w$.

$$\mathcal{M}_w(\varphi) = \{(t, t') \mid w[t, t'] \text{ satisfies } \varphi\}$$
Timed Pattern Matching

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Compute the match set $\mathcal{M}_w(\varphi)$ in the following.
**Skeleton Algorithm**

\[ Z = \text{EVAL}_W(\varphi) \] is the match set of the pattern \( \varphi \) over \( W \).

```plaintext
select (\varphi)
case \, p:
  Z := V(p)
case \bar{\psi}:
  Z := \text{COMPLEMENT}(\text{EVAL}_W(\psi))
case \, \psi_1 \cup \psi_2:
  Z := \text{UNION}(\text{EVAL}_W(\psi_2), \text{EVAL}_W(\psi_2))
case \, \psi_1 \cap \psi_2:
  Z := \text{INTERSECT}(\text{EVAL}_W(\psi_2), \text{EVAL}_W(\psi_2))
case \, \Diamond_l \psi:
  Z := \Diamond\text{-SHIFT}(\text{EVAL}_W(\psi), l)
end select
return Z
```
Shifting Time Periods

- Look at the effect of $\Diamond_I$ on a single period.
- Need to represent and manipulate sets of time periods.
A zone is a convex set of time periods, formed by constraints on begins, ends, and durations.

\[ c_1 \prec x \prec c_2 \]
\[ c_3 \prec y \prec c_4 \]
\[ c_5 \prec y - x \prec c_6 \]

We represent a match set \( Z \) by a finite union \( R_Z \) of zones.
Theorem: Zones are closed under $\Diamond_I$ operators.
## Experiments

<table>
<thead>
<tr>
<th>Test Patterns</th>
<th>Offline Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Input Size</td>
</tr>
<tr>
<td></td>
<td>100K</td>
</tr>
<tr>
<td>(\bar{p})</td>
<td>0.18/12</td>
</tr>
<tr>
<td>(\Diamond_i p)</td>
<td>0.07/16</td>
</tr>
<tr>
<td>(\Box_i p)</td>
<td>0.49/23</td>
</tr>
<tr>
<td>(\Diamond_i \Diamond_j p)</td>
<td>0.08/20</td>
</tr>
<tr>
<td>(\Diamond \Diamond (\Box p \cdot q))</td>
<td>0.40/31</td>
</tr>
<tr>
<td>(\Diamond \Diamond (\Box p \cdot q) \cap \Diamond_i q)</td>
<td>0.43/38</td>
</tr>
</tbody>
</table>

- Diamonds are cheap, complementation is expensive.
- Linear execution time for typical inputs.
Conclusions

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- We should consider more concise, expressive, and elegant formalisms for monitoring and pattern matching even though they are not good for other tasks.

- Expressiveness? (FAQ):
  - ♦, ♦ is not expressible in regular expressions.
  - Concatenation is not expressible in compass logic.

- Online algorithm? (A theoretical and practical challenge)

Thank you!