Parametric Identification of Temporal Properties

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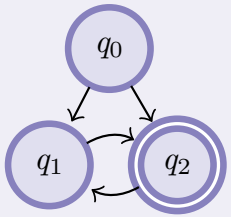
Context

- Properties monitoring of hybrid systems (embedded systems, mixed-signal circuits, etc)
- Behaviours have dense-time and real-valued components
- We specify and monitor properties using Signal Temporal Logic

In this work, we consider *Parametric* STL formulas, i.e., formulas with parameters

Hybrid System

$\dot{x} = f_q(x, t)$ ||



Goal: solving an inverse problem

For which values of parameters (e.g., p, s_1, s_2) does a given signal x satisfies φ ?

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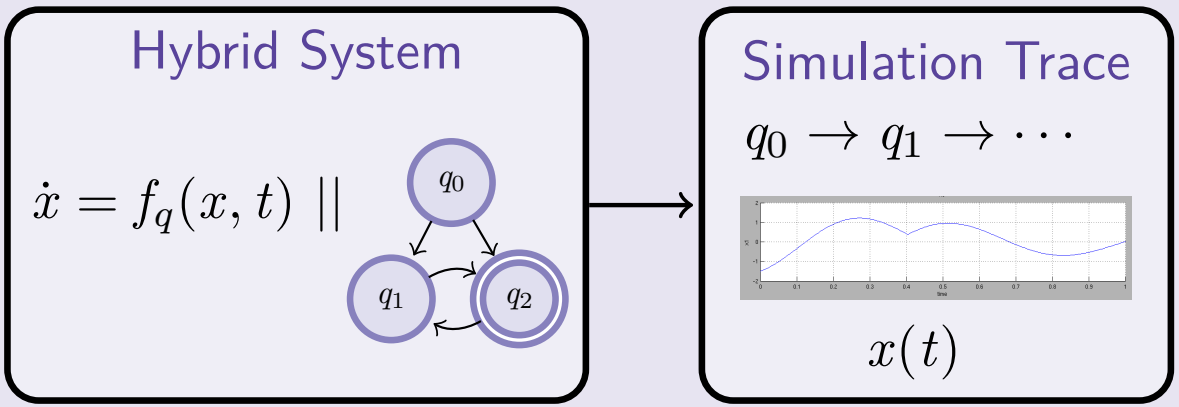

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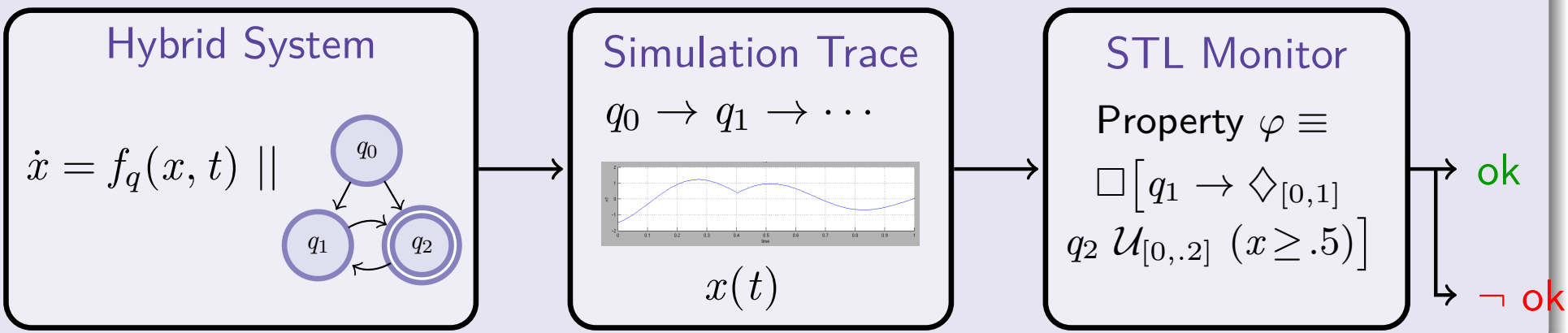

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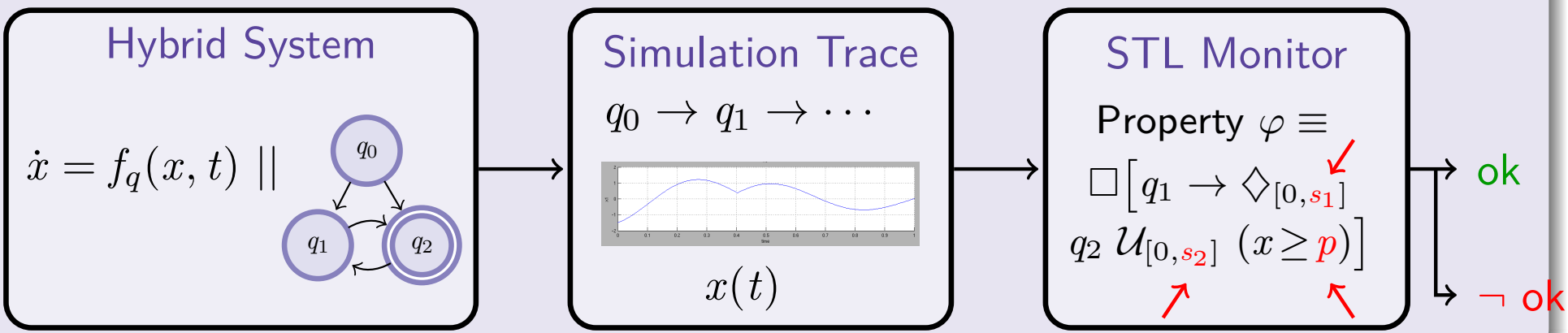

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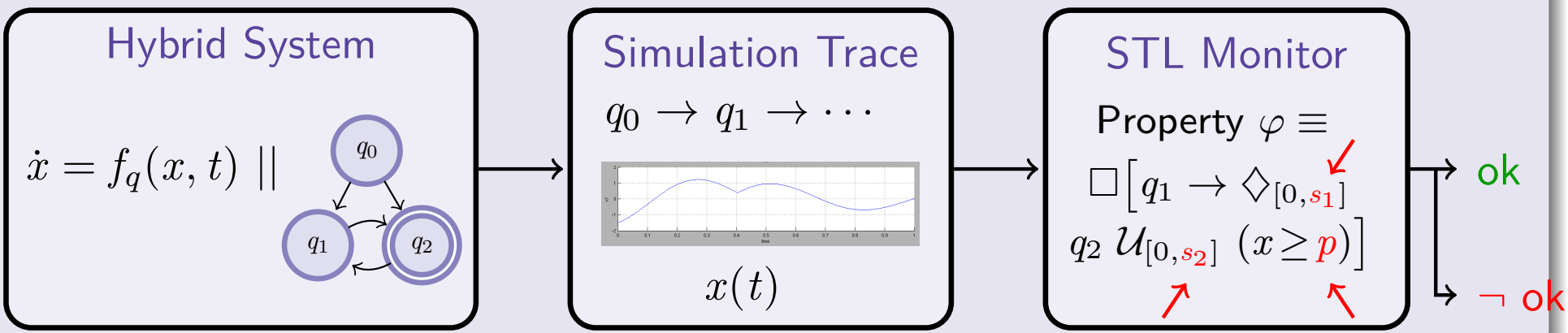

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Outline

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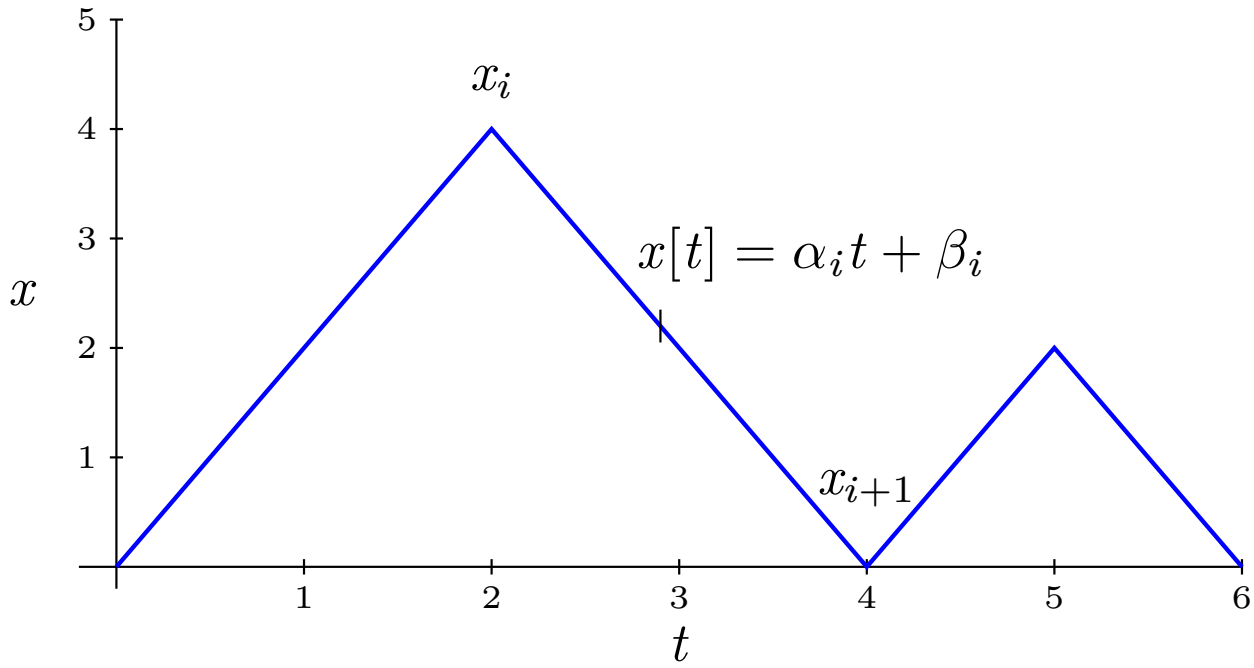
Signals

We consider piecewise affine signals given by sets of points $(x_i = x[t_i])$



In this talk, we consider 1-dimensional signals for simplicity.

Signal Temporal Logic

Goal specifying temporal properties of signals

Definition (STL Syntax)

An STL formula is given by:

$$\varphi := \mu \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \, \mathcal{U}_{[a,b]} \, \varphi_2$$

where μ is a predicate of the form $f(x) > p$.

Definition (STL Semantics)

The validity of a formula φ with respect to a signal x at time t is

$$\begin{aligned}
(x,t) \models f(x) > p \quad &\Leftrightarrow \quad f(x[t]) > p \\
(x,t) \models \varphi \wedge \psi \quad &\Leftrightarrow \quad (x,t) \models \varphi \wedge (x,t) \models \psi \\
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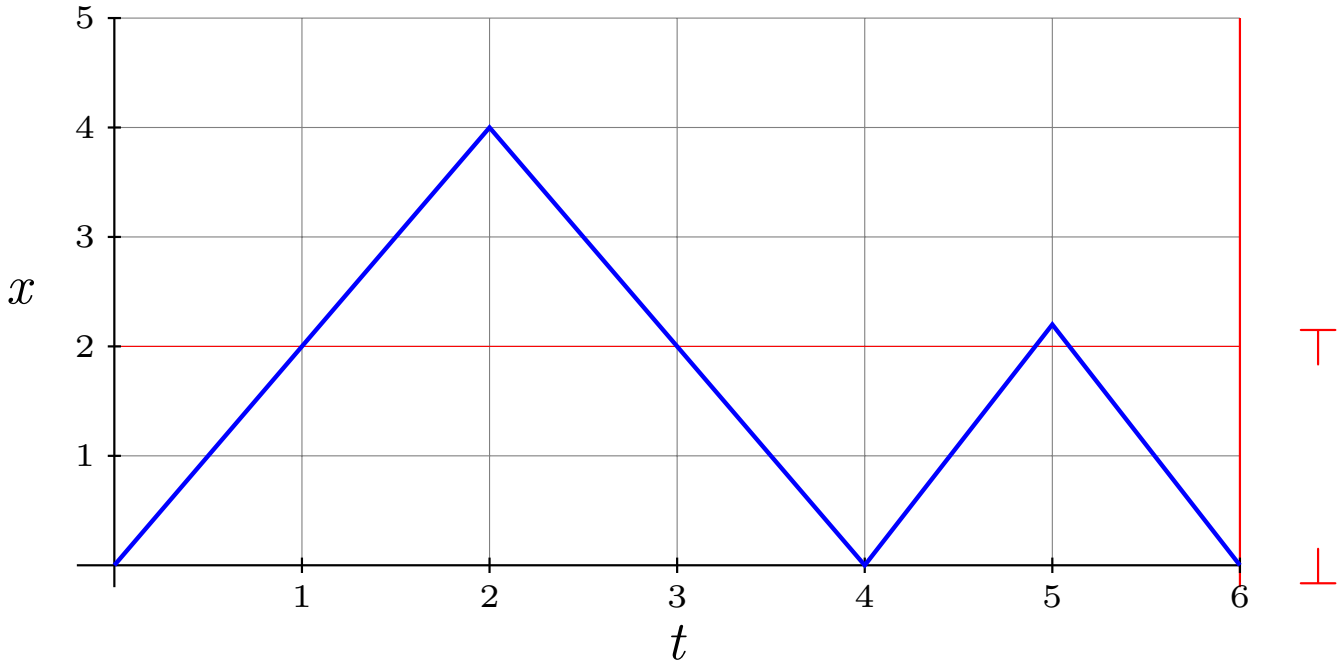
\end{array}$$

Additionally: $\Diamond_{[a,b]}\varphi = \top \, \mathcal{U}_{[a,b]}\varphi$ and $\Box_{[a,b]}\varphi = \varphi \, \mathcal{U}_{[a,b]} \bot$.

Examples



Truth value of :

- $\varphi = x > 2$

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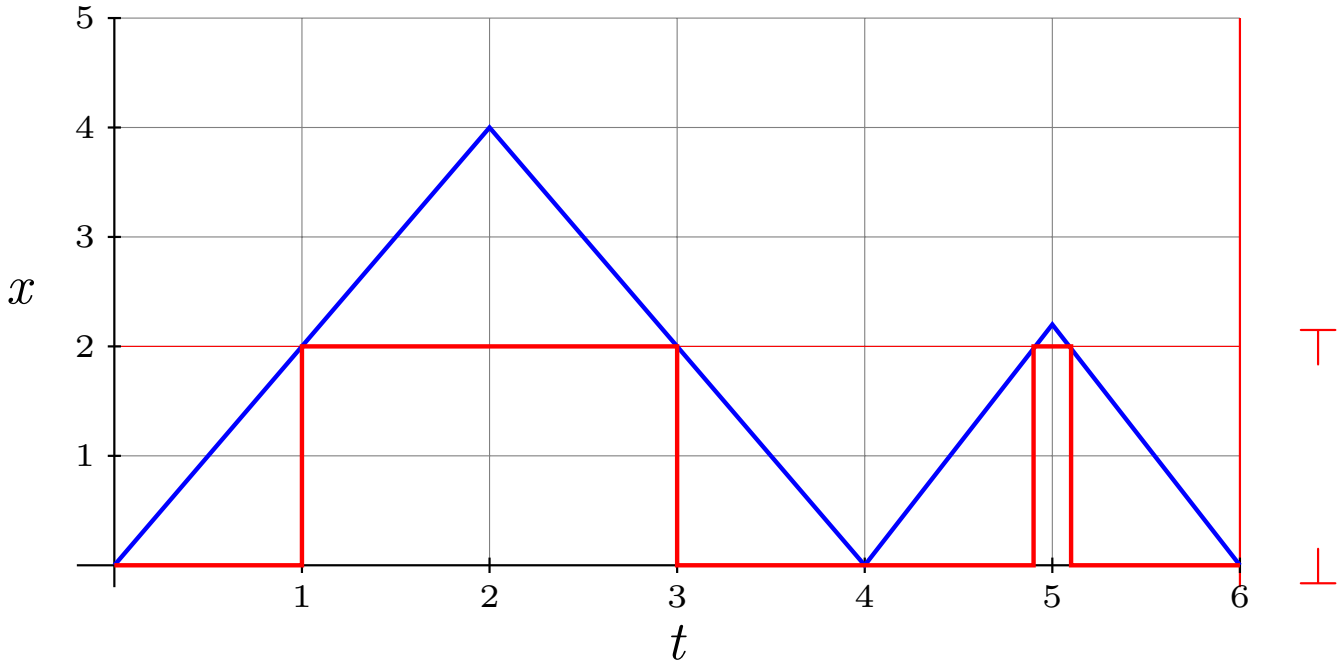

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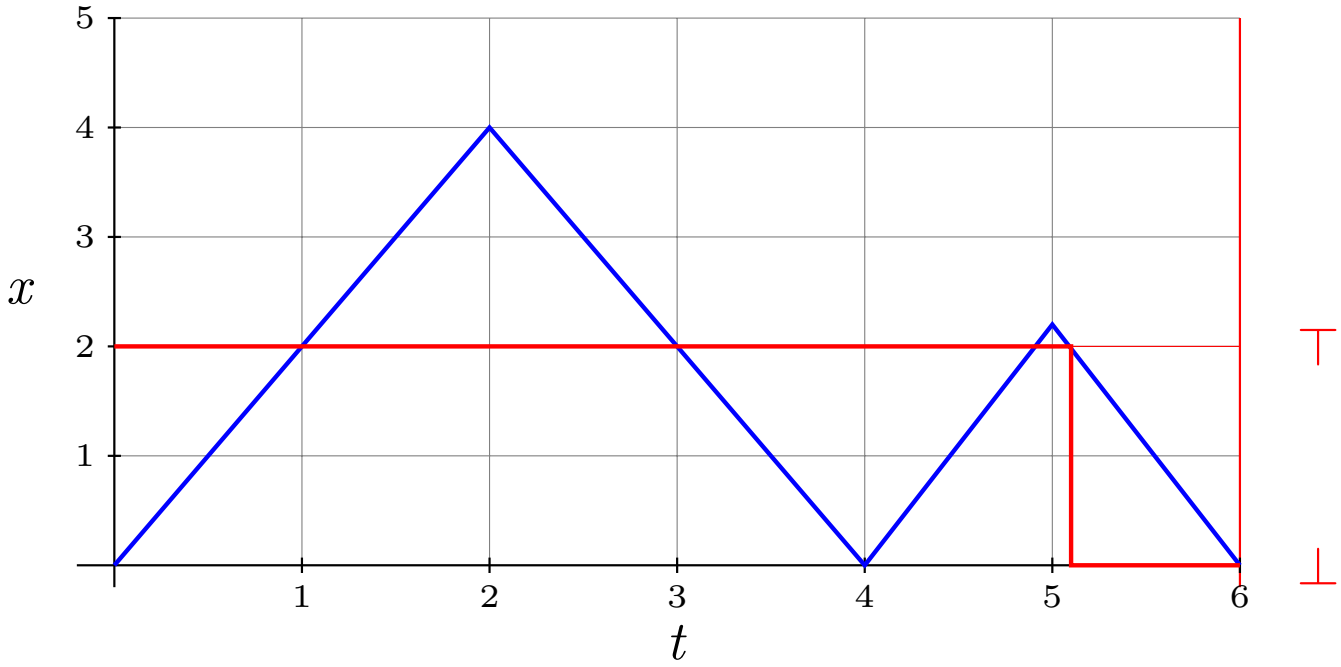

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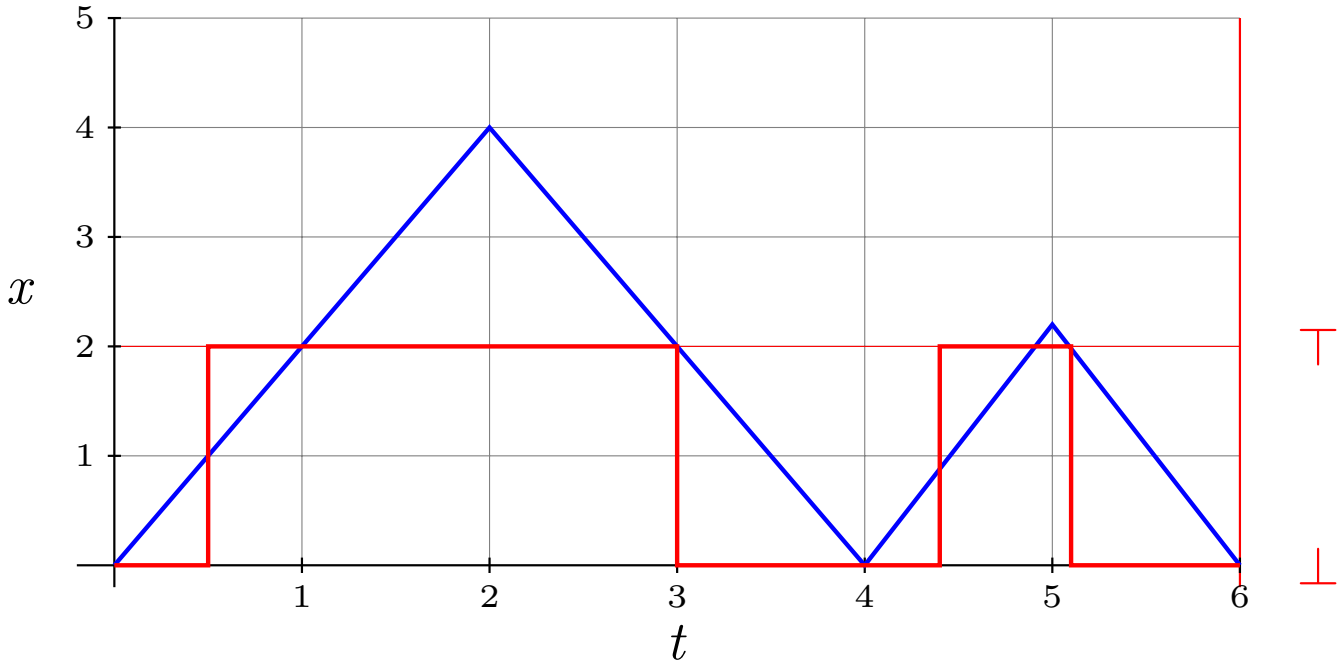

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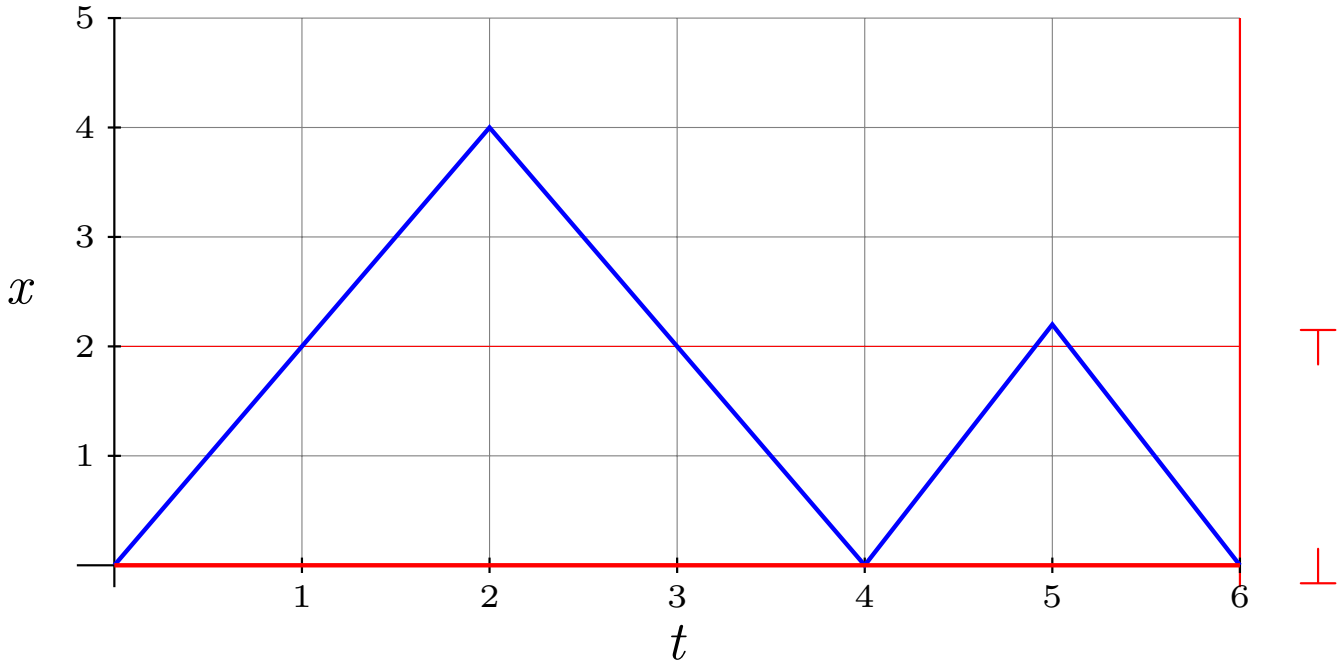

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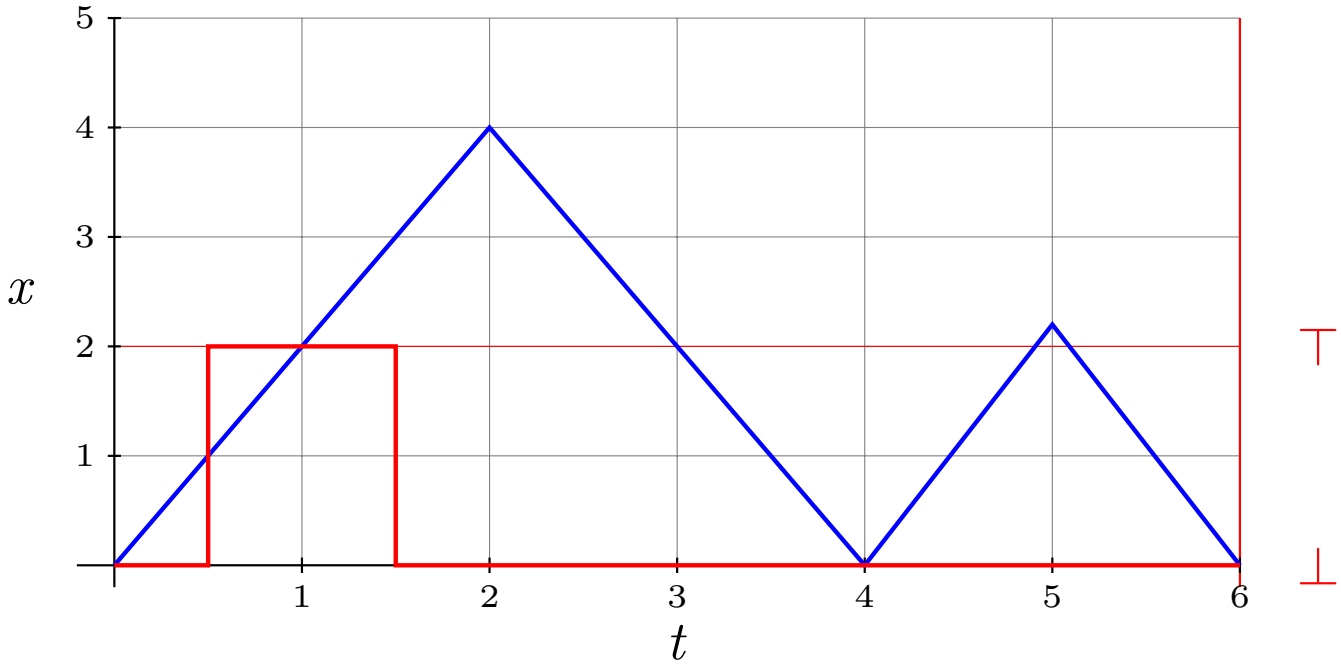


Truth value of :

- $\varphi = x > 2$

- $\varphi = \Box_{[0.5,1.5]}(x > 2)$

Outline

Validity Domains

Definition

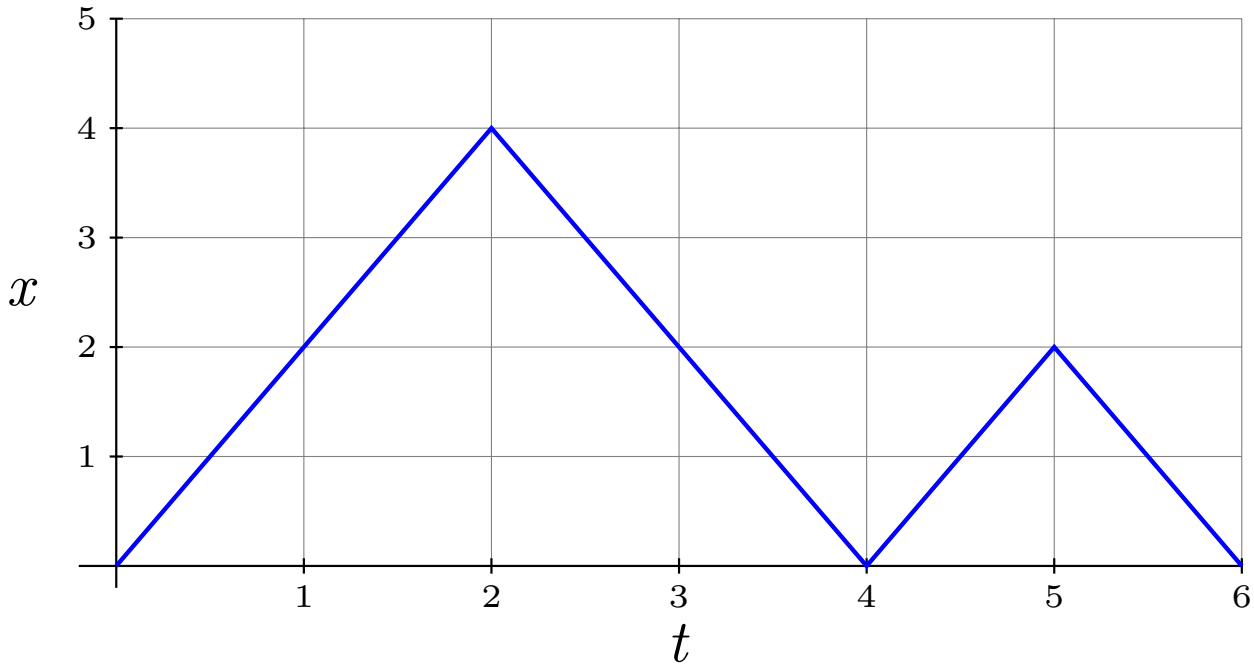
Let φ be a PSTL formula with amplitude parameters $p \in \mathcal{P}$ and timing parameters $s \in \mathcal{S}$.

The validity domain of a signal x for φ is the set of times and parameters values for which φ is satisfied by x

$$d(x, \varphi) = \{(p, s, t) \in \mathcal{P} \times \mathcal{S} \times \mathbb{R}^+ \text{ s.t. } (x, t) \models \varphi_{p,s}\}$$

Validity Domains of Predicates

Consider again signal x and the predicate $x > p$.



$$d(x, x > p) \quad = \quad (t \geq 0 \wedge t < 2 \wedge 2p > 4t) \quad \vee$$
$$(t \geq 2 \wedge t < 4 \wedge 2p + 4t > 16) \quad \vee$$
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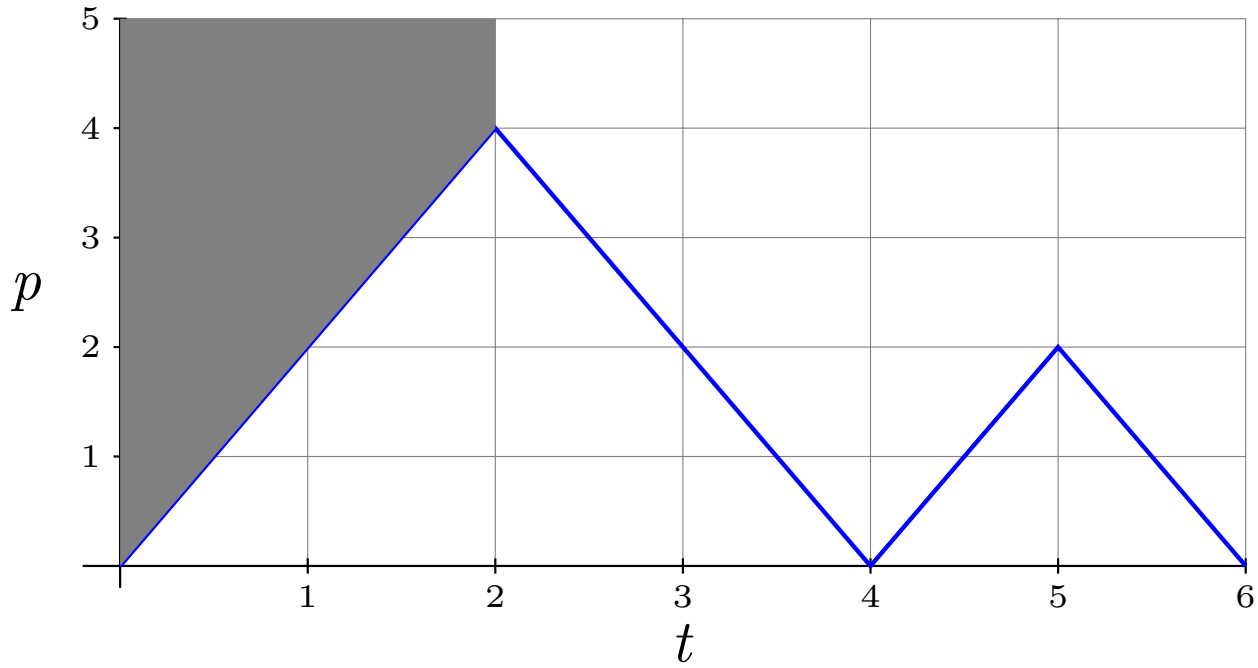

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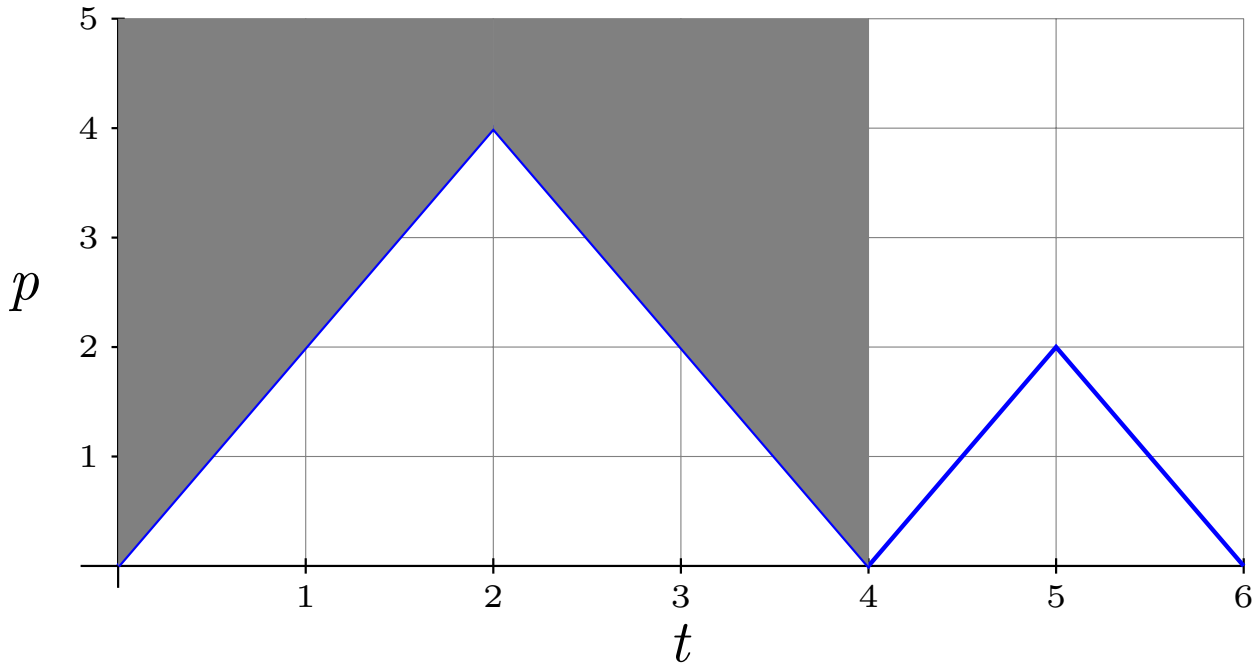

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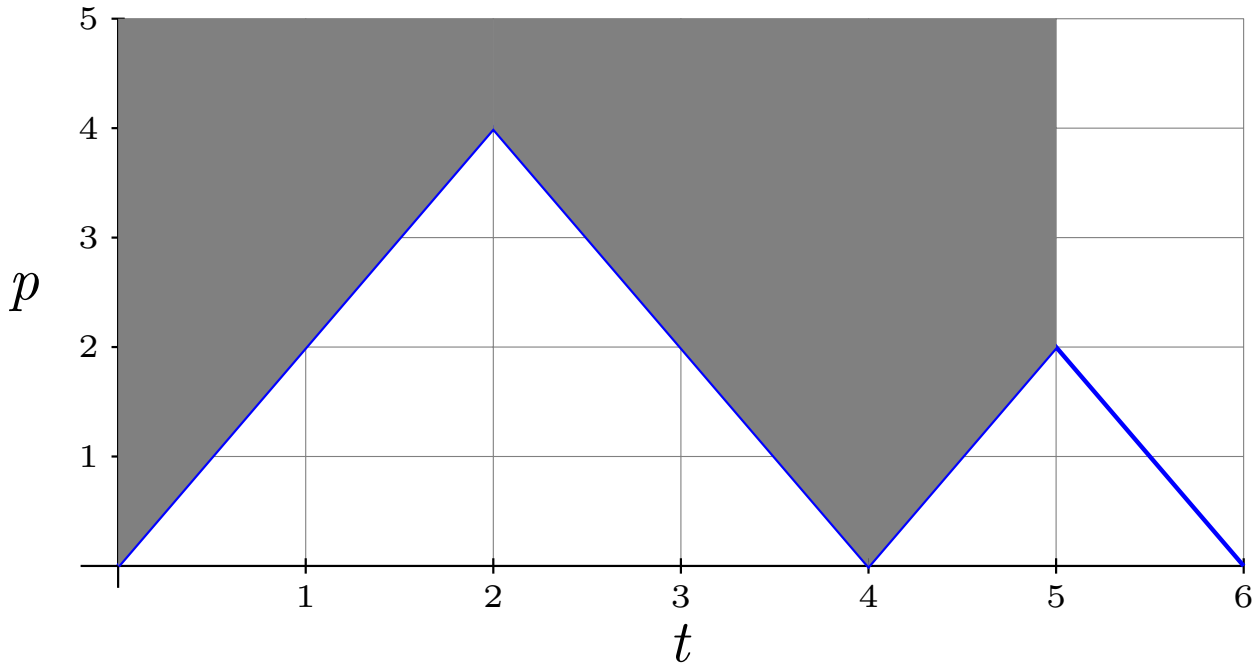

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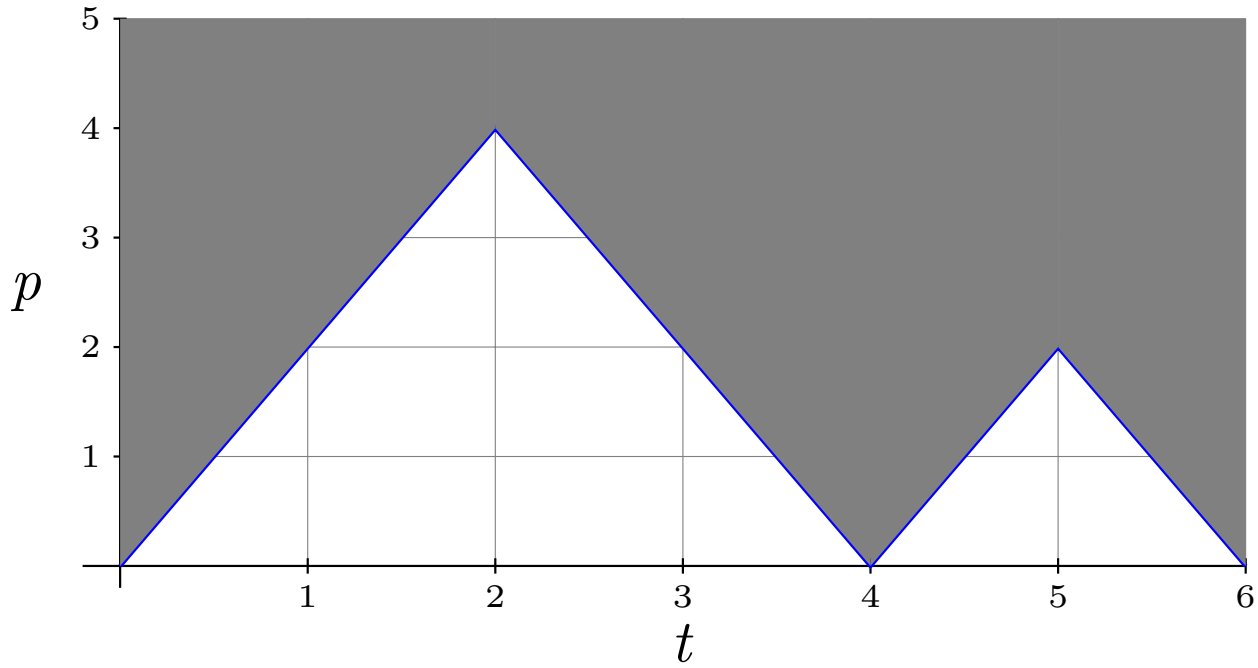

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Inductive Computation of Validity Domains

Validity domains can be computed inductively as follows:

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\begin{aligned}
d(x, f(x) > p) &= \{(t, p) : f(x[t]) > p\} \\
d(x, \varphi \wedge \psi) &= d(x, \varphi) \cap d(x, \psi) \\
d(x, \neg\varphi) &= \overline{d(x, \varphi)} \\
d(x, \varphi\, \mathcal{U}_{[a,b]} \psi) &= \{(t, p, s) : \exists t' \in [t + a, t + b] \text{ s.t. } (t', p, s) \in d(x, \psi) \wedge \\
&\qquad \forall t'' \in [t, t'](t'', p, s) \in d(x, \varphi)\}
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Example

Validity domains of $\varphi = \Box_{[0,s_1]}(x < p)$?

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& d(x, \Box_{[0,s_1]}(x < p)) \\
= \; & \{(t, s_1, p) \text{ s.t. } (x,t) \models \Box_{[0,s_1]}(x < p)\} \\
= \; & \{(t, s_1, p) \text{ s.t. } \forall \tau,\ t \leq \tau \leq t + s_1 \implies (\tau, p) \in d(x, x < p)\}
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Quantifier-free (QF) formula, knowing a QF formula for $d(x, x < p)$:

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= \; & p + 2s_1 + 2t < 12 \vee p + 2t > 12 \vee p > 0 \vee p \leq 0) \wedge \\
& (p + 2s_1 + 2t < 8 \vee p + 2t > 8 \vee p + 4 \leq 0 \vee p > 4) \wedge \\
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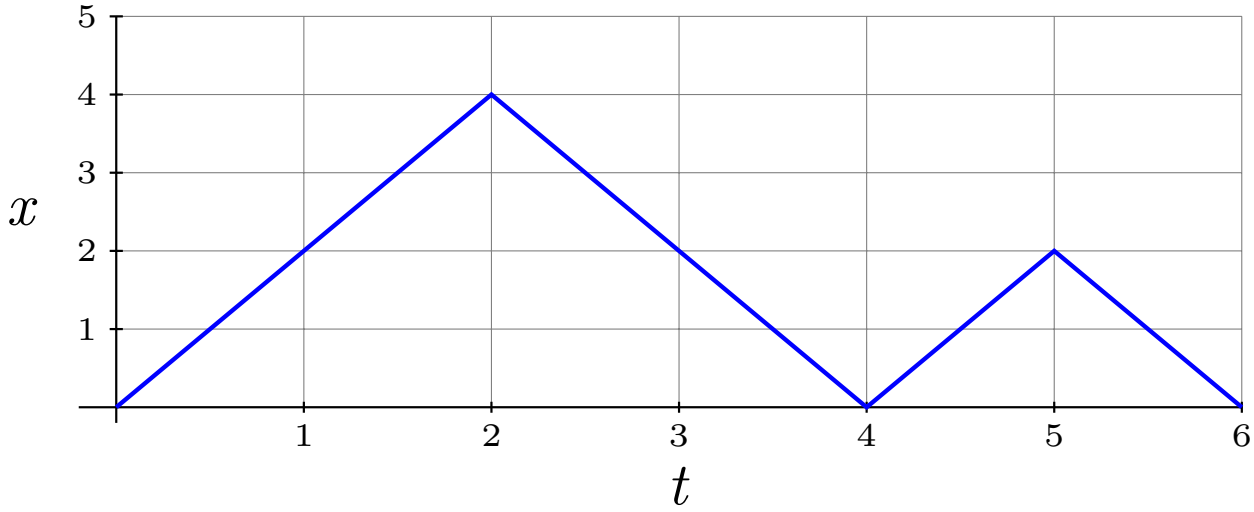
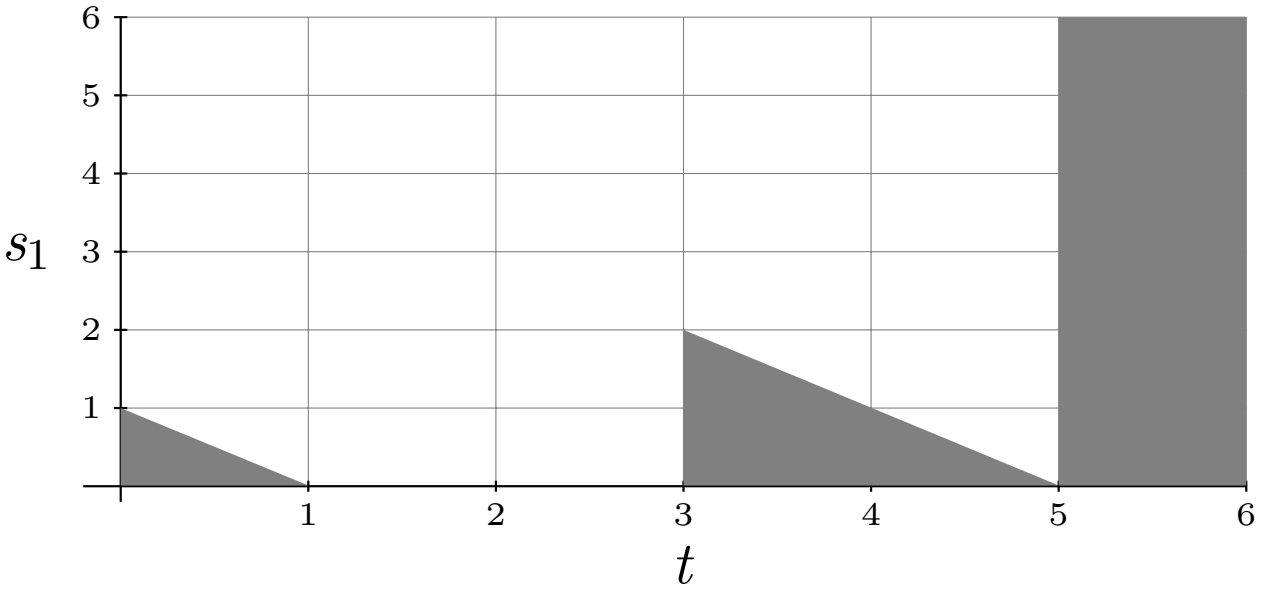


Validity domain $d(x, \Box_{[0,s_1]}(x < 2))$



Experimental Results

Typical stabilization property

$$\varphi_{st} : \Box((x > p) \rightarrow \diamondsuit_{[0,s_2]}\Box_{[0,s_1]}(x < p)).$$

which we decompose into subformulas

$$\begin{aligned}
\varphi_1 : &\quad \Box_{[0,s_1]}(x < p) \\
\varphi_2 : &\quad \diamondsuit_{[0,s_2]}\Box_{[0,s_1]}(x < p) \\
\varphi_3 : &\quad (x \geq p) \rightarrow \diamondsuit_{[0,s_2]}\Box_{[0,s_1]}(x < p)
\end{aligned}$$

We tested the computation of validity domains using QE for a signal with different number of samples.

Experimental Results

	φ_1		φ_2		φ_3		φ_{st}	
k	time(s)	size	time(s)	size	time(s)	size	time(s)	size
8	0.02	38	0.11	197	0.17	207	3	4219
16	0.10	66	0.81	855	0.74	375	83.79	37709
32	0.26	86	19.07	6553	18.27	2885	*	*
64	4.16	144	341.95	23103	308.93	10258	*	*
128	68.29	895	*	*	*	*	*	*
256	386.72	3098	*	*	*	*	*	*

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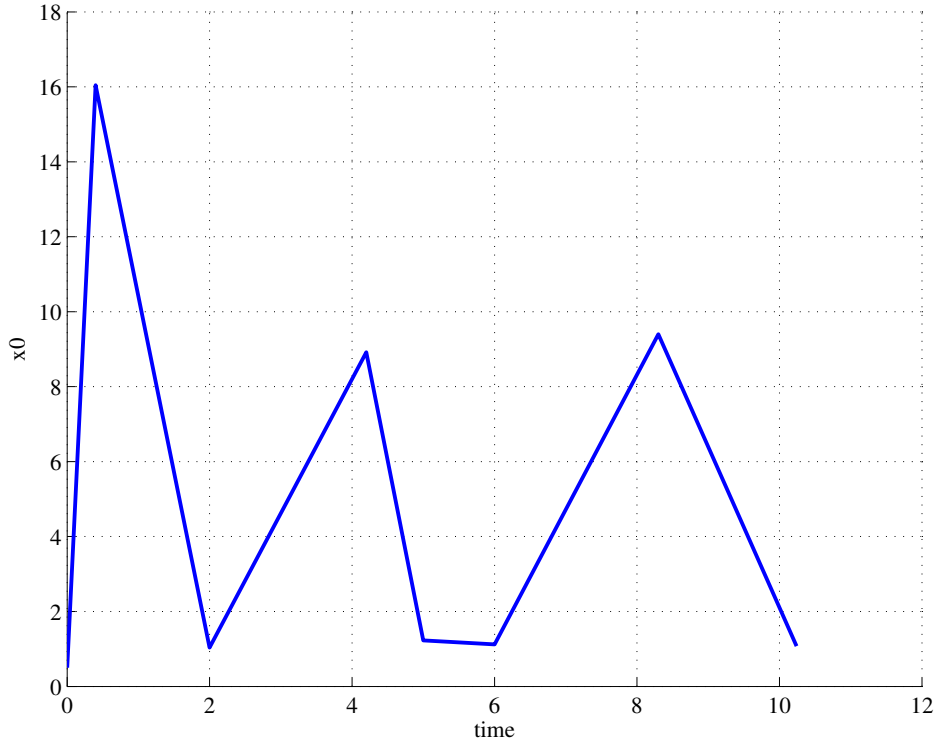

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128	68.29	895	*	*	*	*	*	*
256	386.72	3098	*	*	*	*	*	*

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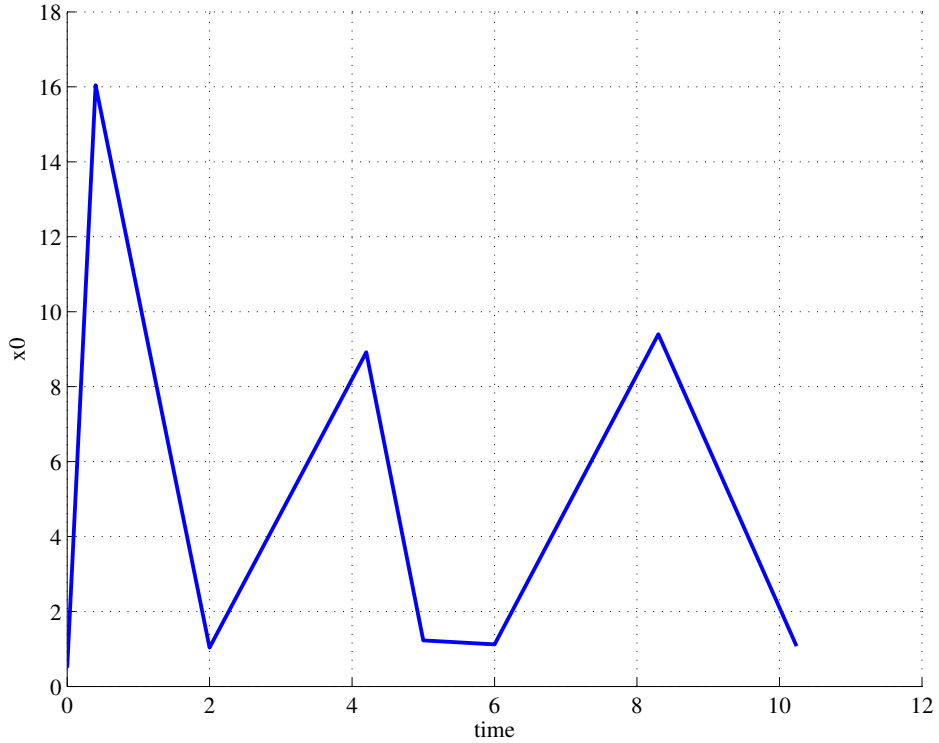

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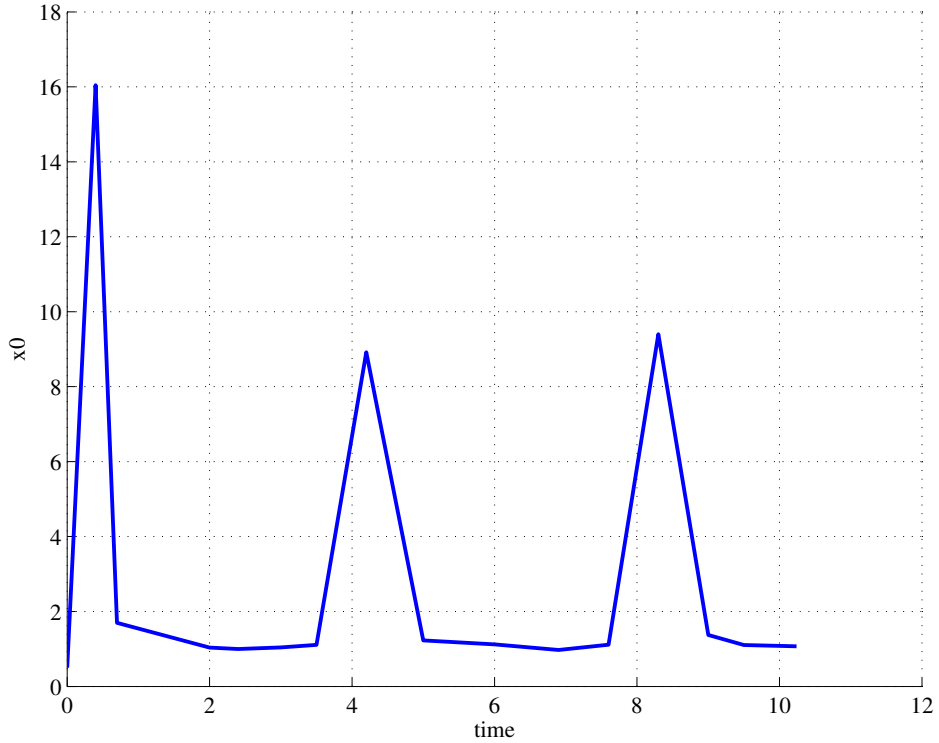

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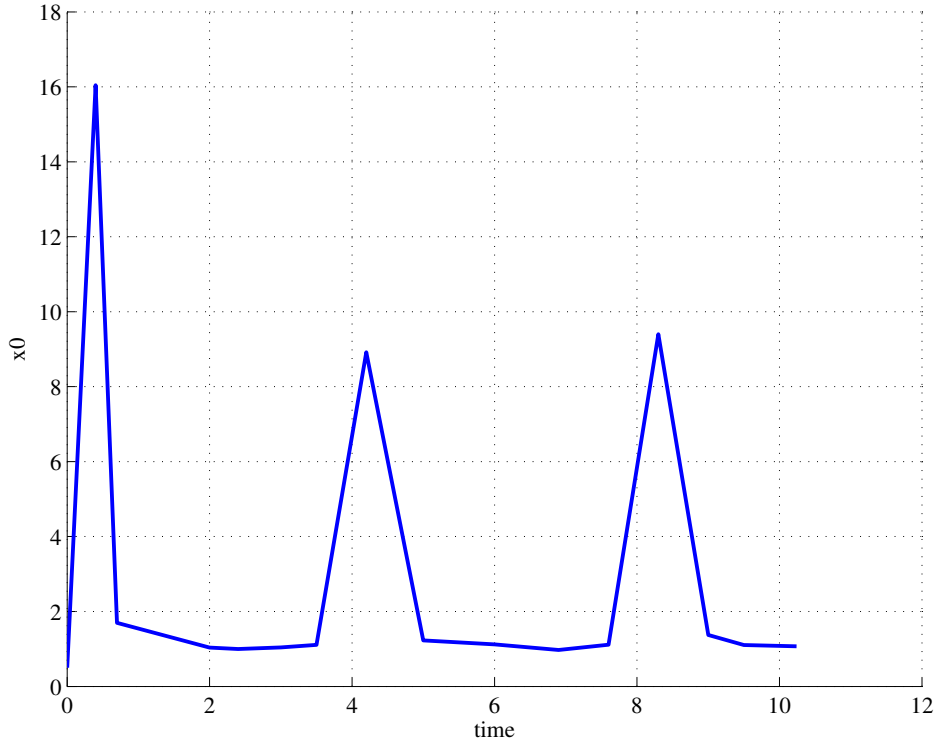

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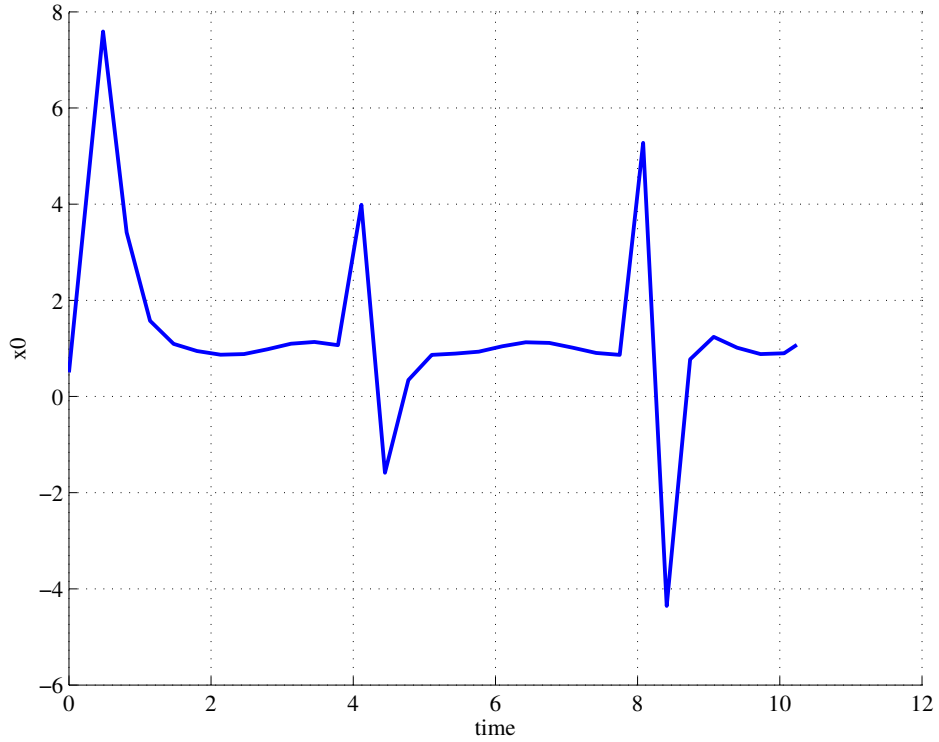

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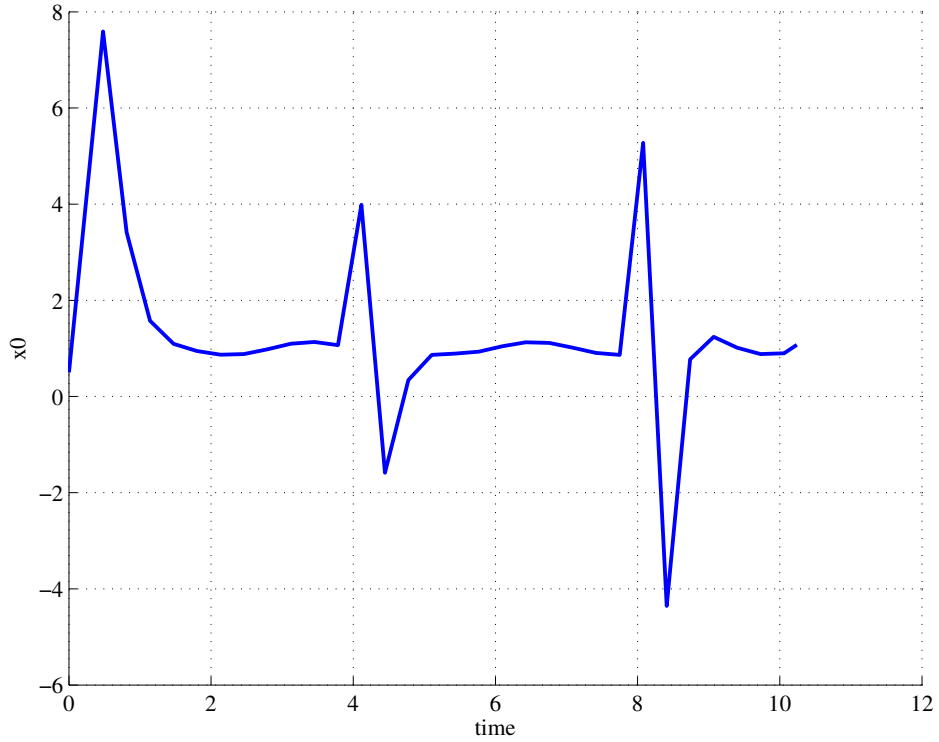

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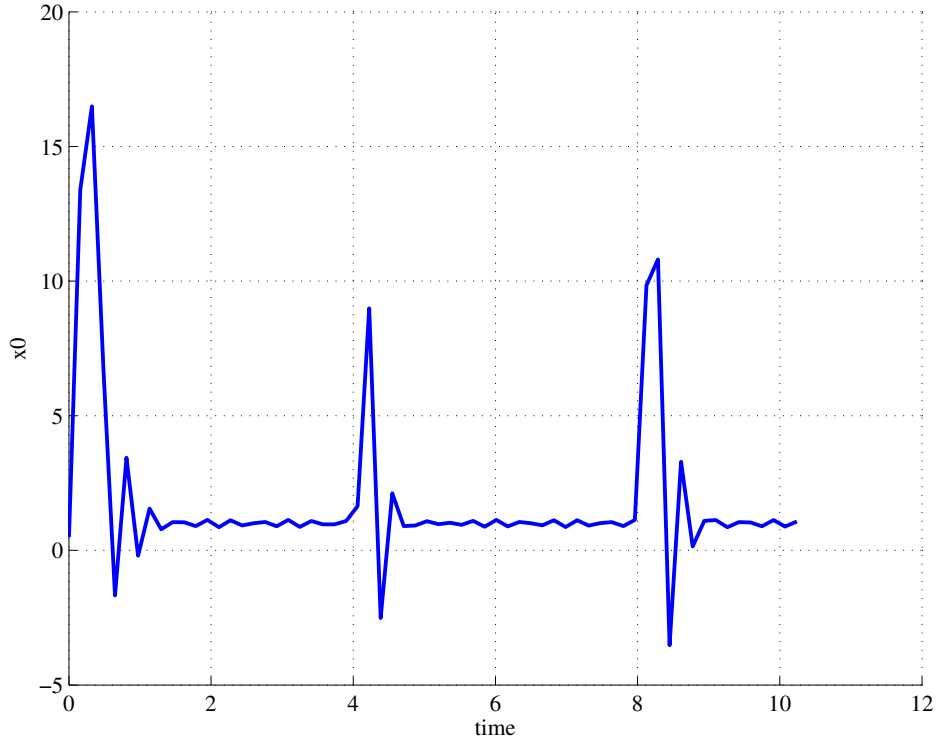

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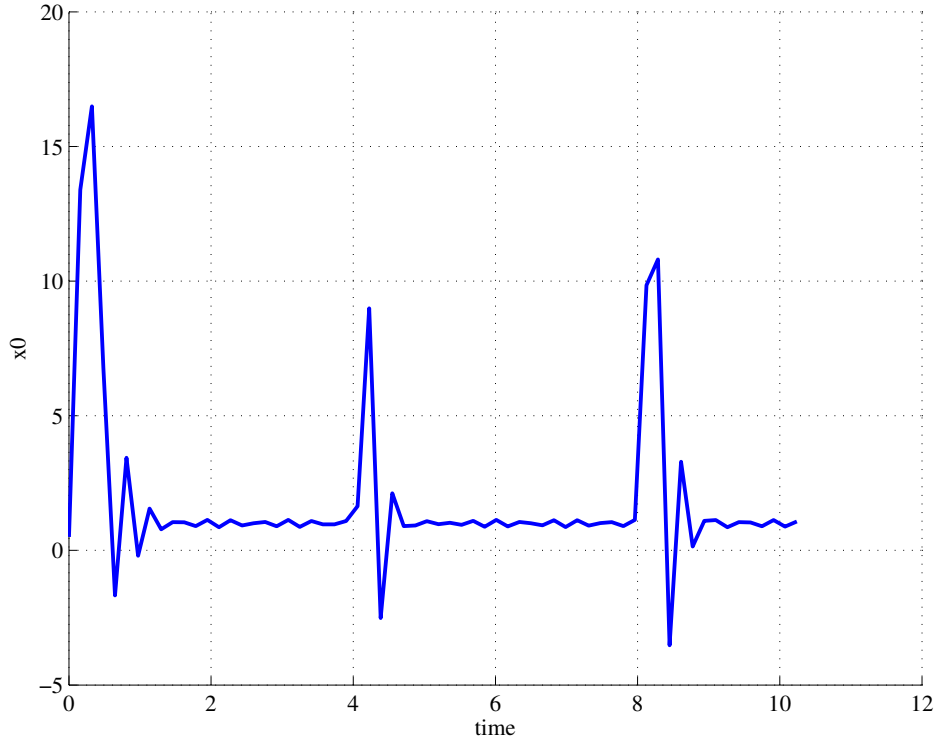

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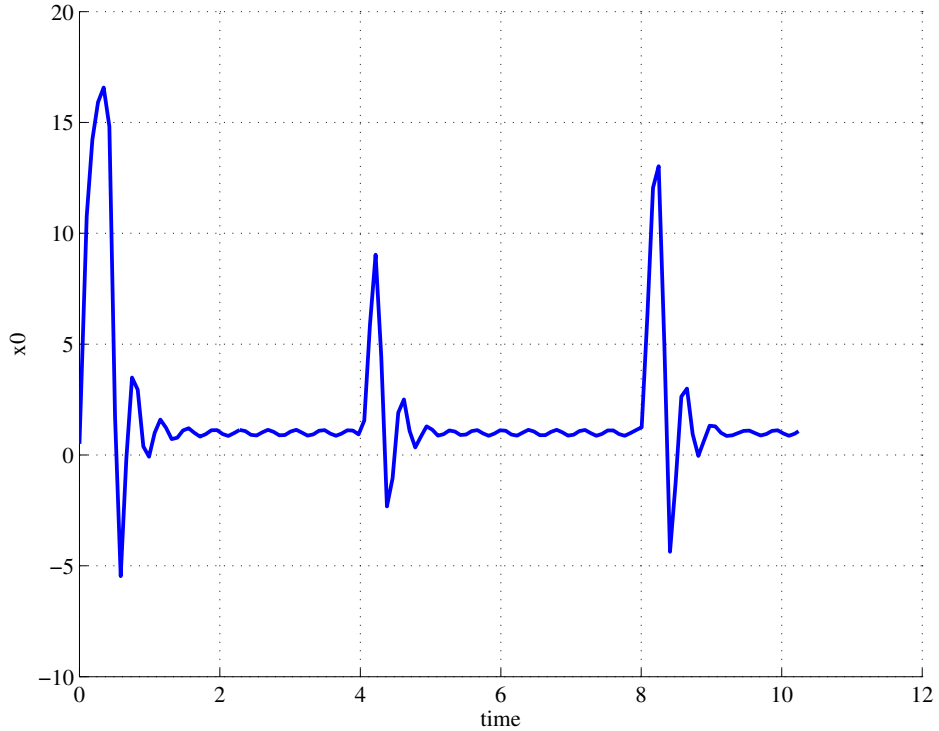

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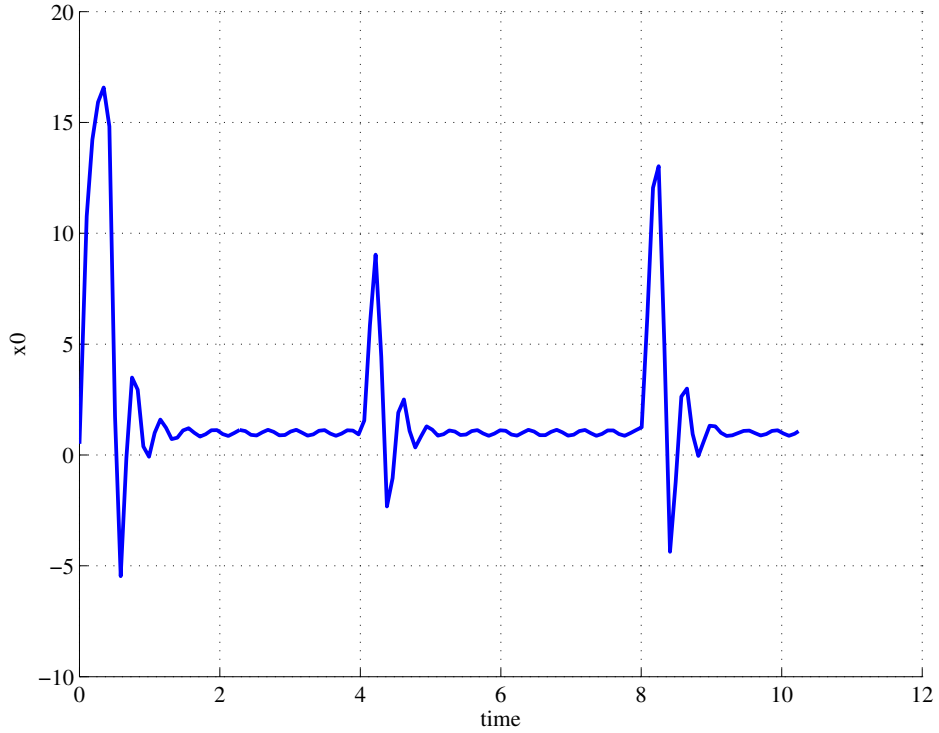

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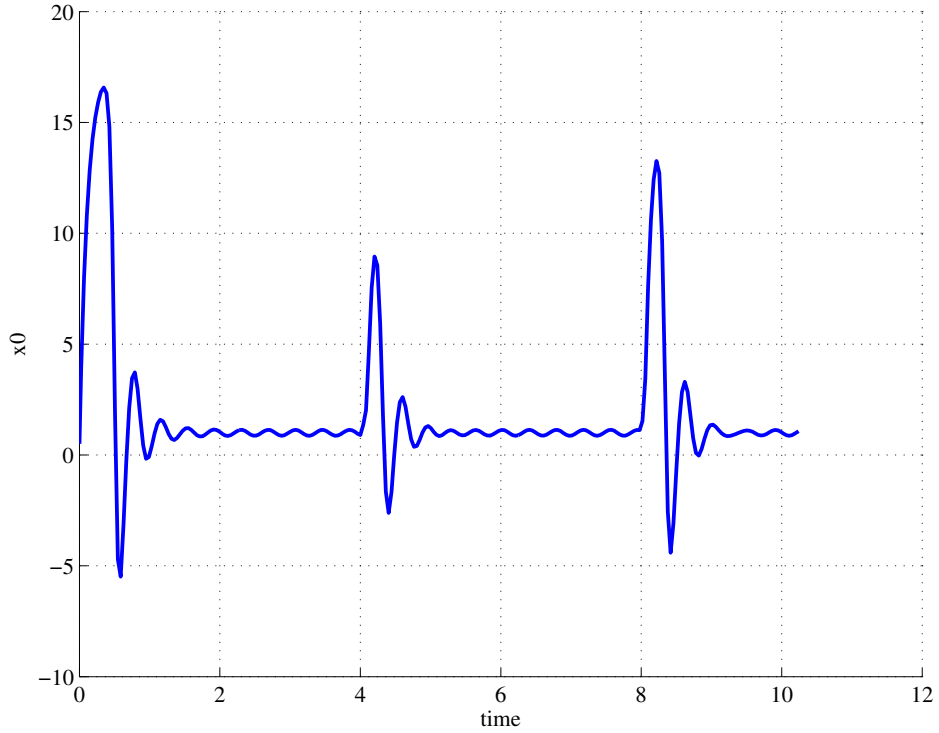

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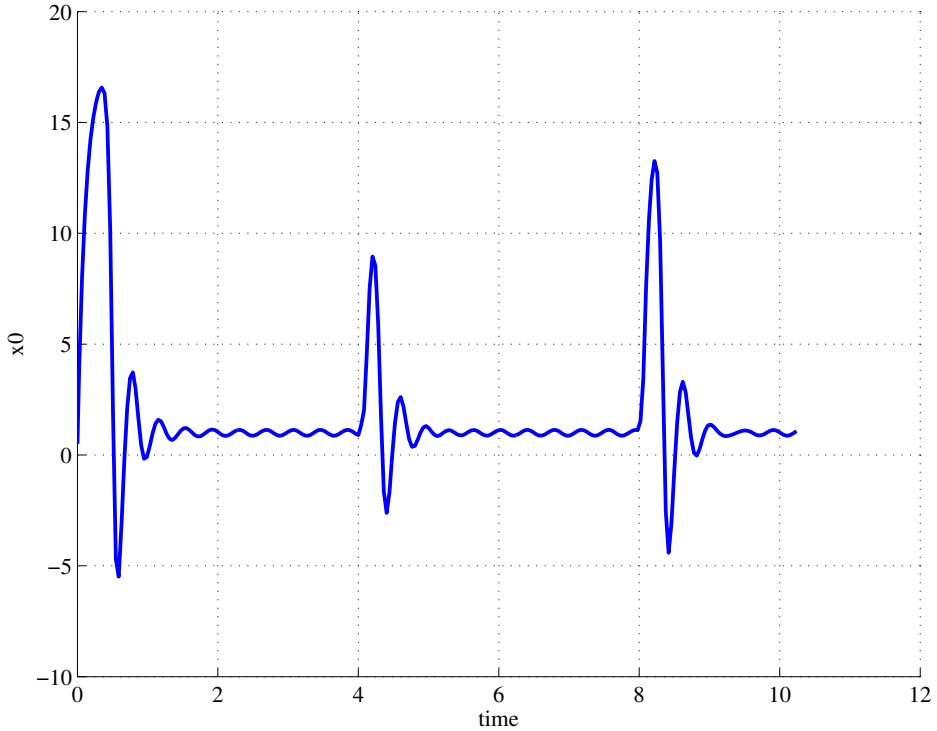

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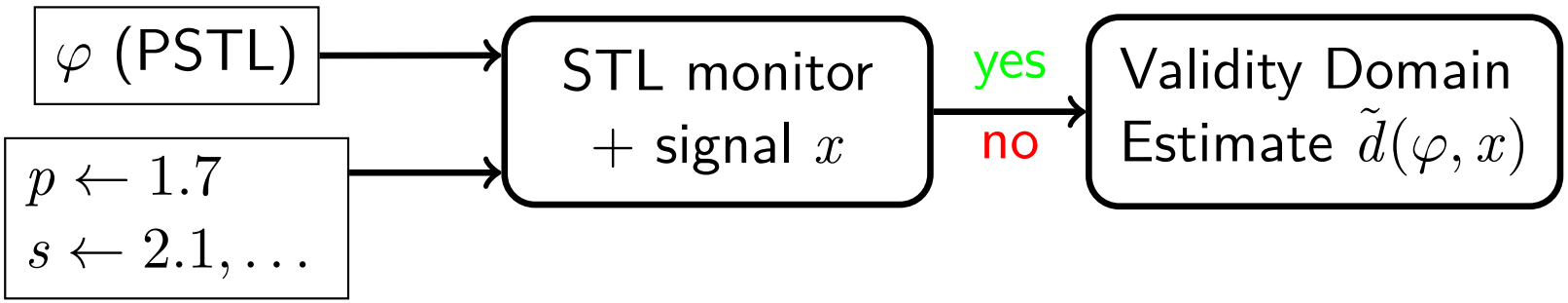
Outline

General Approach

Estimate the set $d(\varphi, x)$ using a finite number of queries to an STL monitor with different values for the parameters



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- Scalability issues with the QE based approach
- STL (i.e., instantiated PSTL), formulas can be monitored efficiently

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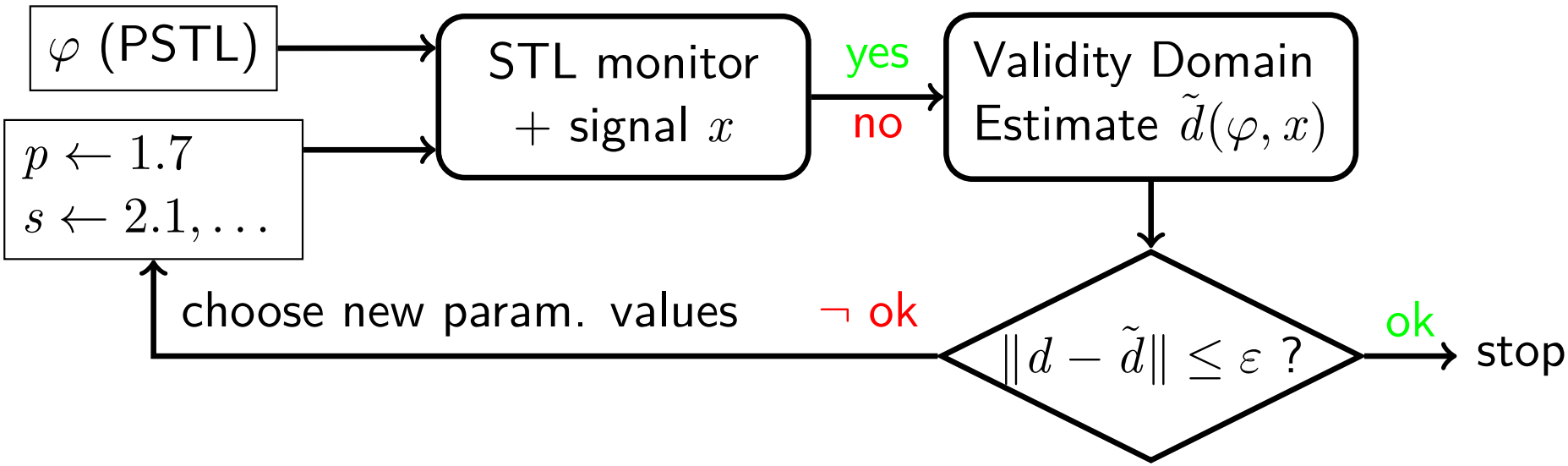

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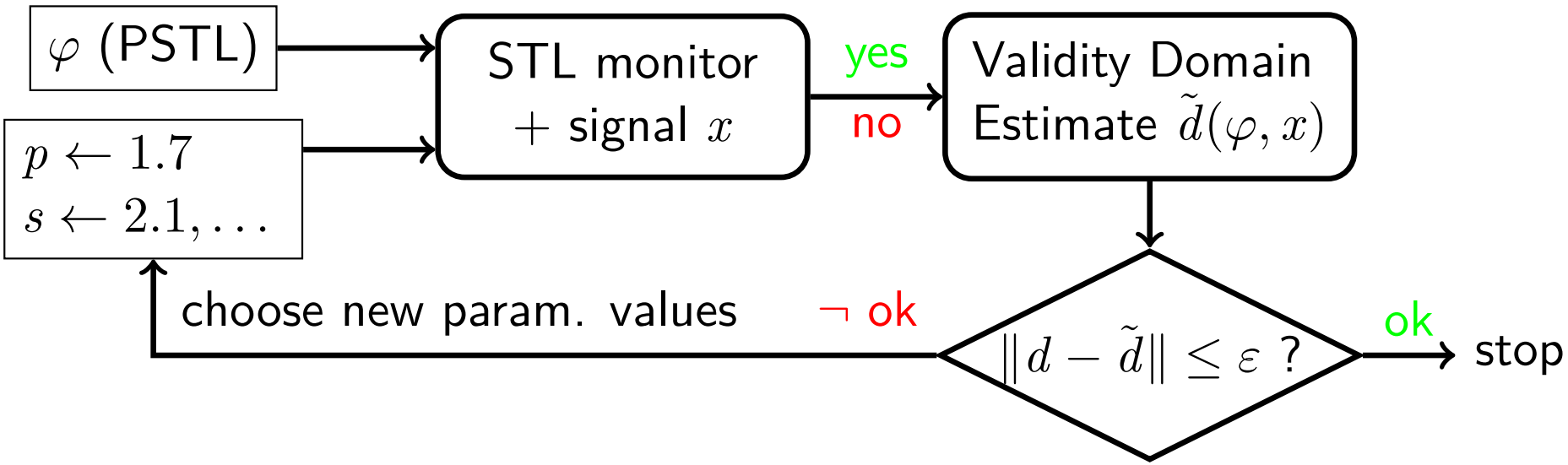

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Monitoring Computational Cost[1]

(a) Signal x, formula $\varphi = (x > 0) \underbrace{\mathcal{U}_{[0,1]}\ (x > 0)\ \mathcal{U}_{[0,1]}\ (x > 0) \ldots}_{i \text{ times}}$

(b) Formula φ_{st} with different number of samples

(a)

i	time(s)
1	0.34747
2	0.46335
3	0.60599
4	0.76067
5	0.89201
6	1.03761

(b)

Nb of samples	time(s)
31416	0.18402
345566	0.40761
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- However, the approximation scales exponentially with the number of queries.
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Polarity

Definition (informal)

The *polarity* $\pi(p, \varphi)$ of a parameter p with respect to a formula φ is positive if it is easier to satisfy φ as we increase the value of p and negative otherwise

Examples and remarks

- Magnitude parameters satisfy:

$$\pi(p, |x| \leq p) = +, \quad \pi(p, |x| \geq p) = -$$

- Temporal parameters satisfy:

$$\pi(p, \Diamond_{[0,p]} \varphi) = +, \quad \pi(p, \Box_{[0,p]} \varphi) = -$$

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- It can be undetermined, e.g. in $\pi(p, |x| \leq p \Leftrightarrow |x| \geq p) = ?$

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Monotonic Validity Domains

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A subset $V \subseteq \mathcal{P} \times \mathcal{S}$ is monotonic if for every i, whenever a parameter valuation $(v_1, \ldots, v_i, \ldots, v_n)$ is in V so is any $(v_1, \ldots, v'_i, \ldots, v_n) \in \mathcal{P} \times \mathcal{S}$ satisfying $v'_i > v_i$ (when $\pi(p_i, \varphi) = +$) or $v'_i < v_i$ (when $\pi(p_i, \varphi) = -$).

- If φ has only one monotonic param. p then $d(x, \varphi) \subset [p^*, \infty)$ for some p^*
- More generally the *boundary* of $d(x, \varphi)$ has the same properties as *Pareto fronts*

Monotonic validity domains can be estimated using heuristics generalizing dichotomic search in dim ≥ 1

In the following, we sketch a methodology adapted from

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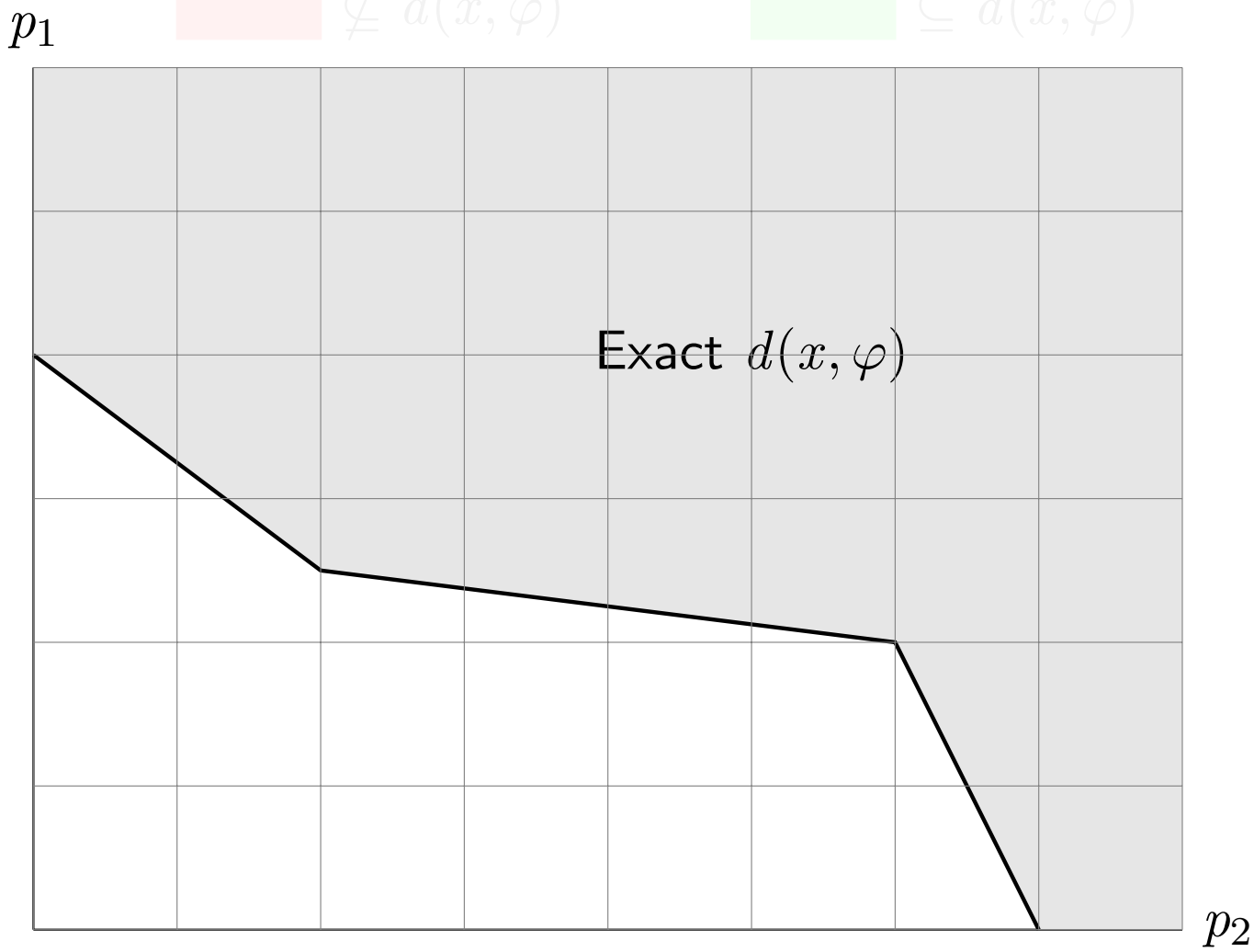
Approximating Monotonic Validity Domains, Illustration



Different heuristics for reducing ϵ, distance between and .

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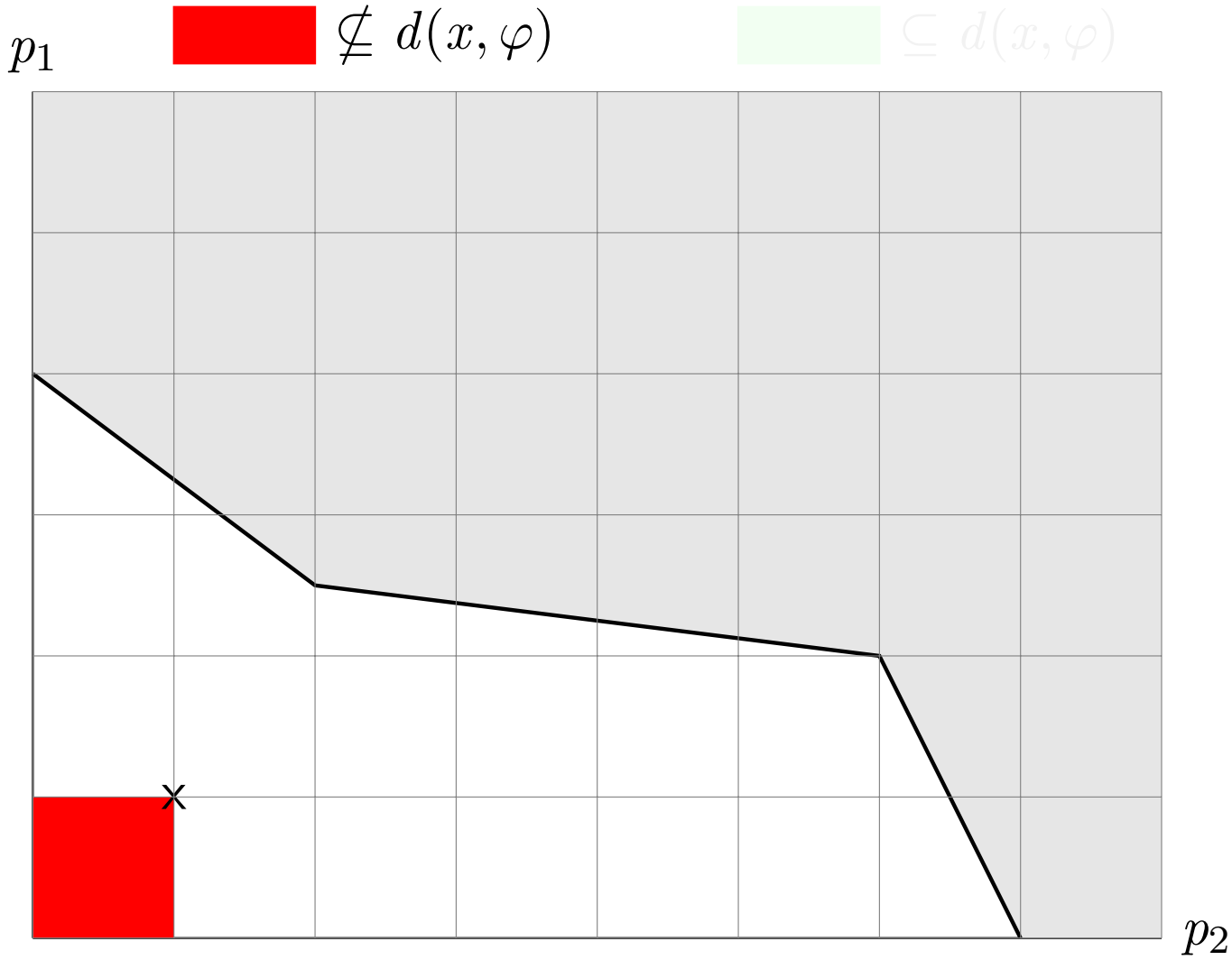

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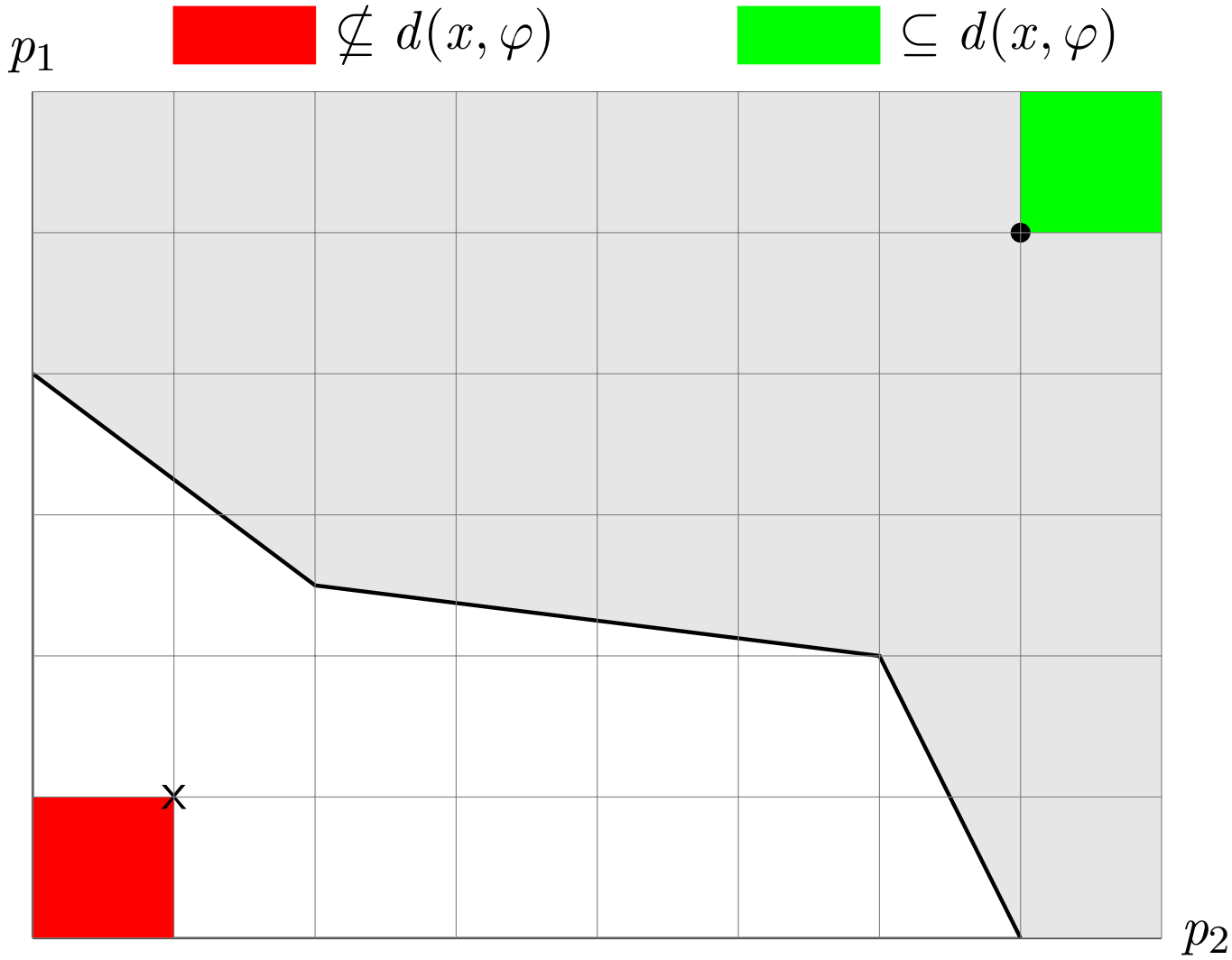

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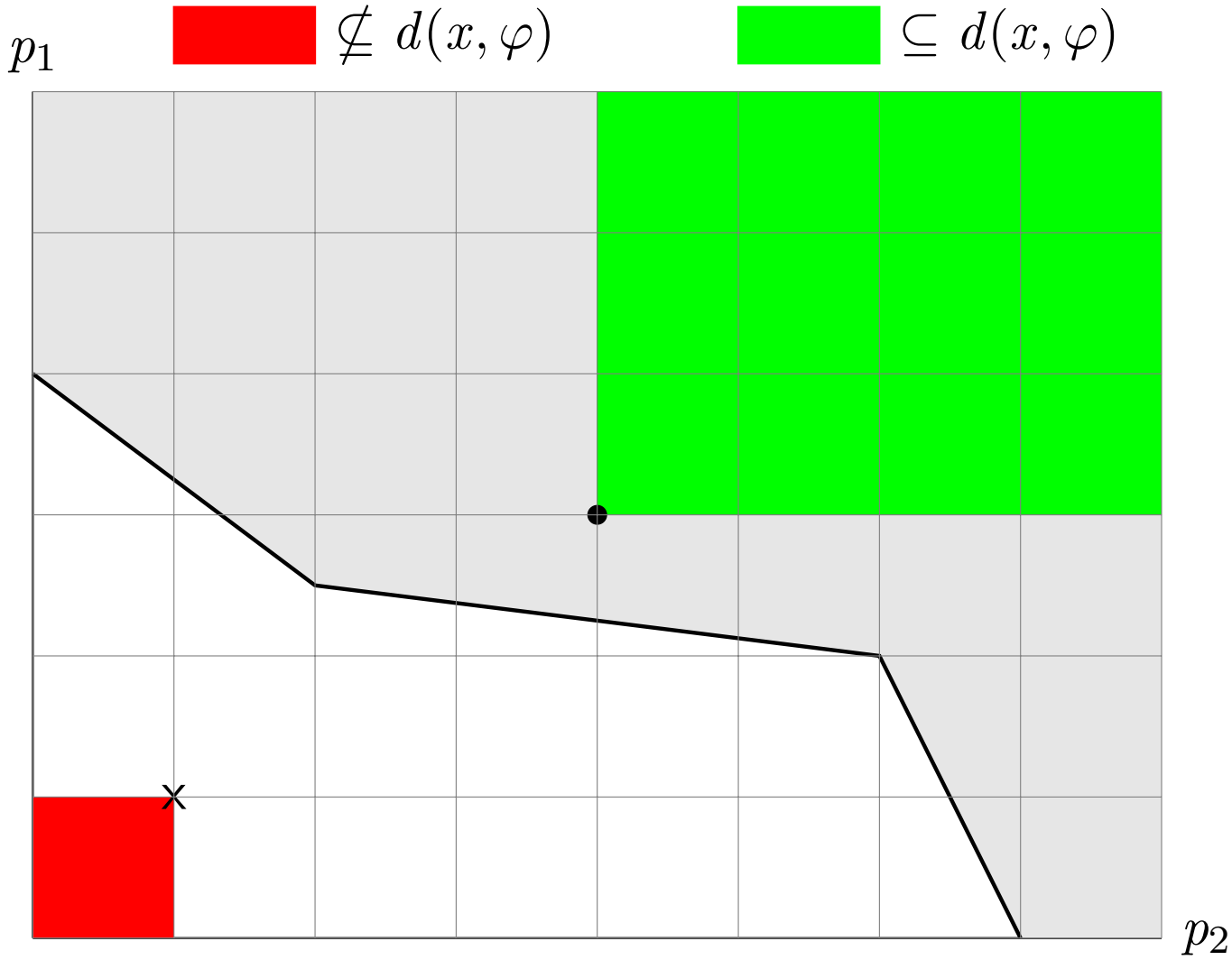

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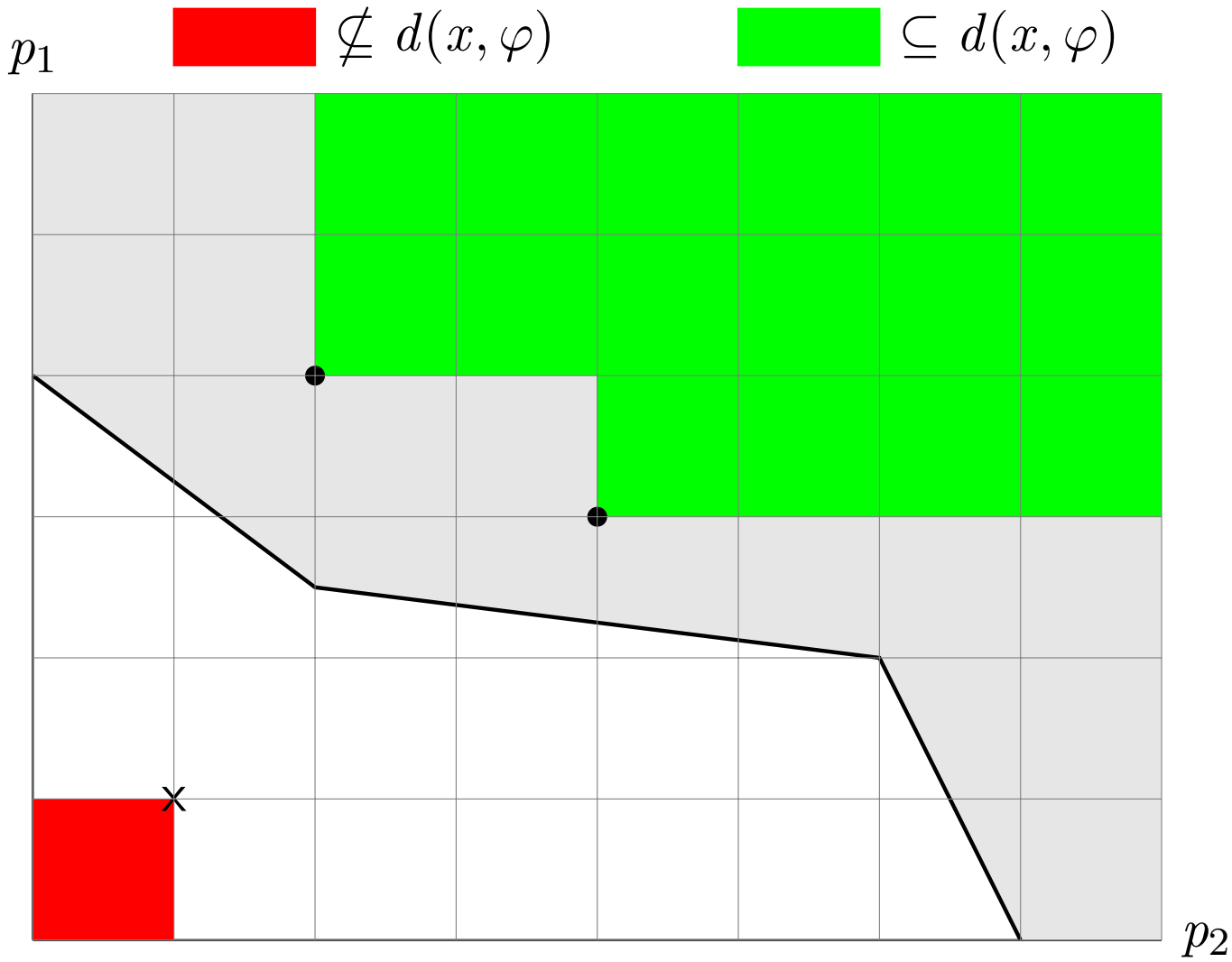

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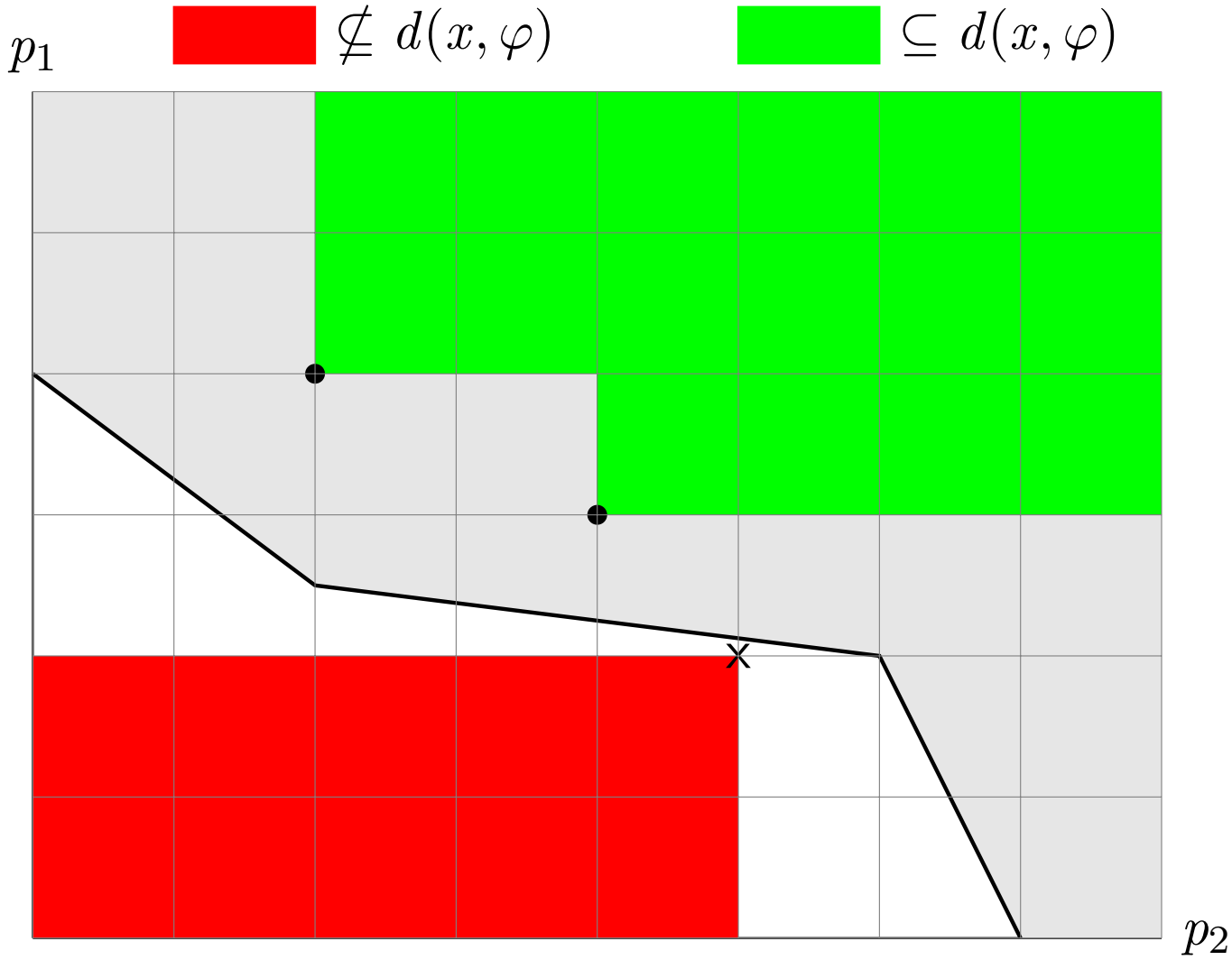

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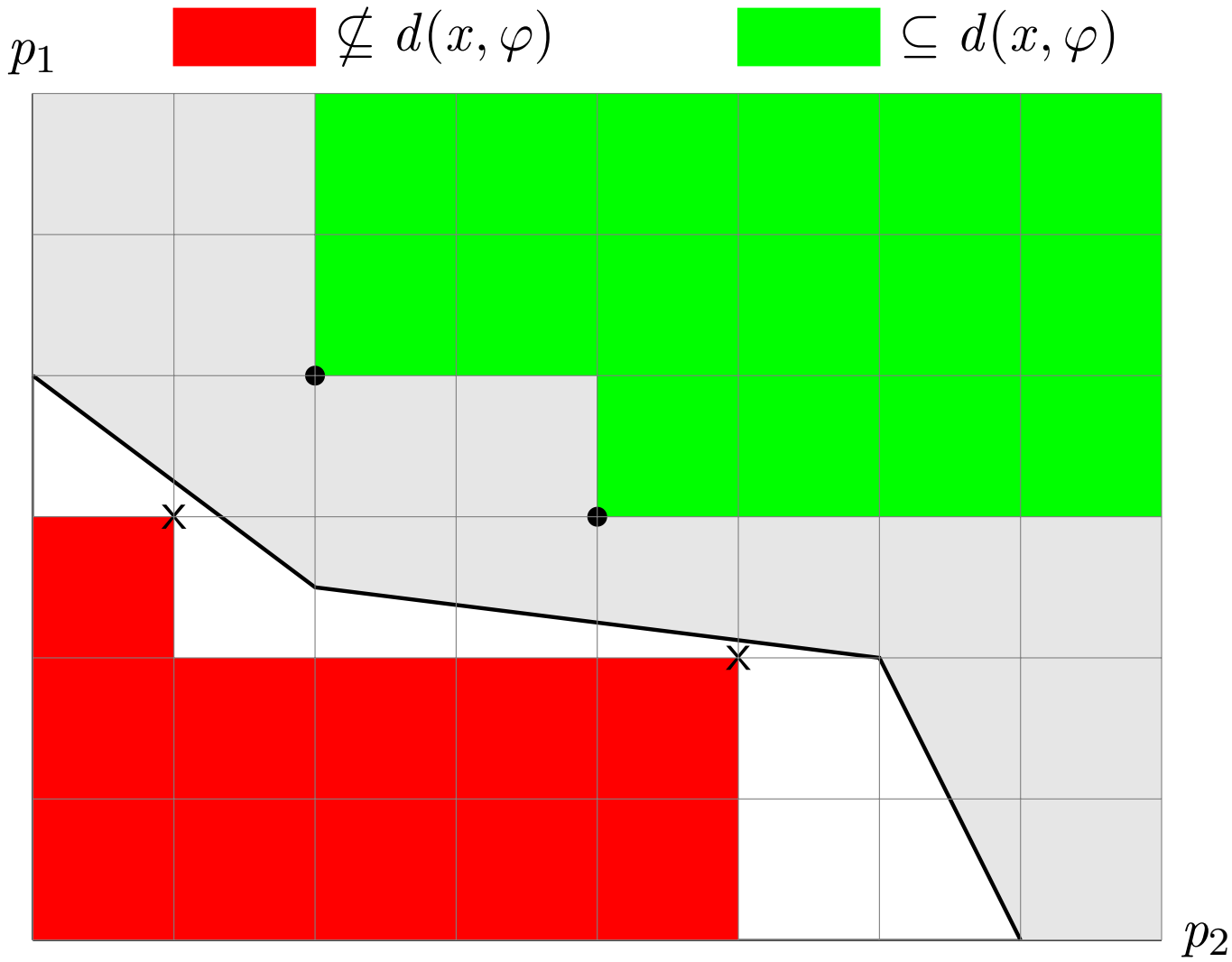

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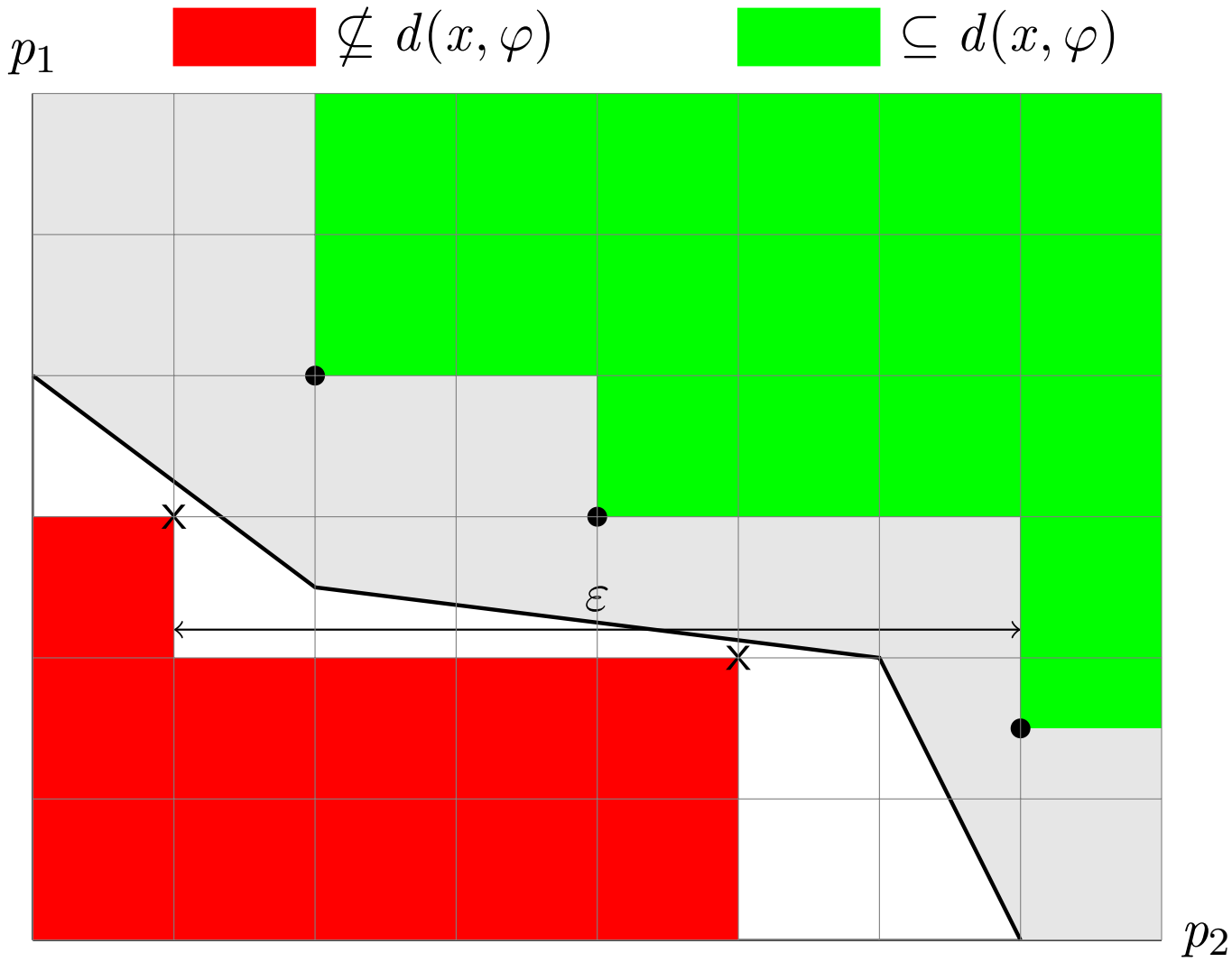
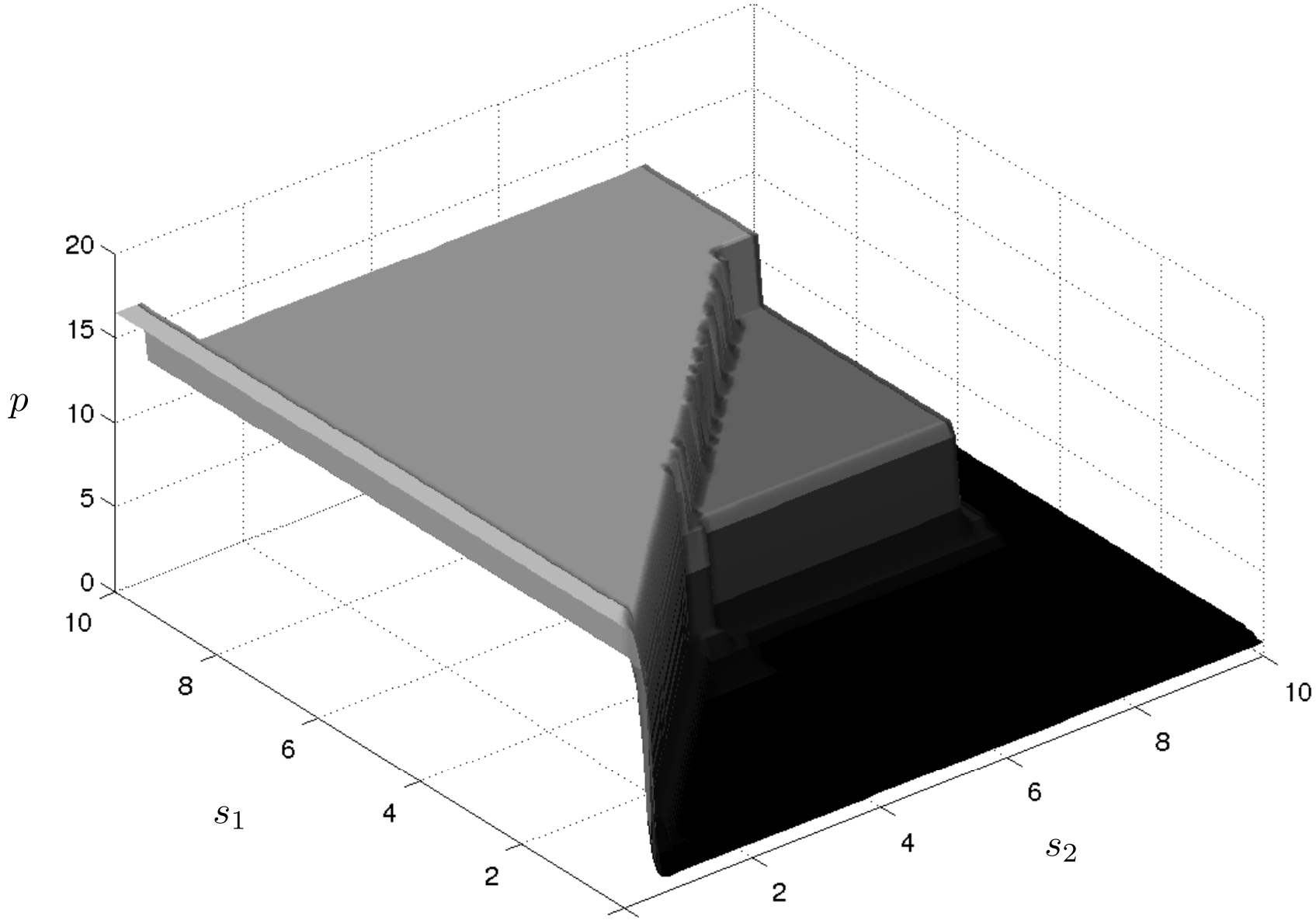


Different heuristics for reducing ϵ, distance between and .

Example

Stabilization property, $d(x, \varphi_{st})$ for x with 1024 samples



Conclusion

A problem potentially useful in many situations : given simulation traces, what properties do the system satisfy ?

Exact computation of validity domains using quantifier elimination

- Elegant but scalability problems with the size of x and φ
- What can be done beyond our "naive" approach ?

Numerical approximation using efficient monitoring

- Scales in $|x|$ and $|\varphi|$ but exponential in number of parameters
- Efficient heuristics in the case monotonic domains
- Applicable in the general case (including generic parameters, non-linear predicates, etc) but only to find *local* boundaries

Thanks for your attention !

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