Efficient Robust Monitoring for STL

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Overview

- Signal Temporal Logic (STL): temporal specifications for continuous and hybrid systems
- Quantitative satisfaction of STL can accomodate noise/approximation, increase coverage

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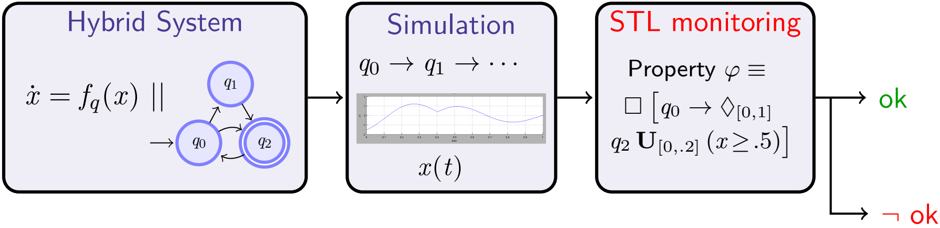

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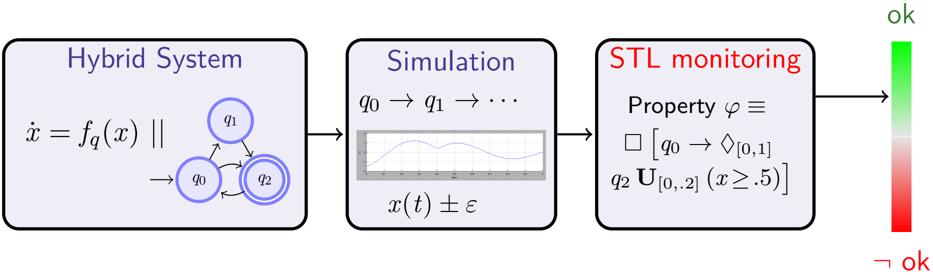

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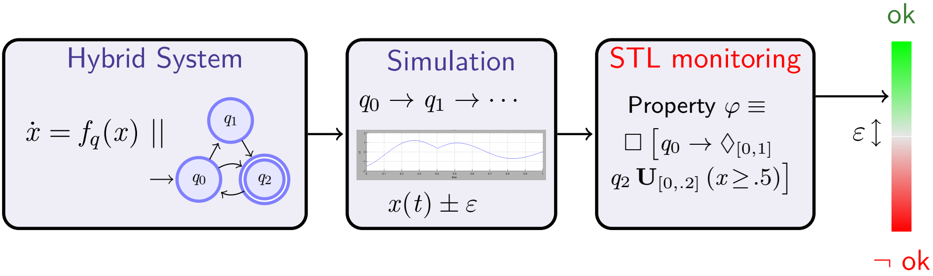

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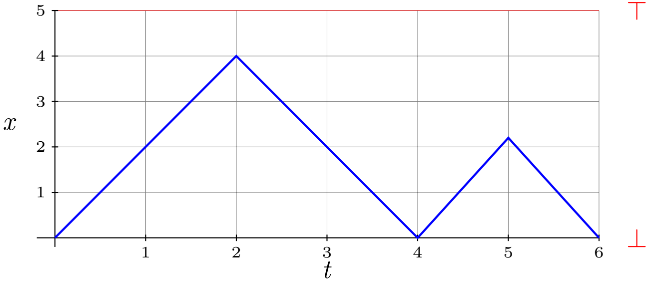
Example: Boolean Monitoring



Satisfaction signal of :

- $\varphi = x \geq 2$
- $\varphi = \Diamond_{[0,0.5]} (x \geq 2)$

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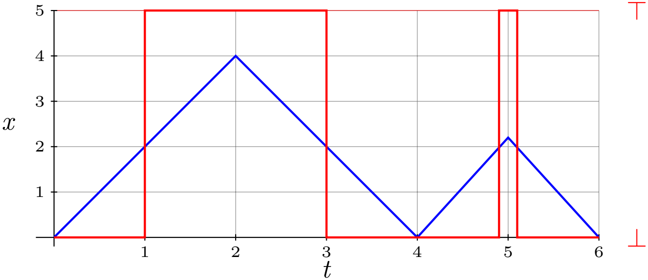

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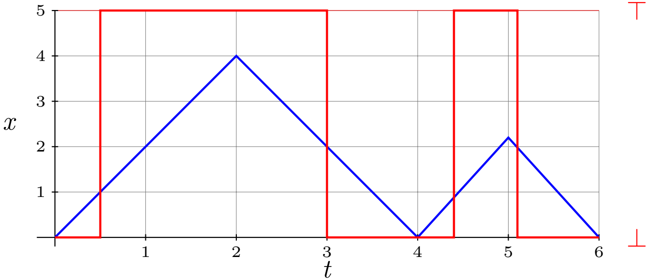

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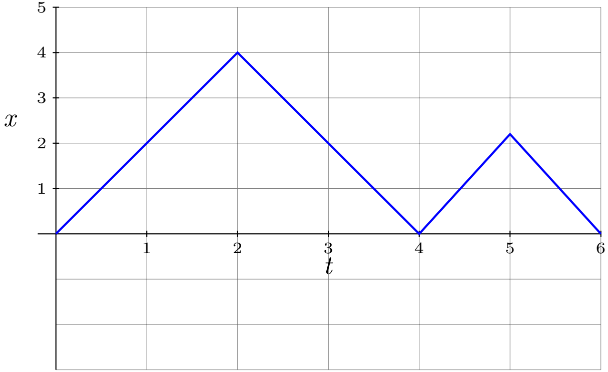

Robustness signal of :

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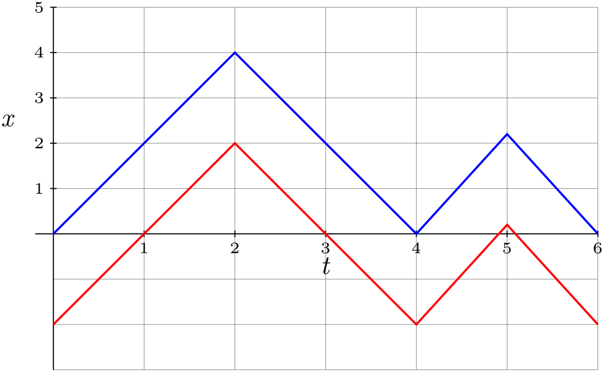

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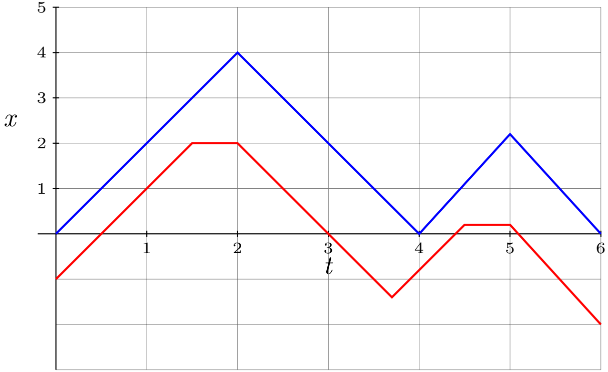

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Outline

1. Signal Temporal Logic
2. Robust Monitoring Algorithms
3. Complexity and Evalutation

Outline

Formal Definitions

Definition (STL Syntax)

$$\varphi := \mathsf{true} \mid x_i \geq c \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \, \mathbf{U}_I \, \varphi$$

with I closed interval of $\mathbb{R}^+$

Definition (STL Semantics)

The validity of a formula φ with respect to a trace w at time t is

$$\begin{array}{lcl}
w, t \vDash \mathsf{true} & & \\
w, t \vDash x_i \geq c & \Longleftrightarrow & x_i^w(t) \geq c \\
w, t \vDash \neg\varphi & \Longleftrightarrow & w, t \nvDash \varphi \\
w, t \vDash \varphi \wedge \psi & \Longleftrightarrow & w, t \vDash \varphi \text{ and } w, t \vDash \psi \\
w, t \vDash \varphi \, \mathbf{U}_I \, \psi & \Longleftrightarrow & \exists t' \in t + I \text{ s.t. } w, t' \vDash \psi \\
& & \text{and } \forall t'' \in [t, t'],\ w, t'' \vDash \varphi
\end{array}$$

Additionally: $\Diamond_I \varphi := \top \, \mathbf{U}_I \, \varphi$ and $\Box_I \varphi := \neg \Diamond_I \neg\varphi$.

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w, t \models x_i \geq c & \iff & x_i^w(t) \geq c \\
w, t \models \neg\varphi & \iff & w, t \not\models \varphi \\
w, t \models \varphi \wedge \psi & \iff & w, t \models \varphi \text{ and } w, t \models \psi \\
w, t \models \varphi \, \mathbf{U}_I \, \psi & \iff & \exists t' \in t + I \text{ s.t. } w, t' \models \psi \\
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Additionally: $\Diamond_I \, \varphi := \top \, \mathbf{U}_I \, \varphi$ and $\Box_I \, \varphi := \neg \Diamond_I \, \neg\varphi$.

Monitoring

Truth value of a formula for a given trace defines a Boolean signal

Definition (Satisfaction Signal)

$$\chi(\varphi, w, .) := t \mapsto \begin{cases} \top \text{ if } w, t \vDash \varphi \\ \bot \text{ otherwise} \end{cases}$$

Procedure: by bottom-up computation of $\chi(\psi, x, .)$ for each subformula $\psi \in \varphi$

From Boolean to quantitative semantics

Boolean algebra $(\{\top, \bot\}, <, -)$	Real algebra $(\mathbb{R} \cup \{\top, \bot\}, <, -)$
$p \vee p \sim p$	–
$p \wedge \mathsf{true} \sim p$	–
$p \vee (q \wedge r) \sim (p \wedge q) \vee (p \wedge r)$	–
$\neg p \wedge \neg q \sim \neg(p \vee q)$	–
$p \vee \neg p \sim \mathsf{true}$	x

Satisfaction Signal

$$
\begin{array}{lcl}
\chi(\textsf{true}, w, t) & = & \top \\
\chi(x_i \geq c, w, t) & = & \left\{ \begin{array}{l} \top \text{ if } x_i(t) \geq c, \\ \bot \text{ otherwise} \end{array} \right. \\
\chi(\neg\varphi, w, t) & = & -\chi(\varphi, w, t) \\
\chi(\varphi \wedge \psi, w, t) & = & \min\{\chi(\varphi, w, t), \chi(\varphi, w, t)\} \\
\chi(\varphi \, \mathbf{U}_I \, \psi, w, t) & = & \sup\limits_{t' \in t+I} \min\{\chi(\psi, w, t'), \inf\limits_{t'' \in [t,t']} \chi(\varphi, w, t'')\}
\end{array}
$$

Quantitative Semantics

$$
\begin{array}{lcl}
\rho(\mathsf{true}, w, t) & = & \top \\[1em]
\rho(x_i \geq c, w, t) & = & x_i(t) - c \\[1em]
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Property of Robustness Estimate

- Sign indicates satisfaction status
- Absolute value indicates tolerance

Theorem (Faneikos and Pappas 2009)

$$\rho(\varphi, w, t) > 0 \Rightarrow w, t \vDash \varphi$$

$$w, t \vDash \varphi \textit{ and } \|w - w'\|_\infty < \rho(\varphi, w, t) \quad \Rightarrow \quad w', t \vDash \varphi$$

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Corollary

$$\rho(\varphi, w, t) < 0 \Rightarrow w, t \not\models \varphi$$

$$w, t \not\models \varphi \textit{ and } \|w - w'\|_\infty < -\rho(\varphi, w, t) \quad \Rightarrow \quad w', t \not\models \varphi$$

Until Rewrite

The rewrite extends from Boolean to quantitative semantics

- unbounded until
  $\varphi \, \mathbf{U}_{[a,+\infty)} \, \psi \;\sim\; \Box_{[0,a]} \, (\varphi \, \mathbf{U} \, \psi)$
- bounded until
  $\varphi \, \mathbf{U}_{[a,b]} \, \psi \;\sim\; \Diamond_{[a,b]} \, \psi \wedge \varphi \, \mathbf{U}_{[a,+\infty)} \, \psi$

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Outline

Preliminaries

- Signals: timed words $(t_i, y(t_i))_{i \leq n_y}$, with linear interpolation
- Procedure: inductive computation of robustness signals $\rho(\varphi, x, .)$ on the formula structure
- Sampling: continuity, piecewise-affine property are preserved

Boolean operators

Negation

- Input signal: $(t_i, y(t_i))_{i \leq n_y}$
- Output signal: $(t_i, -y(t_i))_{i \leq n_y}$

Conjunction

- Input signals: $(t_i, y(t_i))_{i \leq n_y}$, $(t'_i, y'(t'_i))_{i \leq n_{y'}}$
- Output signal: $(r_i, z(r_i))_{i \leq n_z}$
  Time sequence r contains t, t', and punctual intersections $y \cap y'$
  Value $z(r_i) = \min\{y(r_i), y'(r_i)\}$

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Untimed Until

Induction Property: for all $s < t$

- Boolean Semantics $\quad w, s \vDash \varphi \, \mathbf{U} \, \psi \quad \iff$
  $w_{\upharpoonright[s,t)}, s \vDash \varphi \, \mathbf{U} \, \psi$ or $\big(w_{\upharpoonright[s,t)}, s \vDash \Box \, \varphi$ and $w, t \vDash \varphi \, \mathbf{U} \, \psi\big)$

- Quantitative Semantics $\quad \rho(\varphi \, \mathbf{U} \, \psi, w, t) =$
  $\max\{\rho(\varphi \, \mathbf{U} \, \psi, w_{\upharpoonright[s,t)}, s), \min\{\rho(\Box \, \varphi, w_{\upharpoonright[s,t)}, s), \rho(\varphi \, \mathbf{U} \, \psi, w, t)\}\}$

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Timed Eventually

Definition: $\rho(\Diamond_{[a,b]} \varphi, w, t) = \sup\limits_{t' \in [t+a, t+b]} \rho(\varphi, w, t) = \sup\limits_{[t+a, t+b]} y$

The maximum is reached at $t + a, t + b$, or at sample point in $\{t_i \mid t_i \in (t + a, t + b]\}$

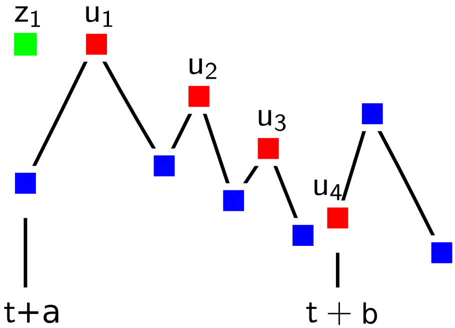
Theorem (Lemire 2006)

The maximum of a sequence of over a shifting window can be computed in linear time

Idea: we maintain an ordered set M such that

$\max\{y(t_i) | i \in M\} = \max\{y(t_i) \mid t_i \in (t + a, t + b]\}$

Timed Eventually: two steps in the algorithm



Maximum candidates $\{y(t_i) | i \in M\} = \{u_1, u_2, u_3, u_4\}$

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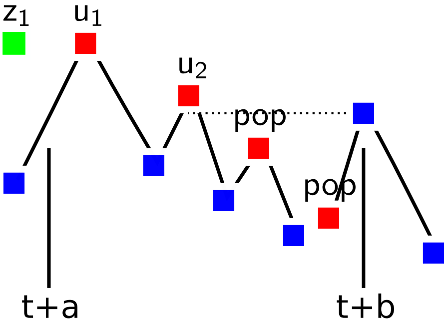

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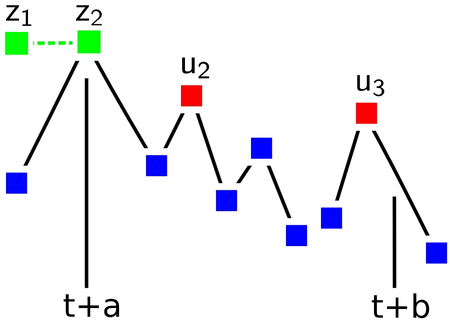


Maximum candidates $\{y(t_i) | i \in M\} = \{u_2, u_3\}$

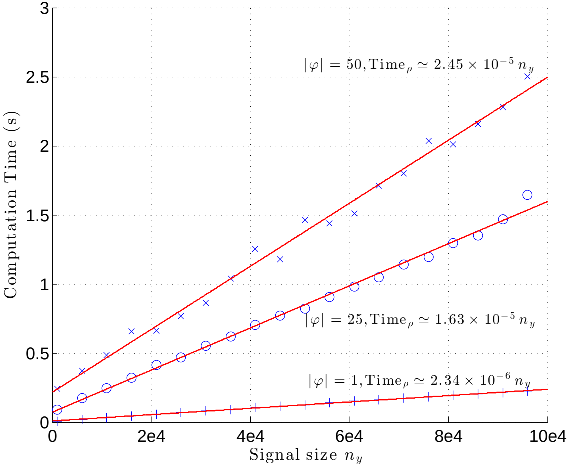
Outline

Worst-case Complexity

- For each subformula $\psi \in \varphi$, computation time linear in the input size
- Problem: size of robustness signal can increase exponentially with formula height
- Computation time in $\mathcal{O}(|\varphi| \cdot d^{\mathrm{h}(\varphi)} \cdot |x|)$

Experimentation: Random Signals

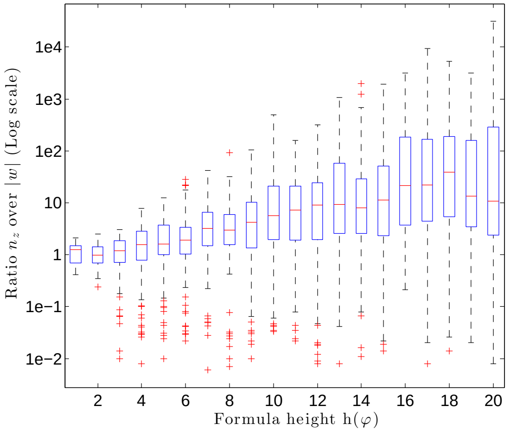
Computation is linear in size of input trace



Experimentation: Random Formulas

Exponential growth rate of robustness signal size with formula height

- average: $d \simeq 1.12$
- worst-case: $d \simeq 1.7$



Conclusion

Summary

- Enhancement to "Boolean" monitoring with reasonable computational overhead
- Piecewise affine signals: practical model for robustness computation

Perspectives

- Simulation-based approaches for verification, parameter synthesis
- Time-robustness as opposed to space-robustness