What is Verification?

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Plan

1. Context: system design and mathematical models

2. Example: how to have a free coffee

3. Major issues in discrete verification

4. New challenges: Timed and Hybrid systems

Context

We want to build something (a "system") that works.

The system should achieve some of our goals, it should make parts of our world behave in certain way.

We want to build a "good" system that works, not a bad one that fails.

Examples:

- a house
- a micro-processor
- a web server
- a political system
- a railway network
- a car, an airplane, a ship
- a mobile phone
- a football team
- a chemical plant
- ...

Major Issues

1) What we want the system to do? How do we specify it?

2) How to design it correctly?

3) How to build it physically?

4) How to check whether it works?

5) How to operate and maintain it?

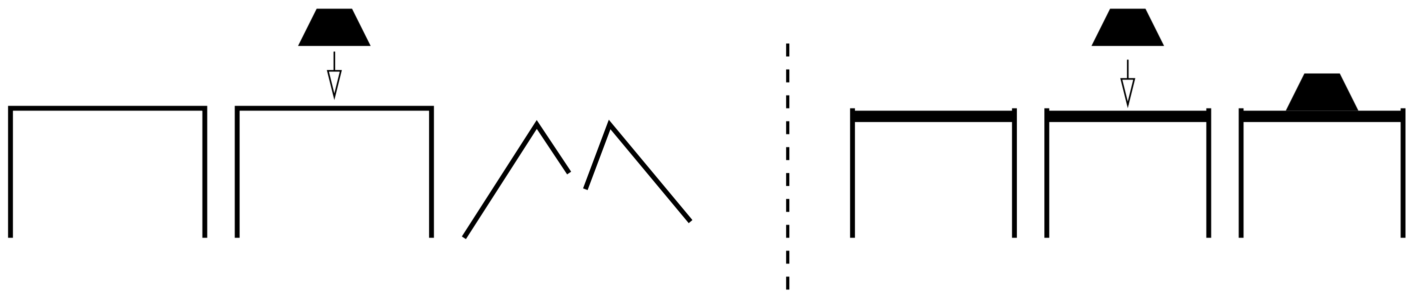
Some of these points are very important also as part of the legal contracts between the provider of the system, sub-contractors and customers: how can we claim in "objective" and observable terms that a product that we have bought does not work properly? Of course, there is a limit to formalisation and human judges are unavoidable.

Example: Building a House
Trial and Error

What do we want from a house? Many things (aesthetics, isolation, functioning of sub-systems, ...)

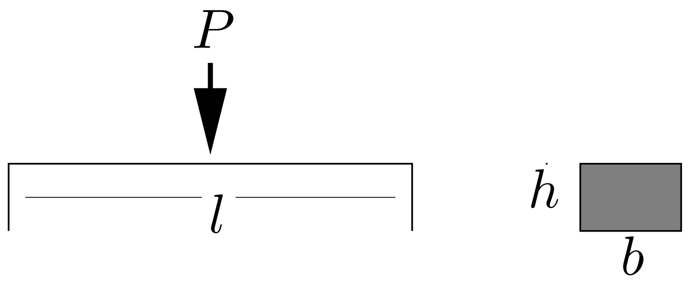
In particular: we want it not to crash under certain loads.

An old-fashioned way to achieve it: build and see (trial and error).



Building a house - using a Model

Based on physical laws and experiments we can build a **model** and use it to predict the behavior (Gedanken experiments).

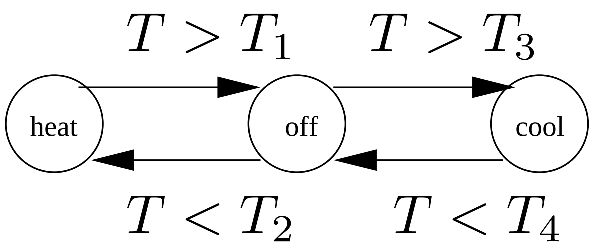


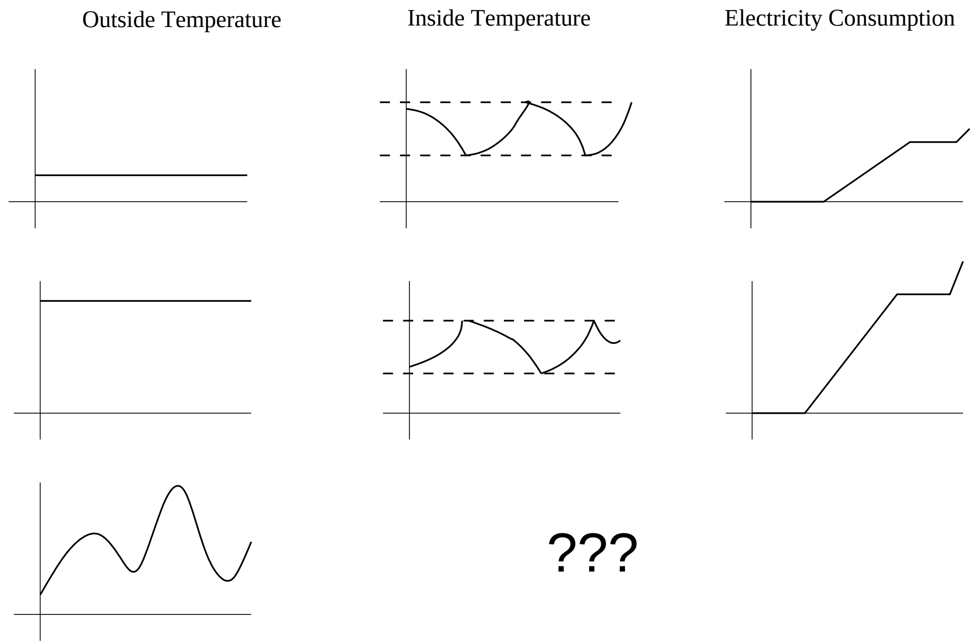
Maximal bending moment on a beam of length l under a load P is $p \cdot l/4$
Module of resistance of a beam with $b \times h$ section is $b \cdot h^2/6$

...

Finally we can predict whether or not the beam will support the load.

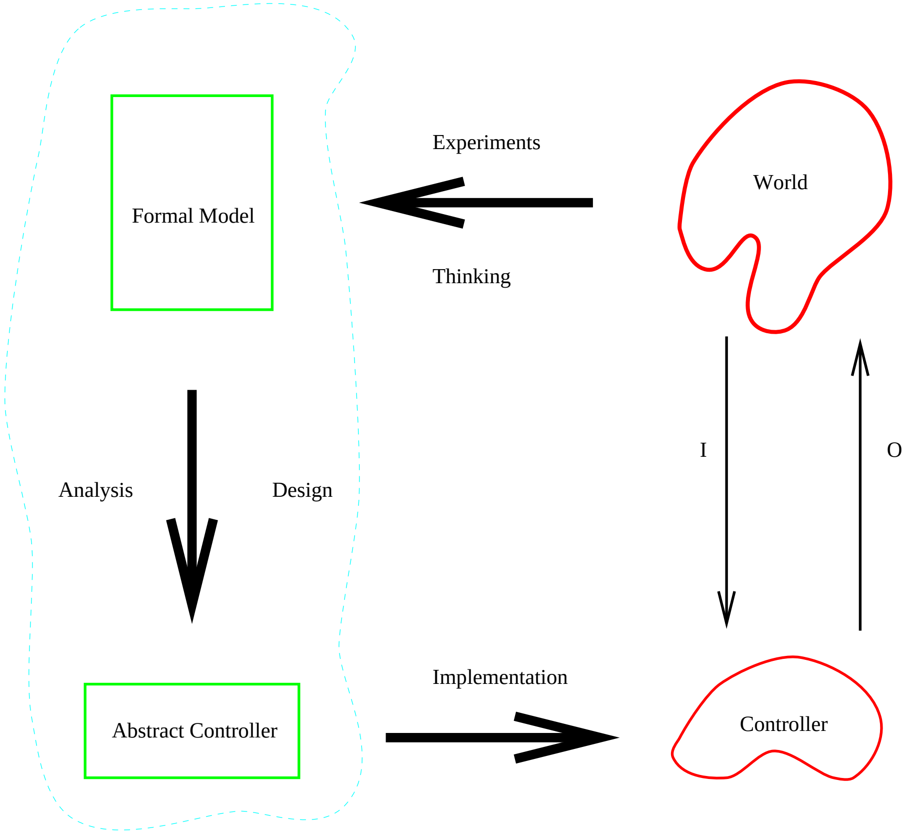
Example: Air-Conditioning





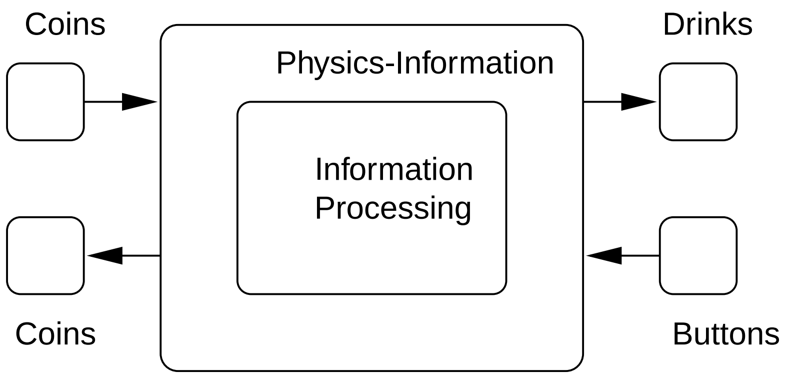
Can we show that the temperature is *always* maintained in a desired range with some bounded cost? For *all* external disturbances?

Model-based System Design

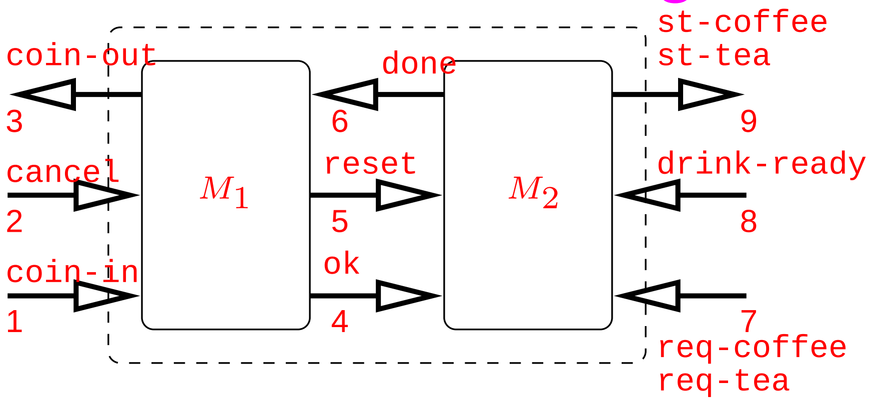


Example: The Coffee Machine

We want to build a machine that gets coins and delivers coffee or tea

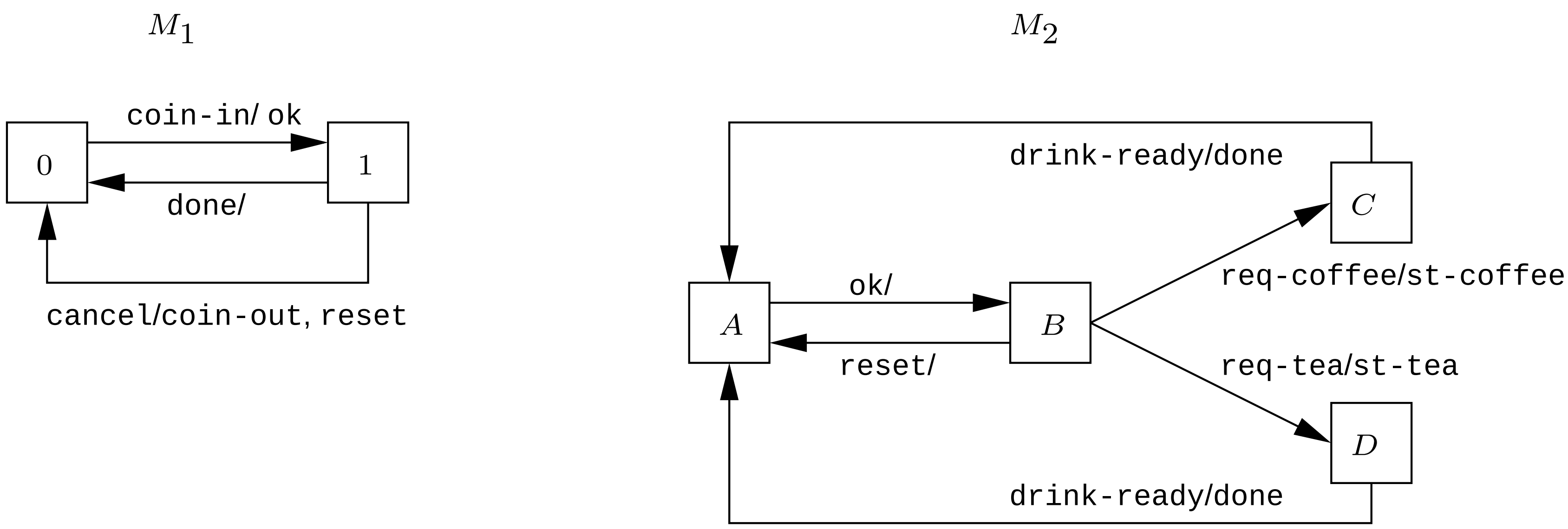


The Coffee Machine: Signals

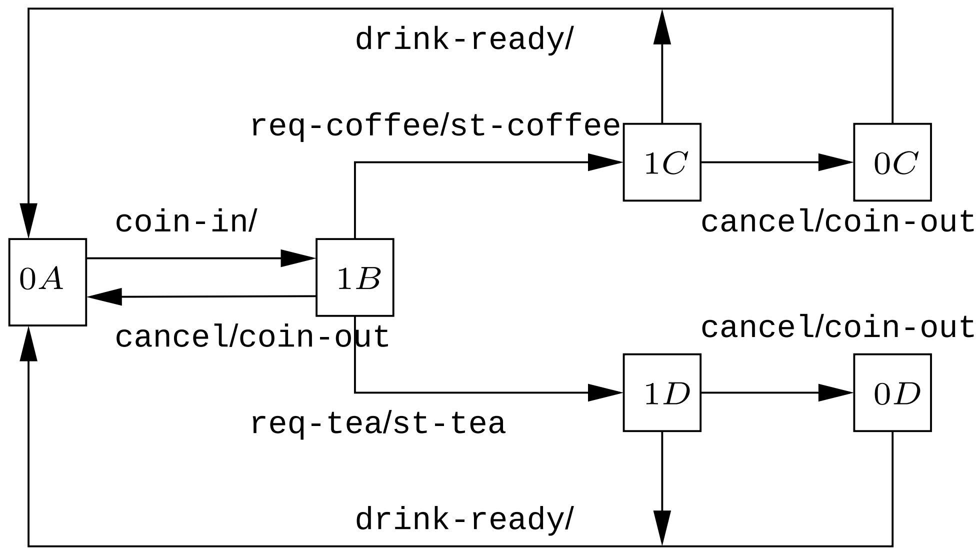


Port	From→To	Event types	Meaning
1	$E \to M_1$	coin-in	a coin was inserted
2	$E \to M_1$	cancel	cancel button pressed
3	$M_1 \to E$	coin-out	release the coin
4	$M_1 \to M_2$	ok	sufficient money inserted
5	$M_1 \to M_2$	reset	money returned to user
6	$M_2 \to M_1$	done	drink distribution ended
7	$E \to M_2$	req-coffee req-tea	coffee button pressed tea button pressed
8	$E \to M_2$	drink-ready	drink preparation ended
9	$M_2 \to E$	st-coffee st-tea	start preparing coffee start preparing tea

The Two Sub-Machines



The Global Model



Normal behaviors:

$0A$ coin-in $1B$ cancel coin-out $0A$

$0A$ coin-in $1B$ req-coffee st-coffee $1C$ drink-ready $0A$

It can be much more Complex

Various means of payment: combinations of coins, notes, credit cards (which require a module for communication with banks).

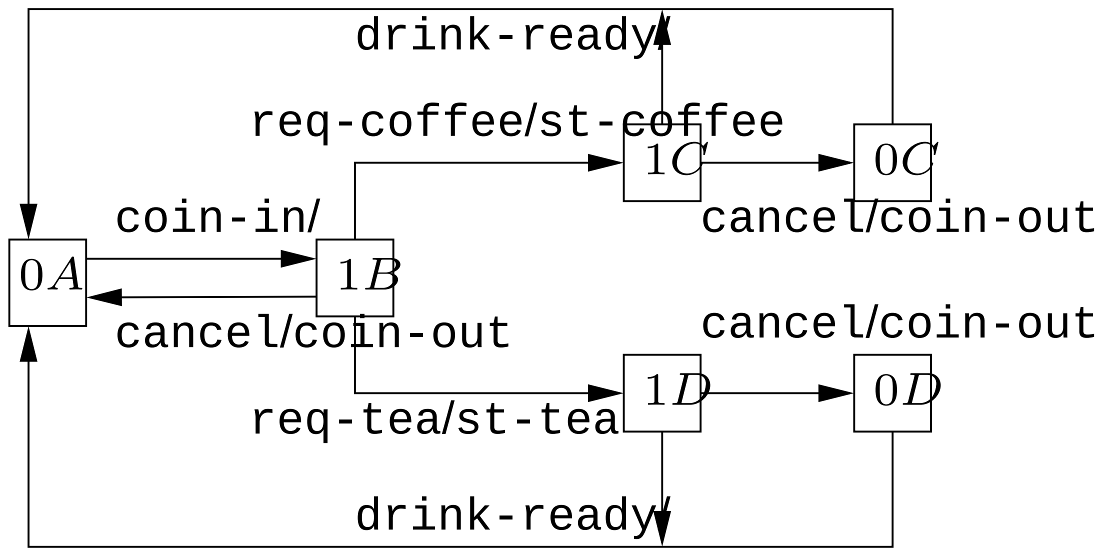
A wider variety of drinks with choices of milk, sugar, grinding, etc.

Consider now a big factory with thousands of components and communication channels.

When you build a large and complex system with many *interacting* components the number of global states is roughly the product of the number of states of the components (exponential growth).

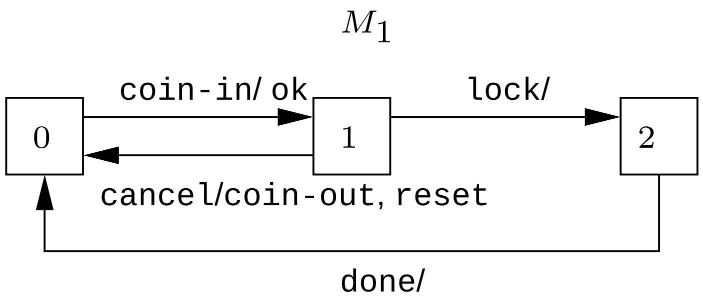
It is practically impossible to predict all the possible behaviors (scenarios) of the system.

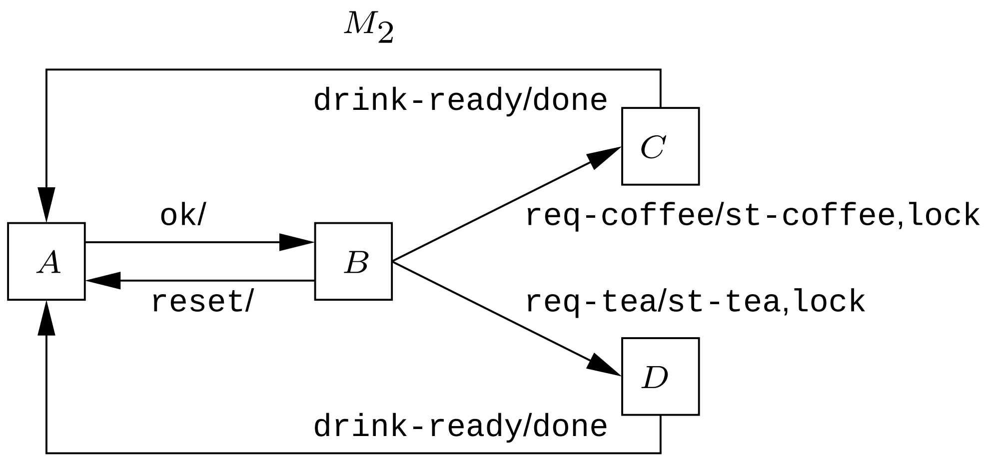
An Unexpected Behavior



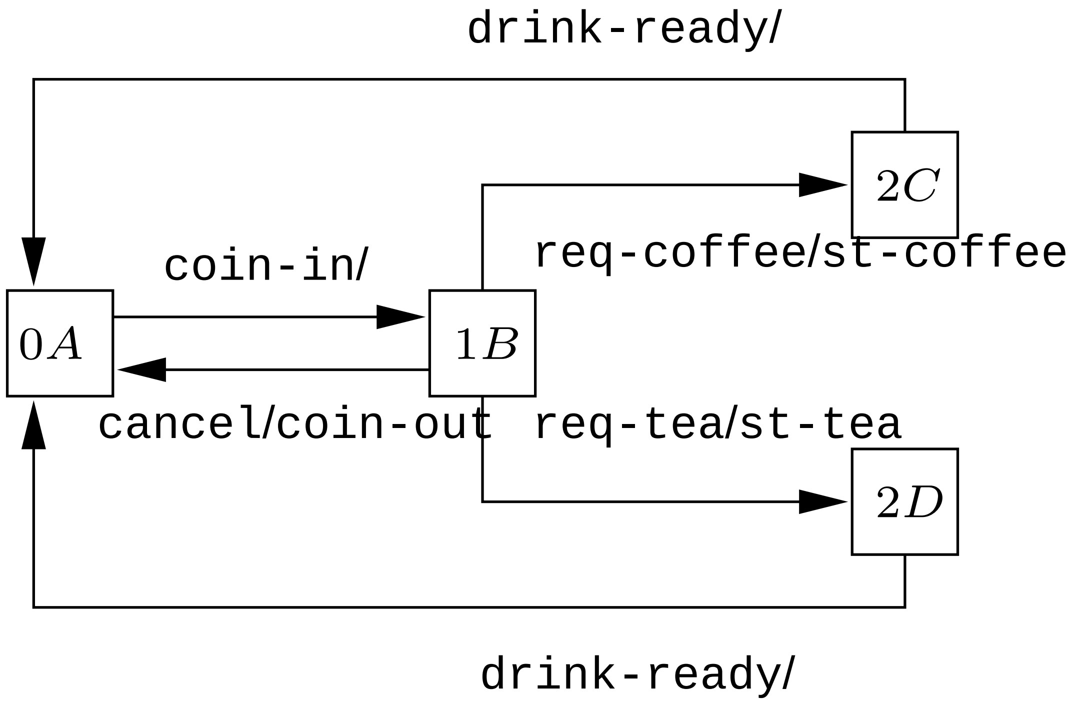
$0A$ `coin-in` $1B$ `req-coffee` `st-coffee` $1C$ `cancel` `coin-out` $0C$ `drink-ready` $0A$

Fixing the Bug





Fixing the Bug – the Global Model



The Moral of the Story I

1) Many systems can be modeled as a **composition of interacting automata** (transition systems, discrete event systems).

2) Potential behaviors of the system correspond to **paths** in the **global transition graph** of the system.

3) These paths are **labeled** by **input events**. Each input sequence might generate a **different behavior**.

4) We want to make sure that a system responds correctly to **all conceivable inputs.**

The Moral of the Story II

5) For every **individual input sequence** we can **simulate** the reaction of the system. But we cannot do it exhaustively due to the huge number of input sequences.

6) Verification is a collection of automatic and semi-automatic methods to analyze **all** the paths in the graph.

7) This is hard for humans to do and even for computers.

The Ingredients of a Verification Methodology

A Specification Language:
A formalism for describing the desired properties of the system. In other words a criterion for classifying event sequences as good and bad (e.g. Temporal Logic).

A Computational Model:
A formalism for describing the designed system (automata, transition systems).

A Verification Technique:
A method to show that the system satisfies the desired properties, i.e. all the behaviors generated by the system are those accepted by the specification (deductive and algorithmic approaches).

Specification Languages

How to specify in a **rigorous** and **precise** manner what are the desired properties of the system.

Temporal Logic is a formalism in which you can express properties of sequence of events, especially about the order of their occurrences.

If a customer puts the right amount of money and chooses a drink then he will **later** get the chosen drink.

If a customer selects a drink and the process has started the `cancel`button is ignored.

If the customer has put money and 30 seconds have passed before a drink is selected, the money is given back.

The Deductive Approach to Verification

Formalization of Human Reasoning:

IF `req-coffee`causes a `lock`message from M_1 to M_2 before `st-coffee`
AND a `lock`message makes M_1 move to state 2
AND in state 2, M_2 ignores `cancel`messages

THEN it is impossible to get a free coffee.

In order to show correctness of the system we have to prove many many small and boring theorems.

Here the computer and the human **cooperate** in the verification process. The human (who has **intuition** about the system) suggests proof directions and the computer checks, does the book-keeping, etc.

The Algorithmic Approach to Verification

Brute-force Search

Graph algorithms are applied to the global transition graph of the system in order to detect bad behaviors (or to prove their absence).

Advantages: you don't need an intelligent user (an endangered species) – in principle you just push a button and the computer answers.

Problem: **state-explosion** – the number of states can be 2^{100} beyond the capabilities of the fastest (present and future) computers.

Most of the work: inventing tricks to treat larger problems, e.g. Symbolic representation of large graphs, compositional reasoning, approximation and abstraction, combination with deductive methods.

Model I: Closed Systems

A transition system is $S = (X, \delta)$ where X is finite and $\delta : X \to X$ is the transition function.

The state-space X has no numerical meaning and no interesting structure.

X^k is the set of all sequences of length k; X^* the set of all sequences.

Behavior: The behavior of S starting from an initial state $x_0 \in X$, is

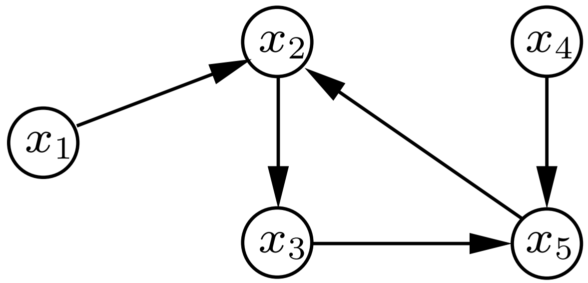
$$\xi = \xi[0], \xi[1], \ldots \in X^*$$

s.t. $\xi[0] = x_0$ and for every i, $\quad \xi[i+1] = \delta(\xi[i])$

Basic Reachability Problem: Given x_0 and a set $P \subseteq X$, does the behavior of S starting at x_0 reach P?

Solution by Forward Simulation

$\xi[0]$:=x_0
F^0 := $\{x_0\}$
repeat
  $\xi[k+1]$:=$\delta(\xi[k])$
  F^{k+1} := $F^k \cup \{\xi[i+1]\}$
until $F^{k+1} = F^k$
F_*:=F^k



$$\{x_1\}, \{x_1, x_2\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_3, x_5\}$$

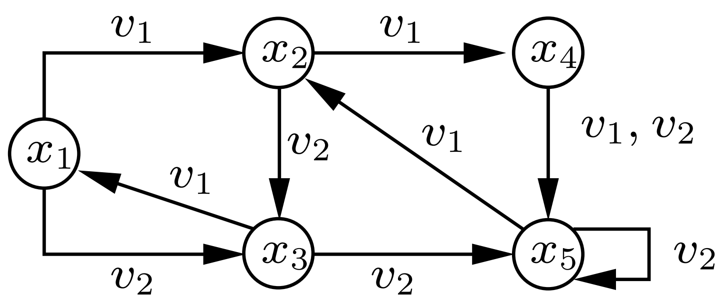
How to do it for continuous system defined by $\dot{x} = f(x)$?

Model II: Systems with One Input

A one-input transition system is $S = (X, V, \delta)$ where X and V are finite and $\delta : X \times V \to X$ is the transition function.

Behavior Induced by Input: Given an input sequence $\psi \in V^*$, the behavior of S starting from $x_0 \in X$ in the presence of ψ is a sequence

$$\xi(\psi) = \xi[0], \xi[1], \ldots \in X^* \text{ such that } \quad \xi[i+1] = \delta(\xi[i], \psi[i]).$$



$$x_1 \xrightarrow{v_1} x_2 \xrightarrow{v_2} x_3 \xrightarrow{v_2} x_5 \xrightarrow{v_1} x_2 \xrightarrow{v_1} x_4$$

Reachability for Open Systems

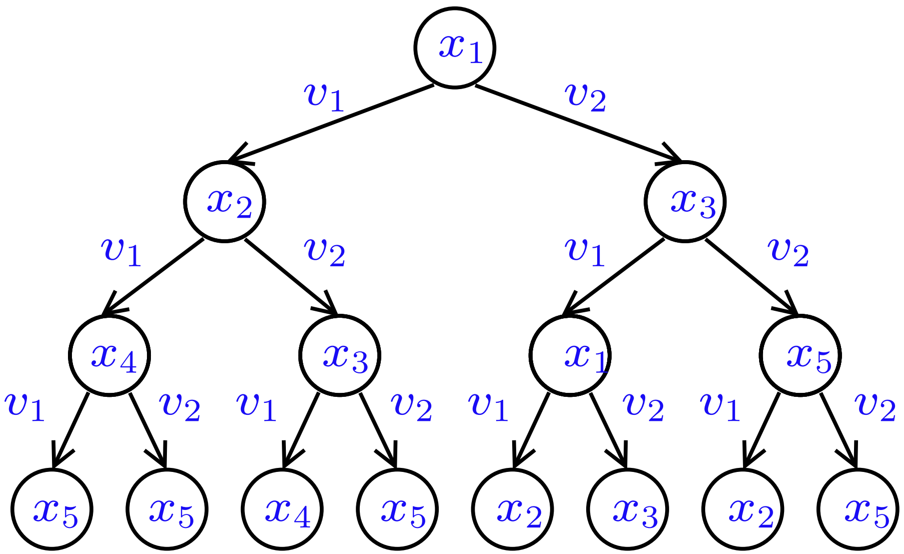
The reachability problem: Is there some input sequence $\psi \in V^*$ such that $\xi(\psi)$ reaches P?

For every given ψ we can use the previous algorithm, simulate and obtain $F_*(\psi)$.

For an automaton with n states all states are reachable by sequences of length $< n$.

$$F_* = \bigcup_{\xi \in V^n} F_*(\psi)$$

Reachability for Open Systems



There are 2^n input sequences to simulate with (and n itself is, typically exponential in the number of system components).

A More Efficient Way

Many different input sequences lead to the same state and if $\delta(x,u) = \delta(x,v)$ then for every w, $\delta(x,uw) = \delta(x,vw)$.

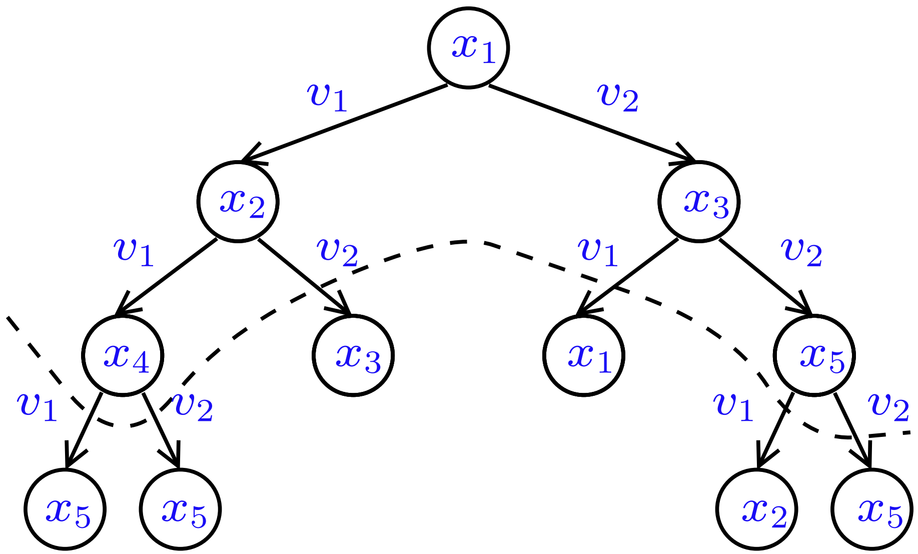
We do not need to “simulate” with both uw and vw.

Since we have access to the transition graph (unlike black box) we can apply graph algorithms.

Immediate successors of a state x: $\delta(x) = \{x' : \exists u \; \delta(x,u) = x'\}$

Successors of a set F: $\delta(F) = \{\delta(x) : x \in F\}$

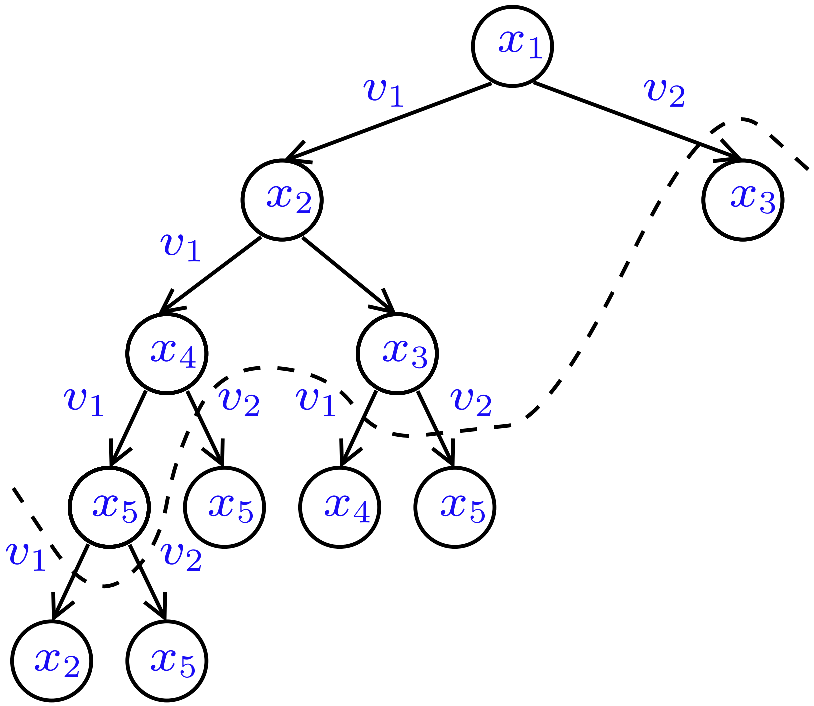
Forward Reachability (breadth-first)



F^0 := $\{x_0\}$
repeat
F^{k+1} := $F^k \cup \delta(F^k)$
until $F^{k+1} = F^k$
F_*:=F^k

Complexity: only $O(n \cdot \log n \cdot |V|)$ $\{x_1\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_3, x_4, x_5\}$

Variation: Depth-First



Variation: Backwards

Backwards: find all states from which there is an input leading to P.

Immediate predecessorsof a state x:
$\delta^{-1}(x) = \{x' : \exists u \; \delta(x', u) = x\}$

F^0 := P
repeat
  F^{k+1} := $F^k \cup \delta^{-1}(F^k)$
until $F^{k+1} = F^k$
F_*:=F^k

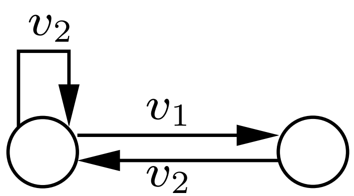
Admissible Inputs

So far we have assumed that the external environment can generate all sequences in V^*.

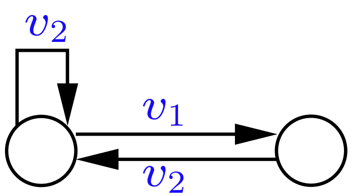
This is as if we modeled the environment as a one-state automaton (the universal generator).

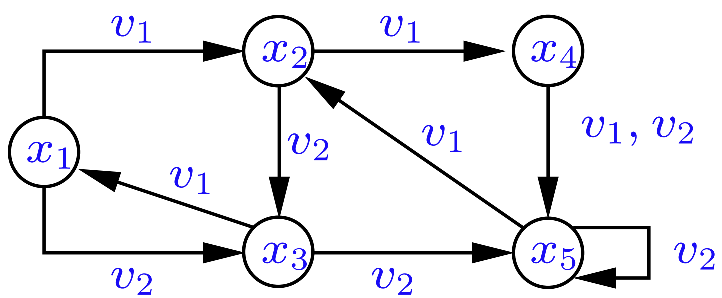
We can have a more restricted environment, e.g. it will never produce $v_1 v_1$.

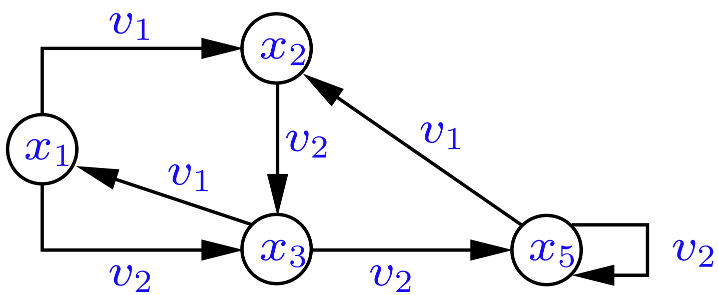
We can build an automaton which models the environment and compose it with the model of the system.



Admissible Inputs - the Composition







Verification: The State-of-the-Art

There are algorithms that take a description of any open system and verify whether any of the admissible inputs drives the system into a set P. Such algorithms always terminate after a finite number of steps.

This is essentially what algorithmic verification ("model checking") is all about.

The result is general: it is valid for every discrete finite-state system. Of course, finite systems can be very large and special tricks are needed to verify them.

The analogue for continuous systems: do the same for a system defined by $\dot{x} = f(x, u)$.

Systems with two Inputs

A two-input transition system is $S = (X, U, V, \delta)$ where X, U and V are finite sets and $\delta : X \times U \times V \rightarrow X$ is the transition function.

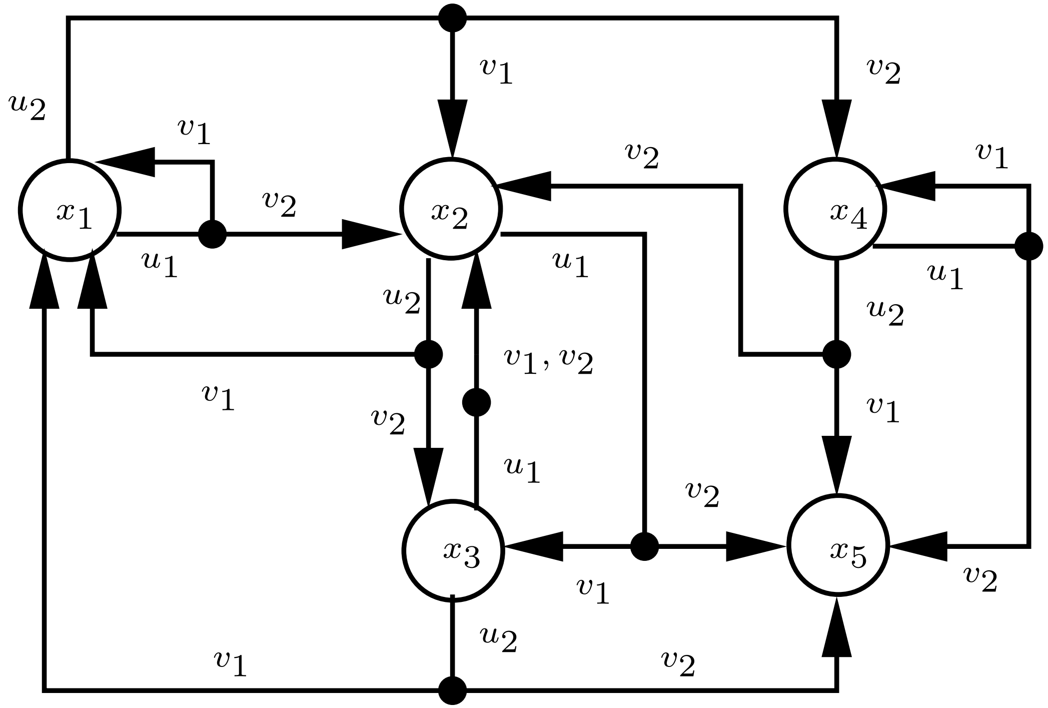
Interpretation of inputs:

U: we, the good guys, the controller.

V: they, the bad guys, disturbances.

An antagonist game situation. Our goal is to choose each time an element of U such that the behaviors induces by all possible disturbances are good.

Systems with two Inputs



$$\delta(x_1, u_1, v_1) = x_1 \qquad \delta(x_1, u_1, v_2) = x_2$$
$$\delta(x_1, u_2, v_1) = x_2 \qquad \delta(x_1, u_2, v_2) = x_4$$

Strategies

Strategy: a function $c : X^* \rightarrow U$

State strategy: a function $c : X \rightarrow U$.

Each strategy c converts a type III system into a type II system $S_c = (X, V, \delta_c)$ s.t. $\delta_c(x, v) = \delta(x, c(x), v)$.

Let $S = (X, U, V, \delta)$ let $P \subseteq X$ be a set of "bad" states. The controller synthesis problem is: find a strategy c such that all the behaviors of the derived system $S_c = (X, V, \delta_c)$ never reach P.

Controllable Predecessors

For $S = (X, U, V, \delta)$ and $F \subseteq X$, the set of controllable predecessors of F is

$$\pi(F) = \{x : \exists u \in U \;\; \forall v \in V \;\; \delta(x, u, v) \in F\}$$

The states from which the controller, by properly selecting u, can force the system into P in the next step.

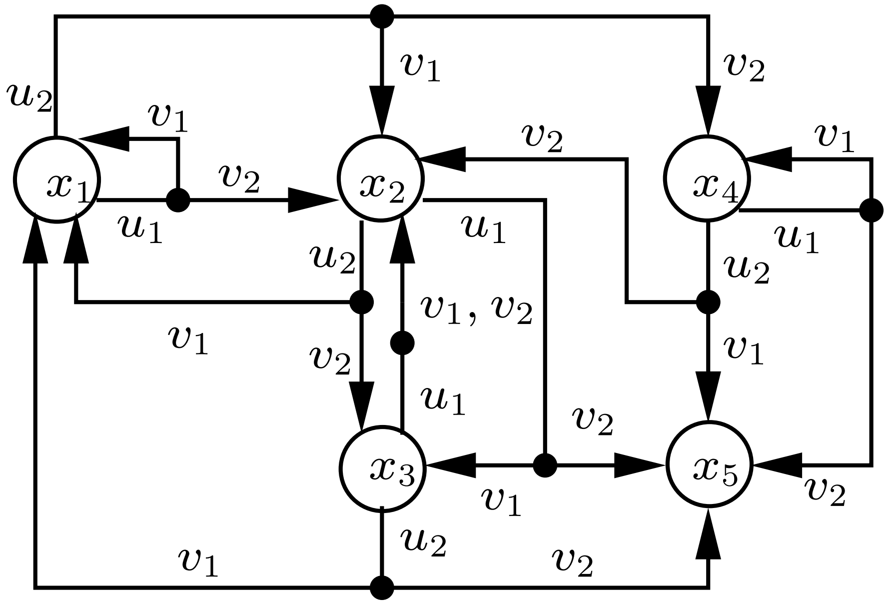
Finding Winning States and Strategies

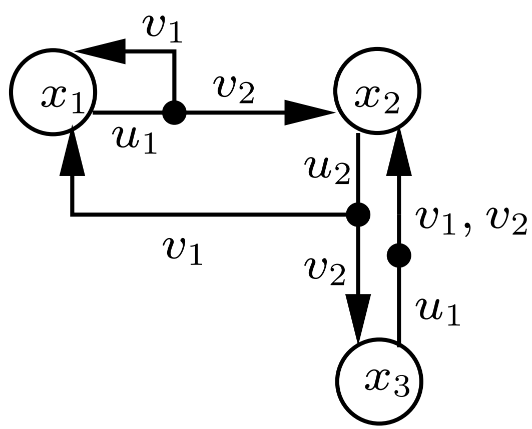
The following backward algorithm finds the set F_* of "winning states" from which P can be avoided forever.

F^0 := $X - P$
repeat
  F^{k+1} := $F^k \cap \pi(F^k)$
until $F^{k+1} = F^k$
F_*:=F^k

Remark: this is similar to the Ramadge-Wonham theory of discrete event control, dynamic programming, min-max, game algorithms, etc.

Synthesis Example





We want to avoid x_5. $F^0 = \{x_1, x_2, x_3, x_4\}$ $F^1 = \{x_1, x_2, x_3\} = F_*$

The resulting “closed-loop” system always remains in $\{x_1, x_2, x_3\}$.

Remark: Quality vs. Quantity

Correctness is a special case of the more general notion of a performance measure: an assignment of a value to each behavior as indication of its goodness.

One can assign to system behaviors numbers the indicate their “cost” or utility and then try to synthesize optimal controllers/schedulers.

Traditionally verification is concerned with estimating the worst-case (over all inputs) for a $\{0, 1\}$ measures.

Discrete Infinite-State Systems

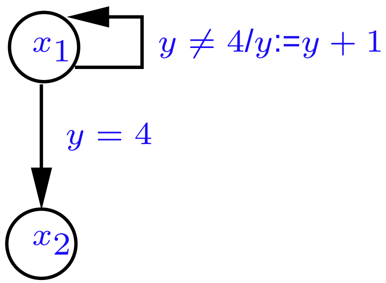
So far we have dealt with finite-state systems (“control” but no “data”).

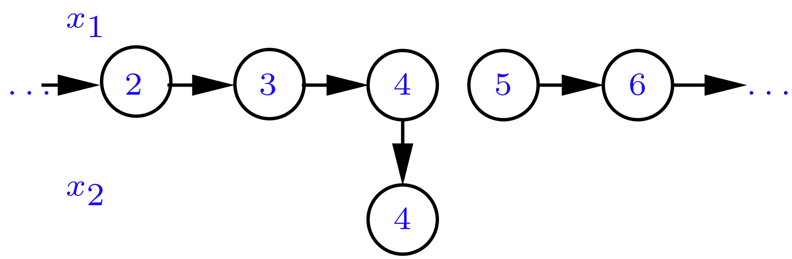
Computer programs can be viewed as syntactic representations of discrete dynamical systems with an infinite state-space.

repeat
  y:=$y + 1$
until $y = 4$

State space is the product of the set of program locations and the domains of the variables: $\{x_1, x_2\} \times \mathbb{Z}$

Verification of Infinite-State Systems





Forward reachability algorithm will terminate if started from $(x_1, 2)$ but not from $(x_1, 5)$.

The reachability problem is unsolvable: there is no **general** algorithm that solves every instance of it.

"Deductive" approach: prove properties "analytically". "Symbolic" approach: reachability using formulae to represent sets of states, e.g. $x = x_1 \wedge y \geq 5$.

Hybrid Systems: Modeling the Physical Environment

Most systems are embedded in the physical environemnt via sensors and actuators.

Sometimes it is sufficient to abstract the dynamics of the environment using discrete events (the physical part of the coffee machine emits `drink-ready`sometime after receiving `st-coffee`).

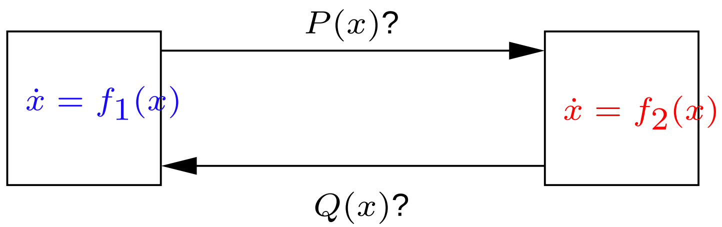
Sometime we want to estimate the **time** between the two events. Sometime we want to look **even closer** and model how the water temperature changes over time.

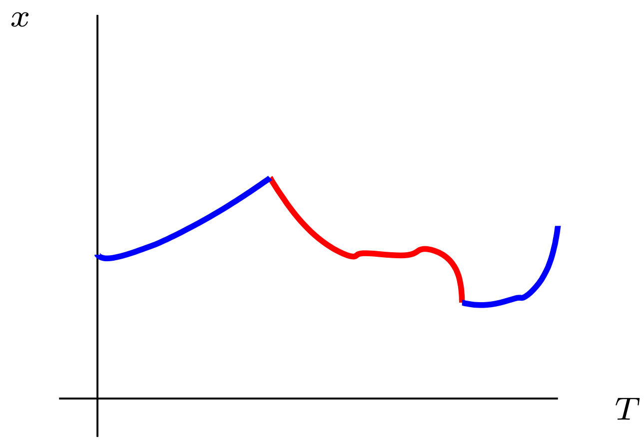
The common models for describing the dynamics of such phenomena are, alas, **continuous** and based on formalisms such as differential equations.

A new model is needed for combining **discrete** and **continuous** dynamics.

Hybrid Automata

Automata augmented with continuous variables and differential equations.





Exporting Verification to Continuous (and Hybrid) Systems

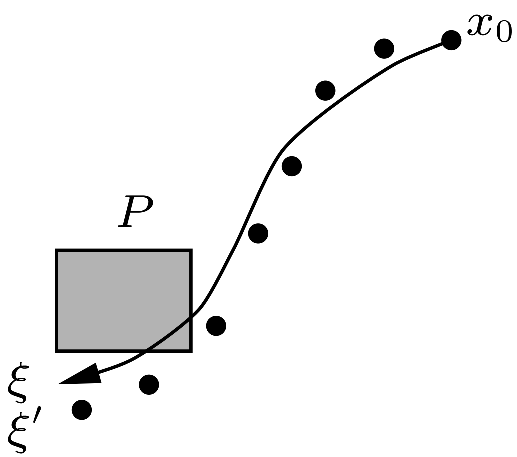
Why? ...

Problems: state space $\mathbb{R}^n$, infinite even when bounded, time domain $\mathbb{R}$. Mathematical $\mathbb{R}$ vs. numerical $\mathbb{R}$ in the computer.

Reachability for $\dot{x} = f(x)$: When we have a closed-form solution, e.g. for $\dot{x} = Ax$, the reachable set can be written as $F_* = \{x_0 e^{At} : t \geq 0\}$ but how to test whether $F_* \cap P = \emptyset$?

Forward Simulation for Closed Continuous Systems

Forward simulation: discretize time and replace the system with $\xi'[(n{+}1)\Delta] = \xi'[n\Delta] + h(\xi'[n\Delta], \Delta)$.



This is not the “real” thing and it is not guaranteed to converge but that’s life.

Continuous Systems with Input

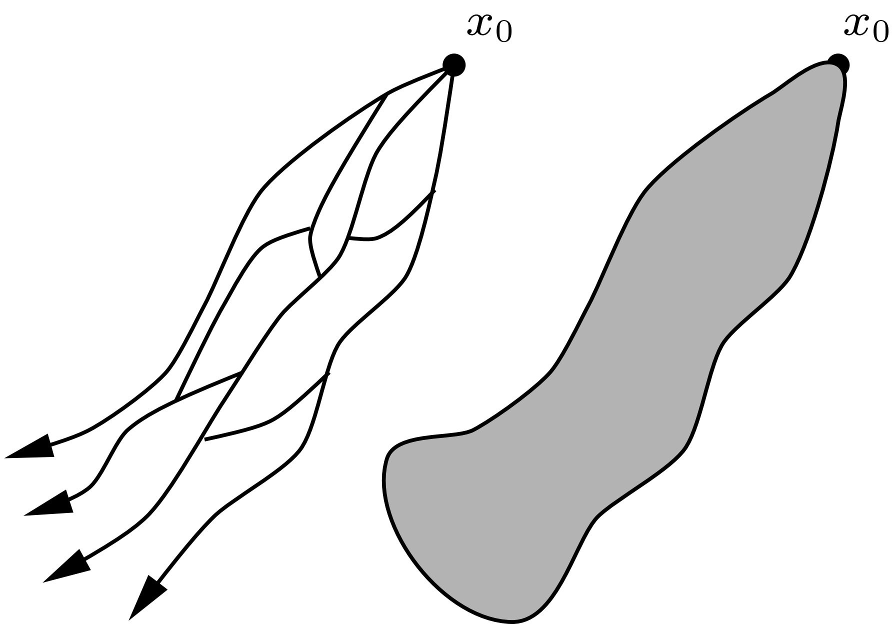
Systems of the form $\dot{x} = f(x, v)$. Admissible inputs are signals of the form $\psi : T \to V$.

Problem: show that no admissible input drives the system into a set P.

For every ψ we can simulate and "compute" $F_*(\psi)$, but there is no finite subset of inputs that covers all reachable states.

The Input Space and its Induced Behaviors

The set of all inputs is a **doubly-dense tree**, both vertically (time) and horizontally (V).



Incremental Reachability Computation

$x \xrightarrow{t} x'$ denotes the existence of an input signal $\psi : [0, t] \to V$ that drives the system from x to x' in t time.

Let F be a subset of X and let I be a time interval. The I-successors of F are all the states that can be reached from F within that time interval, i.e.

$$\delta_I(F) = \{x' : \exists x \in F \; \exists t \in I \; x \xrightarrow{t} x'\}.$$

Semigroup property: $\delta_{[0,r_2]}(\delta_{[0,r_1]}(F)) = \delta_{[0,r_1+r_2]}(F)$.

Breadt-first Reachability Computation

F^0 := $\{x_0\}$
repeat
  F^{k+1} := $F^k \cup \delta_{[0,r]}(F^k)$
until $F^{k+1} = F^k$
F_*:=F^k

Two problems:

1) The algorithm is not guaranteed to converge (like for most classes of infinite-state systems).

2) The operator $\delta_{[0,r]}$ is not more computable than $\delta_{[0,\infty]}$ for most non-trivial systems.

Approximate Reachability Computation

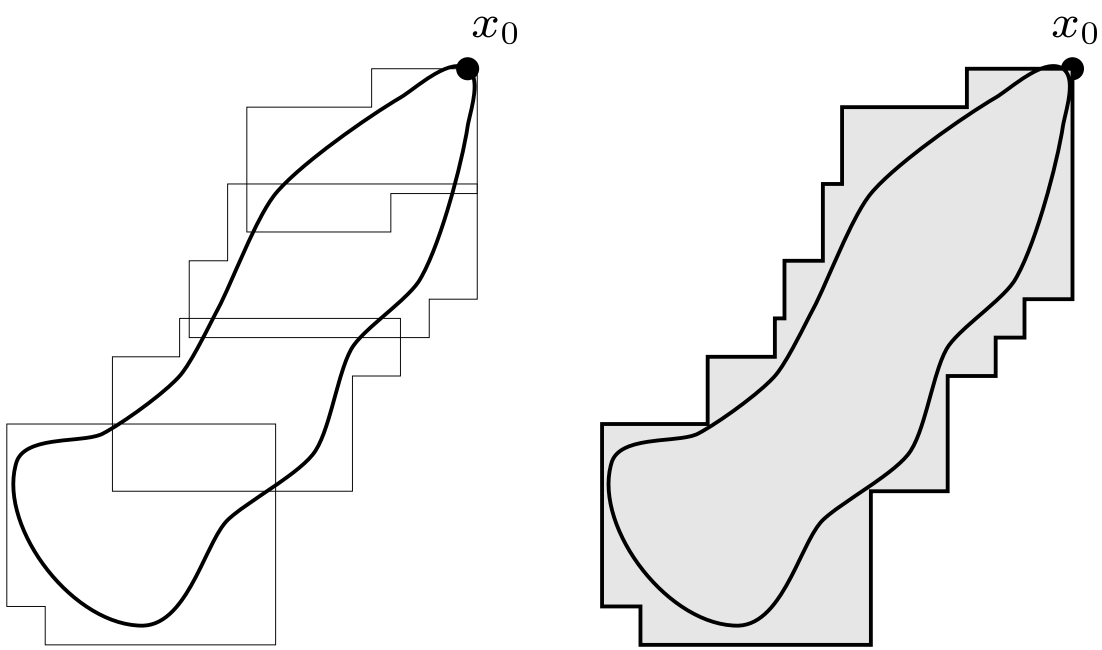
Although $\delta_{[0,r]}(F)$ cannot be computed exactly, we can over-approximate it by δ' such that for every F

$$\delta_{[0,r]}(F) \subseteq \delta'_{[0,r]}(F)$$

and $\delta'_{[0,r]}(F)$ belongs to some effective sub-class of $\mathbb{R}^n$, e.g. polyhedra.

The result of the algorithm is a set F'_* s.t. $F_* \subseteq F'_*$ and hence $F'_* \cap P = \emptyset$ implies the correctness of the system.

Approximate Reachability Computation - Illustration



We have developed a system called **d/dt** which accepts as input a description of a continuous or a hybrid system and computes automatically an over-approximation of the reachable states.

Conclusions

The “right” model to see what’s going on in a system is a model of a dynamical system with state-variables, with a dynamics that describes the possible future evolutions from each state.

Such models generate behaviors, trajectories in the state-space, that can be evaluated according to correctness or other performance measures.

Within these models we can formulate all sorts of system design problems.

Syntax (logic assertions, programming languages) might be important for computational considerations, but it should not obscure the underlying dynamic semantics (as is often the case in AI).