Reachability for Continuous and Hybrid Systems

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Abstract. In this talk I present some past, present and future work concerning reachability computation for continuous and hybrid systems.

The problem of *what may happen in the future* is very naturally phrased as a reachability problem in a dynamical system. Starting from an initial state or a set of states, following some dynamic rules, possibly with some dose of uncertainty, we would like to know whether something will happen, whether a particular state of affairs will occur. In the finite-state case the problem is trivially decidable by graph search algorithms (but is intractable when you have a product of many subsystems), while in the general discrete case it is undecidable as *halting* is just reachability of a particular state.

In the "early" days of hybrid systems research (early 90s), after the the fact that reachability is decidable for automata with real-valued clocks has been reformulated, there was some hope that similar things could be done for hybrid systems having a more complex (but still piecewise-trivial) dynamics, that is, automata where the continuous evolution is linear (the derivative is constant) in each discrete location.[1] But soon it was realized how easy it is to build TM, counter machine or 2PDA gadgets when you have real-valued variables. Consequently, finding the boundary of decidability became a sportive activity like building a universal TM with 2 states, 3 symbols and 1.5 heads. Practically, the best you could provably expect for even those very simple hybrid systems is "semi-decidability": running a verification tool (like the pioneering HyTech) and hoping that the iteration terminates.

My conclusion from all these investigations was that in the context of continuous systems, solving the exact "reachability problem" does not make much sense, except for mathematical amusement. Almost everything in the continuous world is already approximate and the infinite precision used to simulate TMs is unrealistic from the points of view of modeling, measurement and computation. Thus the expectations were lowered once more, from *exact* semi-decidability to approximate computation of reachable sets and then from unbounded time horizon to a finite one. Approximating tubes of trajectories of continuous dynamical systems turned out to be an interesting research domain involving graph algorithms, computational geometry in high dimension, numerical analysis and other

[1] Such systems are very attractive for computer scientists like us because they can be studied with elementary linear algebra and there is no need to understand those horrible differential equations.

branches of mathematics and computer science. It has potential applications in analog circuit verification, in the debugging of biochemical models, in the analysis of control systems and in any other domain which uses differential equations subject to some nondeterminism and in which we want more information than we can get from running arbitrary simulations.

I will start my talk with some decidability and undecidability results for piecewise-constant derivative systems. Then I will explain how reachable states of linear systems are approximated by unions of convex geometric objects such as polytopes, describe an algorithmic scheme that allows this computation to be performed much more efficiently and conclude with two recent works, one that takes us one step closer to simple numerical simulation and one which extends reachability to systems with nonlinear dynamics.