Time Domain Verification of Oscillator Circuit Properties

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Abstract

The application of formal methods to analog and mixed signal circuits requires efficient methods for constructing abstractions of circuit behaviors. This paper concerns the verification of properties of oscillator circuits. Generic *monitor automata* are proposed to facilitate the application of hybrid system reachability computations to characterize time domain features of oscillatory behavior, such as bounds on the signal amplitude and jitter. The approach is illustrated for a nonlinear tunnel-diode circuit model using PHAVer, a hybrid system analysis tool that provides sound verification results based on linear hybrid automata approximations and infinite precision computations.

Key words: verification, oscillators, analog circuits, hybrid systems, hybrid automata

1 Introduction

Formal methods are widely and successfully used to verify the correctness of digital circuits. So far, no sufficiently powerful verification techniques exist in the domain of analog and mixed-signal circuits. The complexity of the verification problem for such circuits is immense for several reasons. Analog designs have high-dimensional nonlinear dynamics. It is difficult to formally

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define the desired behavior of such circuits. Their sensitivity to a variety of physical effects makes it difficult to isolate their behavior and abstract from their interactions. Given this complexity, the initial focus for formal methods should be in the early, block-level design steps. In this paper, we discuss methods to verify design properties of oscillator circuits, which play a fundamental role as a basic design block in many practical applications, by using methods and tools for reachability analysis.

Oscillator circuits are designed to produce a periodic signal with small variations in amplitude and phase even when subject to parameter variations and disturbances. We can guarantee conservative bounds on these variations by applying formal verification techniques that are particularly suited to handle nondeterminism and uncertainties. Our models, so-called *hybrid automata* [1], capture continuous as well as discrete state behavior, and can be used to model nonlinear dynamics. Verification tools exist that guarantee correctness of the results by using exact arithmetic and overapproximations and recently, progress has been made in the efficient computation of reachable states for hybrid systems. We present a set of generic models for measuring and verifying bounds on amplitude variation and (phase) jitter. We provide some experimental results with PHAVer, a hybrid systems tool that compute a guaranteed conservative overapproximations of sets of reachable states [7]. As a benchmark we use a tunnel diode oscillator circuit, as it appears in [10,9].

Model checking of nonlinear analog circuits was first proposed in [10], where the continuous state space is discretized, and an abstract transition relation is computed for the finite, discrete model. Conventional model checking can be applied to this abstraction, but due to the overapproximation only safety properties are preserved. Similarly to the approach in PHAVer, the partitioning of the state space is adapted to the dynamics of the system. The overapproximation of the transition relation in this approach is much larger than the continuous-valued overapproximation of PHAVer. It is not guaranteed to be conservative, and striving for conservativeness in a discretization based approach can quickly lead to an excessive loss of accuracy. In [9], analog circuits were verified using the tool CheckMate [5], which computes an abstract transition relation between user-defined regions of the state space. Optimization is used to guarantee, as much as possible, the conservativeness of polyhedral enclosures of the reachable states, but it is not guaranteed that the global optimum is found. The tool d/dt [2] computes the reachable states for hybrid systems with affine dynamics by discrete time integration, and guarantees conservativeness using an approach that maximizes the normal derivative of the vector field over the faces of the polyhedron. Sets of states are efficiently represented with orthogonal polyhedra. The approach is algorithmically sound because an under- as well as overapproximation is used. Linear analog circuits have been analyzed with d/dt in [6]. All the above tools are not numerically exact, and consequently not formally sound in their overapproximations. While in PHAVer the integration error accumulates, both

CheckMate and d/dt have the advantage that the overapproximation error does not accumulate over time. The formal background for the overapproximations of hybrid automata with nonlinear and affine dynamics originates in the work of Henzinger et al. [11].

In the following section, we define oscillations and derive properties of oscillations that can be checked using reachability, such as amplitude and phaser jitter. We show hybrid automaton models for monitoring such properties using reachability tools. In Sect. 3, we illustrate how PHAVer can be applied to verify these properties, and provide some experimental results for a tunnel diode circuit in Sect. 4. Finally, we draw some conclusions in Sect. 5.

2 Defining and Verifying Oscillations

Our goal is to take a model of the oscillator of the form $\dot{x} = f(x, p, u)$, where p represents parameters and u stands for external disturbances, and check whether it exhibits oscillating behavior that is robust under variations in p and all admissible values of u. As a first step we define oscillations, exactly and approximately. The specification may vary from one application to another. The point of the discussion is to study oscillatory behavior in the vicinity of some periodic limit cycle, so we neglect transient behavior and assume a phase error close to zero.

For the sake of simplicity, we assume that our requirements describe the desired properties of a scalar signal ξ obtained as a projection of our system onto one variable x of interest. This simplifies the representation of threshold crossings and other behaviors. On scalar domains we will use $x \approx y$ to indicate that $|x - y| < \epsilon$ without being specific about ϵ.

2.1 Reference Signal

The most rigid specification is given by a reference oscillatory behavior, say

$$\bar{\xi}(t) = Asin(\omega t + \phi), \tag{1}$$

to which ξ should be close in some metric d:

$$d(\xi, \bar{\xi}) < \epsilon.$$

Using the pointwise maximum distance

$$d(\xi, \bar{\xi}) = \max_{t \in \mathbb{R}_+} |\xi(t) - \bar{\xi}(t)|$$

is equivalent to the property

$$\forall t \in \mathbb{R}_+ : \xi(t) \approx \bar{\xi}(t). \tag{2}$$

This is a reachability property, as it is satisfied if the states of ξ over time remain inside an ϵ-envelope around $\bar{\xi}$, and can be computed in the augmented state space of ξ, $\bar{\xi}$ and t. Since the linear oscillator is only marginally stable, it will be numerically difficult to simulate. Alternatively, one could use an explicit, analytically computed, representation of the envelope. In practise, the verification of this property is rather difficult. The infinite time interval would have to be mapped onto a bounded interval $[0, T]$, e.g., by resetting the reference time for $\bar{\xi}$ at $t = T$, which requires the explicit knowledge of T. A conservative reachability analysis usually depends on some type of overapproximation, which would for many circuits result in period that can vary in some interval unless the circuit is designed to be asymptotically stable with respect to the period T. Since the quantification in (2) ranges over all time, any deviation in the period would result in a prohibitively large distance between the two signals. In the following, we will therefore consider properties that are more amenable to practical purposes.

2.2 Arbitrary T-Periodic Behaviors

A less specific property to consider is the periodic time T of ξ. Suppose we know that all signals in question, clean and noisy alike, stay inside a range for which the set $\bar{X}$ is a good under-approximation.[3] Since x can occur more than once in a period, we consider signals for which x occurs only once per period in some combination with the signs of the higher derivatives, and for which, to avoid pathological cases, at any time for some $k > 0$ the derivative $d^k/dt^k x(t)$ is nonzero. For simplicity, we denote x in the following as a state-like tuple $(x', \bowtie_1, \ldots, \bowtie_m)$ where $x' \in X$, $\bowtie_i \in \{\leq, \geq\}$ and $m + 1$ is the order of a system producing the signal. Furthermore, we write $\xi(t) = x$ if $\xi(t) = x'$ and $d^i/dt^i \xi(t) \bowtie_i 0$ at time t. This will allow us to express properties of ξ in terms of the time points at which values in $\bar{X}$ occur. The classical way to express periodicity is

$$\forall t \in \mathbb{R}_+, n \in \mathbb{N} : \xi(t) \approx \xi(t + nT). \tag{3}$$

An alternative and almost equivalent definition (they are equivalent when strict equality is used) can be made by counting the n^{th} occurrence of a value x of ξ. Let $\tau(x, n) = t$ iff the n^{th} occurence of x in ξ is at time t. We will later use this formulation to build monitor automata. Equipped with this function we can say:

$$\forall x \in \bar{X}, n, m \in \mathbb{N} : \tau(x, n) \approx \tau(x, m) - (m - n)T. \tag{4}$$

The difference between (3) and (4) is that we quantify in (3) over all positive values of t, while in (4) we do so only for values in $\bar{X}$, i.e., only for the values of x that appear in every ξ.

[3] One could define it as the largest set that all signals of interest visit infinitely often.

Both (3) and (4) are not reachability properties on the original state-space of the system, because they require infinite (bounded but dense) memory. Let us first reduce them to their "one step" equivalents:

$$\forall t \in \mathbb{R}_+ : \xi(t) \approx \xi(t+T), \tag{5}$$
$$\forall x \in \bar{X}, n \in \mathbb{N} : \tau(x,n) \approx \tau(x,n+1) - T. \tag{6}$$

The properties above are equivalent to (3) and (4) only if we use strict equality, because approximate equality is not transitive. In the case of (5), the memory consists of a function $z : [0,T) \mapsto X$ such that at any instant t, $z(t') = \xi(t-t')$. For (6), the memory is a function $w : \bar{X} \times \{\leq, \geq\}^m \mapsto \mathbb{R}_+$ with $w(x) = \tau(x, n_x)$, with n_x being the number of times x already occured in ξ.

2.3 Reduction to Finite Memory

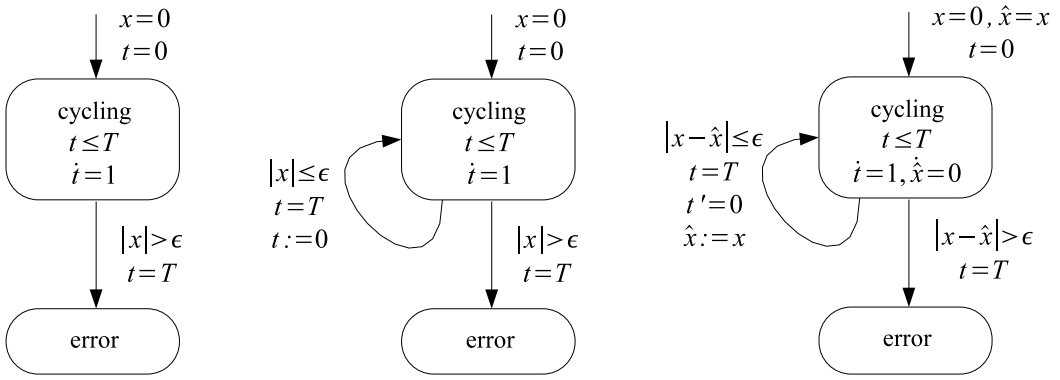
We now introduce approximations with finite memory to characterize these properties. The idea is very simple: just replace the quantification over all t or x by finitely many values for each period. Without loss of generality, let us pick zero for both t and x, and some fixed tuple of signs $\bowtie$ for the derivatives. The two formulae simplify to:

$$\forall n \in \mathbb{N} : \xi(nT) \approx \xi((n+1)T), \tag{7}$$
$$\forall n \in \mathbb{N} : \tau(0,n) \approx \tau(0,n+1) - T. \tag{8}$$

In (7) we say that we roughly get the same value every T time units. In (8) we say that zero crossings are roughly T spaced. Mechanically speaking, these properties can be checked by adding a clock to the system as well as a memory variable which can remember either time or value. For the first property, also called *cycle-to-cycle amplitude variation*, we remember the last value of x at $\xi(nT)$ and when the clock reaches T we compare the current value of ξ with the previous one, reset the clock and update the memory. For the second property, called *period jitter*, we compare the value of the clock with the memory every time we encounter $\xi(t) = 0$ and then reset the clock and update the memory. In order for x to be encountered only once per period, we take into account the derivatives (or other state variables), which are encoded with discrete states.

2.4 Monitor Automata

Figures 1 and 2 show monitor automata for verifying bounds on the amplitude variation and the phase jitter using reachability analysis. They contain a special error location, and the properties are fulfilled if this error location is not reachable. The simplest form of amplitude variation, the variation over a single cycle, is checked by the monitor in Fig. 1(a). It has a clock that measures the elapse of the cycle period T. If at the end of the cycle x can be too far away from zero, an error location is reachable and the property



(a) Single cycle (b) Absolute (c) Cycle-to-cycle

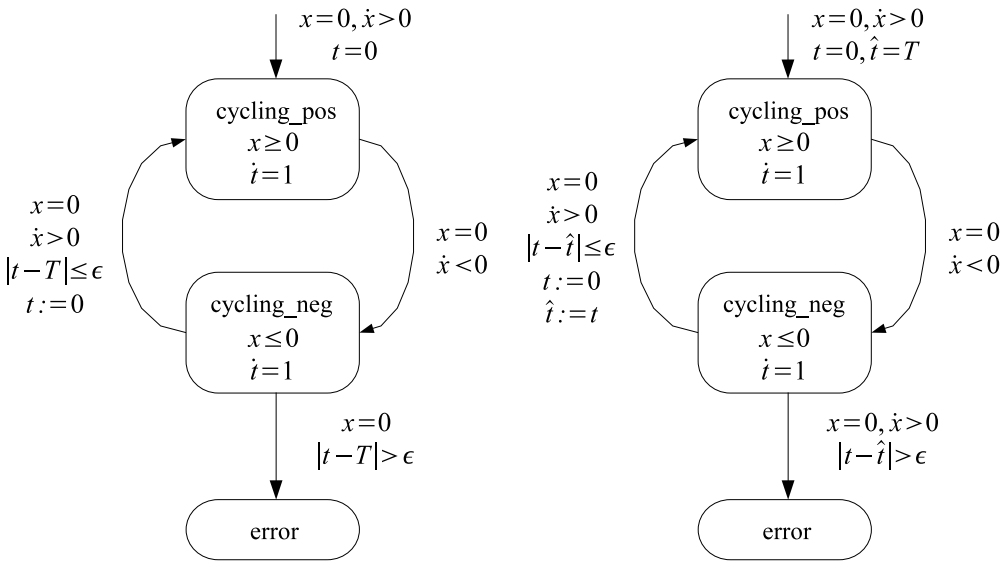
Fig. 1. Monitor automata for the amplitude variation

is violated. The monitor in Fig. 1(b) checks whether for all multiples of the period T the state is in the vicinity of the zero crossing, i.e., whether

$$\forall n \in \mathbb{N} : \xi(nT) \approx 0. \tag{9}$$

The monitor is initially in the state *cycling*. A clock t measures the period, and at $t = T$ two transitions are possible: If x is close to zero, the clock is reset and the monitoring continues. If x is outside of the ϵ-region around zero, the monitor enters the error location and the property is violated. As discussed at the end of Sect. 2.1, the absolute variation with respect to an explicit period T is practical only in special cases. By contrast, the cycle-to-cycle variation of the amplitude, i.e., property (7), is of practical usefulness since it allows for some deviation from the period T. A corresponding monitor is shown in Fig. 1(c). The value of x at $t = T$ is compared against the value $\hat{x}$ memorized at the last period.

Monitoring the jitter is slightly more complicated, because we have to uniquely identify one zero crossing per cycle. We demonstrate this for a second order system, which has at most two zero crossings that can be distinguished by the sign of the derivative $\dot{x}$. The monitor in Fig. 2(a) verifies the period jitter, i.e., property (8). It has a clock t and features three locations, one for positive values and one for negative values of x, and one error location. The automaton is initially in the location *cycling_pos* and assumes a positive derivative for x. The first half of the cycle takes place in this location, until the zero crossing is hit with a negative derivative $\dot{x}$, which triggers a transition to the location *cycling_neg*. There the second half of the cycle occurs, until the zero crossing is hit with a positive derivative. At this point two transitions are possible: If the clock t is within ϵ units of the period T, the clock is reset and the cycle recommences in location *cycling_pos*. Otherwise the monitor enters the error state.



(a) Period (b) Cycle-to-cycle

Fig. 2. Monitor automata for the phase jitter

The monitor in Fig. 2(b) verifies the *cycle-to-cycle jitter*, i.e., the property

$$\forall n \in \mathbb{N} : \tau(0, n+1) - \tau(0, n) \approx \tau(0, n) - \tau(0, n-1). \tag{10}$$

It operates similarly to the monitor in Fig. 2(a), except that the previous cycle time is stored in a variable $\hat{t}$, and after each new cycle the time is compaired against $\hat{t}$ instead of T. We finally turn to the problem of obtaining bounds on the amplitude variation and phase jitter instead of verifying a priori known bounds. The monitor automata from Figs. 1 and 2 are easily adapted by removing the error location. The bounds on the amplitude variation can then be obtained from the states in the intersection of the set of reachable states with the constraint $t = T$. For the bounds on the phase jitter, we regard the reachable states in the location *cycling_neg*, intersected with the constraint $x = 0$.

3 Implementation in PHAVer

We provide experimental results for some of the monitor automata in the previous section. For computing the set of reachable states we use the verification tool PHAVer [7], which can compute exact reachability for *linear hybrid automata* [4] (LHA) [1]. In linear hybrid automata, the invariants and transitions given by linear predicates, and the dynamics by conjuncts of linear constraints

$$a_i^T \dot{x} \bowtie_i b_i, \qquad a_i \in \mathbb{Z}^n, b_i \in \mathbb{Z}, \bowtie_i \in \{<, \leq, =\}, \ i = 1, \ldots, m. \tag{11}$$

[4] The term linear hybrid automata is ambiguously used in literature, sometimes also referring to what we call affine dynamics.

For this class of hybrid systems, the computation is exact algorithmically and, being purely state-based, ranges over infinite time. PHAVer uses polyhedra to represent sets of states, and exact arithmetic based on the Parma Polyhedra Library [3] with robust unbounded integer representations. Because of the exact arithmetic, the complexity of the linear predicates representing states typically increases prohibitively through the course of the analysis. The size of coefficients in the predicates can increase exponentially, and the number of constraints in a predicate polynomially with each iteration of the fixpoint computation. This complexity is managed by fully user-controllable limits on the number of bits used in coefficients and the number of constraints. Predicates that exceeded these limits are overapproximation conservatively with a simpler predicate. In addition to reachability analysis, PHAVer supports compositional and assume-guarantee reasoning with a separate engine for computing simulation relations, which can also be used to verify abstractions [8].

PHAVer can analyze systems with affine dynamics of the form $\dot{x} = Ax + b$ by overapproximating them with LHA. The overapproximation is guaranteed to be conservative and algorithmically as well as numerically sound [11]. Affine dynamics are specified in a relaxed form as a conjunction of constraints

$$a_i^T \dot{x} + \hat{a}_i^T x \bowtie_i b_i, \qquad \hat{a}_i \in \mathbb{Z}^n. \tag{12}$$

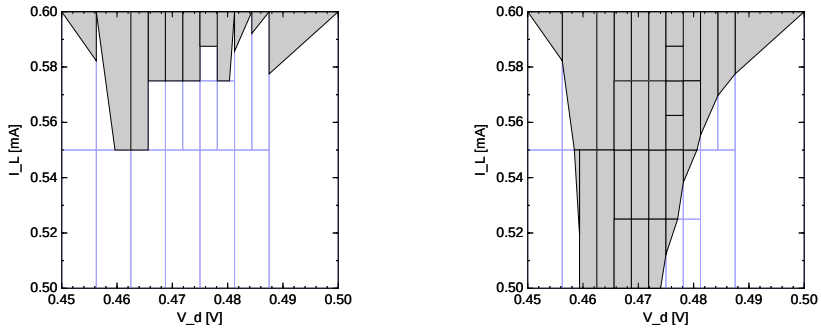
The relaxed form allows us to model uncertainties in the approximation or in the parameters. E.g., for dynamics $\dot{x} = ax$, a parameter range of $a \in [a_l, a_u]$ yields a differential inclusion $a_l x \leq \dot{x} \leq a_u x$ for positive values of x, and $a_u x \leq \dot{x} \leq a_l x$ otherwise. The two cases are modeled by introducing a separate location for each case, thus "hybridizing" the model.

For the analysis, affine dynamics from (12) are overapproximated with LHA dynamics from (11) by linear programming as follows. Let equalities be modeled by the conjunction of two inequalities. If the invariant $Inv(loc)$ of a location loc is bounded, the set of $\dot{x}$ that fulfill (12) is bounded by $a_i^T \dot{x} \bowtie_i b_i - p/q$, where p/q is a rational

$$p/q = \inf_{x \in Inv(loc)} \hat{a}_i^T x, \qquad p, q \in \mathbb{Z}.$$

This overapproximation introduces a loss of accuracy that depends on the size of the location and the angular spread of the derivative vectors in the location, i.e., the spatial angle of the widening of the vector field. To improve the accuracy, locations are recursively split into two along a suitable hyperplane, effectively partitioning the state space with a grid whose shape depends on the choice of the splitting hyperplanes. They are prioritized according to a set of criteria that aims at minimizing the number of partitions, i.e., locations introduced by the splitting. Let $\dot{X}$ be the set of derivatives in a location. We define the *spread* of the derivatives as

$$\sphericalangle(\dot{X}) = \arccos \min_{x,y \in \dot{X}} x^T y / |x||y|.$$



(a) Snapshot during computation

(b) Final result

Fig. 3. Reachable states and partitioning of the invariants

The partitioning can be adapted to the dynamics by choosing the hyperplane that minimizes the spread of the derivatives, and stopping the splitting once a lower threshold of the spread is reached. To restrict the partitioning to the reachable set of locations, the splitting is recursively triggered for each location when it is taken from the waiting list of the reachability algorithm, but before the actual computation of the reachable states in the location. Once a reachable location is split down to the lower thresholds of size and spread, it remains fixed for the remainder of the analysis.

Figure 3 shows an example of the partitioning during the analysis. Starting from a set of initial states at $I_L = 0.6mA$ the region is split until the initial states are partitioned such that each location has a spread $\sphericalangle(\dot{X}) = \arccos 0.85$. Then the set of reachable states is computed inside each location by applying the time elapse operator. The result is shown in Fig. 3(a). The transitions between the locations yield successor states that are put onto a waiting list. The locations on the waiting list are partitioned, and the time elapse operator is applied. The procedure is repeated until the fixpoint in Fig. 3(b) is obtained.

We model nonlinear circuit equations with piecewise affine differential inclusions in the form of (12). Partitioning the one-dimensional characteristics of nonlinear components, e.g., diodes, into convex or concave sections allow us to easily construct a linear envelope for each interval. This is achieved by separating at the minima, maxima and inflexion points. Inaccuracies in finding these points can be compensated by overlapping the sections (more precisely, the invariants of the resulting abstraction), see [11] for more details. Equations that don't have nonlinear elements are modeled directly. The affine model can then be analyzed in PHAVer, which during the analysis overapproximates it with a LHA. The on-the-fly approach decisively improves the speed and memory requirements of the analysis.

The monitor automata from the previous two sections are linear hybrid automata, and are readily modeled in PHAVer's textual input language. For the automata with an error state, the reachability computation can be stopped

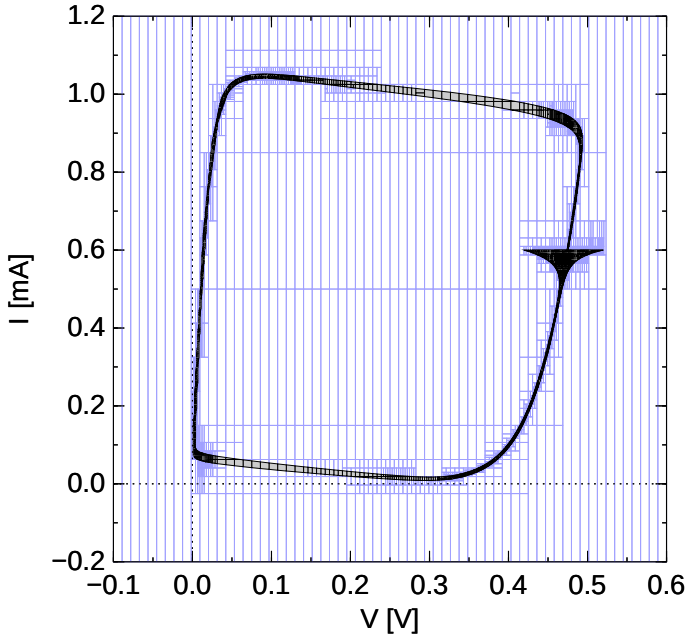


Fig. 4. Reachable states of the tunnel diode circuit

as soon as this state is reachable, which greatly improves the speed of trial-and-error experiments.

4 Experimental Results

In this section, we present results from experiments with the tunnel-diode oscillator circuit from [9]. We model the current I_L through the inductor and the voltage drop V_d of a tunnel diode in parallel with the capacitor of a serial RLC circuit, which are in stable oscillation for the given parameters. The state equations are given by

$$\dot{V}_d = 1/C(-I_d(V_d) + I_L), \tag{13}$$

$$\dot{I}_L = 1/L(-V_d - 1/G \cdot I_L + V_{in}), \tag{14}$$

where $C = 1\ pF$, $L = 1\ \mu H$, $G = 5\ m\Omega^{-1}$, $V_{in} = 0.3\ V$, and the diode current

$$I_d = \begin{cases} 6.01V_d^3 - 0.992V_d^2 + 0.0545V_d & \text{if } V_d \leq 0.055, \\ 0.0692V_d^3 - 0.0421V_d^2 + 0.004V_d + 8.96 \cdot 10^{-4} & \text{if } 0.055 \leq V_d \leq 0.35, \\ 0.263V_d^3 - 0.277V_d^2 + 0.0968V_d - 0.0112 & \text{if } 0.35 \leq V_d. \end{cases}$$

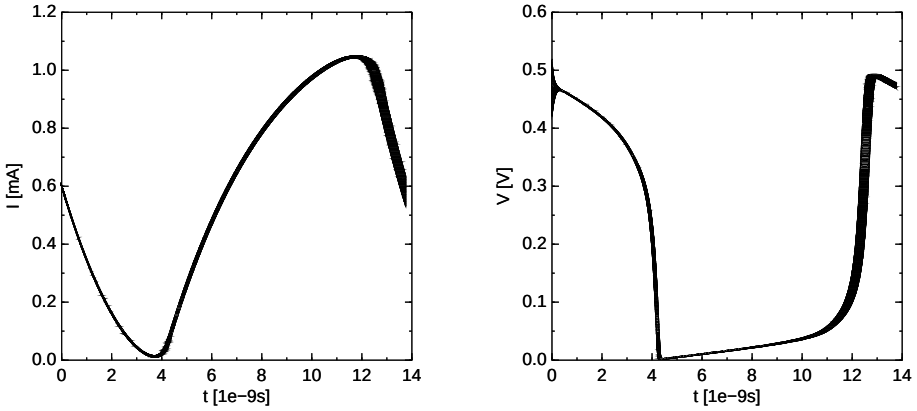
Following the procedure outlined in the previous section, a piecewise affine envelope was constructed for the tunnel diode characteristic $I_d(V)$. We choose 64 intervals for the range $V_d \in [-0.1, 0.6]$ to yield sufficient accuracy and so obtain a piecewise affine model for (13). It is modeled as a hybrid automaton with V_d as an output- and I_L as an input-variable, and consists of 64 locations,

one for each interval. Equation (14) is affine, and is modeled as a hybrid automaton with V_d as an input and a single location. Both models are composed and analyzed in PHAVer. Figure 4 shows the states reachable from a set of initial states given by $V_d \in [0.42V, 0.52V]$, $I_L = 0.6mA$. It also shows the invariants (grey), of which the vertical lines correspond to the 64 intervals of the affine diode characteristic, and the rest of the partitioning was generated during the analysis. The parameters of the algorithm were the direction of the splitting planes, a minimum and maximum size for the locations, here 1/256th and $1/16th$ of the visible region, and a minimum spread of the derivatives in each location of $\sphericalangle_{min} = \arccos(0.999)$. To manage the complexity, the coefficients of polyhedra are limited to 24 bit, and the polyhedra to 32 constraints. On an Intel Xeon processor with 2.8GHz and 4GB RAM running a 32-bit Linux kernel the reachability analysis takes 72.8 s and requires 126.7 MB RAM. The partitioning results in 2998 locations, of which 1892 are reachable.

We now apply reachability analysis to measure the single cycle amplitude variation and the period jitter of $I_L(t)$. As described in Sect. 2.4, we use the monitor automata of Fig. 1(a), respectively Fig. 2(a), with the error locations removed to detect the period and zero crossing. We use an affine approximation of the diode characteristic with 128 partitions. The analysis was carried out with parameters as before, except for a minimum location size of 1/512th and a derivative spread of $\sphericalangle_{min} = \arccos(0.99999)$. For measuring the amplitude variation, we assume a cycle time of $T = 13.75$ μs. The reachability analysis takes 1880 s (1325 MB). The obtained set of reachable states $I_L(t)$ and $V_d(t)$, shown in Fig. 5, guarantees an amplitude variation between 0.532 mA and 0.636 mA. For measuring the phase jitter, the zero crossing was assumed to be at $I_L = 0.6$ mA. There are two crossings per period, which we distinguish according to whether $V_d > 0.25$ V or $V_d < 0.25$ V. The reachable states, shown in Fig. 6, guarantee the period to be between 13.52 μs and 13.86 μs. The computation takes 1997 s (1463 MB).

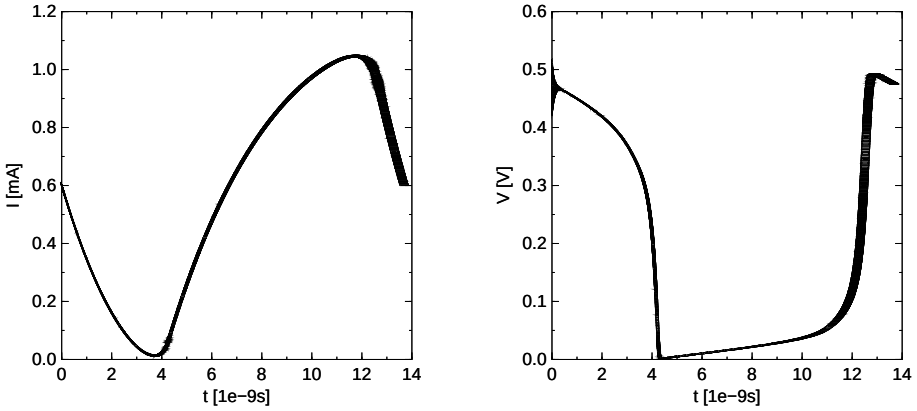
5 Conclusions

This paper presents a method for constructing monitor automata for oscillator circuits based on the analysis of scalar signal properties related to the specifications of interest. Here we consider the amplitude and phase jitter. The objective is to construct hybrid system models that are amenable to finite-time reachability analysis. Typically reachability computations for hybrid systems focus on the transient system behavior starting from a set of initial conditions. The present work aims to verify properties of the asymptotic, "steady state" behavior of a circuit. We are interested in extending this approach to verify other properties that are commonly analyzed using frequency domain techniques.



(a) Current I_L (b) Voltage V_d

Fig. 5. Reachable states of the circuit and the amplitude monitor of Fig.1(a)



(a) Current I_L (b) Voltage V_d

Fig. 6. Reachable states of the circuit and the phase jitter monitor of Fig.2(a)

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