Analysing protocols subject to guessing attacks

Gavin Lowe*

January 7, 2003

Abstract

In this paper we consider guessing attacks upon security protocols, where an intruder guesses one of the values used (typically a poorly-chosen password) and then seeks to verify that guess. We formalise such attacks, and in particular the way in which the guess is verified. We then describe how to model such attacks within the process algebra CSP, so that they can be detected using the model checker FDR, and illustrate our technique on some examples.

1 Introduction

It is well known that people tend to choose poor passwords [MT79]. Such poorly-chosen passwords are vulnerable to *guessing attacks*, where an intruder observes some information based upon the password, tries to guess the password, and is able to verify the guess in some way.

For example, consider the following simple protocol, which aims to authenticate a user (or client) a to a server b using a password $passwd(a, b)$:

Message 1. $a \rightarrow b \ : \ a, b$
Message 2. $b \rightarrow a \ : \ nb$
Message 3. $a \rightarrow b \ : \ \{|nb|\}_{passwd(a,b)}$

Here nb is a nonce; we use the notation $\{|m|\}_k$ to represent m encrypted with k. An intruder could overhear this exchange and attempt a guessing

*Author's address: Oxford University Computing Laboratory, Wolfson Building, Parks Road, Oxford, OX1 3QD, UK; e-mail: gavin.lowe@comlab.ox.ac.uk.

attack: if he guesses that the password is "`aardvark`", he could decrypt the ciphertext from message 3 with `aardvark`, and compare the result with the observed value of nb; if they match, this verifies the guess (with high probability); if they don't match, he can guess again. Of course, this guessing can be automated, trying each word in some list in turn. If the password was chosen by a person, there is a fair probability that it will appear in a suitable dictionary, and so this attack is feasible. On the other hand, if it was generated by a good random number generator from a large space of passwords, then guessing will be infeasible. We will be considering only the former, feasible, form of guessing attack in this paper.

Note that this guessing attack can be performed off-line, to avoid detection of incorrect guesses. Contrast this with an on-line attack, where the intruder sends messages based upon guesses; in such cases, incorrect guesses will be detected and probably logged; repeated incorrect guesses might lead to additional defences being mounted, such as blocking further attempts. We will deal only with off-line guessing attacks in this paper.

The question we are interested in, therefore, is whether the intruder has an efficient off-line procedure whereby he can determine whether or not a candidate password (or "guess") is correct.

Note also that if, in the above protocol, the intruder had not seen nb, then a guessing attack would not be possible: decrypting with either a correct or incorrect guess would just reveal a string of bits, and there would be no way of verifying that the guess was correct. We are assuming—contrary to the usual practice in formal analysis of security protocols—that encryptions do not contain any added redundancy such as MACs (beyond that explicitly included in the message); such redundancy would allow the intruder to verify that he had indeed decrypted with the correct key.

In this paper we study how protocols subject to guessing attacks can be analysed. There has been much work in recent years on protocol analysis, but this aspect has been largely ignored. In Section 2 we produce a general model for such guessing attacks and define how a guess can be verified. In Section 3 we adapt this general model to the CSP/FDR approach to protocol analysis, and thus show how guessing attacks can be detected automatically. In Section 4 we illustrate our approach by applying it to a couple of well-known protocols that aim to be resistant to guessing attacks. We sum up in Section 5.

2 Formalising guessing attacks

A guessing attack consists of the intruder guessing a value g, and then verifying that guess in some way. The verification will be by the intruder using g to produce a value v, which we call the *verifier*; the verifier will demonstrate that the guess was correct, i.e., an incorrect guess would not have lead to this value. This verification can take a number of different forms: (1) the intruder knew v initially or has seen v during the protocol run; (2) the intruder produced v in two distinct ways from g; or (3) v is an asymmetric key, and the intruder knows v's inverse from somewhere. We consider some examples, below, to illustrate these forms of verification:

Example 1 If the protocol directly discloses v and $\{\!|v|\!\}_g$ to the intruder (as in the example in Section 1, with $v = nb$ and $g = passwd(a, b)$), then the guess of g allows the intruder to decrypt the second component to obtain v again, thereby verifying the guess; an incorrect guess would have lead to a different value, enabling the intruder to realise the guess was incorrect.

Alternatively, the guess of g allows the intruder to encrypt v with g to obtain $\{\!|v|\!\}_g$ again. This alternative form of attack succeeds only if the encryption is deterministic, which we assume throughout most of this paper; we briefly discuss probabilistic encryption in the conclusion.

Example 2 If the protocol directly discloses $\{\!|g|\!\}_g$ to the intruder, then if the intruder guesses g, he can encrypt g with itself to obtain $\{\!|g|\!\}_g$ again, thereby verifying the guess; alternatively, the intruder could decrypt $\{\!|g|\!\}_g$ with g to obtain g again.

Example 3 If the protocol allows the intruder to deduce $\{\!|v, v|\!\}_g$ directly, then the guess of g allows the intruder to obtain (v, v), from which he can obtain v in two different ways, thereby verifying the guess. An incorrect guess would have (with high probability) produced a string of bits not representing two copies of the same value, enabling the intruder to realise the guess was incorrect.

Example 4 If the intruder can deduce $PK(a)$ and $\{\!|SK(a)|\!\}_g$,[1] then the guess of g allows the intruder to obtain $SK(a)$; he can then verify the guess by seeing that this key is the inverse of $PK(a)$, by encrypting some arbitrary text with one key and decrypting with the other.

[1] These two terms are included in the file `/etc/publickey` under SunOS-4.0.

Note that the guess and attempted verification can take place at any point during the execution of a protocol: when we search for guessing attacks, we will consider all possible ways the intruder could interact with the protocol, intercepting and sending messages, before the guess itself.

Let $Fact$ contain all submessages of messages in the protocol[2]. Our definition of guessing attacks is largely independent of the form of the set $Fact$, but in the examples of this section we take $Fact$ to be built up from some set of atoms using pairing and encryption.

We assume that the intruder can directly deduce new facts from facts he already knows (without guessing). We model these deductions by a one-step deduction relation:

$$\vdash \;::\; \mathbb{P}(Fact) \times Label \times Fact$$

The idea is that $S \vdash_l f$ represents that f can be deduced from the set S in a single step; the label l indicates how the deduction is done, and is necessary to capture the concept of deriving a fact in two different ways, mentioned above.

Again, our definition of guessing attacks is largely independent of the details of this relation, but for the examples of this section we will use a fairly standard derivation relation, defined as follows, where f, f' range over facts, and k ranges over keys:

$$\{f, f'\} \vdash_{\mathsf{pair}} (f, f'), \qquad \{(f, f')\} \vdash_{\mathsf{fst}} f, \qquad \{(f, f')\} \vdash_{\mathsf{snd}} f',$$
$$\{f, k\} \vdash_{\mathsf{enc}} \{\!|f|\!\}_k, \qquad \{\{\!|f|\!\}_k, k^{-1}\} \vdash_{\mathsf{dec}} f.$$

The first rule represents tupling; the next two represent un-tupling; the final two represent encryption and decryption.

We write $IK \models_{tr} IK'$ if the intruder can deduce the set of facts IK' from the set of facts IK, using zero or more deduction steps ("IK" stands for "intruder's knowledge"); the sequence tr records the trace of deductions the intruder performs. We define $\models$ by the following rules:

$$IK \models_{\langle\rangle} IK,$$
$$S \subseteq IK \wedge S \vdash_l f \wedge IK \cup \{f\} \models_{tr} IK' \;\Rightarrow\; IK \models_{\langle S \vdash_l f \rangle^\frown tr} IK'.$$

[2]We use the word "fact" for these, preferring to reserve the word "message" for an actual message of the protocol.

The second rule reflects the fact that if the intruder can deduce f from some subset S of his knowledge, then he can add f to his knowledge, and use it to deduce more facts.

We now define what it means for the intruder to verify a guess g using verifier v from knowledge IK; this is the case if there are sets of facts IK', S and S', labels l and l', and trace tr such that either conditions (1)–(5) or condition (6), below, hold.

Firstly, from this knowledge and the guess, the intruder must be able to reach some knowledge IK':

$$IK \cup \{g\} \models_{tr} IK'. \tag{1}$$

One of those deductions must produce the verifier v:

$$S \vdash_l v \text{ in } tr. \tag{2}$$

However, this deduction must really depend upon the knowledge of g: it must be impossible to obtain the information S necessary for the deduction without knowing g:

$$\nexists IK'' \bullet (IK \models IK'' \supseteq S). \tag{3}$$

Further, either the intruder can deduce v in a second way (first disjunct below) (the conjunct $(S, l) \neq (S', l')$ ensures this really is a *different* way of deducing v); or the intruder already knew v in the initial state, or guessed it (second disjunct); or v is an asymmetric key whose inverse the intruder knows (third disjunct):

$$\begin{array}{l} S' \vdash_{l'} v \text{ in } tr \wedge (S, l) \neq (S', l') \\ \vee \ \ v \in IK \cup \{g\} \\ \vee \ \ v \in ASYMMETRIC_KEYS \wedge v^{-1} \in IK'. \end{array} \tag{4}$$

We will require a fifth condition, designed to prevent certain false attacks, which we will describe below. First, we illustrate how this definition applies to the above examples. Our descriptions concentrate on conditions (1), (2) and (4), which capture the deductions the intruder makes; we leave it to the reader to check condition (3) in each case.

Example 1 Suppose $IK = \{v, \{|v|\}_g\}$; consider $IK \cup \{g\} \models_{\langle \{\{|v|\}_g, g\} \vdash_{\mathsf{dec}} v \rangle} IK \cup \{g\}$, i.e. where the intruder decrypts $\{|v|\}_g$ to obtain v; this satisfies condition (1); the deduction $\{\{|v|\}_g, g\} \vdash_{\mathsf{dec}} v$ satisfies condition (2); and $v \in IK \cup \{g\}$ so condition (4) is satisfied.

Example 2 Suppose $IK = \{\{\!|g|\!\}_g\}$, and take $v = g$; then $IK \cup \{g\} \models_{\langle \{\{\!|g|\!\}_g, g\} \vdash_{\mathsf{dec}} g \rangle} IK \cup \{g\}$ satisfies condition (1); the deduction $\{\{\!|g|\!\}_g, g\} \vdash_{\mathsf{dec}} g$ satisfies condition (2); and $v \in IK \cup \{g\}$ so condition (4) is satisfied.

Example 3 Suppose $IK = \{\{\!|v, v|\!\}_g\}$; then consider

$$IK \cup \{g\} \models_{\langle \{\{\!|v,v|\!\}_g, g\} \vdash_{\mathsf{dec}} (v,v),\ \{(v,v)\} \vdash_{\mathsf{fst}} v,\ \{(v,v)\} \vdash_{\mathsf{snd}} v \rangle} IK \cup \{g, (v,v), v\},$$

i.e. where the intruder decrypts $\{\!|v, v|\!\}_g$ to obtain (v, v), from which he obtains v as both the first and second component; this satisfies condition (1); the deduction $\{(v, v)\} \vdash_{\mathsf{fst}} v$ satisfies condition (2); and the deduction $\{(v, v)\} \vdash_{\mathsf{snd}} v$ satisfies condition (4), noting that this deduction is different from the one for (2).

Example 4 Suppose $IK = \{PK(a), \{\!|SK(a)|\!\}_g\}$, and take $v = SK(a)$; then $IK \cup \{g\} \models_{\langle \{\{\!|SK(a)|\!\}_g, g\} \vdash_{\mathsf{dec}} SK(a) \rangle} IK \cup \{g, SK(a)\}$ satisfies condition (1); the deduction $\{\{\!|SK(a)|\!\}_g, g\} \vdash_{\mathsf{dec}} SK(a)$ satisfies condition (2); and the fact that $(SK(a))^{-1} = PK(a) \in IK$ satisfies the third disjunct of condition (4).

As mentioned above, we need another condition in order to prevent certain false attacks. Here's an example to show why. Let $IK = \{\{\!|v|\!\}_g\}$; consider

$$IK \cup \{g\} \models_{\langle \{\{\!|v|\!\}_g, g\} \vdash_{\mathsf{dec}} v,\ \{v\} \vdash_{\mathsf{pair}} (v,v),\ \{(v,v)\} \vdash_{\mathsf{fst}} v \rangle} IK \cup \{g, v, (v,v)\},$$

i.e. where the intruder decrypts $\{\!|v|\!\}_g$ to obtain v, pairs it with itself to obtain (v, v), and then extracts the first component to obtain v again; this satisfies condition (1); the deduction $\{\{\!|v|\!\}_g, g\} \vdash_{\mathsf{dec}} v$ satisfies condition (2); and the deduction $\{(v, v)\} \vdash_{\mathsf{fst}} v$ satisfies condition (4). However, this is not a valid guessing attack, because the intruder obtains no verification of the guess: the fact that he derived v from (v, v) does not give him that verification, because he built (v, v) himself.

Our extra condition will prohibit deduction steps that simply reverse previous steps, in the way the step $\{(v, v)\} \vdash_{\mathsf{fst}} v$ reverses $\{v\} \vdash_{\mathsf{pair}} (v, v)$. We define a relation undoes that describes when a deduction step undoes another. The relation is defined by the following rules:

$$\begin{aligned}
\{f, f'\} \vdash_{\mathsf{pair}} (f, f') \ &\mathsf{undoes}\ \{(f, f')\} \vdash_{\mathsf{fst}} f, \\
\{f, f'\} \vdash_{\mathsf{pair}} (f, f') \ &\mathsf{undoes}\ \{(f, f')\} \vdash_{\mathsf{snd}} f', \\
\{(f, f')\} \vdash_{\mathsf{fst}} f \ &\mathsf{undoes}\ \{f, f'\} \vdash_{\mathsf{pair}} (f, f'), \\
\{(f, f')\} \vdash_{\mathsf{snd}} f' \ &\mathsf{undoes}\ \{f, f'\} \vdash_{\mathsf{pair}} (f, f'), \\
\{f, k\} \vdash_{\mathsf{enc}} \{\!|f|\!\}_k \ &\mathsf{undoes}\ \{\{\!|f|\!\}_k, k^{-1}\} \vdash_{\mathsf{dec}} f, \\
\{\{\!|f|\!\}_k, k^{-1}\} \vdash_{\mathsf{dec}} f \ &\mathsf{undoes}\ \{f, k\} \vdash_{\mathsf{enc}} \{\!|f|\!\}_k.
\end{aligned}$$

Our fifth condition, then, says that, neither of the deductions from conditions (2) or (4) undoes one of the deductions in condition (1):

$$\forall (S'' \vdash_{l''} v'') \text{ in } tr \bullet \neg(S \vdash_l v \text{ undoes } S'' \vdash_{l''} v'') \wedge \neg(S' \vdash_{l'} v' \text{ undoes } S'' \vdash_{l''} v'') \tag{5}$$

(where S, S', l and l' are as in conditions (1)–(4)). We leave it to the reader to check that this condition is satisfied by each of the examples above.

It is worth pointing out that the third disjunct of condition (4) is necessary, in particular for Example 4. The reader may think that, having obtained $PK(a)$ and $SK(a)$, the intruder can verify the guess by picking an arbitrary v, encrypting it with $PK(a)$, and then decrypting with $SK(a)$. However, these two deductions, $\{v, PK(a)\} \vdash_{\mathsf{enc}} \{|v|\}_{PK(a)}$ and $\{\{|v|\}_{PK(a)}, SK(a)\} \vdash_{\mathsf{dec}} v$, do not satisfy condition (5).

In fact, the above conditions do not quite cover all possible guessing attacks: there is the additional possibility that the guessed value itself is an asymmetric key, and that its inverse is known initially[3]. (Condition (2) is not satisfied in this scenario; it is frustrating that we have to treat this scenario as a special case, but there seems no elegant way of combining it with the earlier conditions.)

$$g \in ASYMMETRIC_KEYS \wedge g^{-1} \in IK. \tag{6}$$

Hence we define a guessing attack to be successful if either conditions (1)–(5) hold, or condition (6) holds.

Above we have assumed that the intruder guesses only a single value. It is straightforward to adapt the above conditions to the case where the intruder can simultaneously guess multiple values: simply replace the $\{g\}$ above by some set G of guessed values. Of course, the complexity of any such guessing attack is exponential in the number of values that need to be correctly guessed, so it is reasonable to place some small bound upon the size of G.

3 Modelling guessing attacks in CSP

In this section, we describe how guessing attacks can be modelled using the process algebra CSP and discovered using the model checker FDR, by adapting the standard intruder model from [RSG+00].

[3] I am grateful to Ernie Cohen for pointing out this additional possibility.

We take a slightly different structure for the set $Fact$ of facts from that considered in Section 2: we use sequences of facts rather than pairs, for efficiency; and we include a constructor corresponding to Vernam encryption (bit-wise exclusive-or), to extend the scope of our techniques (in particular, the protocol we consider in Section 4.2 uses Vernam encryption).

The model of the intruder is built around a set *Deductions* representing the deductions the intruder can make. Each element $(f, l, S) \in Deductions$ corresponds to a deduction of the form $S \vdash_l f$: if the intruder knows the set of facts S, then he can deduce the fact f using rule l. In fact, for reasons of efficiency, we use slightly different deductions to those in Section 2, combining decryption and splitting into a single rule, and combining sequencing and encryption into a single rule; we also include deductions corresponding to Vernam encryption and decryption. It is worth giving the definition of the undoes relation for these latter deductions:

$$\{f, f'\} \vdash_{\mathsf{vernEnc}} f \oplus f' \ \mathsf{undoes}\ \{f \oplus f', f\} \vdash_{\mathsf{vernDec}} f',$$
$$\{f \oplus f', f\} \vdash_{\mathsf{vernDec}} f' \ \mathsf{undoes}\ \{f, f'\} \vdash_{\mathsf{vernEnc}} f \oplus f',$$
$$\{f \oplus f', f\} \vdash_{\mathsf{vernDec}} f' \ \mathsf{undoes}\ \{f \oplus f', f'\} \vdash_{\mathsf{vernDec}} f.$$

The third term is perhaps not obvious.

The intruder process has two principle states, $INTRUDER$ and $INTRUDER'$, corresponding to whether the intruder has not or has made a guess. If the intruder is able to use the guess to deduce a fact that he already knows—as captured by the earlier conditions—then that will verify the guess.

Each process is parameterised by the intruder's knowledge, IK; this knowledge will change as the intruder sees or deduces new facts. We need to control the deductions performed: we need to prevent the intruder from repeating deductions, to ensure that the two ways in which a fact is deduced are indeed different, as required by condition (4); and we need to prevent the intruder from undoing deductions, as required by condition (5). Therefore the intruder processes will also be parameterised by a set *deds* of deductions, representing those deductions that can still be performed.

In the non-guessing state the intruder can:

- hear a message on the network, and add the components of that message to his knowledge (a message is modelled as a sequence of facts; $components(m)$ gives the corresponding set of facts);

- say a message (i.e. send it on the network) provided he knows all the components;
- perform a deduction (f, l, S), provided he knows all of S; the fact f is added to his knowledge; that deduction and all its undoing deductions are removed from the set *deds* of deductions that are still allowed;
- signal, via the channel *leak*, if he has learnt a secret;
- guess some guessable value g, and enter the guessing state; if g is an asymmetric key and g^{-1} is already known then this verifies the guess, following condition (6).

The following process description captures this:

$$
\begin{array}{l}
INTRUDER(IK, deds) = \\
\quad hear?m \rightarrow INTRUDER(IK \cup components(m), deds) \\
\quad \Box \\
\quad (\Box\, m : Message, components(m) \subseteq IK \bullet \\
\qquad say.m \rightarrow INTRUDER(IK, deds)) \\
\quad \Box \\
\quad (\Box (f, l, S) : deds, S \subseteq IK \bullet \\
\qquad infer.(f, l, S) \rightarrow INTRUDER(IK \cup \{f\}, deds \ominus (f, l, S))) \\
\quad \Box \\
\quad (\Box\, f : SECRETS \cap IK \bullet leak.f \rightarrow INTRUDER(IK, deds)) \\
\quad \Box \\
\quad guess?g \rightarrow \text{if } g \in ASYMMETRIC_KEYS \wedge g^{-1} \in IK \\
\qquad\qquad\qquad \text{then } verify.(g, g^{-1}) \rightarrow INTRUDER'(IK \cup \{g\}, deds, g) \\
\qquad\qquad\qquad \text{else } INTRUDER'(IK \cup \{g\}, deds, g),
\end{array}
$$

where the operator $\ominus$ performs the removal described above:

$$
\begin{array}{l}
deds \ominus (f, l, S) \mathrel{\widehat{=}} \\
\quad deds - (\{(f, l, S)\} \cup \{(f', l', S') \mid (f', l', S') \text{ undoes } (f, l, S)\}).
\end{array}
$$

Recall that we are interested in *off-line* guessing attacks; we will therefore not allow any network events during the guessing period. During the guessing period, the intruder can continue to perform deductions. If one of these

deductions produces a fact that he already knows, or the inverse of a key he already knows, then this verifies his earlier guess:

$$
\begin{array}{l}
INTRUDER'(IK, deds, g) = \\
\quad \Box(f, S) : deds, S \subseteq IK \bullet \\
\qquad infer.(f, l, S) \rightarrow \\
\qquad \text{if } f \in IK \vee f \in ASYMMETRIC_KEYS \wedge f^{-1} \in IK \\
\qquad \text{then } verify.(f, g) \rightarrow INTRUDER(IK \cup \{f\}, deds \ominus (f, l, S)) \\
\qquad \text{else } INTRUDER'(IK \cup \{f\}, deds \ominus (f, l, S), g).
\end{array}
$$

We initialise the intruder's knowledge with some suitable initial value IIK, initialise *deds* with the set of all deductions, and hide the deductions. At many intermediate states, there will be several different deductions available; the intruder does not lose any abilities by performing as many deductions as possible, so it makes sense to force him to perform them all; further, the order of the deductions makes no difference, so picking one particular order will reduce the state space. The function *chase* does this: it forces internal τ events to be performed, picking an arbitrary order for them:

$$chase\,(INTRUDER(IIK, Deductions) \setminus infer)\,.$$

The use of *chase* has a fortunate and necessary consequence: by forcing deductions to occur, the intruder will perform a guess only when no more deductions are available; this means that any subsequent deduction will really depend upon the guess, as required by condition (3).

We can incorporate the above intruder model into the standard CSP model of a system running the protocol under consideration. In order to find out if a guessing attack is possible, we can use FDR to find out if any $verify$ event can occur. Of course, a successful guessing attack can also lead to breaches of secrecy (either of the password or other values disclosed in the attack) or failures of authentication; these can be tested for using the normal secrecy and authentication specifications.

It is straightforward to adapt the above intruder model to allow the intruder to guess multiple values, by allowing further *guess* events from the $INTRUDER'$ state (possibly subject to some maximum number).

Casper [Low98] is a compiler that produces CSP models of security protocols, suitable for checking using FDR, from more concise descriptions. We have extended Casper to model guessing attacks. We have tested it on the earlier examples, and used it in the examples of the following section, obtaining the expected results.

4 Examples

In this section we consider some examples, illustrating the techniques described earlier.

4.1 Encrypted key exchange

In [BM92], Bellovin and Merritt introduce the following key-establishment and mutual authentication protocol, which uses a potentially poorly-chosen password:

Message 1. $a \rightarrow b \ : \ a, \{|pk|\}_{passwd(a,b)}$
Message 2. $b \rightarrow a \ : \ \{|\{|k|\}_{pk}|\}_{passwd(a,b)}$
Message 3. $a \rightarrow b \ : \ \{|na|\}_k$
Message 4. $b \rightarrow a \ : \ \{|na, nb|\}_k$
Message 5. $a \rightarrow b \ : \ \{|nb|\}_k$.

Here pk is an *asymmetric* key, and k is a symmetric session key. We have analysed this protocol using Casper and FDR, and found no attacks.

It is interesting to ask whether pk needs to be an asymmetric key. We changed it to a symmetric key in the model, and discovered an attack. The intruder can observe a normal run of the protocol between A and B, guess $passwd(A, B)$, and then verify the guess using A's nonce Na. FDR produces a long trace containing all the deductions the intruder can make as a result of the guess. However, the relevant events are:

$guess.passwd(A, B)$
$infer.(\{|K|\}_{PK},\ Dec.1,\ \{\{|\{|K|\}_{PK}|\}_{passwd(A,B)}, passwd(A, B)\})$
$infer.(PK,\ Dec.1,\ \{\{|PK|\}_{passwd(A,B)}, passwd(A, B)\})$
$infer.(K,\ Dec.1,\ \{\{|K|\}_{PK}, PK\})$
$infer.(Na,\ Dec.1,\ \{\{|Na, Nb|\}_K, K\})$
$infer.(Na,\ Dec.1,\ \{\{|Na|\}_K, K\})$
$verify.(Na,\ passwd(A, B))$.

The intruder: decrypts the second message with $passwd(A, B)$ to obtain $\{|K|\}_{PK}$; decrypts the first message with $passwd(A, B)$ to obtain PK; decrypts $\{|K|\}_{PK}$ with PK to obtain K (this step is not possible if PK is an asymmetric key); decrypts the third message to obtain Na; and decrypts

the fourth message to obtain Na again; the fact that he obtains the same value Na from both these decryptions verifies the guess. It is also possible to verify the guess by decrypting the fourth and fifth messages to obtain Nb in two different ways.

4.2 Lomas et al.'s protocol

In [LGSN89], Lomas, Gong, Saltzer and Needham introduced the following protocol:

Message 1. $a \rightarrow s \ : \ \{|a, b, na1, na2, ca, \{|ta|\}_{passwd(a)}|\}_{pks}$
Message 2. $s \rightarrow b \ : \ a, b$
Message 3. $b \rightarrow s \ : \ \{|b, a, nb1, nb2, cb, \{|tb|\}_{passwd(b)}|\}_{pks}$
Message 4. $s \rightarrow a \ : \ \{|na1, k \oplus na2|\}_{passwd(a)}$
Message 5. $s \rightarrow b \ : \ \{|nb1, k \oplus nb2|\}_{passwd(b)}$
Message 6. $b \rightarrow a \ : \ \{|rb|\}_k$
Message 7. $a \rightarrow b \ : \ \{|f(rb), ra|\}_k$
Message 8. $b \rightarrow a \ : \ \{|f(ra)|\}_k$.

Here pks is the public key of server s; $na1$, $na2$, $nb1$, $nb2$, ra, rb, ca and cb are all random numbers; ta and tb are timestamps; f is an arbitrary function (we model it as a hash function); and $\oplus$ represents Vernam encryption. a sends two nonces $na1$ and $na2$ to the server in message 1; the server uses these in message 4 to return a session key k to a: $na1$ gives a guarantee of freshness, and $na2$ encrypts the key to prevent the intruder from learning it by guessing the password; we examine the role of ca below. b carries out a similar exchange with s in messages 3 and 5. Finally, a and b carry out a nonce exchange in messages 6–8.

When we modelled this protocol and analysed it using Casper/FDR, we found the following attack.

$$
\begin{array}{lrcl}
\text{Message } \alpha.1. & A \rightarrow S & : & \{|A, I, Na1, Na2, Ca, \{|0|\}_{passwd(A)}|\}_{PKs} \\
\text{Message } \alpha.2. & S \rightarrow I & : & A, I \\
\text{Message } \alpha.3. & I \rightarrow S & : & \{|I, A, Ni, Ni, Ci, \{|0|\}_{passwd(I)}|\}_{PKs} \\
\text{Message } \alpha.4. & S \rightarrow A & : & \{|Na1, K1 \oplus Na2|\}_{passwd(A)} \\
\text{Message } \beta.1. & I_A \rightarrow S & : & \{|A, I, Na1, Na2, Ca, \{|0|\}_{passwd(A)}|\}_{PKs} \\
\text{Message } \beta.2. & S \rightarrow I & : & A, I \\
\text{Message } \beta.3. & I \rightarrow S & : & \{|I, A, Ni, Ni, Ci, \{|0|\}_{passwd(I)}|\}_{PKs} \\
\text{Message } \beta.4. & S \rightarrow I_A & : & \{|Na1, K2 \oplus Na2|\}_{passwd(A)} \ .
\end{array}
$$

The notation I_A on the left of an arrow represents the intruder I imitating A to send a message; the same notation on the right of an arrow represents the intruder intercepting a message intended for A. In the attack, the timestamp 0 is arbitrary. The intruder replays A's messages from the (partial) run α in run β, so as to get the server to issue another message 4 using the same nonces. The intruder guesses the value $passwd(A)$, and verifies the guess using verifier $Na1$; the relevant events of the verification are:

$guess.passwd(A)$
$infer.(Na1,\ Dec.1,\ \{\{|Na1, K1 \oplus Na2|\}_{passwd(A)}, passwd(A)\})$
$infer.(Na1,\ Dec.1,\ \{\{|Na1, K2 \oplus Na2|\}_{passwd(A)}, passwd(A)\})$
$verify.(Na1,\ passwd(A))$

The intruder obtains $Na1$ by decrypting both $\{|Na1, K1 \oplus Na2|\}_{passwd(A)}$ and $\{|Na1, K2 \oplus Na2|\}_{passwd(A)}$.

There are a couple of variations on this attack: the nonce $Na2$ could be used as the verifier; or the intruder could replay the responder's messages from a normal run so as to obtain two instances of message 5 using the same nonces, guess the responder's password, and verify it using either of the responder's nonces.

It turns out that this attack is not new. It is noted in a later paper by the same authors, [GLNS93], which also notes that it can be prevented by the server detecting replays of messages.

The latter paper introduced another protocol, replacing the timestamps with a nonce generated by the server. We have analysed this protocol using Casper and FDR, and found no attacks.

It is worth asking what is the point of the fields *ca* and *cb* in messages 1 and 3. If we change the model to remove these fields, we find the following attack.

Message 1. $A \rightarrow S \ : \ \{|A, I, Na1, Na2, \{|0|\}_{passwd(A)}|\}_{PKs}$
Message 2. $S \rightarrow A \ : \ A, I$
Message 3. $I \rightarrow S \ : \ \{|I, A, Ni, Ni, \{|0|\}_{passwd(I)}|\}_{PKs}$
Message 4. $S \rightarrow A \ : \ \{|Na1, K \oplus Na2|\}_{passwd(A)}$
Message 5. $S \rightarrow I \ : \ \{|Ni, K \oplus Ni|\}_{passwd(I)}$.

The intruder runs the protocol, using his own identity with some initiator A. He then guesses A's password, and verifies it as follows:

$infer.(K \oplus Ni,\ Dec.2,\ \{\{|Ni, K \oplus Ni|\}_{passwd(I)}, passwd(I)\})$
$infer.(K,\ VernDec,\ \{K \oplus Ni, Ni\})$
$guess.passwd(A)$
$infer.(\{|0|\}_{passwd(A)},\ Enc,\ \{0, passwd(A)\})$
$infer.(K \oplus Na2,\ Dec.2,\ \{\{|Na1, K \oplus Na2|\}_{passwd(A)}, passwd(A)\})$
$infer.(Na1,\ Dec.1,\ \{\{|Na1, K \oplus Na2|\}_{passwd(A)}, passwd(A)\})$
$infer.(Na2,\ VernDec,\ \{K \oplus Na2, K\})$
$infer.(\{|A, I, Na1, Na2, \{|0|\}_{passwd(A)}|\}_{PKs},$
$\quad Enc,\ \{A, I, Na1, Na2, \{|0|\}_{passwd(A)}, PKs\})$
$verify.(\{|A, I, Na1, Na2, \{|0|\}_{passwd(A)}|\}_{PKs},\ passwd(A)).$

The intruder obtains the session key in the normal way; he obtains A's nonces from message 4; he then uses the knowledge he has accumulated to create a message of the form of message 1, compares it with the ciphertext he observed for the actual message 1, and so verifies the guess.

This attack depends upon the intruder being able to estimate—or guess—the timestamp A uses in message 1; if the intruder cannot do this, then the attack is prevented. In the original protocol, the field *ca* has the same effect: it prevents the intruder from building the message 1.

We can find a very similar attack against the responder of the protocol, exploiting the absence of *cb*.

5 Conclusions

In this paper we have considered guessing attacks upon security protocols, produced a formal model explaining how an intruder can verify a guess,

shown how to use CSP and FDR to discover such attacks, and illustrated our techniques on a number of examples.

In this final section we consider some related work and a few possible extensions.

5.1 Related work

In [GLNS93], Gong et al. give a definition of guessing attacks. However, this seems less general than our definition. For a start, they assume that the set of facts known to the intruder (IK in our definition) is given to the analyser, whereas we allow IK to range over all obtainable knowledge sets. Further, they only consider the case where the verifying fact is initially known to the intruder, corresponding to the case $v \in IK$ in our formalisation: thus

- they do not consider the case that v is produced in two different ways, corresponding to the first disjunct of our condition (4);
- they do not consider the case where the verifying fact is the same as the guessed fact, corresponding to part of the second disjunct of our condition (4); this possibility is needed for Example 2.

(They have an extra deduction, producing some known value from a key and its inverse, so they do include the case captured by the third disjunct of our condition (4).)

In [Pat97], Patel presents active, number theoretic attacks against particular implementations of EKE. The intruder performs an on-line attack, and exploits, for example, properties of the RSA cryptosystem, so as to allow a fixed proportion of possible passwords to be eliminated; thus the number of possible passwords can be reduced to one after $O(\log W)$ interactions, where W is the original number of possible passwords.

5.2 Extensions

One extension to the work in this paper concerns protocols that contain fields that can be used for verification, but are unguessable. For example, suppose we have a protocol that contains a long piece of English text encrypted with the guessable value. Then a correct guess will reveal the text, verifying the guess; but it seems unreasonable to suppose that the intruder can guess what the text is in advance, because the space of such texts is too large.

Within our framework of Section 2, we could model this scenario by assuming that such fields are not initially known to the intruder, but allow them to be used for verification by adding a disjunct $v \in V$ to condition (4), where V is the set of all such verifying values. We could model them within the CSP model of Section 3 in essentially the same way, adding a disjunct to the test for verification within the $INTRUDER'$ process.

In this paper, we have assumed that encryption is deterministic: encrypting the same message with the same key always produces the same ciphertext. This is important in examples such as the following: suppose the protocol directly discloses $\{|g|\}_{pk}$ to the intruder, where g is guessable, and pk a public key; then the intruder could guess g, encrypt it with pk, and compare this with the observed ciphertext so as to verify the guess.

However, many encryption schemes (particularly public-key encryption schemes) are probabilistic, generating a random value r, and combining this with the plaintext in some way before encrypting (see [MvOV96, Section 8.7] for more details). This would prevent the above attack, because the intruder will presumably not be able to guess r, so will not be able to perform the same encryption.

It is straightforward to adapt the work of this paper to deal with probabilistic encryption: we simply explicitly include the random number, writing, for example, $\{|m|\}_k^r$ for an encryption using random number r; we then adapt the definition of the derivation relation appropriately:

$$\{f, k, r\} \vdash_{\mathsf{enc}} \{|f|\}_k^r, \qquad \{\{|f|\}_k^r, k^{-1}\} \vdash_{\mathsf{dec}} f, \qquad \{\{|f|\}_k^r, k^{-1}\} \vdash_{\mathsf{dec}'} r.$$

(The final clause might not be present, depending on the details of the encryption scheme.)

One interesting question is whether we have modelled all ways in which the intruder can verify his guess. Our formalism captures all guessing attacks that we have considered, or that we have found in the literature, but it is hard to be sure that there are no others.

The standard CSP model is such that honest agents will decrypt a message only if it is encrypted with the expected key; however, we have been assuming that encrypted messages do not contain redundancy to allow the agent to verify this fact. It would therefore be interesting to relax this restriction. (One of the attacks in [Pat97] exploits such an incorrect decryption.)

This paper developed a general definition of guessing attacks, and specialised it to one particular protocol analysis technique. It would be inter-

esting to specialise it to other protocol analysis techniques such as strand spaces [THG99], rank functions [Sch97, RSG+00], theorem provers [Pau98, Coh00], or other model checkers, e.g. [MCJ97, MMS97]. There are two aspects of the search for a guessing attack: the search for a suitable set of knowledge for the intruder, obtained by interacting with the protocol; and the search for a guess that is verifiable using that knowledge. Model checkers can combine these two aspects into a single automated search. It remains to be seen whether other approaches can combine them so easily.

Acknowledgments

I would like to thank Philippa Broadfoot, Ernie Cohen and the anonymous referees for useful comments about this paper.

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