Certification of Large Distributed Computations with Task Dependencies in Hostile Environments

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Presentation Outline

- Motivation: Application and Threat
- Execution Model
- Certification with independent tasks
- Certification with task dependencies
- Results
- Conclusions and Future Work

Target Application

- Large-Scale Global Computing Systems
- Subject Application to Dependability Problems
 - Can be addressed in the design
- Subject Application to Security Problems
 - Requires solutions from the area of survivability, security, fault-tolerance

Global Computing Architecture

- Large-scale distributed systems (e.g. Grid, P2P)
- Transparent allocation of resources





Unbounded Environments

 In the Survivability Community our general computing environment is referred to as

Unbounded Environment

- Lack of physical / logical bound
- Lack of global administrative view of the system.

What risks are we subjecting our applications to?

Typical Application

- Computation intensive parallel application
 - Medical (mammography comparison)



Two kinds of failures (1/2)

1. Node failures





Unreliability in the absence of Fault Tolerance Mechanism

• Computation on Cluster

- MTBF = 2000 days (48,000h, approx. 5 1/2 years)
- Unreliability of one node: $F(t) = 1 R(t) = 1 e^{-\lambda t}$



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Fault Tolerance Approaches

• Simplified Taxonomy for Fault Tolerance Protocols



- Rely on a "stable storage"
 - persistent and assumed to be reliable [Kaapi / Athapascan]
 - If not persistent: only duplication of saved data (checkpoint / message)
 - » probabilistic FT protocols: fault tolerance is guaranteed with good probability

Two kinds of failures (2/2)

2. Task forgery



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How bad is the Problem?

Vulnerabilities reported (CERT/CC statistics)

1995-1999

Year	1995	1996	1997	1998	1999
Vulnerabilities	171	345	311	262	417

2000-2004

Year	2000	2001	2002	2003	2004
Vulnerabilities	1,090	2,437	4,129	3,784	3,780

Total vulnerabilities reported (1995-2004): 16,726

How bad is the Problem?

Incidents reported (CERT/CC statistics)

1988-1989

Year	1988	1989
Incidents	6	132

1990-1999

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
Incidents	252	406	773	1,334	2,340	2,412	2,573	2,134	3,734	9,859

2000-2003

Year	2000	2001	2002	2003
Incidents	21,756	52,658	82,094	137,529

Total incidents reported (1988-2003): 319,992

Fault Models

• Simplified Fault Taxonomy



- Fault-Behavior and Assumptions
 - Independence of faults
 - Common mode faults -> towards arbitrary faults!
- Fault Sources
 - Trojan, virus, DOS, etc.
 - How do faults affect the overall system?

Assumptions





- Malicious act will occur sooner or later
- It is hard or impossible to predict the behavior of an attack

Attacks and their impact

- Attacks
 - single nodes, difficult to solve with certification strategies
 - solutions: e.g. intrusion detection systems (IDS)
- Massive Attacks
 - affects large number of nodes
 - may spread fast (worm, virus)
 - may be coordinated (Trojan)
- Impact of Attacks
 - attacks are likely to be widespread within neighborhood, e.g. subnet
- Our focus: massive attacks
 - virus, trojan, DoS, etc.

Certification Against Attacks

- Mainly addressed for **independent tasks**
- Current approaches
 - Simple checker [Blum97]
 - Voting [SETI@home]
 - Spot-checking [Germain-Playez 2003, based on Wald test]
 - Blacklisting
 - Credibility-based fault-tolerance [Sarmenta 2003]
 - Partial execution on reliable resources (partitioning) [Gao-Malewicz 2004]
 - Re-execution on reliable resources
- Certification of Computation

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Definitions and Assumptions

Dataflow Graph

$$- \mathbf{G} = (\mathcal{V}, \mathcal{E})$$

 \mathcal{V} finite set of vertices v_i

 \mathcal{E} set of edges e_{jk} vertices $v_j, v_k \in \mathcal{V}$



- T_i Tasks in the traditional sense
- D_j Data tasks inputs and outputs



Global Computing Platform (GCP)

• GCP includes workers, checkpoint server and verifiers







- Executions in <u>unreliable</u> environment
 - E execution of workload represented by G
 - i(T,E) input to T in execution E
 - o(T,E) output of T in execution E
- Executions in <u>reliable</u> environment: Verifier
 - \hat{E} execution of workload G on Verifier
 - $\hat{\iota}(T, \hat{E})$ input to T in execution \hat{E}
 - $\hat{o}(T, \hat{E})$ output of T in execution \hat{E}
 - $\hat{o}(T,E)$ output of T with input from E executing on verifier

Note: notations $\hat{o}(T, \hat{E})$ and $\hat{o}(T, E)$ differ!

• If $E = \hat{E}$ then *E* is said to be "correct" otherwise *E* is said to have "failed"

Probabilistic Certification

Monte Carlo certification: (analogy to Miller-Rabin)

- a randomized algorithm that
 - 1. takes as input *E* and an arbitrary ε , $0 < \varepsilon \leq 1$
 - 2. delivers
 - either CORRECT
 - or FAILED, together with a proof that *E* has failed
- certification is with error ε if the probability of answer CORRECT, when *E* has actually failed, is less than or equal to ε .

Probabilistic Certification

- What does the certification really mean?
 - what is the real interpretation of $E = \hat{E}$
 - connection between $E = \hat{E}$ and massive attack
 - use $E \neq \hat{E}$ as a "tool" to determine if a massive attack has occurred
- Monte Carlo certification against massive attacks
 - number of tasks actually failed/attacked n_F
 - consider two scenarios
 - » $n_F = 0$
 - » n_F is large => massive attack

Attack Ratio q

$$n_q = \left\lceil nq \right\rceil \leq n_F$$

Monte Carlo Test

Algorithm MCT

- 1. Uniformly select one task *T* in *G* we know input i(T,E) and output o(T,E) of *T* from checkpoint server
- 2. Re-execute *T* on verifier, using i(T,E) as inputs, to get output $\hat{o}(T,E)$ If $o(T,E) \neq \hat{o}(T,E)$ return FAILED
- 3. Return CORRECT

Certification of Independent Tasks

• How many independent executions of MCT are necessary to achieve certification of *E* with probability of error $\leq \varepsilon$?

$$N \ge \left\lceil \frac{\log \varepsilon}{\log(1-q)} \right\rceil$$

- Prob. that MCT selects a non-forged tasks is

$$\frac{n-n_F}{n} \le 1-q$$

- N independent applications of MCT results in

$$\mathcal{E} \leq (1 - q)^N$$

Certification of Independent Tasks

Relationship between attack ratio and N



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Certification of Independent Tasks

• Relationship between certification error and N



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- What does a re-execution really tell us w.r.t. the result?
 - One can only talk about outputs of tasks, not tasks!
 - If $o(T,E) \neq \hat{o}(T,E)$ we know that an error has occurred
 - If $o(T,E) = \hat{o}(T,E)$ we cannot say much at all!
 - » for independent tasks this indicated a good task/result
 - » what do we know about the inputs?
 - in the presence of error propagation -- not much!
 - » if the verifier uses $\hat{i}(T, \hat{E})$ then $o(T, E) = \hat{o}(T, \hat{E})$ indicates a good result but we don't have \hat{E} , (would require total re-execution on verifier)

- The concept of "Initiator"

 - Definition:

An *initiator* is a falsified tasks that has no falsified predecessors

- Worst case assumption is very conservative
 - » one still might detect a falsified non-initiator
 - » but there is no guarantee

- Certification is now based on initiators
- Using Algorithm MCT we get

$$N \ge \left[\frac{\log \varepsilon}{\log(1 - \frac{n_I}{n})} \right]$$

- $G^{\leq}(V)$ predecessor graph of all tasks in V $k \leq n_F$ be the number of falsified tasks assumedI(F)set of all initiators
- Minimum Number of Initiators

$$\gamma_V(k) = \min |G^{\leq}(V) \cap I(F)|$$

Minimal Initiator Ratio

$$\Gamma_{V}(k) = \frac{\gamma_{V}(k)}{|G^{\leq}(V)|}$$

Extended Monte Carlo Test

- Algorithm EMCT
 - 1. Uniformly select one task T in G
 - 2. Re-execute all T_j in $G^{\leq}(T)$, which have not been verified yet, with input i(T,E) on a verifier and return FAILED if for any T_j we have $o(T_j,E) \neq \hat{o}(T_j,E)$
 - 3. Return CORRECT

Behavior

- disadvantage: the entire predecessor graph needs to be re-executed
- however: the cost depends on the graph
 - » luckily our application graphs are mainly trees

Analysis of EMCT

Results of independent tasks still hold,

- but *N* hides the cost of verification
 - » independent tasks: C = 1
 - » dependent tasks: $C = |G^{\leq}(T)|$



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Reducing the cost of verification

For EMCT the entire predecessor graph had to be verified To reduce verification cost two approaches are considered next:

- 1. Verification with fractions of $G^{\leq}(T)$
- 2. Verification with fixed number of tasks in $G^{\leq}(T)$

Verifying with fractions of $G^{\leq}(T)$

• Algorithm EMCT $\alpha(E)$

- 1. Uniformly choose one task T in G.
- 2. Uniformly select $n_{\alpha} = \lceil \alpha | G^{\leq}(T) | \rceil$ tasks in $G^{\leq}(T)$ and let this set be denoted by A. If for any $T_j \in A$, that has not been verified yet, re-execution on a verifier results in $\hat{o}(T_j, E) \neq o(T_j, E)$ then return FAILED.
- 3. Return CORRECT.

Verifying with fractions of $G^{\leq}(T)$

• For Algorithm EMCT $\alpha(E)$

Lemma 1 Let T be a task randomly chosen by $EMCT_{\alpha}(E)$. Then the probability of error, e_{α} , when $EMCT_{\alpha}(E)$ returns CORRECT is given by

 $e_{\alpha} \leq \begin{cases} (1 - q\alpha\Gamma_{T}(n_{q})) & \text{for} \quad 0 < \alpha \leq 1 - \Gamma_{T}(n_{q}) \\ (1 - q) & \text{otherwise.} \end{cases}$

Verifying with fractions of $G^{\leq}(T)$

• For Algorithm EMCT $\alpha(E)$

Theorem 1 Let E be an execution with dependencies that is either correct or massively attacked with ratio q. Given ϵ and $0 < \alpha \leq 1$, N independent invocations of Algorithm $EMCT_{\alpha}(E)$ provide a certification with error probability

$$\epsilon \leq \begin{cases} (1 - q\alpha \Gamma_G(n_q))^N & for \ 0 < \alpha \leq 1 - \Gamma_T(n_q) \\ (1 - q)^N & otherwise. \end{cases}$$

- We will now modify algorithm EMCT so that only a fixed number of tasks in the predecessors are verified.
 - We limit our investigations to unity, i.e. one task is verified.

• Algorithm $EMCT^1(E)$

- 1. Uniformly choose one task T in G.
- 2. Uniformly select a single T_j in $G^{\leq}(T)$. If reexecution of T_j on a verifier results in $\hat{o}(T_j, E) \neq o(T_j, E)$ then return FAILED.
- 3. Return CORRECT.

• For Algorithm $EMCT^1(E)$

Lemma 2 Let T be a task randomly chosen by $EMCT^{1}(E)$ and let $V = G^{\leq}(T)$. Then the probability of error, e_{1} , when $EMCT^{1}(E)$ returns CORRECT is given by

$$e_1 \le 1 - \frac{n_F}{n} \Gamma_T(n_F) \le 1 - q \Gamma_T(n_q)$$

• For Algorithm $EMCT^1(E)$

Theorem 2 Let E be an execution with dependencies that is either correct or massively attacked with ratio q. Given ϵ then N independent invocations of Algorithm $EMCT^{1}(E)$ provide a certification with error probability

 $\epsilon \le (1 - q\Gamma_G(n_q))^N.$

The cost of certification

• A balance between N and C

- Monte Carlo certification for a given ε:
 - 1. a priori convergence
 - determine up front how many times one has to verify
 - one does not know which tasks are selected
 - 2. run-time convergence
 - run until certain ε is achieved
 - take advantage of knowledge about task selected
 - 3. for general graphs
 - 4. for special graphs (e.g. out-trees)

Note: For independent tasks a priori and run-time convergence are the same.

Number of effective initiators

- this is the # of initiators as perceived by the algorithm
- e.g. for EMCT an initiator in $G^{\leq}(T)$ is <u>always</u> found, if it exists

	MCT(E) [7]	EMCT(E) [7]	$EMCT_{\alpha}(E)$	$EMCT^{1}(E)$
# of effective initiators	$\left\lceil \frac{n_q}{\left(\frac{1-d^h}{1-d}\right)} \right\rceil$	n_q	$n_q \alpha \Gamma_T(n_q) \text{ or } n_q$	$n_q \Gamma_T(n_q)$
Probability of error	$1 - \frac{\left\lceil \frac{n_q}{\left(\frac{1-d^h}{1-d}\right)} \right\rceil}{n}$	1 - q	$1 - q \alpha \Gamma_T(n_q) \text{ or } 1 - q$	$1 - q\Gamma_T(n_q)$
A priori convergence	$\frac{\log \epsilon}{\left\lceil \frac{n_q}{\left(\frac{1-d^h}{1-d}\right)} \right\rceil}$	$\frac{\log \epsilon}{\log(1-q)}$	$\frac{\log \epsilon}{\log(1-q\alpha\Gamma_G(n_q))}$ or $\frac{\log \epsilon}{\log(1-q)}$	$\frac{\log \epsilon}{\log(1 - q\Gamma_G(n_q))}$
q_e a priori	$\frac{\left\lceil \frac{n_q}{\left(\frac{1-d^h}{1-d}\right)} \right\rceil}{n}$	q	$q \alpha \Gamma_G(n_q)$ or q	$q\Gamma_G(n_q)$
q_e run-time	$\frac{\left\lceil \frac{n_q}{\left(\frac{1-d^h}{1-d}\right)} \right\rceil}{n}$	q	$q \alpha \Gamma_T(n_q)$ or q	$q\Gamma_T(n_q)$
Verification cost (exact)	1	$ G^{\leq}(T) $	$\left\lceil \alpha G^{\leq}(T) \right\rceil$	1
Max. cost (out-tree)	1	h	αh	1

Probability of error induced by one invocation

- derived for each algorithm

	MCT(E) [7]	EMCT(E) [7]	$EMCT_{\alpha}(E)$	$EMCT^{1}(E)$
# of effective initiators	$\left\lceil \frac{n_q}{\left(\frac{1-d^h}{1-d}\right)} \right\rceil$	n_q	$n_q \alpha \Gamma_T(n_q)$ or n_q	$n_q \Gamma_T(n_q)$
Probability of error	$1 - \frac{\left\lceil \frac{n_q}{\left(\frac{1-d^h}{1-d}\right)} \right\rceil}{n}$	1 - q	$1 - q \alpha \Gamma_T(n_q) \text{ or } 1 - q$	$1 - q\Gamma_T(n_q)$
A priori convergence	$\frac{\log \epsilon}{\left\lceil \frac{n_q}{\left(\frac{1-d^h}{1-d}\right)} \right\rceil}$	$\frac{\log \epsilon}{\log(1\!-\!q)}$	$\frac{\log \epsilon}{\log(1-q\alpha\Gamma_G(n_q))}$ or $\frac{\log \epsilon}{\log(1-q)}$	$\frac{\log \epsilon}{\log(1 - q\Gamma_G(n_q))}$
q_e a priori	$\frac{\left\lceil \frac{nq}{\left(\frac{1-dh}{1-d}\right)} \right\rceil}{n}$	q	$q lpha \Gamma_G(n_q)$ or q	$q\Gamma_G(n_q)$
q_e run-time	$\frac{\left\lceil \frac{n_q}{\left(\frac{1-d^h}{1-d}\right)} \right\rceil}{n}$	q	$q \alpha \Gamma_T(n_q)$ or q	$q\Gamma_T(n_q)$
Verification cost (exact)	1	$ G^{\leq}(T) $	$\left\lceil \alpha G^{\leq}(T) \right\rceil$	1
Max. cost (out-tree)	1	h	αh	1

• A priori convergence (*N* is determined a priori)

- cannot take advantage of run-time knowledge
- has to use $\Gamma_G(n_q)$ rather than $\Gamma_T(n_q)$
- q_e is the effective attack ratio

$$N \geq \left\lceil \frac{\log \varepsilon}{\log(1-q_e)} \right\rceil$$

	MCT(E) [7]	EMCT(E) [7]	$EMCT_{\alpha}(E)$	$EMCT^{1}(E)$
# of effective initiators	$\left\lceil \frac{n_q}{\left(\frac{1-d^h}{1-d}\right)} \right\rceil$	n_q	$n_q \alpha \Gamma_T(n_q)$ or n_q	$n_q \Gamma_T(n_q)$
Probability of error	$1 - \frac{\left\lceil \frac{n_q}{\left(\frac{1-d^h}{1-d}\right)} \right\rceil}{n}$	1-q	$1 - q \alpha \Gamma_T(n_q)$ or $1 - q$	$1 - q\Gamma_T(n_q)$
A priori convergence	$\frac{\log \epsilon}{\left\lceil \frac{n_q}{\left(\frac{1-d^h}{1-d}\right)} \right\rceil}$	$\frac{\log \epsilon}{\log(1\!-\!q)}$	$\frac{\log \epsilon}{\log(1-q\alpha\Gamma_G(n_q))}$ or $\frac{\log \epsilon}{\log(1-q)}$	$\frac{\log \epsilon}{\log(1 - q\Gamma_G(n_q))}$
q_e a priori	$\frac{\left\lceil \frac{n_q}{\left(\frac{1-dh}{1-d}\right)} \right\rceil}{n}$	q	$q lpha \Gamma_G(n_q)$ or q	$q\Gamma_G(n_q)$
q_e run-time	$\frac{\left\lceil \frac{n_q}{\left(\frac{1-d^h}{1-d}\right)} \right\rceil}{n}$	q	$q \alpha \Gamma_T(n_q)$ or q	$q\Gamma_T(n_q)$
Verification cost (exact)	1	$ G^{\leq}(T) $	$\left\lceil \alpha G^{\leq}(T) \right\rceil$	1
Max. cost (out-tree)	1	h	αh	1

- Run-time convergence (*N* is determined at run-time)
 - takes advantage of run-time knowledge
 - initial verification $\varepsilon_e = 1 q_e$
 - each verification $\varepsilon_e = \varepsilon_e (1 q_e)$
 - until

 $\varepsilon_e \le \varepsilon$

 $N \ge \left[\frac{\log \varepsilon}{\log(1-q_e)}\right]$

	MCT(E) [7]	EMCT(E) [7]	$EMCT_{\alpha}(E)$	$EMCT^{1}(E)$
# of effective initiators	$\left\lceil \frac{n_q}{\left(\frac{1-d^h}{1-d}\right)} \right\rceil$	n_q	$n_q \alpha \Gamma_T(n_q) \text{ or } n_q$	$n_q \Gamma_T(n_q)$
Probability of error	$1 - \frac{\left\lceil \frac{n_q}{\left(\frac{1-d^h}{1-d}\right)} \right\rceil}{n}$	1 - q	$1 - q \alpha \Gamma_T(n_q) \text{ or } 1 - q$	$1 - q\Gamma_T(n_q)$
A priori convergence	$\frac{\log \epsilon}{\left\lceil \frac{n_q}{\left(\frac{1-dh}{r}\right)} \right\rceil}$	$rac{\log \epsilon}{\log(1\!-\!q)}$	$\frac{\log \epsilon}{\log(1-q\alpha\Gamma_G(n_q))}$ or $\frac{\log \epsilon}{\log(1-q)}$	$\frac{\log \epsilon}{\log(1 - q\Gamma_G(n_q))}$
	$\log(1-\frac{1-d}{n})$			
q_e a priori	$\frac{\left\lceil \frac{n_q}{\left(\frac{1-d^h}{1-d}\right)} \right\rceil}{n}$	q	$q lpha \Gamma_G(n_q)$ or q	$q\Gamma_G(n_q)$
q_e run-time	$\frac{\left\lceil \frac{n_q}{\left(\frac{1-d^h}{1-d}\right)} \right\rceil}{n}$	q	$q lpha \Gamma_T(n_q)$ or q	$q\Gamma_T(n_q)$
Verification cost (exact)	1	$ G^{\leq}(T) $	$\lceil \alpha G^{\leq}(T) \rceil$	1
Max. cost (out-tree)	1	h	αh	1

- Verification cost
 - per invocation of the algorithm
 - special case: out-tree

	MCT(E) [7]	EMCT(E) [7]	$EMCT_{\alpha}(E)$	$EMCT^{1}(E)$
# of effective initiators	$\left\lceil \frac{n_q}{\left(\frac{1-d^h}{1-d}\right)} \right\rceil$	n_q	$n_q \alpha \Gamma_T(n_q)$ or n_q	$n_q \Gamma_T(n_q)$
Probability of error	$1 - \frac{\left\lceil \frac{n_q}{\left(\frac{1-d^h}{1-d}\right)} \right\rceil}{n}$	1 - q	$1 - q \alpha \Gamma_T(n_q) \text{ or } 1 - q$	$1 - q\Gamma_T(n_q)$
A priori convergence	$\frac{\log \epsilon}{\lceil \frac{n_q}{1-dh} \rceil}$	$\frac{\log \epsilon}{\log(1\!-\!q)}$	$\frac{\log \epsilon}{\log(1-q\alpha\Gamma_G(n_q))}$ or $\frac{\log \epsilon}{\log(1-q)}$	$\frac{\log \epsilon}{\log(1\!-\!q\Gamma_G(n_q))}$
	$\log(1 - \frac{\left(\frac{1-a}{1-d}\right)}{n})$			
q_e a priori	$\frac{\left\lceil \frac{n_q}{\left(\frac{1-d^h}{1-d}\right)} \right\rceil}{n}$	q	$q lpha \Gamma_G(n_q)$ or q	$q\Gamma_G(n_q)$
q_e run-time	$\frac{\left\lceil \frac{n_q}{\left(\frac{1-d^h}{1-d}\right)} \right\rceil}{n}$	q	$q lpha \Gamma_T(n_q)$ or q	$q\Gamma_T(n_q)$
Verification cost (exact)	1	$ G^{\leq}(T) $	$\lceil \alpha G^{\leq}(T) \rceil$	1
Max. cost (out-tree)	1	h	αh	1

Conclusions

- Certification of large distributed applications
 - hostile environments with no assumptions on fault model
- Considered task dependencies
 - tasks or data may be manipulated
 - allows for error propagation (much more difficult than independent case)
 - very difficult to speculate on the behavior of a falsified task
- Several probabilistic certification algorithms were introduced
 - based on re-execution on verifier (reliable resource)
 - inputs available from dataflow checkpoints
- Certification:
 - very low probability of error can be achieved
 - number of tasks to verify is relatively small, depending on graph
 - relationship between attack rate and probability of error



• The impact of graph G

- Knowing the graph, an attacker may attempt to minimize the visibility of even a massive attack with ration q.
- What is the number of initiators one might have to expect in a graph?
 - » In the worst case we have



Results for MCT and EMTC

Considered

- General graphs
- Out-trees (application domain based on out/in-trees)

Algorithm	MCT	EMCT
Number of effective initiators	$\left\lceil \frac{n_q}{\left(\frac{1-d^h}{1-d}\right)} \right\rceil$	n_q
Probability of error	$1 - \frac{\left\lceil \frac{n_q}{\left(\frac{1-dh}{1-d}\right)} \right\rceil}{n}$	1 - q
Verification cost: general G	1	O(n)
Verification cost: G is out-tree	1	$h - \log_d(n_v)$
Ave. # effective initiators, G is out-tree	$\left\lceil \frac{n_q}{\left(\frac{1-(h+2)d^{h+1}+(h+1)d^{h+2}}{(1-d)(1-d^{h+1})}\right)} \right\rceil$	n_q

Relationship between quantities

• Given a subset V of tasks in G.

What are the relationships between $\gamma_V(k)$, $\gamma_G(k)$ and n_I with respect to $k = n_q$ or $k = n_F$?

By definition $q \le n_F / n$ and thus $n_q \le n_F$ also

 $n_I \leq n_F$

Relationship between quantities

- With respect to n_F we always have $\gamma_V(n_F) \le \gamma_G(n_F) \le n_I \le n_F$
 - But where does n_q fit into this inequality?
 - The only certain relationship is $n_q \le n_F$
- With respect to n_q we always have $\gamma_V(n_q) \le \gamma_G(n_q) \le n_q \le n_F$
 - But where does n_I fit into this inequality?
 - The only certain relationship is $\gamma_G(n_q) \le n_I \le n_F$

Relationship between quantities

• With respect to $n_q \le n_F$ we can compare directly

$$\begin{split} \gamma_{\mathrm{V}}(n_q) &\leq \gamma_{\mathrm{V}}(n_F) \\ \gamma_{\mathrm{G}}(n_q) &\leq \gamma_{\mathrm{G}}(n_F) \end{split}$$

Thus

$$\Gamma_{\rm V}(n_q) \leq \Gamma_{\rm V}(n_F)$$

$$\Gamma_{\rm G}(n_q) \leq \Gamma_{\rm G}(n_F)$$