Solutions Exercise 2: CTL Modelchecking

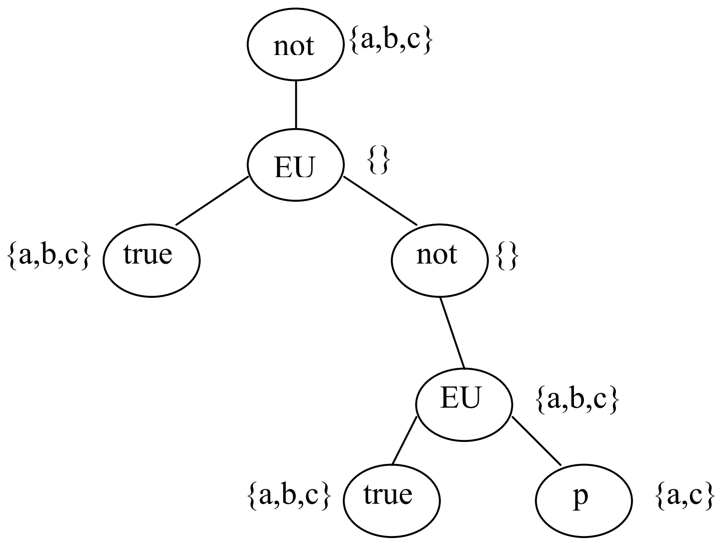
Note that I use '!' for 'not', '$\lor$' for 'or', and '$\land$' for 'and' for simplicity.

1)

(a) $AG(AF\ p \rightarrow AF\ q) = !EF\ !(AF\ p \rightarrow AF\ q) = !EF\ !(!AF\ p \lor AF\ q) =$
$= !EF(AF\ p \land !AF\ q) = !EF(!EG!p \land EG!q) = !E\ true\ U(!EG!p \land EG!q) =$

(b) $AF(p \lor AG\ q) = !EG!(p \lor AG\ q) = !EG(!p \land !AG\ q) = !EG(!p \land EF!q) =$
$= !(EG(!p \land E\ true\ U\ !q))$

2) $!(E\ true\ U\ (!\ E\ true\ U\ p))$



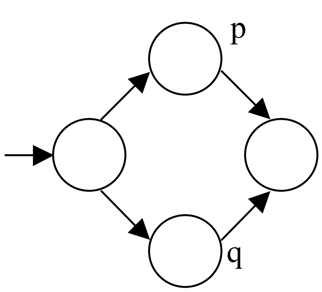
3)

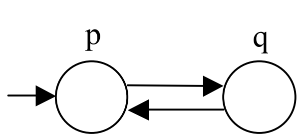
(a) Proof: $EF(p \lor q) = EF\ p \lor EF\ q$ (and $AG\ p \land q = AG\ p \land AG\ q$)

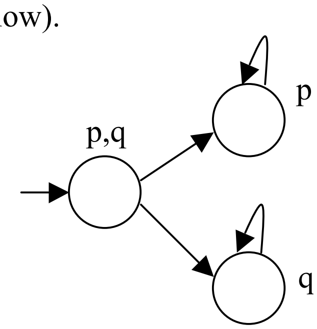
Forward:

Assume $K \models EF(p \lor q)$, then $\exists$ a path $s0,s1,\ldots$ such that $\exists\ i>=0$ with $si \models p \lor q$. It follows that either $si \models p$ or $si \models q$ (or both). So $\exists$ a path $t0, t1,\ldots$ such that $ti \models p$ or $\exists$ a path $t0, t1,\ldots$ such that $ti \models q$ and it follows that $K \models EF(p) \lor EF(q)$

(b), (c), and (d) are not true (see the counterexamples below).







$K \models EF\ p$,	$K \models EG\ p \lor q$	$K \models EG\ p$
$K \models EF\ q$ but	$K \models$ not $EG\ p$	$K \models EG\ q$
$K \models$ not $EF\ p \land q$	$K \models$ not $EG\ q$	$K \models$ not $EG\ p \land q$

4) Proof: $EG\ p = \nu Z.\ p \wedge EX\ Z$

EG p is a fix-point because
$EG\ p = p \wedge EX\ EG\ p$ is true.

EG p is the greatest fix-point, if forall Z s.t. $Z = p \wedge EX\ Z$, we have that $Z <= EG\ p$
From $Z = p \wedge EX\ Z$ is follows that $Z <= p$ and $Z <= EX\ Z$.
So for all s \in Z, $s \models p$ and s has a successor in Z, so s has an infinite path along which p holds, and so $s \models EG\ p$ (and therefore s \in EG p and $Z <= EG\ p$, since s was chosen arbitrarily).

Furthermore, from the exists of a greatest fix-point and since $Z <= EG\ p$ for any arbitrary fix-point, it follows that EG p is the greatest fix-point.

5) To proof EG p = EG EG p.
Note that from fix-point theory, we know that if f is a monotonic function over a finite lattice L and x is an element of L, then $x >= \nu Z.f \Rightarrow \nu Z.f = \nu Z.x \wedge f$ holds. (1)

We note that $EG\ p <= p$, and so $EG\ p = p \wedge EG\ p$.
Furthermore, due to the previous exercise and (1) we know that
$EG\ EGp = \nu Z.\ (EGp) \wedge EX\ Z = \nu Z.\ P \wedge (EGp) \wedge EX\ Z = \nu Z.\ P \wedge EX\ Z = EG\ p$.

6) E p U q
$Z0=\{\}$
$Z1=\{c\} \vee \{b,d\} \wedge EX\{\} = \{c\}$
$Z2=\{c\} \vee \{b,d\} \wedge EX\{c\} = \{c\} \vee \{b,d\} \wedge \{b\} = \{b,c\}$
$Z3=\{c\} \vee \{b,d\} \wedge EX\{b,c\} =\{c\} \vee \{b,d\} \wedge \{a,b\} = \{b,c\}$ fix-point

Bonus)
(a) Zi = S trivial
(b) $Zi = p \vee q$, all states not in $p \vee q$ cannot be in E p U q since either p or q have to be satisfied along a path satisfying E p U q
(c) $Zi >= p$, we know that $Zi <= p \vee q$ and since $Z1=\{q\}$ it follows that if $Zi>= p$, then $Zi = p \vee q$.