Introduction

Course HECS3: Performance and quantitative properties

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September 28, 2016

High-confidence Embedded and Cyber-Physical Systems
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Overview

Introduction

- Embedded and Cyber-Physical Systems
- Key Features of CPS
- Fundamentals of Dynamical Systems
- Specifying and Analyzing Properties
- Model-Based Design

What to Expect in this Lecture

concepts from Embedded and Cyber-Physical Systems

- standard terminology (and some buzzwords)
- informal presentation (formalization in future lectures)
- a rough map of the territory
- what it is all for...

Embedded and Cyber-Physical Systems

original computer: standalone device

embedded system: integrated with non-computational hardware for a specific purpose

- watches, cameras, refrigerators (integrated microcontroller), …

more examples?

Embedded and Cyber-Physical Systems

cyber-physical system[1]: collection of communicating computers, interacting with the physical world via feedback

- using control, computing, communication
- smart buildings, medical devices, cars, …

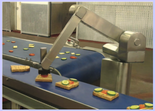
example: team of autonomous robots retrieving target inside house

more examples?

[1] term coined by Helen Gill at the US National Science Foundation (NSF) in 2006

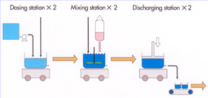
CPS at Verimag's Hybrid Systems Group

assisted and
automated driving

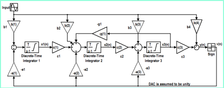


human-robot
interaction

smart buildings



chemical
batch plants



analog mixed-signal
circuits

autonomous drones

FP7 + H2020 projects **NANO 2017 project** **Collaborations**

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Key Features of CPS [Alur'15]

reactiveness

- traditionally: input → computing → output → stop
- mathematically: function: inputs → outputs
- reactive: ongoing computation
- mathematically: function from sequence of inputs to sequence of outputs

examples?

Key Features of CPS

concurrency

- traditionally: sequential computation (Turing machine)
- concurrent: multiple threads of computation, exchanging information
- synchronous computation: all components execute in lock-step
- asynchronous computation: components act independently, communicating via messages
- both can be useful levels of abstraction

examples?

Key Features of CPS

feedback control

- control system interacts with physical world with sensors and actuators
- design requires modeling the dynamics of physical quantities
- traditionally: continuous dynamics
  – a small enough change in the input generates a small change in the output

examples?

Key Features of CPS

real-time

- traditionally: no explicit notion of real time
- CPS: computation needs to finish within a given time frame
- timing delays, timing-dependent coordination protocols, resources allocation $\rightarrow$ study of real-time systems

examples?

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Fundamentals of Dynamical Systems

goal: a unified view of seemingly disparate systems

- using the same concepts
- adapting techniques where necessary
- combining different techniques when systems have heterogeneous components

… which they do in cyber-physical systems!

examples?

Fundamentals of Dynamical Systems

dynamical system

- precisely identified entity
  (we know what is part of the system and what isn't)
- defining behaviors over some notion of time
  (we know what "before" and "after" mean)
- with (possibly) observable outputs
- (possibly) influenced by a given set of inputs

examples for what is *not* a dynamical system?

Fundamentals of Dynamical Systems

behavior

- evolution of states over time
- (possibly) decorated with input or output

formalized as *executions, runs, words, traces, trajectories*,...

examples?

Fundamentals of Dynamical Systems

disturbance

- something that modifies the inputs or outputs of the system

random changes in the environment, electromagnetic interference, sensor noise, quantization error(!)

more examples?

Fundamentals of Dynamical Systems

deterministic system

- if the inputs are known, there is only one future behavior

nondeterministic system

- if the inputs are known, there is a known set of future behaviors
  (actual behavior may be different each time we run the system, but belong to the same set)

examples?

stochastic system

- if the inputs are known, the future behavior is known with a certain probability
  (it's the same behavior xyz% of the times we run the system)

examples?

state

- set of values that suffices to predict the (sets of) future behavior of the system if the inputs are known

state-space

- the set of states of the system

example: motion of a car (with accelerator and brake pedals)

Fundamentals of Dynamical Systems

transition

- relates a state to a successor state
- may depend on time and inputs

transition relation

- defines for each state the possible **successor** states
- a subset of states $\times$ time $\times$ inputs $\times$ states

$$\text{state} \xrightarrow{\text{time,input}} \text{state}'$$

Fundamentals of Dynamical Systems

reachable states

- states in the closure of the transition relation
- starting from a given set of initial states

finite state system

- the state space and the inputs are finite sets

What is maximum size of the transition relation (deterministic/nondeterministic)?

state-space exploration (enumerative)

- starting from a given initial state, visit all reachable states, trying out all possible inputs
- = graph traversal, e.g., breadth-first search

always **terminates** if the state space is **finite**

example: check if the system can go to a given "bad" state infinite state system

- the state space is an infinite set (enumerable or not)

state-space exploration no longer terminates

symbolic state-space exploration

- like state-space exploration, but using sets of states
- terminate if successors $\subseteq$ visited states or bad states overlap visited states
- often uses **overapproximation** to operate on sets with simple descriptions (intervals…)

may terminate even if state space is **infinite**

Fundamentals of Dynamical Systems

Example: A program computing $\sqrt{x_0}$ using the babylonian method

$$x_{k+1} = \tfrac{1}{2}\left(x_k + \frac{x_0}{x_k}\right).$$

implemented using int,float,rationals,reals,...

- state-space? initial state? inputs? outputs? time?
- transition relation? behaviors?
- deterministic? finite?

Exercises:

Given an implementation using int,float,rationals,reals,…

1. When is enumerative state-space exploration applicable?
2. What are the "bad states" for checking if the sequence converges to $\sqrt{x_0}$?
3. Apply symbolic state-space exploration starting from $x_0 = 8$. Use integer intervals to describe sets of states. Overapproximate if necessary.
4. Start from $x_0 = 9$. How can the precision be increased?
5. Does always rounding up or always rounding down cover all possibilities?

Fundamentals of Dynamical Systems

Discrete-Time Dynamical System:

$$x_{k+1} = f(x_k, u_k) \, .$$

- state-space? initial state? inputs? outputs? time?
- why "discrete-time"?
- transition relation?
- deterministic? finite?

Examples: Finite state machine (digital computer)

Fundamentals of Dynamical Systems

Discrete-Time Continuous Dynamical System:

$$x_{k+1} = f\left(x_k, u_k\right).$$

- f is a continuous function of x and u:
- a small enough change in the input (or in time) generates an arbitrarily small change in the output

Examples: Digital controller (considering floating point as real numbers); sun position at noon every day

Fundamentals of Dynamical Systems

Discrete-Time Continuous Dynamical System:

$$x_{k+1} = f(x_k, u_k)\,.$$

two main categories:

- f is linear: $x_{k+1} = Ax_k + Bu_k$
  either converging, diverging, or periodic
- f is nonlinear:
  possibly chaotic behavior

Linear Map

scalar case:

$$x_{k+1} = ax_k$$

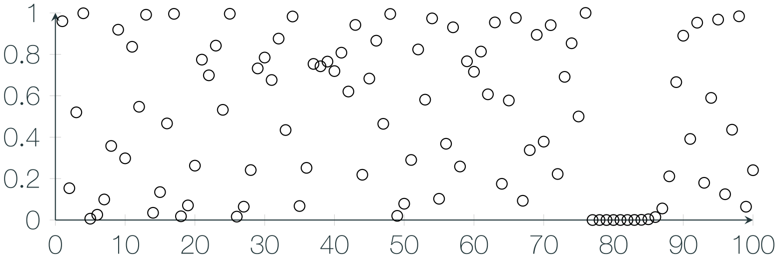
for which values of a:

- converging,
- diverging,
- periodic?

Logistic Map

demographic model with reproduction and starvation
[R. May, 1976]

$$x_{k+1} = r x_k (1 - x_k)$$



$x_0 = 0.6$, $r = 4$

Fundamentals of Dynamical Systems

Discrete-Time **Piecewise Continuous** Dynamical System:

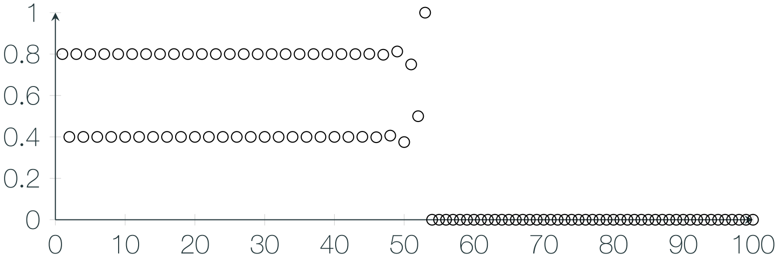
$$x_{k+1} = \begin{cases} f_1\left(x_k, u_k\right), & x_k \leq c_1 \\ \vdots & \\ f_i\left(x_k, u_k\right), & c_{i-1} < x_k \leq c_i \\ \vdots & \\ f_m\left(x_k, u_k\right), & x_k > c_m \end{cases}$$

- may exhibit complex behavior even for simple f_i

Example: continuous systems with saturation of signals

Tent Map

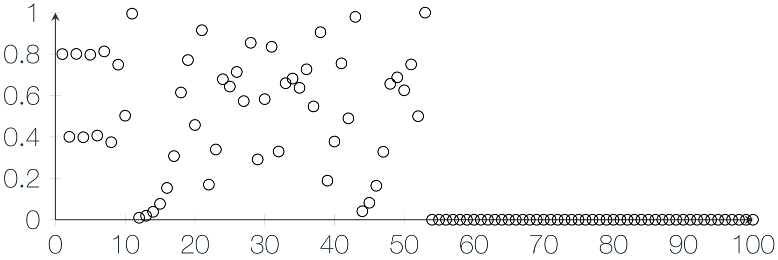
$$x_{k+1} = \begin{cases} \mu x_k, & x_k < \frac{1}{2}, \\ \mu(1 - x_k), & x_k \geq \frac{1}{2} \end{cases}$$



$x_0 = 0.6$, $\mu = 2$

Tent Map

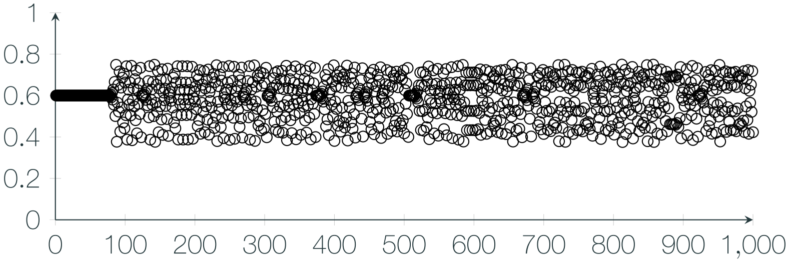
$$x_{k+1} = \begin{cases} \mu x_k, & x_k < \frac{1}{2}, \\ \mu(1 - x_k), & x_k \geq \frac{1}{2} \end{cases}$$



$x_0 = 0.6001$, $\mu = 2$

Tent Map

$$x_{k+1} = \begin{cases} \mu x_k, & x_k < \frac{1}{2}, \\ \mu(1 - x_k), & x_k \geq \frac{1}{2} \end{cases}$$



$x_0 = 0.6$, $\mu = 1.5$

Tent Map vs Logistic Map

tent map:

$$x_{k+1} = \begin{cases} \mu x_k, & x_k < \frac{1}{2}, \\ \mu(1 - x_k), & x_k \geq \frac{1}{2} \end{cases}$$

logistic map:

$$y_{k+1} = r y_k (1 - y_k)$$

for $\mu = 2$ and $r = 4$:

$$x_k = \frac{2}{\pi} \sin^{-1} \sqrt{y_k}$$

relation between *piecewise linear* and *nonlinear* system

Fundamentals of Dynamical Systems

Continuous-Time (Continuous) Dynamical System:

Typically given by a differential equation system:

$$\dot{x}(t) = f(x(t), u(t)) \, .$$

- can be converted to discrete-time system by sampling at time points, e.g., $t = k\delta$

Example: Motion of a car

Fundamentals of Dynamical Systems

Discrete-Continuous Dynamical System (Hybrid System):

- mix of discrete and continuous dynamics
- discrete state changes are considered instantaneous
- discrete state determines continuous dynamics

Example: Motion of a car with gear shift

Levels of Abstraction

discrete (state) system (discrete dynamics)

continuous system (continuous dynamics)

- discrete or continuous time

discrete-continuous (hybrid) system

What is the ”right” model?

Example: Robot in a Maze

- robot turning at exactly 90 degrees, timing irrelevant:
  discrete system
  Can the robot leave the maze?
- maze door opens and closes at specific times: timed system
  Can the robot leave the maze while the door is open?
- robot not turning exactly 90 degrees:
  hybrid (discrete-continuous) system
  accumulation of deviations!
  Can the robot leave the maze while the door is open?

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Specifying and Analyzing Properties

boolean properties

- state property: state $\rightarrow \{\text{true}, \text{false}\}$
  e.g., predicate over the state variables
- behavior property: behavior $\rightarrow \{\text{true}, \text{false}\}$
  e.g., all states along the behavior satisfy the property
- system property: system $\rightarrow \{\text{true}, \text{false}\}$
  e.g., all behaviors from initial states satisfy property

these generalizations to behaviors over time are called
temporal logics

examples?

Specifying and Analyzing Properties

probabilistic properties

- behavior probability: behavior $\rightarrow [0, 1]$
  e.g., probability that the behavior is taken (from given initial state)
- probabilistic property: system $\rightarrow [0, 1]$
  e.g., probability that any behavior from the initial states satisfies the property

these generalizations to behaviors over time are called **probabilistic temporal logics**

examples?

Specifying and Analyzing Properties

safety: nothing bad ever happens

analysis techniques:

- inductive invariants,
- state-space exploration (enumerative, symbolic)

examples?

Specifying and Analyzing Properties

liveness: something good eventually happens

analysis techniques:

- temporal logics,
- model checking (generalization of state-space exploration),
- ranking functions

examples?

Specifying and Analyzing Properties

probabilistic safety and liveness: nothing bad/something good happens with a certain probability

analysis techniques:

- probabilistic temporal logics,
- model checking,
- fault-tree analysis

examples?

Specifying and Analyzing Properties

quantitative semantics of safety and liveness: distance to nothing bad/something good happening

- e.g., min distance of any behavior to violating the property

Example: $x(t) \leq c$ for all $t \geq 0$ (boolean safety)

- quantitative semantics $q = \min_{t \geq 0} c - x(t)$
  property satisfied iff $q \geq 0$.

measure of robustness[2]

[2] A. Donzé, T. Ferrere, and O. Maler, "Efficient robust monitoring for STL," , in *Computer Aided Verification*, Springer, 2013, pp. 264–279.

Specifying and Analyzing Properties

real-time scheduling: system achieves given tasks in given time frame

analysis techniques:

- model checking,
- worst-case execution time (WCET) analysis

quantitative property:

- computing worst-case execution time

examples?

Specifying and Analyzing Properties

stability: system will remain close to its steady state if disturbances (inputs) small enough

analysis techniques:

- linear algebra,
- Lyapunov functions (continuous ranking functions)

quantitative property:

- *stability (gain) margin*: amount that feedback can be increased while remaining stable

examples?

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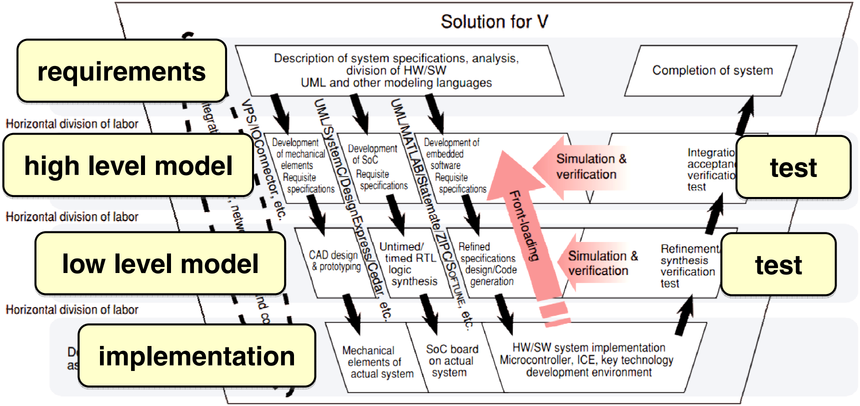
Model-Based Design

- traditionally: design, implementation, testing, validation
- model-based: formal (mathematically precise) requirements, models of the system and its environment, analysis tools for checking requirements on the model
- detect design errors earlier, ensure higher reliability

Model-Based Design

- different from programming: may incorporate nondeterminism and environment behavior
- structured design: complex tasks accomplished by composing simple components (and conversely for properties)
- requirements-based design: requirements are specified up front and guide the design (choice between design alternatives) and debugging

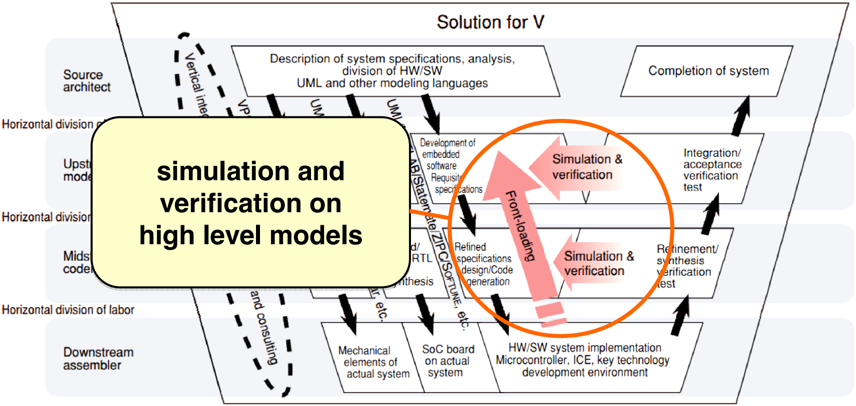
Model-Based Design



Development Vision for Systems Mixing Software, Circuits and Mechanics (Fujitsu 2006)

http://www.fujitsu.com/downloads/EDG/binary/pdf/find/24-1e/2.pdf

Model-Based Design



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Formal Models

- unambiguous – not open to interpretation
- mathematically precise, can be analyzed rigorously
- block diagrams, code, state machines, differential equations

Formal Verification

check if a formal model satisfies a property using mathematical reasoning

- rigorous (sound)
- exhaustive (all behavior is covered)
- (possibly) algorithmic or with computer support (theorem prover)

Formal Verification

drawbacks:

- generally: not scalable (or not even decidable)
- a suitable model needs to be constructed first
- typically requires expert knowledge

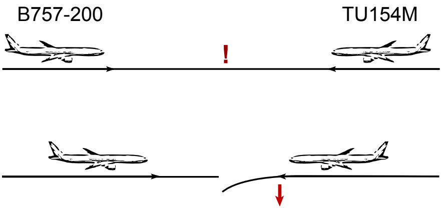
hard questions:

- Does the model match reality?
- Who verifies the verifier?

Boeing & Tupolew Collision

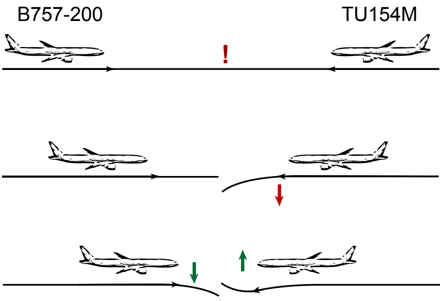
- **Überlingen, July 1, 2002**
- **21:33:03**
  - Alarm from Traffic Collision Avoidance System (TCAS)

Boeing & Tupolew Collision



- **Überlingen, July 1, 2002**
- **21:33:03**
  - Alarm from Traffic Collision Avoidance System (TCAS)
- **21:34:49**
  - Human air traffic controller command

Boeing & Tupolew Collision



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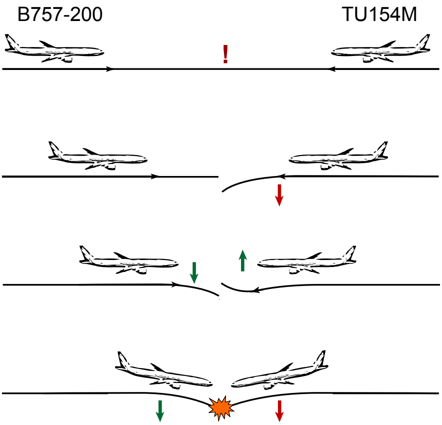
- **21:33:03**
  - Alarm from Traffic Collision Avoidance System (TCAS)
- **21:34:49**
  - Human air traffic controller command
- **21:34:56**
  - TCAS recommendation

Boeing & Tupolew Collision



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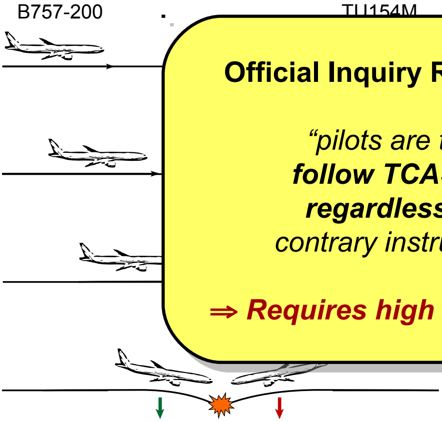
- **21:34:49**
  - Human air traffic controller command
- **21:34:56**
  - TCAS recommendation
- **21:35:32**
  - Collision

Boeing & Tupolew Collision



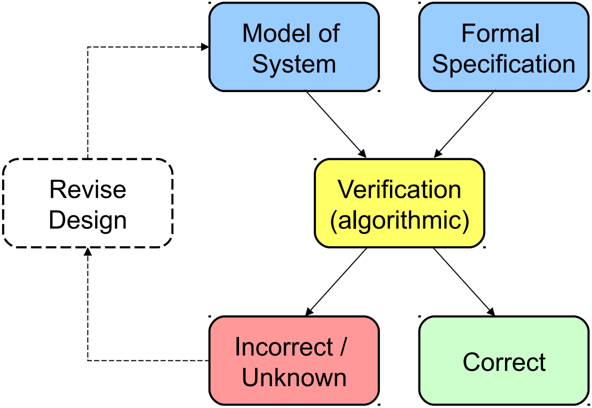
- **Überlingen, July 1, 2002**
  - …c Collision
  - …em (TCAS)
  - … controller
  - …ndation
- **21:35:32**
  - Collision

Official Inquiry Recommendation:

“pilots are to obey and ***follow TCAS*** *advisories,* ***regardless*** *of whether contrary instruction is given”*

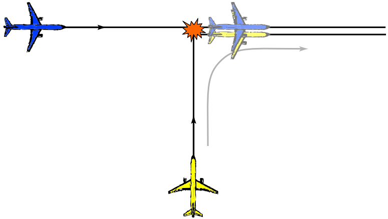
⇒ Requires high confidence design

Formal Verification



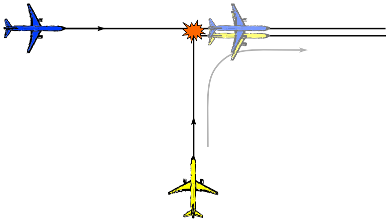
TCAS verified in part
[Livadas, Lygeros, Lynch, '00]

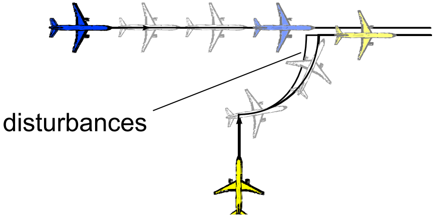
Join Manoeuvre [Tomlin et al.]



- **Traffic Coordination Problem**
  - join paths at different speed
- **Goals**
  - avoid collision
  - join with sufficient separation

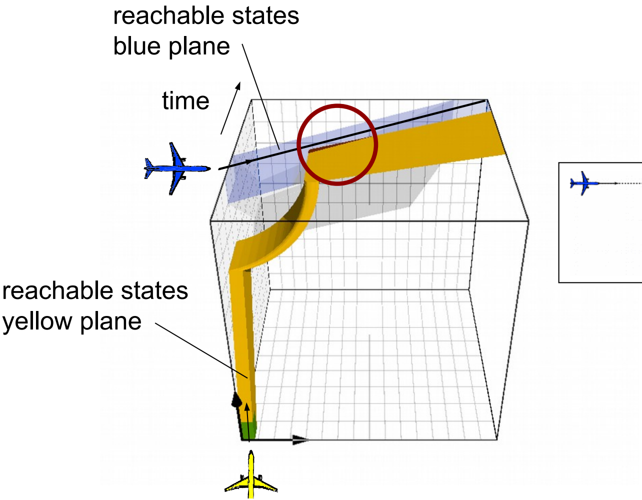
Join Manoeuvre

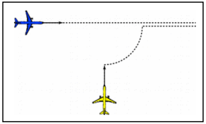




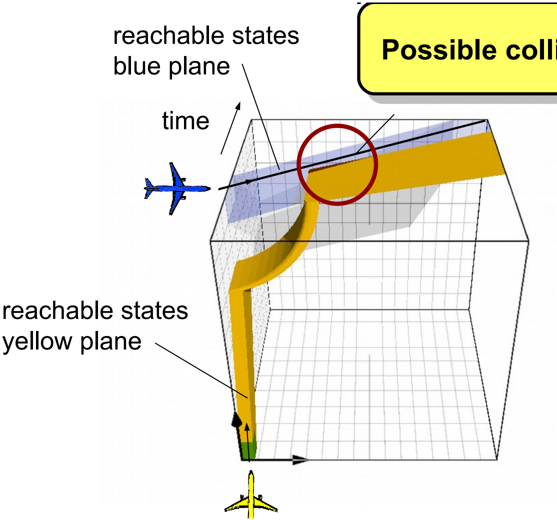
- **Traffic Coordination Problem**
  - join paths at different speed
- **Goals**
  - avoid collision
  - join with sufficient separation
- **Models**
  - Environment: Planes
  - Software: Controller
    - switches fast/slow
- **Specification**
  - keep min. distance

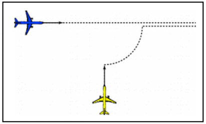
Join Manoeuvre





Join Manoeuvre





Formal Verification

”systems that users can bet their life on”

-D. Corman, NSF

cars…

airplanes…

pacemakers…