

FoCaLiZe: Inside an F-IDE

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For years, **I**ntegrated **D**evelopment **E**nvironments have demonstrated their usefulness in order to ease the development of software. High-level security or safety systems require proofs of compliance to standards, based on analyses such as code review and, increasingly nowadays, formal proofs of conformance to specifications. This implies mixing computational and logical aspects all along the development, which naturally raises the need for a notion of *Formal* IDE. This paper examines the **FoCaLiZe** environment and explores the implementation issues raised by the decision to provide a single language to express specification properties, source code and machine-checked proofs while allowing incremental development and code reusability. Such features create strong dependencies between functions, properties and proofs, and impose an particular compilation scheme, which is described here. The compilation results are runnable **OCaml** code and a checkable **Coq** term. All these points are illustrated through a running example.

1 Introduction

Thanks to Wikipedia, an Integrated Development Environment is a “software application that provides comprehensive facilities to computer programmers for software development”. Such environments do not provide in general all the necessary tools for critical software developments as safety, security and privacy standards require the use of formal methods all along the development cycle to achieve high levels of assurance. Every critical system must be submitted to an assessment process before commissioning. It consists in a rigorous examination of the demonstration, given by the developer, that the implementation complies with the specification requirements. Specifications, source and object codes, automated verifications and mechanically-checked (or not) proofs are scrutinized and, if needed, validation is re-done with other tools. This assessment process takes a lot of time and is expensive.

The **FoC** environment was created ten years ago ([14]) by T. Hardin and R. Rioboo as a laboratory language to study “how to provide comprehensive facilities” to developers of critical systems, complying to high-level rates of standards like [16, 15, 7]. At this moment, only **B**, **Z** and some tools based on algebraic data types were used in such developments. The idea was to couple the programming facilities of **OCaml** with the capabilities of the theorem prover **Coq** while avoiding the use of some complex features of these languages. The help provided to developers by object-oriented features like inheritance and late-binding, by abstraction features like modules, by parametrisation features like functors was already widely recognized in the software development world. Such features were required in the **FoC** specification to manage not only source code but also requirements and proofs. Their possible codings in **Coq** were studied[5] and some possibilities of logical inconsistencies brought by mixing these features were identified. Some of them were avoided by using a dedicated concrete syntax. The remaining ones require a dedicated static analysis to be eliminated. This was the first version of the “dependency calculus”, studied by V. Prevosto[10], who also designed the first compiler of the language (which name evolved in **FoCaLiZe**). At that time, the programming language was a pure functional strongly typed kernel, proofs were done directly in **Coq**. Then, a language for proofs was introduced to enable the use of the automated theorem prover **Zenon**[1] and automatically translate them into proof terms of **Coq**.

FoCaLiZe is the current version of Foc. It provides a logical language to express requirements as first-order formulas depending on identifiers present in the context, powerful data-types and pattern-matching, object-oriented and modular features, a language of proofs done by Zenon, which may use unfolding of functions defined by pattern-matching. From the whole development, the compiler issues a source file written in OCaml and a proof term submitted to Coq. The two files are kept close to the FoCaLiZe source to ease traceability and the compiler guarantees that, if the FoCaLiZe source can be compiled, then its OCaml translation will be runnable and its Coq translation will be checkable. This is a way to avoid returning errors from target languages, which would be unacceptable for the user.

In this paper, we present the main features of the compiler, trying to explain why they permit to improve the adaptability of FoCaLiZe to the needs of formal developments. More returns on user experience using FoCaLiZe and help provided by this environment can be found in [4, 3, 12].

The first specificity of FoCaLiZe is the mixing of logical and computational aspects which create a lot of dependencies between elements of a development. Definitions are considered as terms of the logical language, properties can embed names of functions being unfolded in proofs. Late-binding allows to change definitions thus altering proofs done with the old version. The second specificity of the current version of FoCaLiZe is the maximizing of sharing between computational and logical codes, at the source level, through the use of inheritance, late-binding and modularity which also adds dependencies. To keep this maximal sharing at object levels, a rather new usage of lambda-lifting relying on a static analysis of dependencies is used by the compiler and presented here.

The first section presents the paradigms FoCaLiZe is based on and introduces a running example to support further discussions. In the next section, the notion of dependency is introduced, followed by the formal description of their computation. The last section sketches the code generation model on the basis of the example.

2 From the FoCaLiZe Point of View

2.1 Semantical Framework

Specification requirements and implementations should be together expressible in any FIDE, the first one by logical language, the second one by a programming language. If these two languages can be related through a single semantical framework, then the demonstration of the conformance of developments to requirements can be facilitated. Choosing an imperative style related to a Hoare logical language leads to some difficulties when composing specifications and unfolding function bodies, due to the lack of referential transparency. So FoCaLiZe is built upon a pure functional language. Then, whatever is the logic, functions can be consistently unfolded in proofs and thus, can be used without restriction to express requirements (called here *properties*) and proofs.

Side-effects are however possible but they have to be confined into dedicated modules, separated from the rest of the development. Properties of functions making side-effects can be separately demonstrated and rendered as contracts.

Static strong typing, rich data-types and pattern-matching ease source coding and error detection. This is why the FoCaLiZe programming (sub-)language is very close to a functional kernel of the OCaml language. The logical sub-language offers first-order quantifiers and the ability to use identifiers present in the context in formulas. So formulas can contain free variables, which however are bound in the development context. These formulas are indeed formulas of a dependent type theory and are translated as such in Coq, relying on the Curry-Howard isomorphism : data types are translated onto types, properties onto types and functions on terms. Having our own logical syntax instead of the one of a

dependent theory like Coq allows (not only) to restrict formation rules of terms. For example, properties can use function names but functions cannot use property names.

2.2 Incremental Specification and Development through a Running Example

To support further discussions, this section will gradually introduce a running example. It represents a “monitor” which inputs a value and outputs a validity flag indicating the position of the value with regards to two thresholds, and this value or a default one in case of problem. A fleshed-out version of this example was used to develop a generic voter used in critical software [4].

Object-oriented features are often offered by IDE as they have the reputation to help software development. Here, assessment also should be helped and some of these features like complex rules of visibility (tags private, friend, etc.) and possibilities of runtime late-binding can weight down this process. FoCaLiZe proposes two object-oriented features: inheritance and late-binding. We call here *inheritance* the ability of introducing, at any stage of the development, properties, functions and proofs (called *methods* in FoCaLiZe) that these new items contribute to fulfill previously stated requirements. As usual, *late-binding* is the possibility of introducing the name, and here the type, of a function while deferring its definition. This allows to adapt definitions along the inheritance tree and enhances reusability.

```

species Data =
  let id = "default" ;
  signature fromInt : int -> Self ;
end ;;

species OrdData =
  inherit Data ;
  signature lt : Self -> Self -> bool ;
  signature eq : Self -> Self -> bool ;
  let gt (x, y) = ~~ (lt (x, y)) && ~~ (eq (x, y)) ;
  property ltNotGt : all x y : Self, lt (x, y) -> ~ gt (x, y) ;
end ;;

```

The component (*species*) Data simply says that the input is coded by an integer, converted by `from_int` to a value of the internal representation, which is denoted by `Self`. The species `OrdData` inherits from `Data`, it declares two functions (`lt` and `eq`), defines a derived function `gt` and states a property `ltNotGt` using both declared and defined methods.

As function types are parts of specifications, late-binding cannot change them. But late-binding can invalidate proofs done with an “old” definition and the compiler must manage this point. Late-binding also allows stating properties and delay their proofs, while the former are already usable to write other proofs. Restricting the access to some elements is needed and handled at the *component* level.

A FoCaLiZe component collects not only types, declarations and definitions but also related properties and proofs. Inside a component, the manipulated data-types have a double role: a programming one used in type verification, a logical one when translated to Coq (this double view has to be managed by the compiler and it is not always straightforward as seen further). To simplify the FoCaLiZe model, all data-types introduced in a component are grouped (by the way of product and union types) into a single data-type called *representation*. It gives to this notion of component a flavor of Algebraic Data Type, a notion which has proven its usefulness in several IDE (e.g. [2]). The representation can be just a type variable (i.e. be not yet given an effective implementation), whose name serves in declarations. It can be instantiated by inheritance (but not redefined) to allow typing of definitions.

```

species TheInt =
  inherit OrdData;
  representation = int ;
  let id = "native_int" ;
  let fromInt (x) : Self = x ;
  let lt (x, y) = x <0x y ;
  let eq (x, y) = x =0x y ;
  proof of ltNotGt = by definition of gt property int_ltNotGt ;
end ;;

```

The species `TheInt` defines the representation (`int`) and functions already declared, then proves the property by unfolding `gt` and using a property found in the file `basics` of the standard library (this

proof, which could be done in `OrdData`, will be used later).

Functors allow parametrisation by components. Here a component may not only use functions provided by other components, but also their properties and theorems. Parametrisation by values of these parameter components is of course needed. The link between a parameterised component and its parameters will be reflected in `Coq` via the notion of `Coq` dependencies.

```

type statut_t = | In_range | Too_low | Too_high ;;

species IsIn (V is OrdData, minv in V, maxv in V) =
  representation = (V * statut_t) ;
  let getValue (x : Self) = fst (x) ;
  let getStatus (x : Self) = snd (x) ;

  let filter (x) : Self =
    if V!lt (x, minv) then (minv, Too_low)
    else
      if V!gt (x, maxv) then (maxv, Too_high)
      else (x, In_range) ;

  theorem lowMin : all x : V, getStatus (filter (x)) = Too_low -> ~ V!gt(x, minv)
  proof =
    <1>1 assume x : V,
      hypothesis H: snd (filter (x)) = Too_low,
      prove ~ V!gt (x, minv)
    <2>1 prove V!lt (x, minv) by definition of filter type statut_t hypothesis H
    <2>2 qed by step <2>1 property V!ltNotGt
    <1>2 qed by step <1>1 definition of getStatus ;
  end ;;

```

The species `IsIn` has a collection parameter `V` and two value parameters `minv` and `maxv` from `V`. It mainly shows a proof decomposition into several steps (see 2.3), including a step of induction (here a simple case split) on the union type `statut_t`.

Should the definition of the representation be exposed or encapsulated by the modularity mechanism? Inheritance and late-binding require its exposure, as total encapsulation can cumbersome the development task. On the contrary parametrisation asks for its abstraction. Indeed, a component seeing the data representation of its parameters can manipulate this data without using the provided functions, hence breaking invariants and structural assumptions made by parameters and invalidating the theorems they provide. Abstract definitions of types is not sufficient as properties can still reveal the exact definition of a type (a bad point when security properties are considered). Thus `FoCaLiZe` has two notions of components. *Species* which expose their representation are only used along inheritance and late-binding during design and refinement. *Collections* which encapsulate the representation are used as species parameters during the integration process.

To avoid link-time errors, any call of an effective species parameter must be ensured that all functions exported by this parameter are really defined. To preserve consistency, all exported properties must have already received proofs. Thus collections can only be created by encapsulation of species, called *complete species*, in which all declarations have received a definition and all properties have a proof. Encapsulation builds an *interface* exposing only the name of the representation, declarations of (visible) functions and (visible) properties of this species. The compiler guarantees that all exposed definitions and theorems have been checked and that the interface is the only way to access collection items.

```

collection IntC = implement TheInt ; end ;;
collection In_5_10 = implement IsIn (IntC, IntC!fromInt (5), IntC!fromInt (10)) ; end ;;
collection In_1_8 = implement IsIn (IntC, IntC!fromInt (1), IntC!fromInt (8)) ; end ;;

```

The species `TheInt` being *complete*, it is submitted to encapsulation (`implement`) to create the collection `IntC`. This latter is then candidate to be effective argument of `IsIn`'s parameter `V` and to apply its method `fromInt` to provide effective values for the `minv` and `max` parameters. Hence it can be used to create other collections, `In_5_10` and `In_1_8`.

From a developer's point of view, species serve to make an incremental development. Collections, mostly used as effective parameters of species, are used to assemble separate parts of the development.

As writing species also needs some primitive notions like booleans, integers, FoCaLiZe has a standard library which provides these low-level bricks, possibly along with proved properties but without encapsulation. The absence of encapsulation is wanted in such a case and allows manipulating basic data-structures as native constructs while however having properties. As long as the considered datatypes (i.e. type definitions here) do not have any invariant to preserve, there is no risk of inconsistency by revealing their effective structure. The standard library consists in source files that can be accessed by an opening mechanism and is not concerned by the species/collections mechanism. From the development and assessment points of view, files of the standard library are just on-the-shelf components granted by the system (or their provider).

2.3 Properties and Proofs

A proof, intended to be mechanically checked, must be decomposed into small enough steps to be checkable by the prover. Our first try (Foc) to directly use the script language of Coq, revealed several drawbacks. First, it required the user to have a deep knowledge of Coq to powerfully use it and to understand error messages. Second, the user had to be aware of the compilation of FoCaLiZe elements to Coq. Third, the proofs too deeply depended on Coq. In FoCaLiZe an intermediate step to do proofs has been introduced. It is based on natural deduction which, being reminiscent of mathematical reasoning, is accessible to a non-specialist without too much effort. It uses the automated theorem prover Zenon. A proof is a hierarchical decomposition into intermediate steps[9] unfolding definitions, introducing subgoals and assumptions in the context until reaching a leaf, that is, a subgoal which can be automatically handled by Zenon. When all the leaves have received proofs, the compiler translates the whole proof to Coq and builds the context needed for checking this proof. The following example shows this decomposition into a list of steps, always ended by a `qed` step, whose goal is the parent goal.

```
theorem t : all a b c : bool, a -> (a -> b) -> (b -> c) -> c
proof =
  <1>1 assume a b c : bool,
    hypothesis h1: a, hypothesis h2: a -> b, hypothesis h3: b -> c,
    prove c
  <2>1 prove b by hypothesis h1, h2
  <2>2 qed by step <2>1 hypothesis h3
  <1>2 qed by step <1>1
```

The proof has two outer steps <1>1 and <1>2. Step <1>1 introduces hypotheses h1, h2, h3 and the subgoal c. It is proved by a 2-step subproof. Step <2>1 uses h1 and h2 to prove b. Step <2>2 uses <2>1 and h3 in order to prove c. Step <1>2 ends the whole proof.

Proofs done in a given species are shared by all species inheriting this one. As late-binding allows redefinition of functions, proofs using the “old” definition are no longer correct, must be detected as soon as the redefinition is compiled, reverted to the property status and should be done again. This link between proofs and definitions is a particular case of *dependencies* between elements of the user development. The section 3.1 intuitively introduces this concept while the section 3.2 formally describes the related calculus.

Specification requirements such as the safety/security ones can be split into finer ones (see [3]) and proved under the hypotheses that these finer properties hold[12]. Thus, FoCaLiZe allows to do proofs *just in time* using not already proved properties and guarantees that they will be proved later in a derived complete species. This can help early detection of specification errors. Moreover, some properties can be granted by other means (contracts, verification tools, etc.). They can be considered as theorems by giving them a proof reduced to the keyword `admitted`. This is a needed but dangerous feature as admitted properties can lead to logical inconsistencies. The use of this keyword is recorded in the automatically

done documentation file provided by the compilation process. The assessment process must ultimately check that any occurrence of this keyword is harmless.

2.4 The Compilation Process

In a nutshell, the compilation process guarantees that the whole development leads to a runnable code satisfying the requirements. Its *code generation* translates user code to a source code (called here simply *computational code*) in OCaml and to a term (called here *logical code*) of Coq. Data types and properties are translated to Coq types while definitions and proofs are translated as terms. Computational code only contains declarations and definitions since they are the only material contributing to an executable.

The translations of the user code are type-checked by OCaml and Coq. But these checkings arrive too late and error diagnostics may be difficult to convey to the user. Thus the compiler has its own typing mechanism, to early detect typing errors and emit comprehensive diagnostics.

First preliminary tries were done to translate inheritance and late-binding using oriented-object features of OCaml([6]). But, as Coq does not propose these features, a different way of compilation was needed to produce logical code (see [10]). These two different compilation schemes jeopardize traceability as the object codes differ a lot. The current compiler uses a single method of compilation to both target languages, inspired by Prevosto’s work and presented in the next section. It resolves inheritance and late-binding before code generation while controlling impacts of redefinitions onto proofs.

2.5 Zenon

The automatic theorem prover Zenon is based on a sequent calculus for first-order logic and has FoCaLiZe-specific features such as unfolding of definitions and inductive types. Boolean values and logical propositions are strictly distinct notions in Coq but this complexity is hidden to the FoCaLiZe user: explicit conversions (done by `Is_true`) between `bool` and `Prop` are inserted in the formula sent to Zenon. They could be axiomatized but this would blow up the search-space; instead, Zenon has inference rules dedicated to these conversions.

A proof written in FoCaLiZe is compiled into a tree of Coq *sections*, which translates the natural deduction style into a natural deduction proof in Coq. Each section first introduces the hypotheses of its corresponding step, then includes the sections of its substeps and ends with the proof of its goal, which is generated by Zenon from the facts of the `qed` step. Each leaf proof is just temporarily compiled into a “hole” later filled with a Zenon proof by invocation of `ZvToV`. This tool translates each leaf of the user’s proof into a Zenon input file that contains the goal and assumptions and definitions listed by the user. Then Zenon outputs a Coq proof which fits right in the corresponding Coq section because every assumption it was given is available at this point in the Coq file. Once all “holes” are filled, the whole Coq source file is sent to Coq for verification.

The only acceptable errors from Zenon are simple: “*out of memory*”, “*time out*”, “*no proof found*”. They mean that Zenon didn’t find any proof, either because some hypotheses were missing (in this case the user may add lemmas to split the proof) or because the proposition is false. If a proof is found, Coq must raise no error when checking it otherwise there is a bug in Zenon or FoCaLiZe.

3 Toward Effective Code

This section begins by an informal presentation of the notion of dependencies since they strongly impact the form of the target codes generated by the FoCaLiZe compiler. Next comes the formal computation

of these dependencies. Finally, the code generation model is depicted on the basis of the above example.

3.1 Dependencies in User Code

A method *depending* on the definition of another method m is said to have a *def-dependency* on m . Some def-dependencies are eliminated by a good choice of syntax. For example, function bodies cannot contain property names nor keywords for proofs, thus a function cannot def-depend on a proof. There are only two possibilities of def-dependencies. First, proofs with a *by definition* of m step (which unfolds the definition of m) def-depend on m . If m is redefined, these proofs must be invalidated. Second, functions and proofs can def-depend on the representation. Properties must not def-depend on the representation, as explained by the following example. The complete species `Wrong` may be encapsulated into a collection, whose interface contains the statement `theo`. This one should be well-typed in the logical code. But typing `x + 1` in `theo` requires to identify (unify) **Self** with `int`. The encapsulation of the representation prevents it. Thus the species `Wrong` must be rejected by the compiler.

```
species Wrong =
  representation = int ;
  let incr (x) : Self = x + 1 ;
  theorem theo : all x : Self, incr (x) = x + 1 ;
end ;;
collection Bad = implement Wrong ;;
```

```
Bad...bad interface

representation : self
incr (x) : Self -> int
theorem theo :
  all x : Self, incr (x) = x + 1 ;
```

Note that function calls do not create def-dependencies and that encapsulation of collections prevents any def-dependency on methods of collection parameters. Thus analysis of def-dependencies must ensure that proofs remain consistent despite redefinitions and that properties have no def-dependencies on the representation (in other words, interfaces of collections should not reveal encapsulated information).

Apart from the def-dependencies of proofs on definitions and of properties on representations, there are other dependencies called *decl-dependencies*. Roughly speaking, a method m_1 decl-depends on the method m_2 if m_1 depends on the declaration of m_2 . The following example gives a first motivation for their analysis.

```
species S =
  signature odd : int -> bool ;
  let even (n) =
    if n = 0 then true else odd (n - 1) ;
end ;;
```

```
species T =
  inherit S ;
  let odd (n) =
    if n = 0 then false else even (n - 1) ;
end ;;
```

In `S`, `even` is at once declared and defined, so its type can be inferred by the type-checker, using the type of `odd`. Thus `even` decl-depends on `odd` but `odd` does not depend on `even`. In `T`, defining `odd` creates a decl-dependency of `odd` on `even` and an implicit recursion between them. To keep logical consistency, such an implicit recursion must be rejected. Recursion between entities must be declared (keyword `rec`). The compiler has to detect any cycle in dependencies through the inheritance hierarchy.

More generally, a function m decl-depends on p if m calls p , a property m decl-depends on p if typing of m in the logical theory requires p 's type, a proof decl-depends on p if it contains a step by property p or an expression whose typing needs p and, recursively, m decl-depends on any method upon which p decl-depends and so on. Def-dependencies are also decl-dependencies. These cases are not the only cases of decl-dependencies. The first version of this calculus is described in [11].

3.2 Dependencies Computation

To support generation of well-formed code, the notion of dependencies must be reinforced and formally studied. The theorem `ltNotGt` syntactically decl-depends on `gt`, `lt`, `rep` and def-depends on `gt`. Thus, its proof is ultimately compiled to a **Coq** term, where `gt` is unfolded, making arising the identifier

eq . The type of eq is needed to **Coq**-typecheck gt and must be provided through the λ -lift abst_eq of eq . Only λ -lifting syntactic def- and decl- dependencies would lead to a generated code looking like:

```
Theorem ltNotGt (abst_T : Set) (abst_lt := lt) (abst_gt := OrdData.gt abst_T abst_eq abst_lt) :
  forall x y : abst_T, Is_true ((abst_lt x y)) -> ~Is_true ((abst_gt x y)).
apply "Large Coq term generated by Zenon".
```

where the $:=$ construct binds def-dependencies, and where abst_eq is unbound ! Moreover, raising eq also exhibits a def-dependency on the carrier through the one of eq . Dependencies over collection parameters methods suffer from the same incompleteness. Hence, a process of “completion” of syntactically found dependencies has to be applied before λ -lifting.

It requires to compute the *visible universe* of a method m which is the set of methods of `Self` needed to analyze m . Then, the minimal (**Coq**) typing environment of m is built by deciding, for each method p of its visible universe if only its type must be kept (issuing λ -lift of p) or if its body is also needed (hence “ $:=$ -binding” of p). Finding the minimal set is especially important since this allows to reduce the amount of λ -liftings of method and collection generators. A last completion of the set of dependencies for parameter methods achieves the building.

In the following, we assume that inheritance has been processed, leading to a *normal form* of a species in which all its methods are present, in their most recent version and well-typed. Although this process is not trivial ([11, 13]), it is out of the scope of this paper.

3.2.1 On Methods of `Self`

Let S be a species. We denote by $\mathcal{D}_S(x)$ (resp. $\mathcal{D}_S^{\text{def}}(x)$) the set of methods of S on which the method x decl-depends (resp. def-depends). This set can be obtained by walking the Abstract Syntax Tree looking for apparition of methods names in expressions (resp. apparition of **by definition** in proofs). Only dependencies on the carrier have to be checked differently, by typechecking. As proofs and properties names are syntactically forbidden inside property expressions and function bodies, typechecking of properties and functions requires only the types of the function names appearing in them (i.e. appearing in \mathcal{D}_S).

Considering theorems proofs (i.e. bodies), def-dependencies can arise forcing the need to keep some definitions (not only types) to be able to typecheck. Then, one also needs to make sure that these definitions can themselves be typechecked. Moreover, proofs may involve decl-dependencies on logical methods, whose types are logical statements (i.e. expressions). Methods appearing in such types must also be typechecked. For all these reasons, the context needs to keep trace of the methods belonging to the transitive closure of the def-dependency relation, plus the methods on which these latter decl-depend. This context is called the *visible universe* of a method x and is noted $|x|$. In the same spirit than in [10], $|x|$ is defined as follows:

$$\frac{y \in \mathcal{D}_S(x)}{y \in |x|} \quad \frac{y <_S^{\text{def}} x}{y \in |x|} \quad \frac{z <_S^{\text{def}} x \quad y \in \mathcal{D}_S(z)}{y \in |x|} \quad \frac{z \in |x| \quad y \in \mathcal{T}_S(z)}{y \in |x|}$$

where $x <_S^{\text{def}} y$ stands for y def-depends on x by transitivity and $\mathcal{T}_S(x)$ stands for the type of x in the species S .

From the notion of visible universe, it is possible to define, for a method x of a species, what are the other methods needed to have x well-typed.

- Methods not present in the visible universe are not required.
- Methods present in the visible universe on which x doesn't def-depend are required but only their type is needed.

- Methods present in the visible universe on which x def-depends are required with both their type and body.

Let S be a species containing the set of methods $\{y_i : \tau_i = e_i\}$. Let x being one of the y_i , the minimal typing environment of x is defined as follows:

$$\begin{array}{c} \emptyset \mathbin{\text{\tiny \mathbb{M}}} x = \emptyset \qquad \frac{y \notin |x| \quad \{y_i : \tau_i = e_i\} \mathbin{\text{\tiny \mathbb{M}}} x = \Sigma}{\{y : \tau = e ; y_i : \tau_i = e_i\} \mathbin{\text{\tiny \mathbb{M}}} x = \Sigma} \\[10pt] \frac{y \in |x| \quad y <_S^{def} x \quad \{y_i : \tau_i = e_i\} \mathbin{\text{\tiny \mathbb{M}}} x = \Sigma}{\{y : \tau = e ; y_i : \tau_i = e_i\} \mathbin{\text{\tiny \mathbb{M}}} x = \{y : \tau = e ; \Sigma\}} \qquad \frac{y \in |x| \quad y <_S^{def} x \quad \{y_i : \tau_i = e_i\} \mathbin{\text{\tiny \mathbb{M}}} x = \Sigma}{\{y : \tau = e ; y_i : \tau_i = e_i\} \mathbin{\text{\tiny \mathbb{M}}} x = \{y : \tau ; \Sigma\}} \end{array}$$

Using the minimal typing environment it is possible to generate the method generators for non-parametrised species only since collection parameters are not taken into account. *A fortiori* it is not possible to generate collection generators.

3.2.2 On Methods from Parameters

Following the same principle than in the previous section, we note the direct dependencies of an expression e in a species S on a parameter C by $\mathcal{D}\mathcal{O}\mathcal{P}_{[\text{EXPR}]}(S, C)[e]$ and define it by a simple search on the AST (looking for occurrences of the form $C ! x$ for any x). In the coming rule, $\mathcal{E}(S)$ stands for the parameters of the species S and $\mathcal{B}_S(x)$ returns the body of the method x in the species S (i.e. an expression for a `let`-definition and a proof for a theorem).

The challenge is to find the minimal set of parameters methods required to typecheck a method. We now present the five first rules driving the calculus since they do not have any order of application. A last one will be exposed after.

$$\begin{array}{l} \mathcal{D}\mathcal{O}\mathcal{P}_{[\text{BODY}]}(S, C)[x] = \mathcal{D}\mathcal{O}\mathcal{P}_{[\text{EXPR}]}(S, C)[\mathcal{B}_S(x)] \qquad \mathcal{D}\mathcal{O}\mathcal{P}_{[\text{TYPE}]}(S, C)[x] = \mathcal{D}\mathcal{O}\mathcal{P}_{[\text{EXPR}]}(S, C)[\mathcal{T}_S(x)] \\[10pt] \mathcal{D}\mathcal{O}\mathcal{P}_{[\text{DEF}]}(S, C)[x] = \mathcal{D}\mathcal{O}\mathcal{P}_{[\text{EXPR}]}(S, C)[\mathcal{B}_S(z)] \quad \forall z \text{ such as } z <_S^{def} x \qquad \mathcal{D}\mathcal{O}\mathcal{P}_{[\text{UNIV}]}(S, C)[x] = \mathcal{D}\mathcal{O}\mathcal{P}_{[\text{EXPR}]}(S, C)[\mathcal{T}_S(z)] \quad \forall z \in |x| \\[10pt] \frac{\mathcal{E}(S) = (\dots, C_p \text{ is } \dots, \dots, C_{p'} \text{ is } S'(\dots, C_p, \dots)) \quad \mathcal{E}(S') = (\dots, C'_k \text{ is } I'_k, \dots) \quad \begin{array}{l} z \in \mathcal{D}\mathcal{O}\mathcal{P}_{[\text{TYPE}]}(S, C_{p'})[x] \vee z \in \mathcal{D}\mathcal{O}\mathcal{P}_{[\text{BODY}]}(S, C_{p'})[x] \quad (y : \tau_y) \in \mathcal{D}\mathcal{O}\mathcal{P}_{[\text{TYPE}]}(S', C'_k)[z] \\ (y : \tau_y[C'_k \leftarrow C_p]) \in \mathcal{D}\mathcal{O}\mathcal{P}_{[\text{PRM}]}(S, C_p)[x] \end{array}}{(y : \tau_y[C'_k \leftarrow C_p]) \in \mathcal{D}\mathcal{O}\mathcal{P}_{[\text{PRM}]}(S, C_p)[x]} \end{array}$$

The rule [BODY] (resp. [TYPE]) takes into account dependencies on a method explicitly stated in the body (resp. type) of a definition.

The rules [DEF] and [UNIV] serve to collect dependencies on a parameter induced by the dependencies its methods have together in their hosting species. Note that methods z introduced by the rule [DEF] are obviously included in those introduced by [UNIV]. In effect, the visible universe is wider than only transitive def-dependencies and their related decl-dependencies: if there is no def-dependencies then the relation $<_S^{def}$ will be empty although decl-dependencies may lead to a non-empty visible universe. The rule [DEF] allows to inspect bodies. The rule [UNIV] allows to inspect types. Hence, any z introduced by [DEF] will also has its type inspected by [UNIV].

Finally, the rule [PRM] applies to take into account dependencies of a method on a previous parameter C_p used as argument to build the current parameter $C_{p'}$. It differs from the previous rules, since the result of the calculus is not only a set of names: types are explicit. This is due to the fact that the type of the methods of this set differs from the one computed during the typechecking of the species used as parameter.

If C is an entity parameter, we set: $\mathcal{D}\mathcal{O}\mathcal{P}_{[\text{XXX}]} = \{C\}$, i.e. that the identifier of the parameter is considered as it's only method.

None of these rules took into account decl-dependencies that methods of parameters have inside their own species and that are visible through types. The following example show that using $P!th0$ to prove $th1$ which only deals with $P!f$ however needs to have $P!g$ in the context.

```
species A =
  signature f : Self -> int ;
  signature g : Self -> int ;
  property th0: all x : Self, f (x) = 0 /\ g (x) = 1 ;
end ;;

species B (P is A) =
  theorem th1 : all x : P, P!f (x) = 0
  proof = by property P!th0 ;
end ;;
```

Note that because of the encapsulation process applied to collection parameters, def-dependencies are never possible and only types are visible. Hence the following rule serves to complete an initial set of dependencies \mathcal{D} .

$$\frac{\mathcal{E}(S) = (\dots, C_p \text{ is } I_p, \dots) \quad z \in \mathcal{D}(S, C_p)[x] \quad (y : \tau_y) \in \mathcal{T}_{I_p}(z) \int_{I_p}}{(y : \tau_y[\text{Self} \leftarrow C_p]) \in \mathcal{D}^+(\mathcal{D}, S, C_p)[x]} \text{CLOSE}$$

Note that new dependencies brought by this rule cannot themselves require applying this rule again to make them typecheckable. In effect, only logical methods can introduce new dependencies and they only can depend on computational methods whose types are “ML-like” ones, hence cannot introduce methods names.

3.3 Code generation

Providing detailed algorithms implementing code generation is out of the scope of this paper. Instead, we illustrate their expected behaviour by showing the output obtained by compiling our above example.

Code generation starts after resolution of inheritance and late-binding, typing and dependency analyses. Note that issuing very similar target codes should ease assessment. Thus the code generation phase should be common to the two targets until concrete syntaxes are produced. Moreover a good sharing of code alleviates the assessment task and eases control of code size and reuse. Therefore we try to maximize sharing.

As inheritance and late-binding must be resolved at compile-time to ensure proof validity, it would be possible to generate code for collections only, but this prevents any sharing as shown below. Code generation of `In_5_10` and `In_1_8` use the last definition of methods of `IsIn` but they do not share their bodies. Some possible sharing of the methods of the parameter `IntC` are also lost. Not only code size increases but assessment takes longer since any collection should be checked *in extenso*. Thus, code has to be generated for all species.

Method Generators In the following example, the species `IsInE` redefines `filter`.

```
species IsInE (X is OrdData, low in X, high in X) =
  inherit IsIn (X, low, high) ;
  let filter = ...
end ;;
collection ExtIn_3_8 = implement IsInE (...) ; end ;;
```

`lowMin` def-depends on `filter`. Its body, that is, the proof of `lowMin` in `IsIn`, can be shared by all species inheriting `IsIn`, if they do not redefine `filter`. But it cannot be shared with `IsInE` as the proof must be re-done. Thus, redefinitions cancel sharing of def-dependencies.

Assume that a method `IsIn!m` decl-depends on `filter`. Then all species inheriting `IsIn!m` and not redefining `m` can share its body, up to the calls of `filter`. Similarly the collections `In_5_10`, `In_1_8` and `ExtIn_1_8` can share the methods of `IntC`, up to the calls of methods on which `IntC` decl-depends. Thus, the body of a definition `m` (function or proof) of a species `S` can be shared along inheritance, up to the calls to methods of `S` and of its parameters upon which `m` decl-depends (thanks to the absence of cycles in decl-dependencies). The sharing is done by abstracting, in `m`'s body, the

names of these decl-dependencies, using the standard technique of λ -lifting[8]. The result of this λ -lifting is called the *method generator* of m . To obtain the final code of m , this generator will have to be applied to the most recent values of its decl-dependencies, provided by the species (or its descendants) (*spec-arguments*) or by the effective parameters (*param-arguments*).

Moreover, if the logical target language (like Coq) requires explicit polymorphism, representations of **Self** and of parameters on which a *method* decl-depends are also λ -lifted (leading to parameters of type `Set` in Coq and named with `_T` in the example below). As the methods and representation of a species can depend on representations and methods of collection parameters, λ -liftings of decl-dependencies upon parameters must be the outermost abstractions.

The generated codes for species are grouped into modules of the target languages, to enforce modularity and to benefit of a convenient namespace mechanism. Now either these modules contain all the inherited and non-redefined method generators of the species (but this leads to code duplication) or a method generator is created once, when a method is (re)defined and appears only in the species that defines it. We use the latter solution.

Example continued. The example of section 2.2 is pursued, using a Coq-like syntax. OCaml files are just obtained by removing some types, properties and proofs and are not listed here. In method generators, any occurrence of the name of a decl-dependency m is replaced by `abst_m` and abstracted. If by adding a definition, this decl-dependency is turned into a def-dependency, then `abst_m` is bound to the definition of m by the construct `abst_m := Coq term`.

Generating code for species Data: The defined method `id` has no dependencies, hence trivially leads to a simple definition.

```
Module Data.
  Definition id : basics.string__t := "default".
End Data.
```

Generating code for species OrdData: The body of `gt` decl-depends on `eq`, `lt` and the representation as its inferred type is `Self → Self → bool`. Note that this body remains close to the source one. Inherited and only declared methods induce no code.

```
Module OrdData.
  Definition gt (abst_T : Set) (abst_eq : abst_T → abst_T → basics.bool__t)
    (abst_lt : abst_T → abst_T → basics.bool__t) (x : abst_T) (y : abst_T) : basics.bool__t :=
    basics.not (abst_lt x y) && basics.not (abst_eq x y)
End OrdData.
```

Generating code for species TheInt: This species redefines `id` and defines `eq`, `fromInt` and `lt`. `id` has no dependencies whereas `eq`, `fromInt` and `lt` only have a def-dependency on the representation, whose value (the built-in type `int`) is bound (`:=` construct) to `abst_T` in the corresponding method generators.

The proof of `ltNotGt` decl-depends on the representation. It def-depends on `gt` because it unfolds its definition. Note that `lt` and `eq` do not appear in this proof : unfolding of `gt` does not unfold them recursively. But typing `ltNotGt` requires typing `gt` and thus typing `lt` and `eq`. Hence, `ltNotGt` has decl-dependencies on `lt` and `eq`, coming from the def-dependency on `gt`. So, `lt` and `eq` must be λ -lifted in `ltNotGt`, to build the value of `abst_gt` by applying the method generator `gt` found in the module `OrdData` to its three arguments. Note that only the types of `lt` and `eq` are used by `ltNotGt`: these methods can be redefined without impacting this theorem.

```
Module TheInt.
  Definition id : basics.string__t := "native_int".
  Definition eq (abst_T := basics.int__t) (x : abst_T) (y : abst_T) : basics.bool__t := (basics._equal_0x x y).
  Definition fromInt (abst_T := basics.int__t) (x : basics.int__t) : abst_T := x.
  Definition lt (abst_T := basics.int__t) (x : abst_T) (y : abst_T) : basics.bool__t := (basics._lt_0x x y).
  Theorem ltNotGt (abst_T : Set) (abst_eq : abst_T → abst_T → basics.bool__t)
    (abst_lt : abst_T → abst_T → basics.bool__t) (abst_gt := OrdData.gt abst_T abst_eq abst_lt) :
    forall x y : abst_T, Is_true ((abst_lt x y)) → ~Is_true ((abst_gt x y)).
```

```

    apply "Large Coq term generated by Zenon" ;
End TheInt.

```

As illustrated by this example, the dependency calculus cannot be reduced to a simple “grep”. For any method, the analysis must compute the sets of methods of the species and of the parameters which are needed to elaborate the type and the value of its logical code (in Coq, they are called visible universe and minimal typing environment). This is the price to pay for having no errors in target codes.

Collection generators Code generation for collections must create computational runnable code and checkable logical code. Suppose that a collection C is built by encapsulating a complete species S . The code of a method $S!m$ is obtained by applying its method generator, say G_m , to its effective arguments. The right version of this method generator G_m comes from the last definition of m in the inheritance path ending on S . The effective spec-arguments of G_m can be retrieved from the species having created G_m and from the instantiations of formal parameters done during inheritance. These applications can be safely done as the species S is complete and as the dependency analysis provides an ordering on all methods of a species, such that the n^{th} method depends only on the $(n-1)$ first ones. The effective param-arguments of G_m come from the application of the species S to effective collection parameters.

Thus a simple solution to generate collection code is to do it method by method, by applying method generators to their effective arguments. These applications are computed at runtime in the target languages. This solution allows us to generate only the needed applications for a given method. But a possibility of sharing is lost, when collections are issued from the same parameterized species, applied to different effective collection parameters (case of `In_5_10` and `In_1_8`). Then the applications of the method generators to the spec-arguments can be shared between all these collections. Regarding memory use, the gain is small. But regarding assessment processes, such an intermediate step of applications represents a large gain as it avoids having to review several copies of the same code, applied to different param-arguments. We retain this solution. To ease code review, the applications to the spec-arguments are grouped into a record (we assume that target languages offer records) called *collection generator* while the λ -liftings of all parameters decl-dependencies are moved outside the record body. Thus the material coming from the species is found in the body while the effective parameters contribution appears in the applications of the body.

It is possible to go further by replacing the bunch of λ -liftings of parameter dependencies with a unique λ -lifting abstracting the collection parameter. Then the target modules should be first-class values of the target languages with a certain kind of subtyping as interfaces inclusion provides a simple but useful subtyping. Even if our target languages have such features, it seems better to leave the code generation model open to a wide range of potential targets.

Example ended We continue the example of 2.2 by completing the module `TheInt` with the collection generator and its record type.

```

Record me_as_species :=
mk_record {
  rf_T : Set ;
  rf_id : basics.string__t ;
  rf_eq : rf_T -> rf_T -> basics.bool__t ;
  rf_fromInt : basics.int__t -> rf_T ;
  rf_lt : rf_T -> rf_T -> basics.bool__t ;
  rf_gt : rf_T -> rf_T -> basics.bool__t ;
  rf_ltNotGt : forall x y : rf_T, Is_true (rf_lt x y) ->
    "Is_true (rf_gt x y)
}.

Definition collection_create :=
let local_rep := basics.int__t in
let local_id := id in
let local_eq := eq in
let local_fromInt := fromInt in
let local_lt := lt in
let local_gt := OrdData.gt local_rep local_eq local_lt in
let local_ltNotGt := ltNotGt local_rep local_eq local_lt in
mk_record
  local_rep local_id local_eq local_fromInt local_lt
  local_gt local_ltNotGt.

```

The type of each method of the species is recorded into a record field labeled `rf_`, its value `local_` (obtained by a future application of the collection generator to all the param-arguments) has no more λ -lift. Here, as this collection has no parameter, there is no λ -lifting on the record itself. The value of `local_gt` for example is obtained by applying the method generator coming from `OrdData` to its spec-

arguments, whose values have already been generated, thanks to the absence of cycles in dependencies. The function `mk_record` builds the record out of these values.

Generating code for species `IsIn`. The types of the fields are translations of the types of the methods of the collection. The dependency calculus shows that the record depends on the carrier of the parameter `V`, on the two value parameters `minv` and `maxv` and on the method `V!gt`. These decl-dependencies are λ -lifted. For example, the type of `rf_lowMin` is the translation of the property `lowMin`, which decl-depends on `V!gt`. The abstraction on `V!gt` needed for `lowMin` is also done on the other fields.

```
Module IsIn.
  Record me_as_species (V_T : Set) (_p_minv_minv : V_T) (_p_maxv_maxv : V_T)
    (_p_V_gt : V_T -> V_T -> basics.bool__t) : Type :=
  mk_record {
    rf_T : Set ;
    rf_filter : V_T -> rf_T ;
    rf_getStatus : rf_T -> statut_t__t ;
    rf_getValue : rf_T -> V_T ;
    rf_lowMin :
      forall x : V_T,
        Is_true (basics.__equal__ (rf_getStatus (rf_filter x)) Too_low) -> ~Is_true (_p_V_gt x _p_minv_minv)
  }.
```

Next, methods generators are created for methods defined in `IsIn`.

```
Definition getValue (_p_V_T : Set) (abst_T := (_p_V_T * statut_t__t)) (x : abst_T) : _p_V_T := basics.fst __ x.
Definition getStatus (_p_V_T : Set) (abst_T := (_p_V_T * statut_t__t)) (x : abst_T) : statut_t__t := basics.snd __ x.
Definition filter (_p_V_T : Set) (_p_V_lt : _p_V_T -> _p_V_T -> basics.bool__t)
  (_p_V_gt : _p_V_T -> _p_V_T -> basics.bool__t) (_p_minv_minv : _p_V_T) (_p_maxv_maxv : _p_V_T)
  (abst_T := (_p_V_T * statut_t__t)) (x : _p_V_T) : abst_T :=
  (if (_p_V_lt x _p_minv_minv) then (_p_minv_minv, Too_low)
   else (if (_p_V_gt x _p_maxv_maxv) then (_p_maxv_maxv, Too_high) else (x, In_range))).
Theorem lowMin (_p_V_T : Set) (_p_V_lt : _p_V_T -> _p_V_T -> basics.bool__t)
  (_p_V_gt : _p_V_T -> _p_V_T -> basics.bool__t)
  (_p_V_ltNotGt : forall x y : _p_V_T, Is_true ((_p_V_lt x y)) -> ~Is_true ((_p_V_gt x y)))
  (_p_minv_minv : _p_V_T) (_p_maxv_maxv : _p_V_T) (abst_T := (_p_V_T * statut_t__t))
  (abst_filter := filter _p_V_T _p_V_lt _p_V_gt _p_minv_minv _p_maxv_maxv) (abst_getStatus := getStatus _p_V_T) :
  forall x : _p_V_T,
    Is_true ((basics.__equal__ (abst_getStatus (abst_filter x)) Too_low)) -> ~Is_true ((_p_V_gt x _p_minv_minv)).
apply "Large Coq term generated by Zenon" ;
```

Methods have no decl-dependencies on methods of `IsIn`, except `lowMin` which has a def-dependency on `filter`. The other decl-dependencies are on the representation of `IsIn` (and through it, on the one of `V`), on `V`'s methods and on values `minv` and `maxv`. The def-dependency of `lowMin` leads to the binding `(:=)` of `abst_filter` to the application of the method generator `filter` to all its arguments, represented by abstracted variables in the context.

The body of the collection generator `IsIn!collection_create` are the applications of the method generators to their param-arguments (no spec-arguments here). Then these param-arguments are λ -lifted.

```
Definition collection_create (_p_V_T : Set) _p_minv_minv _p_maxv_maxv
  _p_V_lt _p_V_gt _p_V_ltNotGt :=
  let local_rep := (_p_V_T * statut_t__t) in
  let local_filter := filter _p_V_T _p_V_lt _p_V_gt _p_minv_minv _p_maxv_maxv in
  let local_getStatus := getStatus _p_V_T in
  let local_getValue := getValue _p_V_T in
  let local_lowMin := lowMin _p_V_T _p_V_lt _p_V_gt _p_V_ltNotGt _p_minv_minv _p_maxv_maxv in
  mk_record
    (_p_V_T : Set) _p_minv_minv _p_maxv_maxv _p_V_gt local_rep local_filter local_getStatus local_getValue local_lowMin.
End IsIn.
```

Generating code for collection `IntC`: The module generated from `IntC` contains all the definitions obtained by just extracting the fields of the collection generator `TheInt.collection_create` as there are no parameters.

```
Module IntC.
  Let effective_collection := TheInt.collection_create.
  Definition me_as_carrier := basics.int__t.
  Definition id := effective_collection.(TheInt.rf_id).
  Definition eq := effective_collection.(TheInt.rf_eq).
  Definition fromInt := effective_collection.(TheInt.rf_fromInt).
  Definition lt := effective_collection.(TheInt.rf_lt).
  Definition gt := effective_collection.(TheInt.rf_gt).
  Definition ltNotGt := effective_collection.(TheInt.rf_ltNotGt).
End IntC.
```

Generating code for collection In_5_10: Here, `IsIn.collection_create` is applied to effective arguments found in the module `IntC` and definitions are extracted as above. The four underscores are just arguments inferred by `Coq`, which denote the four parameters of the record.

```
Module In_5_10.
  Let effective_collection :=
    IsIn.collection_create IntC.me_as_carrier (IntC.fromInt 5) (IntC.fromInt 10) IntC.lt IntC.gt IntC.ltNotGt.
  Definition filter := effective_collection.(IsIn.rf_filter _ _ _ _).
  Definition getStatus := effective_collection.(IsIn.rf_getStatus _ _ _ _).
  Definition getValue := effective_collection.(IsIn.rf_getValue _ _ _ _).
  Definition lowMin := effective_collection.(IsIn.rf_lowMin _ _ _ _).
End In_5_10.
```

3.4 Summarizing Dependencies Usage in λ -lifting

As previously introduced, dependencies are subject to be λ -lifted to define record types, method generators and collection generators. In a symmetrical fashion, they determine the effective methods to provide to these generators, which can only be achieved taking care of the instantiations of formal collection and entity parameters by effective arguments along the inheritance. The detailed mechanism of this is out of the scope of the present paper. Instead, we summarize here the material to λ -lift for each category of code generated element, in order of apparition for a species:

- **Parameters carriers** For record type and collection generator: all those of the parameters. For method generators: per method, only those of the used parameters.
- **Parameters methods**
 - For record type: “union” of all the dependencies of all the methods got by `[CLOSE] ([TYPE])`.
 - For method generators: the dependencies of the related method obtained by rules: `[BODY] + [TYPE] + [CLOSE] ([DEF] + [UNIV] + [PRM])`.
 - For collection generator: “union” of methods dependencies abstracted in each of the method generators.
- **Methods of `Self`** Only needed for method generators: those belonging to the minimal typing environment that are only declared.

Note that because of relative dependencies between methods of parameters inside their own species, λ -lifts of methods must be ordered for a same parameter to ensure they only refer to previously λ -lifted elements. Moreover, parameters are processed in order of apparition in the species: this way, all the methods of a same parameter are λ -lifted in sequence.

4 Conclusion

Building FoCaLiZe, a lot of questions, choice points, etc. arose from the will to avoid dissociating the computational and logical aspects in a formal development while keeping the FIDE palatable. The mix of inheritance, late binding, encapsulation, inductive data types and unfolding of definitions in proofs creates complex dependencies, which have to be analyzed to ensure development correctness. This analysis gives the basis of the compilation process through the notions of method and collection generators, that we introduced to allow code sharing. The code generation model producing computable and logical target codes is outlined through an example. The formal computation of dependencies was presented with an short summary of their usage in the λ -lifting process.

Several other FIDEs mentioned in the introduction have followed other choices. It should be very interesting to compare their compilation models with ours, particularly on the method used to establish correspondence between runnable and logical code and on their semi-automation of proofs.

Zenon greatly contributes to FoCaLiZe since it brings proof automation. This point is especially crucial to keep proofs simpler. We plan to extend it to handle recursion termination, arithmetic, temporal properties and, as it targets other provers than Coq, to experiment with other target type theories.

A huge amount of work remains to do in order to enhance FoCaLiZe. But, with all its weaknesses, it has already proved to be efficient in non-trivial developments. While safety domains already have a large number of good tools, security domains are much less well endowed, and the recent interest in combining safety and security requirements will increase demand for such tools.

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