EXAM INF242, 2011-2012

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Duration: 2h00

Authorized documents: one two-sided, handwritten, A4 format cheat sheet.

The number of points given for each problem corresponds loosely to the time, in minutes, necessary to answer the question.

IMPORTANT:
− Readability is important. We may take off points for illegible or badly written solutions.
− As a rule, answers without justification will not receive any points (for example, indicate the rule you use when using the unification algorithm, in a proof by natural deduction, etc).

Exercice 1 (Formalization: Cluedo (20 points)) Sherlock Holmes and doctor Watson are investigating a murder. Sherlock finds the following clues:

1. The murder occurred either in the study or in the pool.
2. If the murder occurred in the study, then the murderer is either doctor Olive or Mrs. Rose.
3. If the murder occurred in the pool, then the murderer is colonel Mustard.
4. If the murderer is doctor Olive or Mrs. Rose, then the murder weapon is neither the wrench nor the candlestick.
5. The murder weapon is either the wrench or the candlestick.

From these clues, Sherlock Holmes tells doctor Watson: “Elementary my dear Watson, the murderer is colonel Mustard!”

Questions:
− (6 points) Formalize the clues and Sherlock Holmes’ conclusion using the following notation:
  − s: the murder occurred in the study.
  − p: the murder occurred in the pool.
  − o: the murderer is doctor Olive.
  − r: the murderer is Mrs. Rose.
  − m: the murderer is colonel Mustard.
  − w: the murder weapon is the wrench.
  − c: the murder weapon is the candlestick.
  Note that, for this problem, “either x, or y” means “x or y, but not both”, and is formalized as \( x \lor y \).
− (6 points) Put the conjunction of these clues and the negation of the conclusion in conjunctive normal form.
− (8 points) Prove, using the method of your choice, that Sherlock’s reasoning is correct.
Exercice 2 (Unification (Poly) (15 points)) Are the following terms unifiable? If so, give their most general unifier, if not, justify your answer.

- (4 points) \( h(g(x), f(a, y), z) \) and \( h(y, z, f(u, x)) \).
- (4 points) \( h(g(x), f(a, y), z) \) and \( h(y, z, f(u, g(x))) \).
- (7 points) The equation \( f(g(y), y) = f(u, z) \) has two most general unifiers (recall: they are therefore equivalent). Give both solutions.

Exercice 3 (\( n \)-expansion (20 points)) Find, using the finite expansion method, a model AND a counter-model to each of the following formulas:

1. (6 points) \( \exists x P(x) \lor \forall x \exists y Q(x, y) \Rightarrow \exists x P(x) \wedge \exists y P(x, y) \land \neg \exists x Q(x) \).
2. (7 points) \( \forall x \exists y Q(x, y) \lor \exists x \forall y \neg Q(x, y) \Rightarrow \forall x \forall y (x = y \Rightarrow Q(x, y)) \).
3. (7 points) \( \forall x \exists y P(x, y) \lor \forall x \exists y \neg P(x, y) \).

Exercice 4 (Skolemization (15 points)) Let \( A = \forall x((\exists y P(x, y) \Rightarrow \exists x Q(x)) \land \exists y P(x, y) \land \neg \exists x Q(x)) \).

1. (2 points) Skolemize \( A \).
2. (3 points) Give the clausal form \( F(A) \) of \( A \).
3. (5 points) Give a set of contradictory instances of \( F(A) \), and establish the contradiction using propositional resolution.
4. (5 points) Prove that \( F(A) \) is contradictory using first order resolution.

Exercice 5 (First Order Resolution (20 points)) Prove that the following set of clauses is contradictory. First give a contradictory set of contradictory instances, then do a second proof by first order resolution.

1. \( \neg Q(x) \lor R(x) \lor S(y) \)
2. \( Q(a) \lor Q(x) \lor P(x, f(x)) \)
3. \( \neg R(x) \lor \neg Q(a) \)
4. \( Q(y) \lor \neg P(y, z) \)
5. \( \neg S(z) \lor \neg Q(z) \)

Exercice 6 (First Order Natural Deduction (20 points)) Prove the following formulas using natural deduction: Prouver les formules suivantes par déduction naturelle au premier ordre.

1. (5 points) \( \neg \forall x P(x) \lor \neg \exists y Q(y) \Rightarrow \neg (\forall x P(x) \land \exists y Q(y)) \)
2. (5 points) \( \forall x (\forall y P(y) \Rightarrow R(x)) \Rightarrow (\exists y P(y) \Rightarrow \forall x R(x)) \)
3. (10 points) \( (\neg \forall x P(x)) \Rightarrow \exists x P(x) \)

Exercice 7 (Recurrence (10 points)) Prove by recurrence that for all \( n > 1 \) and any set of variables \( a_1, \ldots, a_{n-1}, a_n \), we have:

\[
a_1 \Rightarrow \ldots \Rightarrow a_{n-1} \Rightarrow a_n = a_1 + \ldots + a_{n-1} + a_n
\]