Self-Stabilizing Leader Election in Polynomial Steps

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February 16, 2015. LaBRI

1This work has been partially supported by the LabEx PERSYVAL-Lab (ANR-11-LABX-0025-01) and the AGIR project DIAMS.
Context
Distributed Systems

- Autonomous
- Interconnected
Distributed Systems

- Autonomous
- Interconnected
Distributed Systems

Process
- Autonomous
- Interconnected

Hypotheses
- Connected
- Bidirectional
- Identified
Distributed Systems

- Process: Autonomous, Interconnected
- Hypotheses: Connected, Bidirectional, Identified
- Expected Property: Fault-tolerance
Self-Stabilization

Edsger W. Dijkstra. Self-stabilizing systems in spite of distributed control. 1974
Self-Stabilization

Correct behavior          Transient faults          Stabilization          Correct behavior

Illegitimate configurations

Legitimate configurations

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Edsger W. Dijkstra. Self-stabilizing systems in spite of distributed control. 1974
Locally Shared Memory Model

Configuration

- Reading of the variables of the neighbors
- Enabled nodes
- Daemon election: models the asynchronism
- Update of the local states
Locally Shared Memory Model

Atomic Step

- Reading of the variables of the neighbors
Locally Shared Memory Model

Atomic Step

- Reading of the variables of the neighbors
- Enabled nodes
Locally Shared Memory Model

Atomic Step

- Reading of the variables of the neighbors
- Enabled nodes
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Locally Shared Memory Model

Atomic Step

- Reading of the variables of the neighbors
- Enabled nodes
- Daemon election: models the asynchronism
- Update of the local states
Daemons

- Synchronous
- Central / Distributed
- Fairness: Strongly Fair, Weakly Fair, Unfair
Complexity

- In space: memory requirement in bits
- In time (mainly stabilization time)
  - In (atomic) steps
  - In rounds (execution time according slowest processes)
Rounds

1st round

2nd round

Key: Enabled 🌟 Activated ⭐ Neutralized ⭐️

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Leader Election

- Distinguish a process: the leader
Leader Election

- Distinguish a process: the leader
Leader Election

- Distinguish a process: the leader
- Every process eventually knows the identifier of the leader
Leader Election

- Distinguish a process: the leader
- Every process eventually knows the identifier of the leader
Problem

- **Silent Self-stabilizing Leader Election**

  **Model:**
  - Locally shared memory model
  - Read/write atomicity
  - Distributed unfair daemon

  **Network:**
  - Any connected topology
  - Bidirectional
  - Identified

- **No global knowledge on the network**
# State of the Art

<table>
<thead>
<tr>
<th>Model</th>
<th>Paper</th>
<th>Knowledge</th>
<th>Daemon</th>
<th>Complexity</th>
<th>Silent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Message Passing</td>
<td>Afek, Bremler, 1998</td>
<td>x</td>
<td></td>
<td>$\Theta(\log n)$</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>Awerbuch et al, 1993</td>
<td>x</td>
<td></td>
<td>$O(D)$</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>Burman, Kutten, 2007</td>
<td>x</td>
<td></td>
<td>$O(D)$</td>
<td>✓</td>
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<td>x</td>
<td></td>
<td>$O(D)$</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>Dolev, Herman, 1997</td>
<td>x</td>
<td>Fair</td>
<td>$\Theta(N \log N)$</td>
<td>?</td>
</tr>
<tr>
<td></td>
<td>Arora, Gouda, 1994</td>
<td>x</td>
<td>Weakly Fair</td>
<td>$\Theta(\log N)$</td>
<td>?</td>
</tr>
<tr>
<td></td>
<td>Datta et al, 2010</td>
<td>x</td>
<td>Unfair</td>
<td>unbounded</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>Kravchik, Kutten, 2013</td>
<td>x</td>
<td>Synchronous</td>
<td>$\Theta(\log n)$</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>Datta et al, 2011</td>
<td>x</td>
<td>Unfair</td>
<td>$\Theta(\log n)$</td>
<td>✓</td>
</tr>
</tbody>
</table>

$D$: Diameter
$D \geq D$: Upper bound on the diameter

$n$: Number of nodes
$N \geq n$: Upper bound on the number of nodes

$B$: Upper bound on the link-capacity
Our Contribution

Algorithm \( \mathcal{LE} \)

- **Memory requirement asymptotically optimal**: \( \Theta(\log n) \) bits/process
- **Stabilization time (worst case):**
  - 3\( n + D \) rounds
  - Lower Bound: \( \frac{n^3}{6} + \frac{3}{2} n^2 - \frac{8}{3} n + 2 \) steps,
  - Upper Bound: \( \frac{n^3}{2} + 2n^2 + \frac{n}{2} + 1 \) steps
Our Contribution

Algorithm $\mathcal{LE}$

- Memory requirement asymptotically optimal: $\Theta(\log n)$ bits/process
- Stabilization time (worst case):
  - $3n + D$ rounds
  - Lower Bound: $\frac{n^3}{6} + \frac{3}{2} n^2 - \frac{8}{3} n + 2$ steps,
  - Upper Bound: $\frac{n^3}{2} + 2n^2 + \frac{n}{2} + 1$ steps

Analytical Study of Datta et al, 2011

- Stabilization time not polynomial in steps:
  - $\forall \alpha \geq 3$, $\exists$ networks and executions in $\Omega(n^{\alpha+1})$ steps.

---

3 Datta, Larmore, and Vemula. Self-stabilizing Leader Election in Optimal Space under an Arbitrary Scheduler. 2011
Design of the Leader Election Algorithm
Join a Tree

3 variables per process $p$

- $p.idR \in \mathbb{N}$: ID of the root
- $p.par \in \mathcal{N}_p \cup \{p\}$: Parent pointer
- $p.level \in \mathbb{N}$: Level

Initial Configuration

$\langle 1, 0 \rangle$

- $p.idR = 1$
- $p.par = 1$
- $p.level = 0$

Key: $\langle idR, level \rangle$
Simplified Algorithm (Non Self-stabilizing)

Join a Tree

3 variables per process $p$
- $p.idR \in \mathbb{N}$: ID of the root
- $p.par \in \mathcal{N}_p \cup \{p\}$: Parent pointer
- $p.level \in \mathbb{N}$: Level

Initial Configuration
- $p.idR = p$
- $p.par = p$
- $p.level = 0$

Key: $\langle idR, level \rangle$

- $\langle 1, 0 \rangle$
- $\langle 3, 0 \rangle$
- $\langle 5, 0 \rangle$
- $\langle 6, 0 \rangle$
- $\langle 7, 0 \rangle$
- $\langle 4, 0 \rangle$
- $\langle 2, 0 \rangle$
Simplified Algorithm (Non Self-stabilizing)

Join a Tree

3 variables per process $p$

- $p.idR \in \mathbb{N}$: ID of the root
- $p.par \in \mathcal{N}_p \cup \{p\}$: Parent pointer
- $p.level \in \mathbb{N}$: Level

Initial Configuration

- $p.idR = p$
- $p.par = p$
- $p.level = 0$

Key: $\langle idR, level \rangle$
Simplified Algorithm (Non Self-stabilizing)

Join a Tree

3 variables per process $p$
- $p.idR \in \mathbb{N}$: ID of the root
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- $p.level \in \mathbb{N}$: Level

Initial Configuration
- $p.idR = p$
- $p.par = p$
- $p.level = 0$

Key: $\langle idR, level \rangle$

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Join a Tree

3 variables per process $p$
- $p.idR \in \mathbb{N}$: ID of the root
- $p.par \in \mathcal{N}_p \cup \{p\}$: Parent pointer
- $p.level \in \mathbb{N}$: Level

Initial Configuration
- $p.idR = p$
- $p.par = p$
- $p.level = 0$

Key: $\langle idR, level \rangle$
Simplified Algorithm (Non Self-Stabilizing)

Self-stabilization $\implies$ Arbitrary initialization

$\langle 1, 1 \rangle$, $\langle 3, 0 \rangle$, $\langle 4, 0 \rangle$, $\langle 1, 1 \rangle$

Key: $\langle idR, level \rangle$
Self-stabilization $\implies$ Arbitrary initialization $\implies$ Fake ids
Simplified Algorithm (Non Self-Stabilizing)

Self-stabilization $\Rightarrow$ Arbitrary initialization $\Rightarrow$ Fake ids

Key: $\langle idR, level \rangle$

Diagram:

- Node 2: $\langle 1, 1 \rangle$
- Node 3: $\langle 1, 2 \rangle$
- Node 4: $\langle 1, 2 \rangle$
- Node 5: $\langle 1, 1 \rangle$
Simplified Algorithm: Removal of Fake Ids

Reset

\[ p. idR = p. par = 0 \]

\[ \langle idR, level \rangle \]

Key:

\[ \langle 1, 1 \rangle \leftrightarrow \langle 1, 2 \rangle \rightarrow \langle 1, 2 \rangle \rightarrow \langle 1, 1 \rangle \]

Inconsistency

\[ \langle 1, 1 \rangle \]

\[ \langle 1, 2 \rangle \]

\[ \langle 1, 2 \rangle \]

\[ \langle 1, 1 \rangle \]
Simplified Algorithm: Removal of Fake Ids

Reset

- \( p.idR = p \)
- \( p.par = p \)
- \( p.level = 0 \)

Key: \( \langle idR, level \rangle \)
Simplified Algorithm: Removal of Fake Ids

Reset

- $p.idR = p$
- $p.par = p$
- $p.level = 0$

Key: $\langle idR, level \rangle$
Simplified Algorithm: Removal of Fake Ids

Reset

- \( p.idR = p \)
- \( p.par = p \)
- \( p.level = 0 \)

Inconsistency

\[ \langle 2, 0 \rangle \quad \langle 1, 2 \rangle \quad \langle 1, 2 \rangle \quad \langle 5, 0 \rangle \]

Key: \( \langle idR, level \rangle \)
Simplified Algorithm: Removal of Fake Ids

Reset

- \( p.idR = p \)
- \( p.par = p \)
- \( p.level = 0 \)

\[
\begin{align*}
\langle 2, 0 \rangle & \quad \langle 3, 0 \rangle & \quad \langle 4, 0 \rangle & \quad \langle 5, 0 \rangle \\
2 \quad 3 \quad 4 \quad 5
\end{align*}
\]

Key: \( \langle idR, level \rangle \)
Simplified Algorithm: Removal of Fake Ids

Key: \( \langle idR, level \rangle \)
Simplified Algorithm: Removal of Fake Ids

Reset

Key: \((idR, level)\)
Simplified Algorithm: Removal of Fake Ids

Reset

Key: \( \langle idR, level \rangle \)
Simplified Algorithm: Removal of Fake Ids

Reset

Graph:

- Node 3 with key $\langle 1, 7 \rangle$
- Node 5 with key $\langle 1, 6 \rangle$
- Node 4 with key $\langle 4, 0 \rangle$
- Node 2 with key $\langle 1, 5 \rangle$
- Node 6 with key $\langle 6, 0 \rangle$

Key: $\langle idR, level \rangle$
Simplified Algorithm: Removal of Fake Ids

Reset

Key: \( \langle idR, level \rangle \)
Simplified Algorithm: Removal of Fake Ids

Reset

Key: \langle idR, level \rangle
Simplified Algorithm: Removal of Fake Ids

Reset

Key: \( \langle idR, level \rangle \)
Abnormal Trees

Key: $\langle idR, level \rangle$

Diagram showing nodes with their labels: 1. Node 2 with label $\langle 3, 1 \rangle$, 2. Node 3 with label $\langle 3, 0 \rangle$, 3. Node 4 with label $\langle 1, 2 \rangle$, 4. Node 5 with label $\langle 3, 2 \rangle$, 5. Node 7 with label $\langle 3, 1 \rangle$, 6. Node 8 with label $\langle 1, 1 \rangle$, and 7. Node 6 with label $\langle 1, 0 \rangle$.
Abnormal Trees

Key: \( \langle \text{idR}, \text{level} \rangle \)
Abnormal Trees

Key: \( \langle \text{idR}, \text{level} \rangle \)
Abnormal Trees

Abnormal root

KinshipOk

3

2

5

4

8

6

7

⟨3, 1⟩

⟨3, 2⟩

⟨3, 0⟩

⟨3, 1⟩

⟨1, 2⟩

⟨1, 0⟩

⟨3, 0⟩

⟨3, 1⟩

⟨1, 1⟩

⟨1, 2⟩

⟨3, 2⟩

Key: \(\langle idR, level \rangle\)
Abnormal Trees

Key: $\langle idR, level \rangle$

Abnormal root

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Abnormal Trees

Key: \( \langle idR, level \rangle \)
Abnormal Trees

Key: \( \langle \text{idR}, \text{level} \rangle \)
Abnormal Trees

\( T_2 \)

\( T_1 \)

\( T_3 \)

Key: \( \langle idR, level \rangle \)
Abnormal trees removal

Freeze before Remove

Add a variable $Status \in \{C, EB, EF\}$

- $C$: means “not involved in a tree removal”:
  - Only process of status $C$ can join a tree and
  - only by choosing a process of status $C$ as parent

- $EB$: Error Broadcast

- $EF$: Error Feedback
Abnormal trees removal

Freeze before Remove

Add a variable $\textbf{Status} \in \{C, EB, EF\}$

- $C$ means “not involved in a tree removal”:
  - Only process of status $C$ can join a tree and
  - only by choosing a process of status $C$ as parent
- $EB$: Error Broadcast
- $EF$: Error Feedback

KinshipOk should be modified to take possible inconsistencies of variables $\textbf{Status}$ into account!
Freeze before remove

Key: \( \langle \text{idR, level} \rangle \)

- Clean
- EBroadcast
- EF eedback
Freeze before remove

Key: \( \langle \text{idR, level} \rangle \)

- \( \text{Clean} \)
- \( \text{EBroadcast} \)
- \( \text{EF eedback} \)
Freeze before remove

Key: $\langle idR, \text{level} \rangle$

- Clean
- $EB$-broadcast
- $EF$-feedback

$C$ - $EB$-action

Graph:
- Node 6 with labels $\langle 1, 0 \rangle$
- Node 2 with labels $\langle 1, 1 \rangle$
- Node 8 with labels $\langle 1, 1 \rangle$

Diagram showing the relationship between nodes and $EB$-actions.
Freeze before remove

Key: $\langle idR, level \rangle$  
- **Clean**  
- **EBroadcast**  
- **EF eedback**

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Freeze before remove

Key: \(\langle \text{idR}, \text{level} \rangle\)

- Green: Clean
- Blue: EBroadcast
- Red: EF eedback

\[\langle 1, 0 \rangle \quad \langle 1, 1 \rangle \quad \langle 1, 1 \rangle\]

\[\langle 1, 5 \rangle \quad \langle 1, 6 \rangle\]
Freeze before remove

Key: \(\langle idR, \text{level} \rangle\)  
- Green: Clean  
- Blue: EBroadcast  
- Red: EF eedback

\(\langle 1, 5 \rangle\) \rightarrow 7 \rightarrow \langle 1, 6 \rangle

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Freeze before remove

Key: \langle idR, level \rangle

- Green: Clean
- Blue: EBroadcast
- Red: EF eedback

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Freeze before remove

Key: \(\langle \text{idR}, \text{level} \rangle\)

- Clean
- EBroadcast
- EF eedback

\[\langle 1, 0 \rangle\]
\[\langle 1, 1 \rangle\]
\[\langle 1, 1 \rangle\]
Freeze before remove

Key: \(\langle \text{idR}, \text{level} \rangle\)

- \(\text{Clean}\)
- \(\text{EBroadcast}\)
- \(\text{EF eedback}\)
Stabilization Time in Rounds

- No alive abnormal tree created
- Height of an abnormal tree: at most $n$
Stabilization Time in Rounds

- No alive abnormal tree created
- Height of an abnormal tree: at most $n$

**Cleaning:**
- EB-wave: $n$
- EF-wave: $n$
- R-wave: $n$
Stabilization Time in Rounds

- No alive abnormal tree created
- Height of an abnormal tree: at most \( n \)

**Cleaning:**
- EB-wave: \( n \)
- EF-wave: \( n \)
- R-wave: \( n \)

**Building of the Spanning Tree:** \( D \)
Stabilization Time in Rounds

- No alive abnormal tree created
- Height of an abnormal tree: at most $n$
- Cleaning:
  - EB-wave: $n$
  - EF-wave: $n$
  - R-wave: $n$
- Building of the Spanning Tree: $\mathcal{D}$

$O(3n + \mathcal{D})$ rounds
Lower Bound on the Worst Case Stabilization Time in Rounds (synchronous execution)

- $k$ links
- $j = k + 3$
- $D = n - k$

Key: $\langle \text{idR}, \text{level} \rangle$

- Clean
- EBroadcast
- EF eedback

Diagram:
- Nodes labeled with $\langle 0, 0 \rangle$, $\langle 0, 1 \rangle$, $\langle 0, 2 \rangle$, $\langle 0, 3 \rangle$, $\langle 0, n-2 \rangle$, $\langle 0, n-1 \rangle$
- Edges connecting nodes
- Node $j$ is labeled $\langle 0, j-2 \rangle$
Lower Bound on the Worst Case Stabilization Time in Rounds (synchronous execution)

- $k$ links
- $j = k + 3$
- $D = n - k$

Key:

- $\langle idR, level \rangle$
- $\text{Clean}$
- $\text{EBroadcast}$
- $\text{EFeedback}$
Lower Bound on the Worst Case Stabilization Time in Rounds (synchronous execution)

- $k$ links
- $j = k + 3$
- $\mathcal{D} = n - k$

$\mathcal{D} = n - k$

Key:

- Clean
- EBroadcast
- Feedback

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Lower Bound on the Worst Case Stabilization Time in Rounds (synchronous execution)

- $k$ links
- $j = k + 3$
- $D = n - k$

Key: $(idR, level)$

- Clean
- EBroadcast
- EF eedback

$n$
Lower Bound on the Worst Case Stabilization Time in Rounds (synchronous execution)

- \( k \) links
- \( j = k + 3 \)
- \( \mathcal{D} = n - k \)

\[
\begin{align*}
D & = n - k \\
j & = k + 3
\end{align*}
\]

Key:

\[\langle idR, level \rangle\]

- **Clean**
- **EBroadcast**
- **EFeedback**

\[
n + n
\]
Lower Bound on the Worst Case Stabilization Time in Rounds (synchronous execution)

- $k$ links
- $j = k + 3$
- $D = n - k$

$R$-wave

$\langle 0, n-1 \rangle$
$\langle 0, n-2 \rangle$
$\langle 0, 0 \rangle$
$\langle 0, j-2 \rangle$
$\langle 0, j-1 \rangle$
$\langle 0, j \rangle$
$\langle 0, j+1 \rangle$
$\langle 0, j+2 \rangle$
$\langle 0, j+3 \rangle$

Key:

- $\langle idR, level \rangle$
- $\text{Clean}$
- $\text{EBroadcast}$
- $\text{EFeeback}$

$n + n$

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Lower Bound on the Worst Case Stabilization Time in Rounds (synchronous execution)

- $k$ links
- $j = k + 3$
- $D = n - k$

$n + n + n$
Lower Bound on the Worst Case Stabilization Time in Rounds (synchronous execution)

- $k$ links
- $j = k + 3$
- $D = n - k$

$$n + n + n$$
Lower Bound on the Worst Case Stabilization Time in Rounds (synchronous execution)

- $k$ links
- $j = k + 3$
- $D = n - k$

Key:
- $\langle idR, level \rangle$
- $Clean$
- $EBroadcast$
- $EFeedback$

\[
\begin{align*}
n + n + n + (n - k)
\end{align*}
\]
Lower Bound on the Worst Case Stabilization Time in Rounds (synchronous execution)

- \(k\) links
- \(j = k + 3\)
- \(D = n - k\)

\[
D = n - k
\]

\[
\begin{align*}
\langle 1, 0 \rangle & \quad 1 \\
\langle 1, 1 \rangle & \quad n \\
\langle 1, n-k-2 \rangle & \quad j \\
\langle 1, n-k-1 \rangle & \quad \ldots \\
\langle 1, n-k \rangle & \quad 2 \\
\langle 1, n-k \rangle & \quad 4 \\
\langle 1, n-k \rangle & \quad 5
\end{align*}
\]

Key: \(\langle idR, level \rangle\)

- Clean
- EBroadcast
- EF eedback

\[
n + n + n + (n - k) = \text{exactly } 3n + D \text{ rounds}
\]
Stabilization Time in Steps

Death of an abnormal tree

At most $n$ alive abnormal trees + No alive abnormal tree created $\rightarrow$ At most $n + 1$ segments

In a segment $idR$: $[7, 5, 3, 2, 7, 3, J]\text{-action} [J]\text{-action} [EB]\text{-action} [EF]\text{-action} [R]\text{-action}\]

Death of an abnormal tree = End of the segment

• $n - 1 J\text{-actions}$
• 1 $EB\text{-action}$
• 1 $EF\text{-action}$
• 1 $R\text{-action}$

$\Rightarrow O(n)$ actions per process

$O(n^3)$ steps

Lower Bound: $n^3 6 + 3 2 n^2 - 8 3 n + 2$ steps

Upper Bound: $n^3 2 + 2 n^2 + n + 1$ steps

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Stabilization Time in Steps

A segment

Another segment

Death of an abnormal tree

At most $n$ alive abnormal trees $+$ No alive abnormal tree created

$\Rightarrow \mathcal{O}(n)$ actions per process

$\mathcal{O}(n^3)$ steps

Lower Bound: $n^3 + 3n^2 - 8n + 2$ steps

Upper Bound: $n^3 + 2n^2 + n + 1$ steps
Stabilization Time in Steps

At most $n$ alive abnormal trees $+$ No alive abnormal tree created $\rightarrow$ At most $n + 1$ segments
Stabilization Time in Steps

At most $n$ alive abnormal trees + No alive abnormal tree created \[\rightarrow\] At most $n + 1$ segments

In a segment

\begin{align*}
  \text{idR} : 7 & \xrightarrow{J\text{-action}} 5 \xrightarrow{J\text{-action}} 3 \xrightarrow{J\text{-action}} 2 \xrightarrow{EB\text{-action}} 7 \xrightarrow{J\text{-action}} 3 \\
  \text{Death of an abnormal tree} & = \text{End of the segment}
\end{align*}
At most $n$ alive abnormal trees + No alive abnormal tree created \[\rightarrow\] At most $n + 1$ segments

In a segment

- $n - 1$ $J$-actions
- 1 $EB$-action
- 1 $EF$-action
- 1 $R$-action

Death of an abnormal tree = End of the segment
Stabilization Time in Steps

At most \( n \) alive abnormal trees + No alive abnormal tree created → At most \( n + 1 \) segments

In a segment

\[
\begin{align*}
\text{idR} : 7 & \xrightarrow{J\text{-action}} 5 \quad 3 \xrightarrow{J\text{-action}} 2 \quad 7 \xrightarrow{J\text{-action}} 3 \\
\text{Death of an abnormal tree} & = \text{End of the segment}
\end{align*}
\]

- \( n - 1 \) \( J \)-actions
- 1 \( EB \)-action
- 1 \( EF \)-action
- 1 \( R \)-action

\( \Rightarrow O(n) \) actions per process

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Stabilization Time in Steps

At most $n$ alive abnormal trees + No alive abnormal tree created $\rightarrow$ At most $n + 1$ segments

In a segment

$\text{idR : 7} \xrightarrow{\text{J-action}} 5 \xrightarrow{\text{J-action}} 3 \xrightarrow{\text{J-action}} 2 \xrightarrow{\text{EB-action}} \xrightarrow{\text{EF-action}} \xrightarrow{\text{R-action}} \text{7} \xrightarrow{\text{J-action}} 3$

Death of an abnormal tree = End of the segment

- $n - 1$ J-actions
- 1 EB-action
- 1 EF-action
- 1 R-action

$\Rightarrow O(n)$ actions per process

$O(n^3)$ steps

Lower Bound: $\frac{n^3}{6} + \frac{3}{2}n^2 - \frac{8}{3}n + 2$ steps

Upper Bound: $\frac{n^3}{2} + 2n^2 + \frac{n}{2} + 1$ steps

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Lower Bound on the Worst Case Stabilization Time in Steps

\[ \Theta(n) \text{ reset} \Rightarrow \sum_{i=1}^{n} i \approx \frac{n^2}{2} \text{ steps} \]

Key:
- \[\langle \text{id}_R, \text{level} \rangle\]
- \[\text{Clean}\]
- \[\text{EBroadcast}\]
- \[\text{EFeedback}\]
Lower Bound on the Worst Case Stabilization Time in Steps

\[ \frac{1}{2} n^2 \quad \frac{1}{2} n^2 - 1 \quad \frac{1}{2} n^2 - 2 \quad \frac{1}{2} n^2 - 3 \quad \frac{1}{2} n^2 - 4 \quad \ldots \]

\[ \langle n - 1, 1 \rangle \quad \langle n - 2, 1 \rangle \quad \langle n - 3, 1 \rangle \quad \langle n - 4, 1 \rangle \quad \langle 1, 1 \rangle \]

Key: \( \langle idR, level \rangle \)

- **Clean**
- **EBroadcast**
- **EFeedback**
Lower Bound on the Worst Case Stabilization Time in Steps

\[
\begin{align*}
\langle 2n, 1 \rangle \\
\langle 2n-2, 1 \rangle \\
\langle 2n-3, 1 \rangle \\
\langle 2n-4, 1 \rangle \\
n \sum_{i=1}^{n} i 
\end{align*}
\]

\[
\begin{align*}
\langle n-1, 1 \rangle \\
\langle n-2, 1 \rangle \\
\langle n-3, 1 \rangle \\
\langle n-4, 1 \rangle \\
n \sum_{j=1}^{n-1} j 
\end{align*}
\]

\[\Theta(n)\] reset \Rightarrow \Theta(n^3)\] steps

Key: \(\langle idR, level \rangle\)
- Clean
- EBroadcast
- EFeedback

Stéphane Devismes (VERIMAG)
Lower Bound on the Worst Case Stabilization Time in Steps

Key: \[\langle \text{idR}, \text{level} \rangle\]

- Green: Clean
- Blue: EBroadcast
- Red: Feedback

\[\Theta(n) \text{ reset } \Rightarrow \sum_{i=1}^{n} \sum_{j=1}^{i} \Rightarrow \Theta(n^3) \text{ steps}\]
Lower Bound on the Worst Case Stabilization Time in Steps

\[ \Theta(n^3) \text{ steps} \]

Key:

- \langle idR, level \rangle
- \langle idR, level \rangle
- \langle idR, level \rangle

\( \langle 2n-1, 3 \rangle \)

\( \langle 2n-2, 3 \rangle \)

\( \langle 2n-3, 3 \rangle \)

\( \langle 2n, 1 \rangle \)

\( \langle n+1, 1 \rangle \)

\( \langle n-3, 2 \rangle \)

\( \langle n-3, 1 \rangle \)

\( \langle n-4, 1 \rangle \)

\( \Theta(n^3) \text{ reset} \Rightarrow n \sum_{j=1}^{j} \sum_{i=1}^{i} \Rightarrow \Theta(n^3) \text{ steps} \)
Lower Bound on the Worst Case Stabilization Time in Steps

\[ \sum_{j=1}^{n-2} j \sum_{i=1}^{2n-j-1} i \Rightarrow \Theta(n^3) \]
Lower Bound on the Worst Case Stabilization Time in Steps

\[ 2n^2 \cdot n - \frac{1}{2} \cdot n - 1 \]

\[ 2n^2 - 2n^2 - 3n^2 - 4n^2 - \ldots \]

\[ \langle 2n-2, 0 \rangle \]

\[ \langle 2n-3, 0 \rangle \]

\[ \langle 2n-2, 0 \rangle \]

\[ \langle 2n-3, 1 \rangle \]

\[ \langle 2n, 1 \rangle \]

\[ \langle n-4, 1 \rangle \]

\[ \langle n-3, 3 \rangle \]

\[ \langle n-1, 1 \rangle \]

\[ \ldots \]

\[ \Theta(n) \]

key: \( \langle \text{idR}, \text{level} \rangle \)

- Clean
- EBroadcast
- EFeedback

Case of the reset of \( 2n - 4 \) processes:

\[ 2n - 1 \]

\[ 2n - 2 \]

\[ 2n - 3 \]

\[ 2n - 4 \]

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Lower Bound on the Worst Case Stabilization Time in Steps

\[ \sum_{i=1}^{2n-1} 2^{-i} \leq \sum_{j=1}^{n} \frac{1}{j} \leq \Theta(n) \]

Key:
- \( \langle idR, level \rangle \)
- Clean
- EBroadcast
- EF eedback

Case of the reset of 2

\[ 2n - 4 \]

Steps

\[ \sum_{i=1}^{2n-1} 2^{-i} \leq \sum_{j=1}^{n} \frac{1}{j} \leq \Theta(n) \]
Lower Bound on the Worst Case Stabilization Time in Steps

\[ \Theta(n^3) \text{ steps} \]

Case of the reset of \(2n - 4\) processes:

\[ \langle 2n-2, 1 \rangle \quad \langle 2n-3, 0 \rangle \quad \langle 2n-4, 1 \rangle \]

Key:

- \(\langle idR, level \rangle\)
  - \(Clean\)
  - \(EBroadcast\)
  - \(EFeedback\)
Lower Bound on the Worst Case Stabilization Time in Steps

Case of the reset of $2n - 4$

processes: $2n - 1$ $2n - 2$ $2n - 3$ $2n - 4$ $\cdots$

idR = $2n - 2$

idR = $2n - 3$

\[ \Theta(n^3) \]

Key: \[ \langle idR, level \rangle \]

- $Clean$
- $EBroadcast$
- $EFeeback$

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Lower Bound on the Worst Case Stabilization Time in Steps

Case of the reset of $2n - 4$

processes: $2n - 1$ $2n - 2$ $2n - 3$ $2n - 4$ ...

$\langle 2n-3, 2 \rangle$

$\langle 2n-3, 1 \rangle$

$\langle 2n-3, 0 \rangle$

$\langle n-4, 1 \rangle$

Key: $\langle idR, level \rangle$

- Clean
- EBroadcast
- EF eedback

$\langle 1, 1 \rangle$

$\langle 2n, 1 \rangle$

$\langle 2n-1, 1 \rangle$

$\langle 2n-2, 1 \rangle$

$\langle 2n-3, 1 \rangle$

$\langle 2n-4, 1 \rangle$

$\theta(n)$ reset $\Rightarrow \sum_{j=1}^{n} i_{j} \sum_{i=1}^{n} i_{i} \Rightarrow \theta(n^3)$ steps
Lower Bound on the Worst Case Stabilization Time in Steps

Case of the reset of $2n - 4$

processes: $2n - 1$  $2n - 2$  $2n - 3$  $2n - 4$  \ldots

\begin{align*}
\langle idR, 0 \rangle = 2n - 2 \\
\langle idR, 0 \rangle = 2n - 3 \\
\langle idR, 0 \rangle = 2n - 3 \\
\langle idR, 0 \rangle = n - 4 \\
\langle 1, 1 \rangle \\
\langle 1, 1 \rangle \\
\langle 1, 1 \rangle \\
\langle 1, 1 \rangle \\
\end{align*}

$\Theta(n)$

$\Theta(n^3)$

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Lower Bound on the Worst Case Stabilization Time in Steps

Case of the reset of $2n - 4$

processes: $2n - 1$, $2n - 2$, $2n - 3$, $2n - 4$, ...

Key: $\langle idR, level \rangle$

- $Clean$
- $EBroadcast$
- $EFeedback$

Stéphane Devismes (VERIMAG)
Lower Bound on the Worst Case Stabilization Time in Steps

Case of the reset of $2n - 4$

processes: $2n - 1$ $2n - 2$ $2n - 3$ $2n - 4$ ...

idR = 2n - 2
idR = 2n - 3
idR = 2n - 3
idR = n - 4
idR = n - 4
idR = n - 4

...
Lower Bound on the Worst Case Stabilization Time in Steps

Case of the reset of $2n - 4$

Key: $\langle idR, level \rangle$
- $Clean$
- $EBroadcast$
- $EFeedback$

Processes:
- $2n - 1$
- $2n - 2$
- $2n - 3$
- $2n - 4$

$\sum_{i=1}^{j} i = \Theta(n^3)$

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Self-Stabilizing Leader Election
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Lower Bound on the Worst Case Stabilization Time in Steps

Case of the reset of $2n - 4$ processes:

$$j = 3$$

$$\sum_{i=1}^{j} i = \sum_{i=1}^{3} i = 1 + 2 + 3 = 6$$

$$\Theta(n) \text{ reset } \Rightarrow \sum_{j=1}^{n} \sum_{i=1}^{j} i \Rightarrow \Theta(n^3) \text{ steps}$$

Key:
- $\langle \text{idR}, \text{level} \rangle$
- $\langle \text{Clean} \rangle$
- $\langle \text{EBroadcast} \rangle$
- $\langle \text{EFeedback} \rangle$
Analytical Study of Datta et al, 2011

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4 Datta, Larmore, and Vemula. Self-stabilizing Leader Election in Optimal Space under an Arbitrary Scheduler. 2011
Principles

Join a tree

Key: \(\langle idR, level \rangle\)

- Can be joined
- Cannot be joined
Principles

Join a tree

Key: \( \langle idR, level \rangle \)

- Blue circle: Can be joined
- Red circle: Cannot be joined
Principles

Change of color

Key: $\langle idR, level \rangle$
- Can be joined
- Cannot be joined

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Principles

Change of color

Key: $\langle idR, level \rangle$

- Can be joined
- Cannot be joined
Principles
Change of color

Key: \(\langle idR, level \rangle\)  
- Can be joined
- Cannot be joined
Principles

Color Waves Absorption

Normal tree

Abnormal tree

Key: \( \langle idR, level \rangle \)

- Can be joined
- Cannot be joined
Principles
Color Waves Absorption

Key: \( \langle idR, level \rangle \)

- Can be joined
- Cannot be joined
Principles
Color Waves Absorption

Normal tree

Abnormal tree

Key: $\langle idR, level \rangle$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$\blackstar (i, j).idR = 0$

- Can be joined
- Cannot be joined

Can be joined

Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$\star (i, j).idR = 0$

- Can be joined
- Cannot be joined

Stéphane Devismes (VERIMAG) Self-Stabilizing Leader Election February 16, 2015. LaBRI 29 / 32
Datta et al, 2011
Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
- $(i,j)\cdot ID = (i - 1)\beta + j$
- $(i,j)\cdot idR = 0$
- Can be joined
- Cannot be joined

Can be joined

Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

(i, j).ID = (i - 1)\beta + j

\( (i, j).idR = 0 \)

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$\star (i, j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n^8}{8}$

Key:

$\langle i, j \rangle . ID = (i - 1)\beta + j$

$\star \langle i, j \rangle . idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
$(i, j).ID = (i - 1)\beta + j$

- $(i, j).idR = 0$

- Can be joined
- Cannot be joined

Can be joined

Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$\star (i, j).idR = 0$

- Can be joined
- Cannot be joined

Stéphane Devismes (VERIMAG)
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$i, j).idR = 0$

- Can be joined
- Cannot be joined

Stéphane Devismes (VERIMAG)

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Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
$(i, j).ID = (i - 1)\beta + j$

- *(i, j).idR = 0*
- Can be joined
- Cannot be joined

Stéphane Devismes (VERIMAG)
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$\blackstar (i, j).idR = 0$

- Can be joined
- Cannot be joined

Can be joined

Cannot be joined
Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
$(i, j).ID = (i - 1)\beta + j$
$(i, j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$\blackstar (i, j).idR = 0$

- Can be joined
- Cannot be joined

Stéphane Devismes (VERIMAG)
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
$(i,j).ID = (i - 1)\beta + j$
$(i,j).idR = 0$
- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

- Can be joined
- Cannot be joined

Stéphane Devismes (VERIMAG)
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
- $\langle i, j \rangle.ID = (i - 1)\beta + j$
- $\langle i, j \rangle.idR = 0$
- Can be joined
- Cannot be joined

\[\beta\]
Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
- $(i,j).ID = (i - 1)\beta + j$
- $(i,j).idR = 0$

Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$$\beta^2$$

Key:

- $(i, j).ID = (i - 1)\beta + j$
- $(i, j).idR = 0$

- Can be joined
- Cannot be joined

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Execution in $\Omega(n^4)$ steps: $\beta = \frac{n^8}{8}$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$$\beta^2$$

Key:

$(i, j).ID = (i - 1)\beta + j$

$\star (i, j).idR = 0$

Can be joined

Cannot be joined

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Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
$(i,j).ID = (i - 1)\beta + j$

$(i,j).idR = 0$

Can be joined

Cannot be joined

Stéphane Devismes (VERIMAG)
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

(i, j).ID = (i - 1)\beta + j

(i, j).idR = 0

- Can be joined
- Cannot be joined

Stéphane Devismes (VERIMAG)
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Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

\[(i, j).ID = (i - 1)\beta + j\]

\*(i, j).idR = 0

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

- $(i, j).ID = (i - 1)\beta + j$
- $(i, j).idR = 0$

- Can be joined
- Cannot be joined

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Datta et al, 2011
Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
(i, j).ID = (i - 1)$\beta$ + j
(i, j).idR = 0

Can be joined
Cannot be joined

$\beta^2$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$\beta^2$

Key:

$(i, j).ID = (i - 1)\beta + j$

$\star (i, j).idR = 0$

- Can be joined
- Cannot be joined

Can be joined

Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$\beta^2$

**Key:**

- $(i, j).ID = (i - 1)\beta + j$
- $(i, j).idR = 0$

- Can be joined
- Cannot be joined

- Can be joined
- Cannot be joined

Stéphane Devismes (VERIMAG)

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Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

- $(i, j).ID = (i - 1)\beta + j$
- $(i, j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$\beta^2$

Key:
$(i, j).ID = (i - 1)\beta + j$

- $(i, j).idR = 0$ can be joined
- $(i, j).idR \neq 0$ cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

- $(i, j).ID = (i - 1) \beta + j$
- $(i, j).idR = 0$
- Can be joined
- Cannot be joined

Can be joined

1,1
2,1
3,1
4,1
5,1
6,1
7,1
8,1

1,2
2,2
3,2
4,2
5,2
6,2
7,2
8,2

...
Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

- Can be joined
- Cannot be joined

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Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
- $(i, j).ID = (i - 1)\beta + j$
- $(i, j).idR = 0$
- Blue: Can be joined
- Red: Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

- $(i, j).ID = (i - 1)\beta + j$
- $(i, j).idR = 0$
- Can be joined
- Cannot be joined

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Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$\beta^2$

Key:

$(i,j).ID = (i - 1)\beta + j$

$(i,j).idR = 0$

- **Can be joined**
- **Cannot be joined**

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Self-Stabilizing Leader Election  
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Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
- $i,j).ID = (i - 1)\beta + j$
- $i,j).idR = 0$

* Can be joined
- Cannot be joined

Stéphane Devismes (VERIMAG)
Self-Stabilizing Leader Election
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Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

- Can be joined
- Cannot be joined

Stéphane Devismes (VERIMAG)  
Self-Stabilizing Leader Election  
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Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

\[(i,j) \text{. ID} = (i - 1)\beta + j\]

\[(i,j) \text{. idR} = 0\]

- Can be joined
- Cannot be joined

Stéphane Devismes (VERIMAG)
Self-Stabilizing Leader Election
February 16, 2015. LaBRI
Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

- $(i, j).ID = (i - 1)\beta + j$
- $(i, j).idR = 0$
- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$$\beta^2$$

Key:

- $(i, j).ID = (i - 1)\beta + j$
- $(i, j).idR = 0$
- Can be joined
- Cannot be joined

Can be joined

Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

- $\star (i, j).idR = 0$
- Can be joined
- Cannot be joined

Stéphane Devismes (VERIMAG)
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$\star (i, j).idR = 0$

- Can be joined
- Cannot be joined

$\beta^2$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$$\beta^2$$

Key:

$(i, j).ID = (i - 1)\beta + j$

- $\star (i, j).idR = 0$
- Can be joined
- Cannot be joined

Stéphane Devismes (VERIMAG)

Self-Stabilizing Leader Election

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Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$$(i, j).ID = (i - 1)\beta + j$$

$(i, j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
(i, j).ID = (i - 1)$\beta + j$

(i, j).idR = 0
- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i,j).ID = (i-1)\beta + j$

- $\star$: Can be joined
- $\bigstar$: Cannot be joined

Stéphane Devismes (VERIMAG)
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Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i,j).ID = (i - 1)\beta + j$

$\star (i,j).idR = 0$

- Can be joined
- Cannot be joined

- $\beta^2$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

\[
\beta^2
\]

Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

- Can be joined
- Cannot be joined

Can be joined

Cannot be joined
Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).\text{ID} = (i - 1)\beta + j$

- $(i, j).\text{idR} = 0$
- Can be joined
- Cannot be joined

Stéphane Devismes (VERIMAG)
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

\[ \beta^2 \]

Key:

- $(i, j).ID = (i - 1)\beta + j$
- $(i, j).idR = 0$

- Can be joined
- Cannot be joined

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Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$\star (i, j).idR = 0$

- Can be joined
- Cannot be joined

$\beta^2$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

(i, j).ID = (i − 1)$\beta$ + j

(i, j).idR = 0

Can be joined

Cannot be joined

Stéphane Devismes (VERIMAG)
Self-Stabilizing Leader Election
February 16, 2015. LaBRI
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i,j).ID = (i - 1)\beta + j$

$\star (i,j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$\beta^2$

Key:

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Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$\beta^3$

Key:

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Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

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- Cannot be joined
Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$$\beta = \Omega(n) \Rightarrow \Omega(n^4)$$

Key:

$(i, j).ID = (i - 1)\beta + j$

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∀α ≥ 3, ∃ networks and executions in Ω(\(n^{α+1}\)) steps.

Worst Case: \(Ω\left(\left(2n\right)^{\frac{1}{4}\log_2(2n)}\right)\) steps

Nodes from \(Ω(n^4)\)

New nodes
Goal

Design a self-stabilizing leader election algorithm that stabilizes in $O(D)$ rounds.
Goal

Design a self-stabilizing leader election algorithm that stabilizes in $O(\mathcal{D})$ rounds.

Hypotheses

- Unfair daemon
- Memory requirement of $\Theta(\log n)$ bits/process
Perspectives

Goal
Design a self-stabilizing leader election algorithm that stabilizes in $O(D)$ rounds.

Hypotheses
- Unfair daemon
- Memory requirement of $\Theta(\log n)$ bits/process

With the knowledge of $D \geq D$, ($D = O(D)$) : ✓
Perspectives

Goal
Design a self-stabilizing leader election algorithm that stabilizes in $O(D)$ rounds.

Hypotheses
- Unfair daemon
- Memory requirement of $\Theta(\log n)$ bits/process

- With the knowledge of $D \geq D$, ($D = O(D)$): √
- Without any global knowledge: ??
Thank you for your attention.

Do you have any questions?
Rounds

Processes

1\textsuperscript{st} round

2\textsuperscript{nd} round

Key: Enabled ★ Activated ★ Neutralized ★

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Experimental Results

Average stabilization time in rounds in UDGs ($n = 1000$)
Experimental Results

Average stabilization time in steps in UDGs ($D = 15$)