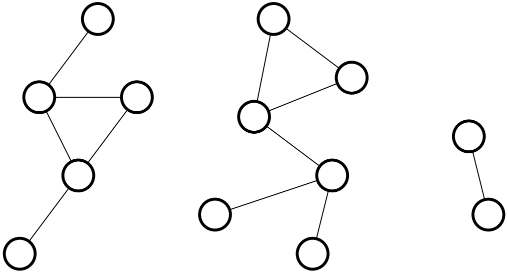
Self-Stabilizing Silent Disjunction in an Anonymous Network

Ajoy K. Datta   Stéphane Devismes   Lawrence L. Larmore

University of Nevada Las Vegas
Université Joseph Fourier, Grenoble

The Disjunction Problem



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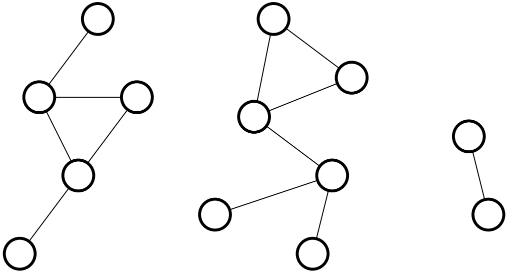
- **Anonymous** Network of **Processes**.



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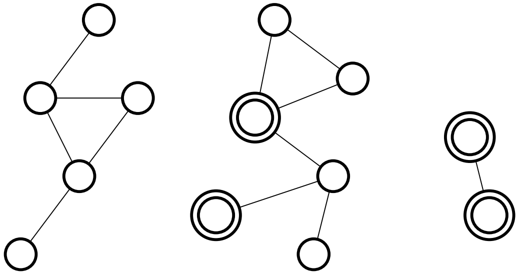
  - Each Process has **Input Bit**.



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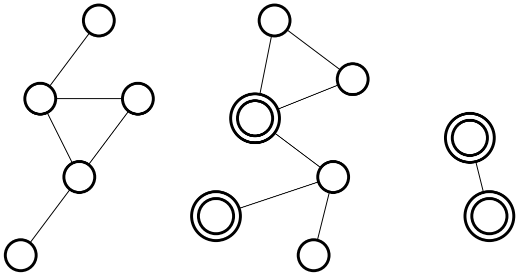

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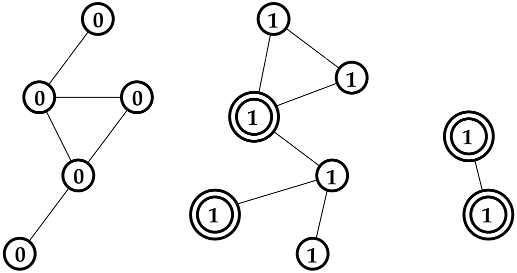

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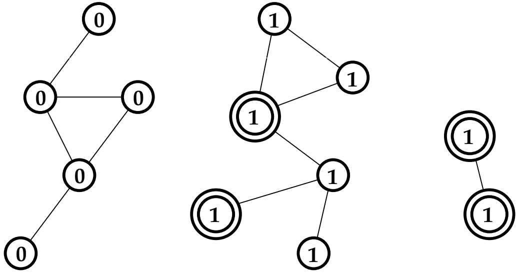

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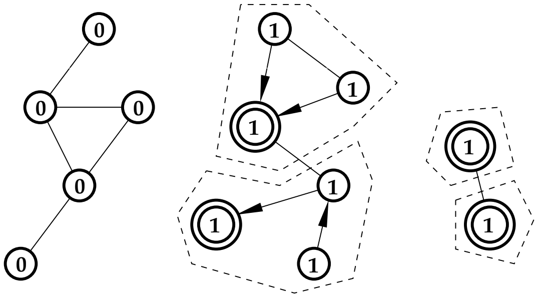
- (Construct **BFS Forest** Rooted at Processes with Input 1.)



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- Makes the Disjunction Problem **Easy**.

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- **Impossible** in an **Anonymous** Network.

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- Each Process with Input Bit 1 is a **Clusterhead**.

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Build Clusters!

- Each Process with Input Bit 1 is a Clusterhead.
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- **Local BFS Tree** in Every Cluster, Rooted at Clusterhead. (Defined by Parent Pointers and Levels.)

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  - All Processes have **Input 0:** might go into **Endless Loop!**

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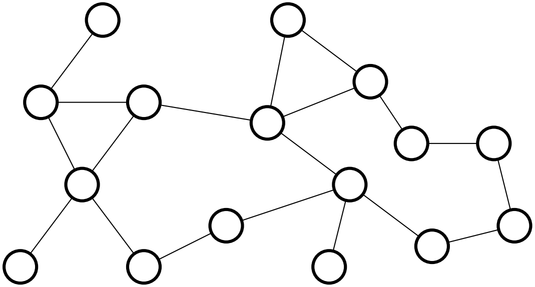
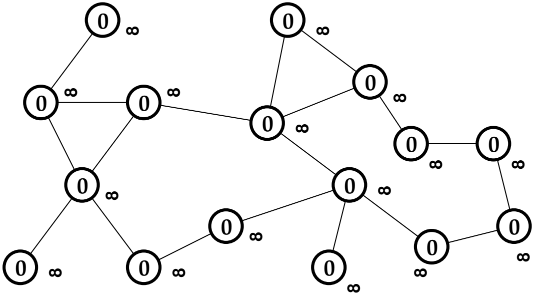


Clean Start

- All Input Bits 0.
- Clean Configuration is **Final.**



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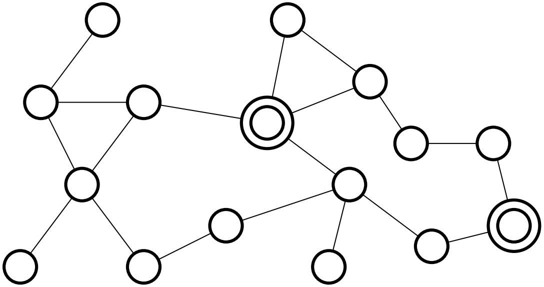

Clean Start

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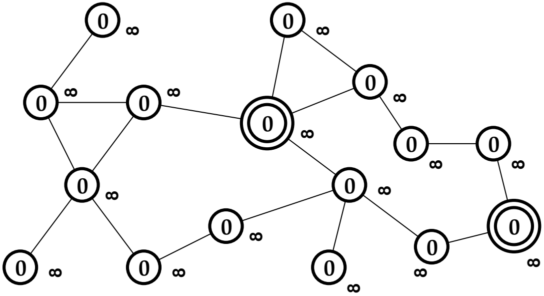
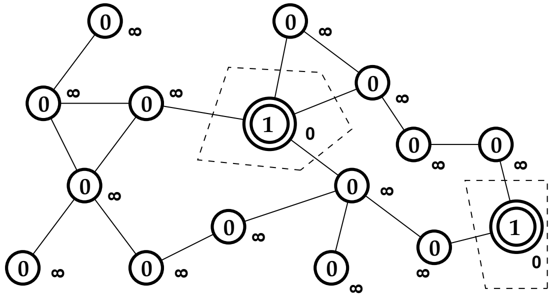


Clean Start

- Some Input Bits = 1.
- Clean Configuration is not Final.
- Clusterheads Execute **Reset.**



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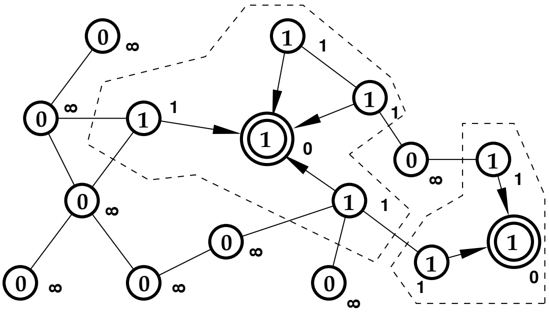

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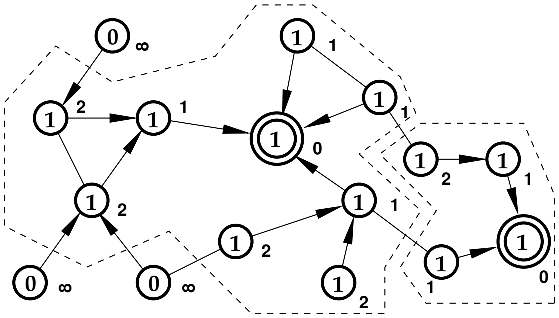

Clean Start

- Some Input Bits = 1.
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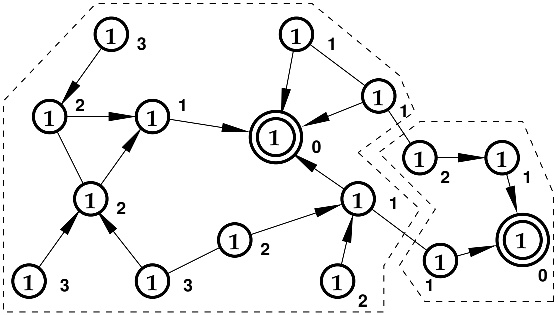
- Adjacent Processes Execute Join.
- **Flooding:** All Parent Pointers and Levels are Computed in at Most $(1 + \mathit{Diam})$ **Rounds.**



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- No Problem if Some Process has Input 1.
- Otherwise, might go into **Endless Loop.**

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VIII

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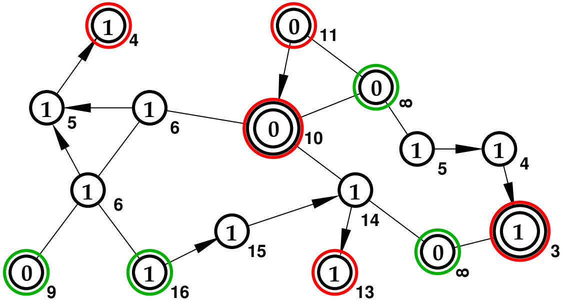

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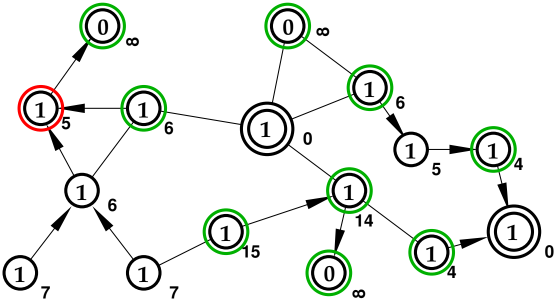

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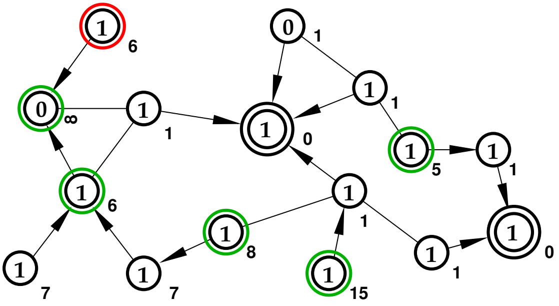

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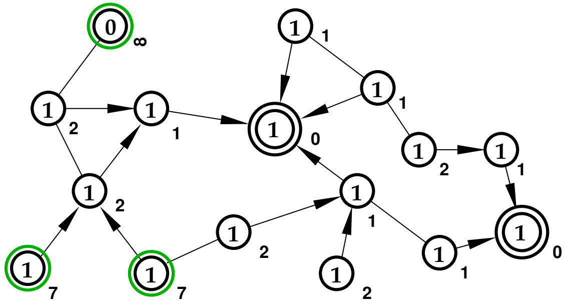

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- Some Input Bits 1.
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- Clusterheads Reset.
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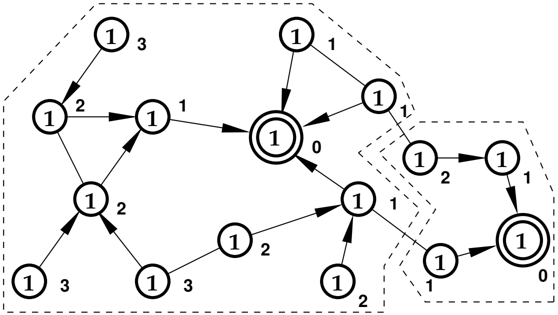
- **Convergence** within $1 + \mathit{Diam}$ **Rounds**.



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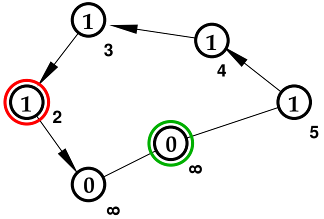

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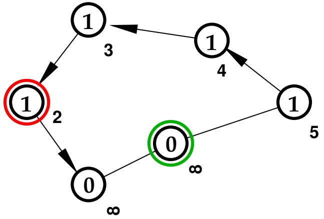

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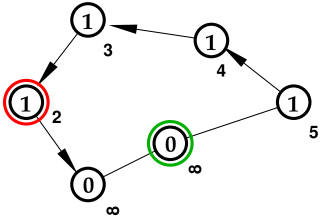

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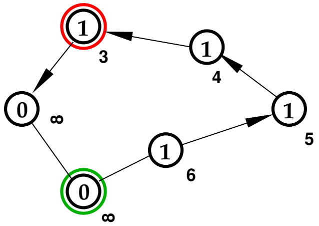

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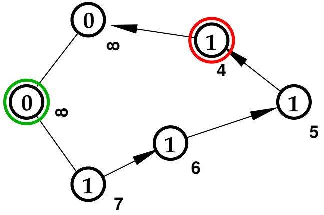

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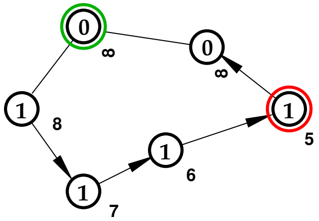

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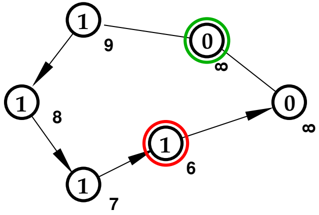

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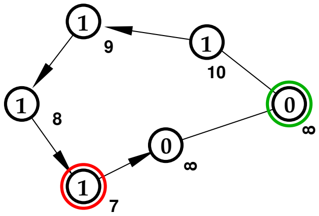

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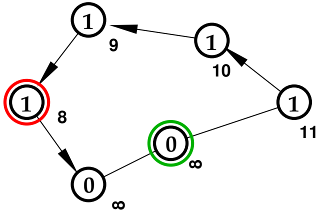

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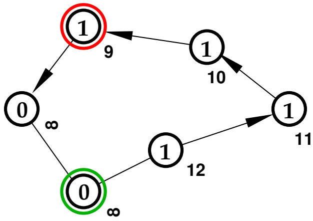

Color Wave Algorithm

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Self-Stabilizing and Silent

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- **Space Complexity.**
  - $O(\log Diam + Degree)$ **Per Process.**

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Side Effects of Color Waves

XIV

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- Color Lock Results Within $O(\mathit{Diam})$ Rounds.
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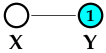

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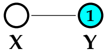
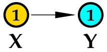
    - **Y** must have Color 1: 

    - Color of **X** Becomes 0: 

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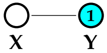
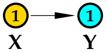
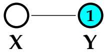
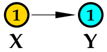
  - **Y** must have Color 1: 

  - Color of **X** Becomes 0: 

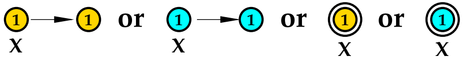
- Process **X** can **Change Color** if the **Following Conditions Hold:**

Color Wave Details:

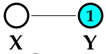
- Process with Output Bit = 1 has Color: 0 = , 1 = .
- If Process **X** Executes Join, Attaching to Process **Y**:
  - **Y** must have Color 1: 

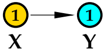
  - Color of **X** Becomes 0: 

- Process **X** can **Change Color** if the **Following Conditions Hold:**
  - **X** has **Same Color** as its **Parent**, or is **Clusterhead:**

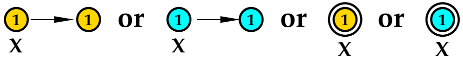


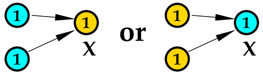
Color Wave Details:

- Process with Output Bit = 1 has Color: 0 = ①, 1 = ①.
- If Process **X** Executes Join, Attaching to Process **Y**:
  - **Y** must have Color 1: 

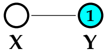
  - Color of **X** Becomes 0: 

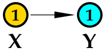
- Process **X** can **Change Color** if the **Following Conditions Hold:**
  - **X** has Same Color as its Parent, or is Clusterhead:



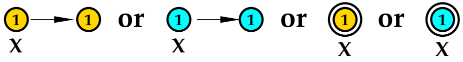
  - **Children** have **Opposite Color**: 

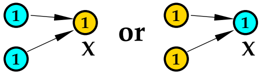
Color Wave Details:

- Process with Output Bit = 1 has Color: $0 =$ ①, $1 =$ ①.
- If Process **X** Executes Join, Attaching to Process **Y**:
  - **Y** must have Color 1: 

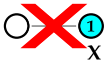
  - Color of **X** Becomes 0: 

- Process **X** can **Change Color** if the **Following Conditions Hold:**
  - **X** has Same Color as its Parent, or is Clusterhead:

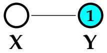


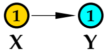
  - Children have Opposite Color: 

  - If **Color = 1, X** **Cannot Change Color** if Any Neighbor is

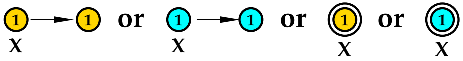
    Enabled to Attach to **X:** 

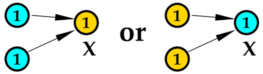
Color Wave Details:

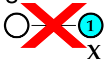
- Process with Output Bit = 1 has Color: 0 = , 1 = .
- If Process **X** Executes Join, Attaching to Process **Y**:
  - **Y** must have Color 1: 

  - Color of **X** Becomes 0: 

- Process **X** can Change Color if the Following Conditions Hold:
  - **X** has Same Color as its Parent, or is Clusterhead:

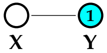


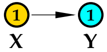
  - Children have Opposite Color: 

  - If Color = 1: **X** Cannot Change Color if Any Neighbor is Enabled to Attach to **X**: 

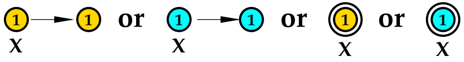
- When **Clusterhead** Changes Color, a **Color Wave** is **Absorbed.**

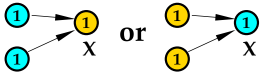
Color Wave Details:

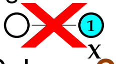
- Process with Output Bit = 1 has Color: 0 = , 1 = .
- If Process **X** Executes Join, Attaching to Process **Y**:
  - **Y** must have Color 1: 

  - Color of **X** Becomes 0: 

- Process **X** can Change Color if the Following Conditions Hold:
  - **X** has Same Color as its Parent, or is Clusterhead:



  - Children have Opposite Color: 

  - If Color = 1: **X** Cannot Change Color if Any Neighbor is Enabled to Attach to **X**: 

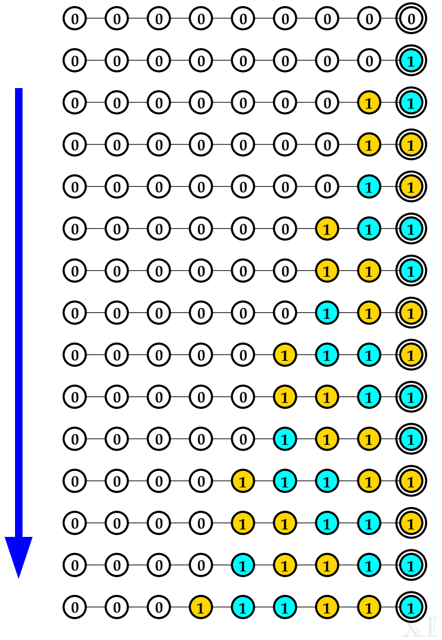
- When **Clusterhead** Changes Color, a **Color Wave** is **Absorbed.**
  - **False Roots Cannot Absorb Color Waves**.

Color Waves:

XI

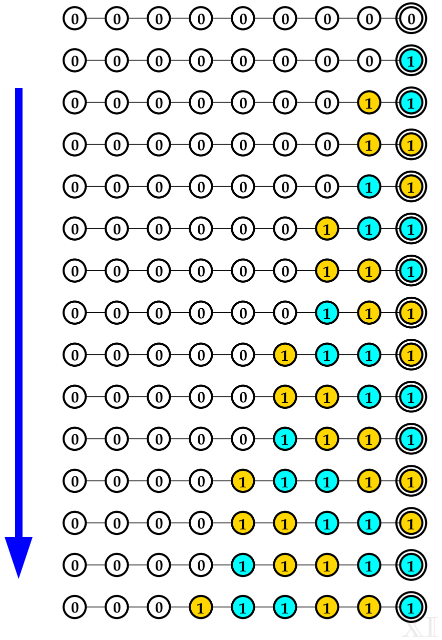
Color Waves:

- **Chain Example.**



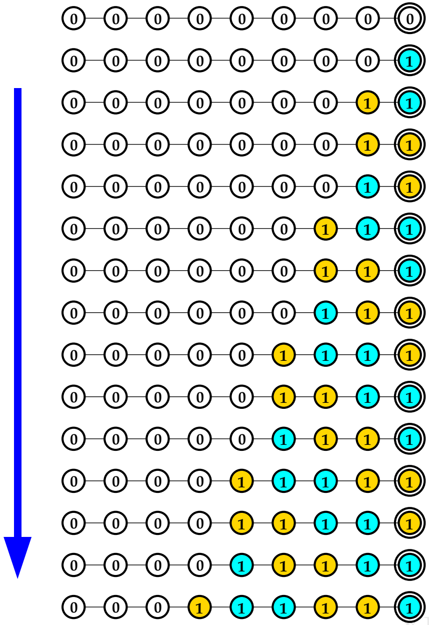
Color Waves:

- Chain Example.
- **One Process has Input = 1.**



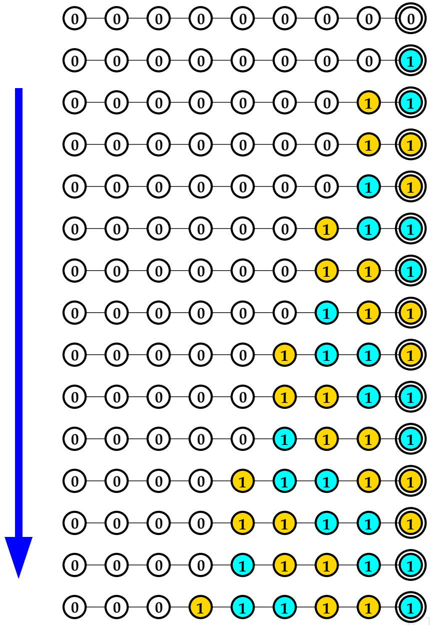
Color Waves:

- Chain Example.
- One Process has Input = 1.
- **Arrow** Shows Flow of Time.



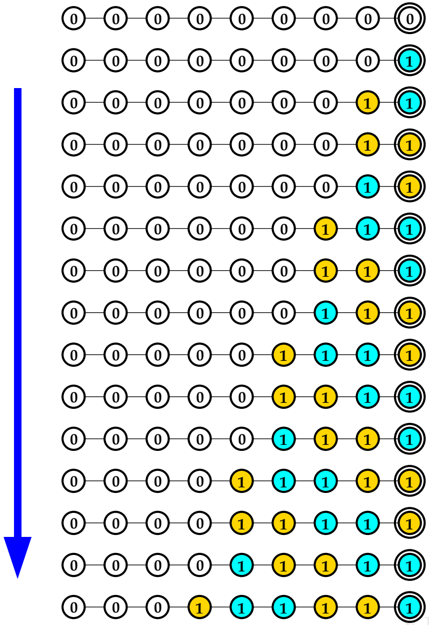
Color Waves:

- Chain Example.
- One Process has Input = 1.
- Arrow Shows Flow of Time.
- **Color Waves** Move
  Toward Clusterhead



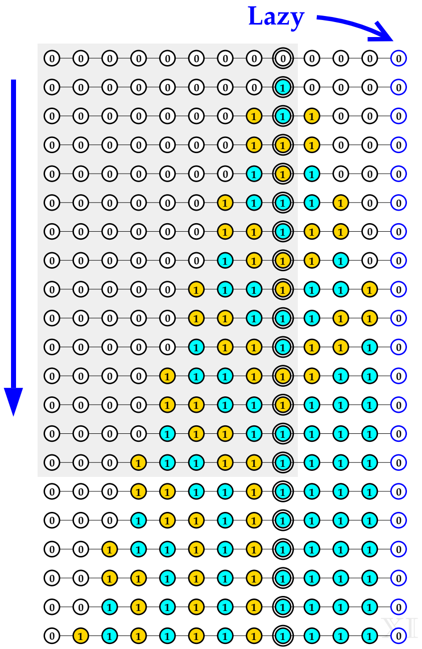
Color Waves:

- Chain Example.
- One Process has Input = 1.
- Arrow Shows Flow of Time.
- Color Waves Move
  Toward Clusterhead
- **Growth Rate = 1/3**.



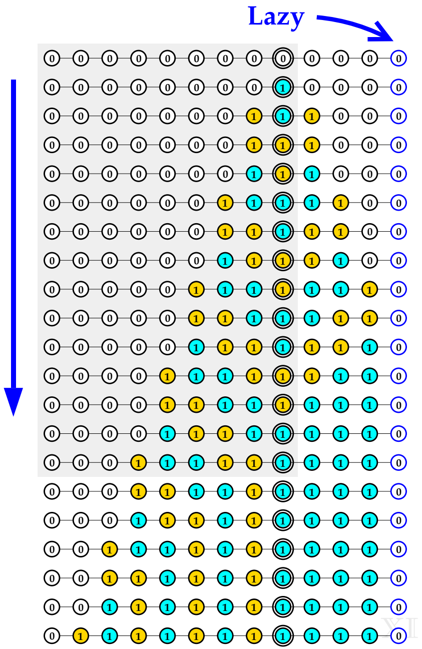
Color Waves:

- Chain Example.
- One Process has Input = 1.
- Arrow Shows Flow of Time.
- Color Waves Move
  Toward Clusterhead
- Growth Rate = 1/3.
- **If Any Process is “Lazy":**



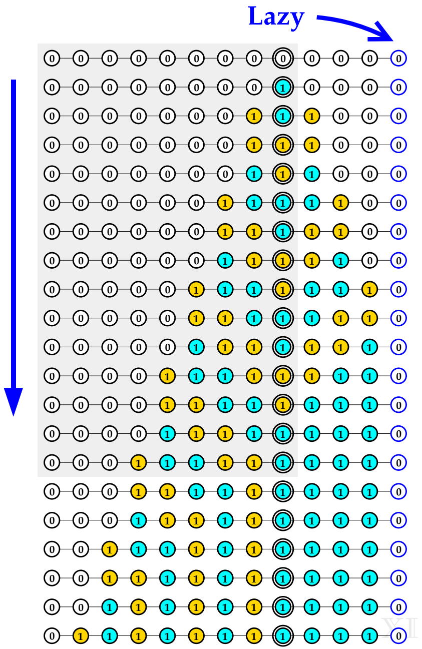
Color Waves:

- Chain Example.
- One Process has Input = 1.
- Arrow Shows Flow of Time.
- Color Waves Move
  Toward Clusterhead
- Growth Rate = 1/3.
- **If Any Process is "Lazy":**
  Eventual **Color Lock**:



Color Waves:

- Chain Example.
- One Process has Input = 1.
- Arrow Shows Flow of Time.
- Color Waves Move
  Toward Clusterhead
- Growth Rate = 1/3.
- **If Any Process is “Lazy":**
  Eventual **Color Lock**:
  That is, Lazy Process =
  Only Enabled Process.



Getting Rid of False Roots:

Getting Rid of False Roots:

- **If Any Process Stops Executing:**

Getting Rid of False Roots:

- **If Any Process Stops Executing:**
    - Eventual **Color Lock**:

Getting Rid of False Roots:

- If Any Process Stops Executing:
    - Eventual Color Lock:
- Consider a **False Root**, **R.**

Getting Rid of False Roots:

- If Any Process Stops Executing:
    - Eventual Color Lock:
- Consider a **False Root**, **R**.
    - **R** is Enabled Only to **Reset**, and thus Cannot Change Color.

Getting Rid of False Roots:

- If Any Process Stops Executing:
  - Eventual Color Lock:
- Consider a **False Root**, **R**.
  - **R** is Enabled Only to Reset, and thus Cannot Change Color.
  - **Tree Rooted at R Cannot Grow Forever.**

XIII

Getting Rid of False Roots:

- If Any Process Stops Executing:
  - Eventual Color Lock:
- Consider a **False Root**, **R**.
  - **R** is Enabled Only to Reset, and thus Cannot Change Color.
  - Tree Rooted at **R** Cannot Grow Forever.
  - **How can you Prove That?**

XIII

Getting Rid of False Roots:

- If Any Process Stops Executing:
  - Eventual Color Lock:
- Consider a **False Root**, **R**.
  - **R** is Enabled Only to Reset, and thus Cannot Change Color.
  - Tree Rooted at **R** Cannot Grow Forever.
  - How can you Prove That?
  - **Use Energy!**

XIII

Energy

Energy

- **Energy(X): Positive Integer** for **X** of **Output = 1**.

Energy

- Energy(**X**): Positive Integer for **X** of Output = 1.
- Defined **Recursively.**

Energy

- Energy(**X**): Positive Integer for **X** of Output = 1.
- Defined **Recursively.**
  - **Energy(X)** $= 1$ if **X** is **Leaf** of **Color = 0.**

Energy

- Energy(**X**): Positive Integer for **X** of Output = 1.
- Defined **Recursively.**
  - Energy(**X**) $= 1$ if **X** is Leaf of Color = 0.
  - **Energy(X)** $= 2$ if **X** is **Leaf** of **Color = 1.**

Energy

- Energy(**X**): Positive Integer for **X** of Output = 1.
- Defined **Recursively.**
  - Energy(**X**) $= 1$ if **X** is Leaf of Color = 0.
  - Energy(**X**) $= 2$ if **X** is Leaf of Color = 1.
  - **X Not Leaf: Energy**(**X**) = **Maximum** of:

Energy

- Energy(**X**): Positive Integer for **X** of Output = 1.
- Defined **Recursively.**
    - Energy(**X**) $= 1$ if **X** is Leaf of Color = 0.
    - Energy(**X**) $= 2$ if **X** is Leaf of Color = 1.
    - **X Not Leaf: Energy**(**X**) = **Maximum** of:
        - **1 + Energy** of any **Child of Opposite Color**.

Energy

- Energy(**X**): Positive Integer for **X** of Output = 1.
- Defined **Recursively.**
    - Energy(**X**) $= 1$ if **X** is Leaf of Color = 0.
    - Energy(**X**) $= 2$ if **X** is Leaf of Color = 1.
    - **X Not Leaf: Energy**(**X**) = **Maximum** of:
        - 1 + Energy of any Child of Opposite Color.
        - **2 + Energy** of any **Child of Matching Color**.

Energy

- Energy(**X**): Positive Integer for **X** of Output = 1.
- Defined Recursively.
  - Energy(**X**) $= 1$ if **X** is Leaf of Color = 0.
  - Energy(**X**) $= 2$ if **X** is Leaf of Color = 1.
  - **X** Not Leaf: Energy(**X**) = Maximum of:
    - 1 + Energy of any Child of Opposite Color.
    - 2 + Energy of any Child of Matching Color.
- **Theorem:** No Action of a **Process of Input = 0** can **Increase Maximum Energy** of the Network.

Energy

- Energy(**X**): Positive Integer for **X** of Output = 1.
- Defined Recursively.
  - Energy(**X**) $= 1$ if **X** is Leaf of Color = 0.
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  - **X** Not Leaf: Energy(**X**) = Maximum of:
    - 1 + Energy of any Child of Opposite Color.
    - 2 + Energy of any Child of Matching Color.
- Theorem: No Action of a Process of Input = 0 can Increase Maximum Energy of the Network.
- **Theorem:** If **All Processes have Input = 0:** **Maximum Energy** of the Network Decreases every **Round.** Hence **Convergence** After $O(n)$ **Rounds.**

Energy

- Energy(**X**): Positive Integer for **X** of Output = 1.
- Defined Recursively.
  - Energy(**X**) $= 1$ if **X** is Leaf of Color = 0.
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- Theorem: No Action of a Process of Input = 0 can
  Increase Maximum Energy of the Network.
- Theorem: If All Processes have Input = 0:
  Maximum Energy of the Network Decreases every Round,
  Hence Convergence After $O(n)$ Rounds.

Silence

XIV

Silence

- **Legitimate Configuration** After Finitely Many **Rounds:** $O(Diam)$ if **Output = 0**, $O(n)$ if **Output = 1.**

Silence

- Legitimate Configuration After Finitely Many Rounds:
  $O(Diam)$ if Output = 0, $O(n)$ if Output = 1.
- How do we Stop **Color Waves** from **Continuing Forever?**

Silence

- Legitimate Configuration After Finitely Many Rounds:
  $O(Diam)$ if Output = 0, $O(n)$ if Output = 1.
- How do we Stop Color Waves from Continuing Forever?
- **Done Waves:**

Silence

- Legitimate Configuration After Finitely Many Rounds:
  $O(Diam)$ if Output = 0, $O(n)$ if Output = 1.
- How do we Stop Color Waves from Continuing Forever?
- **Done Waves:**
  - **Leaf** Initiates when it **Detects** (Local) **Legitimacy.**

Silence

- Legitimate Configuration After Finitely Many Rounds:
  $O(Diam)$ if Output = 0, $O(n)$ if Output = 1.
- How do we Stop Color Waves from Continuing Forever?
- **Done Waves:**
  - Leaf Initiates when it Detects (Local) Legitimacy.
  - Done Wave **Convergecast.**

Silence

- Legitimate Configuration After Finitely Many Rounds: $O(Diam)$ if Output = 0, $O(n)$ if Output = 1.
- How do we Stop Color Waves from Continuing Forever?
- **Done Waves:**
  - Leaf Initiates when it Detects (Local) Legitimacy.
  - Done Wave Convergecast.
  - **Clusterhead** (That is, Process with Input = 1) Becomes **Color Frozen.** Cannot Change Color.

Silence

- Legitimate Configuration After Finitely Many Rounds:
  $O(Diam)$ if Output = 0, $O(n)$ if Output = 1.
- How do we Stop Color Waves from Continuing Forever?
- **Done Waves:**
  - Leaf Initiates when it Detects (Local) Legitimacy.
  - Done Wave Convergecast.
  - Clusterhead (That is, Process with Input = 1) Becomes Color Frozen. Cannot Change Color.
  - **Color Lock:** Within $O(Diam)$ Rounds: **Silence** is Achieved.

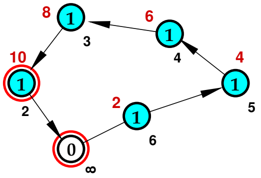
Arbitrary Start: Output = 0: Problem Solved!

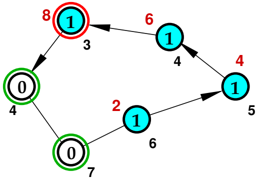
Arbitrary Start: Output = 0: Problem Solved!

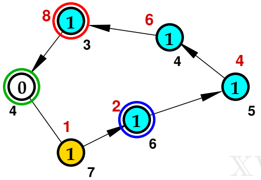
- **Asynchronous Example Computation**

Arbitrary Start: Output = 0: Problem Solved!

- **Asynchronous Example Computation**
- **All Input Bits 0.**

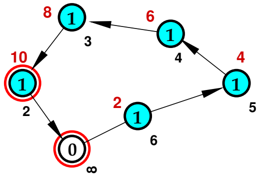


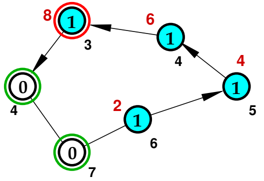


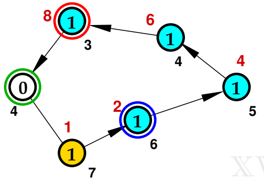


Arbitrary Start: Output = 0: Problem Solved!

- **Asynchronous Example Computation**
- All Input Bits 0.
- Red = Enabled to Reset.

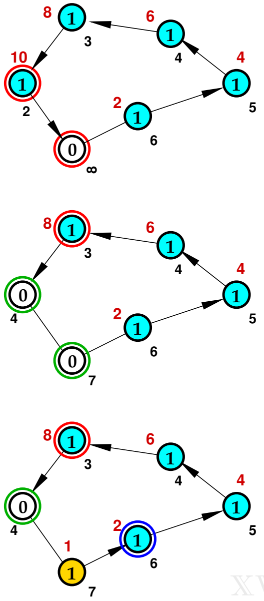






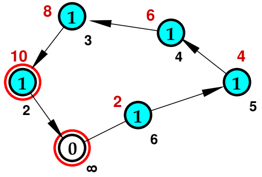
Arbitrary Start: Output = 0: Problem Solved!

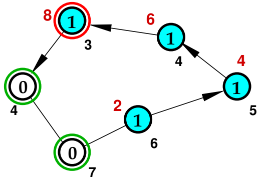
- **Asynchronous Example Computation**
- All Input Bits 0.
- Red = Enabled to Reset.
- Green = Enabled to Join.

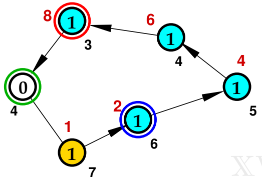


Arbitrary Start: Output = 0: Problem Solved!

- **Asynchronous Example Computation**
- All Input Bits 0.
- Red = Enabled to Reset.
- Green = Enabled to Join.
- Blue = Enabled to Change Color.

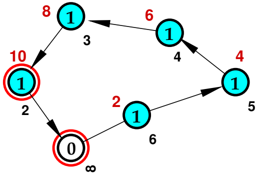


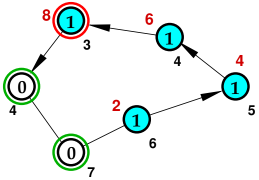


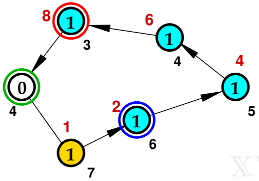


Arbitrary Start: Output = 0: Problem Solved!

- **Asynchronous Example Computation**
- All Input Bits 0.
- Red = Enabled to Reset.
- Green = Enabled to Join.
- Blue = Enabled to Change Color.
- Maximum **Energy** Initially = 10.

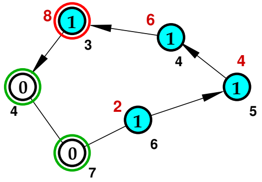


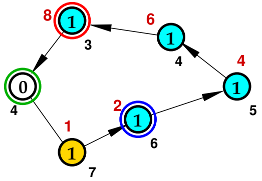


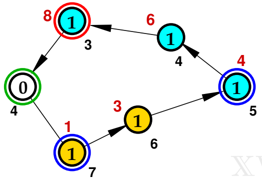


Arbitrary Start: Output = 0: Problem Solved!

- **Asynchronous Example Computation**
- All Input Bits 0.
- Red = Enabled to Reset.
- Green = Enabled to Join.
- Blue = Enabled to Change Color.
- Maximum **Energy** Initially = 10.

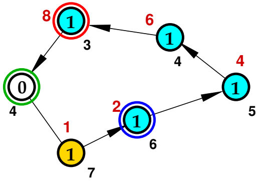


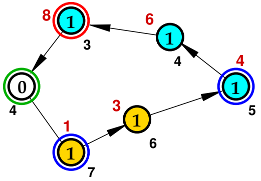


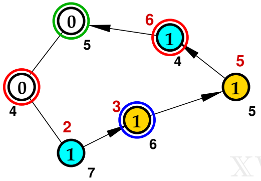


Arbitrary Start: Output = 0: Problem Solved!

- **Asynchronous Example Computation**
- All Input Bits 0.
- Red = Enabled to Reset.
- Green = Enabled to Join.
- Blue = Enabled to Change Color.
- Maximum Energy Initially = 10.
- Maximum **Energy Decreases** each **Round.**







Arbitrary Start: Output = 0: Problem Solved!

- **Asynchronous Example Computation**
- All Input Bits 0.
- Red = Enabled to Reset.
- Green = Enabled to Join.
- Blue = Enabled to Change Color.
- Maximum Energy Initially = 10.
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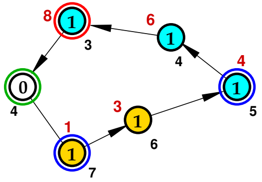
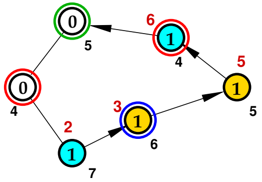
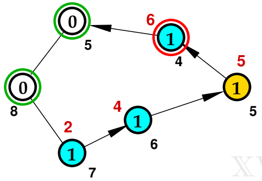
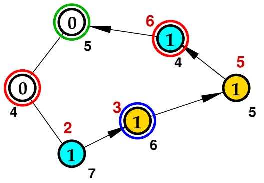
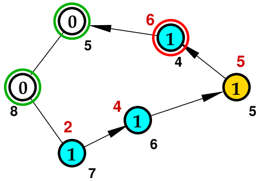
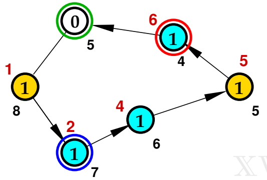




Arbitrary Start: Output = 0: Problem Solved!

- **Asynchronous Example Computation**
- All Input Bits 0.
- Red = Enabled to Reset.
- Green = Enabled to Join.
- Blue = Enabled to Change Color.
- Maximum Energy Initially = 10.
- Maximum **Energy Decreases** each **Round.**

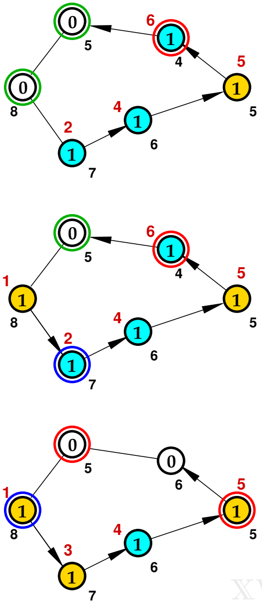






Arbitrary Start: Output = 0: Problem Solved!

- **Asynchronous Example Computation**
- All Input Bits 0.
- Red = Enabled to Reset.
- Green = Enabled to Join.
- Blue = Enabled to Change Color.
- Maximum Energy Initially = 10.
- Maximum **Energy Decreases** each **Round.**



Arbitrary Start: Output = 0: Problem Solved!

- **Asynchronous Example Computation**
- All Input Bits 0.
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- Blue = Enabled to Change Color.
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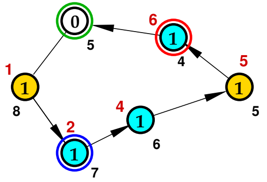
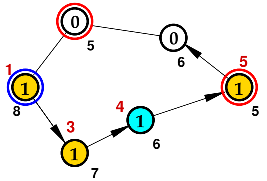
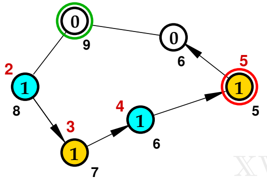
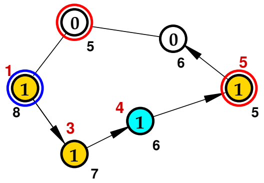


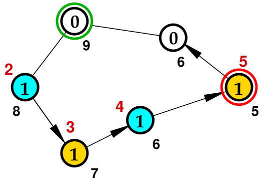


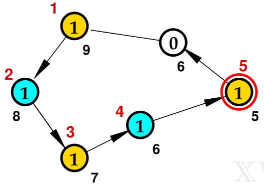


Arbitrary Start: Output = 0: Problem Solved!

- **Asynchronous Example Computation**
- All Input Bits 0.
- Red = Enabled to Reset.
- Green = Enabled to Join.
- Blue = Enabled to Change Color.
- Maximum Energy Initially = 10.
- Maximum Energy Decreases each Round.
- **No Further Growth Possible.**







Arbitrary Start: Output = 0: Problem Solved!

- **Asynchronous Example Computation**
- All Input Bits 0.
- Red = Enabled to Reset.
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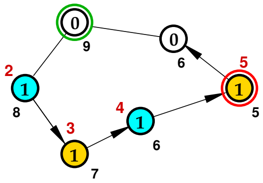
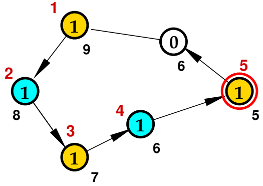
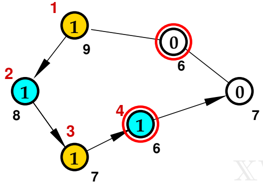
- Blue = Enabled to Change Color.
- Maximum Energy Initially = 10.
- Maximum Energy Decreases each Round.
- **No Further Growth Possible.**







Arbitrary Start: Output = 0: Problem Solved!

- **Asynchronous Example Computation**
- All Input Bits 0.
- Red = Enabled to Reset.
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- Maximum Energy Initially = 10.
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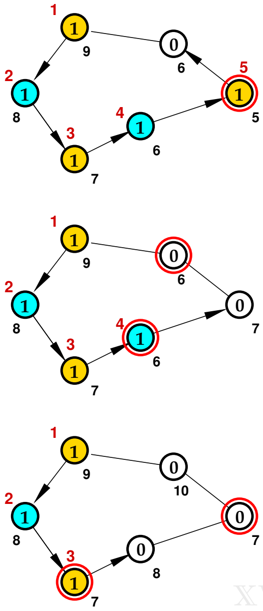
- **No Further Growth Possible.**



Arbitrary Start: Output = 0: Problem Solved!

- **Asynchronous Example Computation**
- All Input Bits 0.
- Red = Enabled to Reset.
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- Blue = Enabled to Change Color.
- Maximum Energy Initially = 10.
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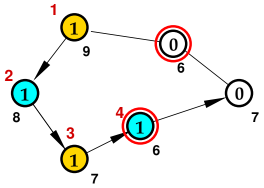
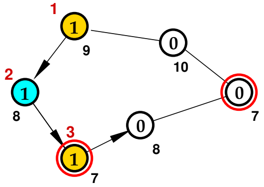
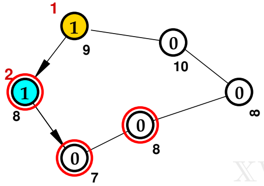
- **No Further Growth Possible.**

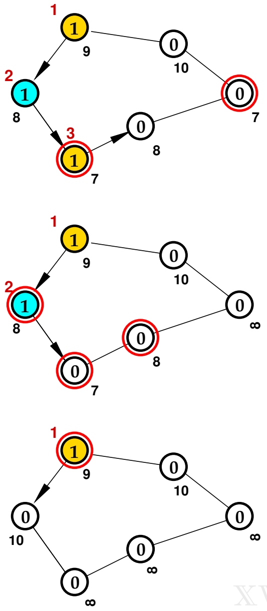






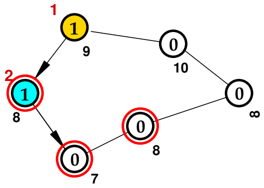
Arbitrary Start: Output = 0: Problem Solved!

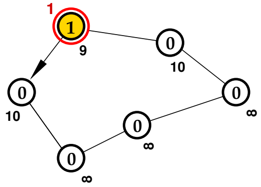
- **Asynchronous Example Computation**
- All Input Bits 0.
- Red = Enabled to Reset.
- Green = Enabled to Join.
- Blue = Enabled to Change Color.
- Maximum Energy Initially = 10.
- Maximum Energy Decreases each Round.
- **No Further Growth Possible.**

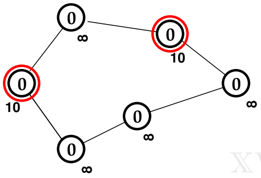


Arbitrary Start: Output = 0: Problem Solved!

- **Asynchronous Example Computation**
- All Input Bits 0.
- Red = Enabled to Reset.
- Green = Enabled to Join.
- Blue = Enabled to Change Color.
- Maximum Energy Initially = 10.
- Maximum Energy Decreases each Round.
- No Further Growth Possible.
- Configuration is Now **Legitimate.**

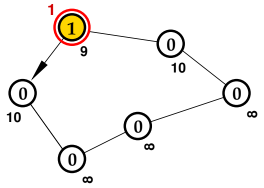


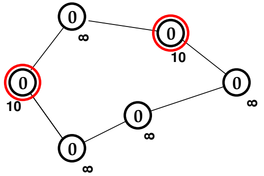


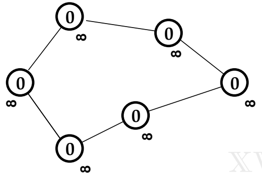


Arbitrary Start: Output = 0: Problem Solved!

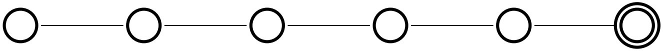
- **Asynchronous Example Computation**
- All Input Bits 0.
- Red = Enabled to Reset.
- Green = Enabled to Join.
- Blue = Enabled to Change Color.
- Maximum Energy Initially = 10.
- Maximum Energy Decreases each Round.
- No Further Growth Possible.
- Configuration is Now Legitimate.
- Configuration is Now **Final. Silent.**



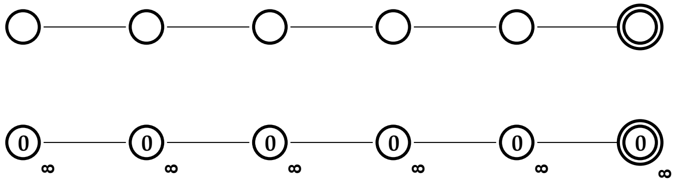




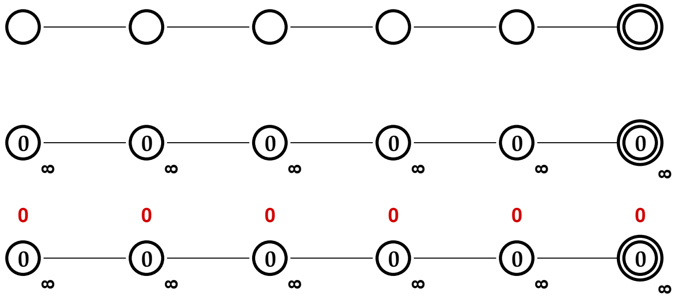
- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.



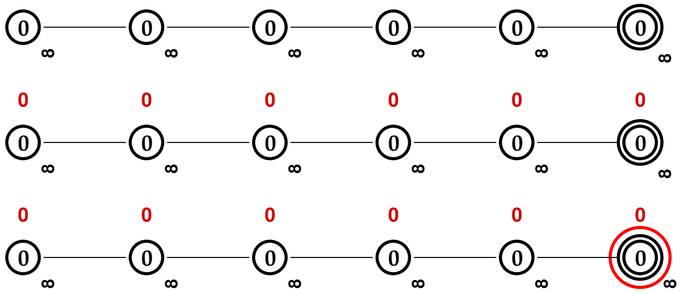
- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Output Bit:** Inside Circle. **Black Numeral:** Level.



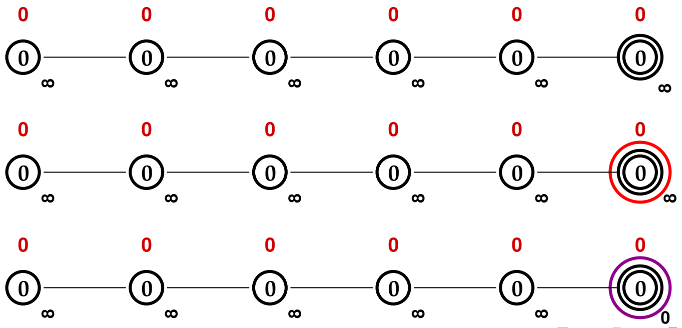
- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Output Bit:** Inside Circle. **Black Numeral:** Level.
- **Red Numeral:** Energy.



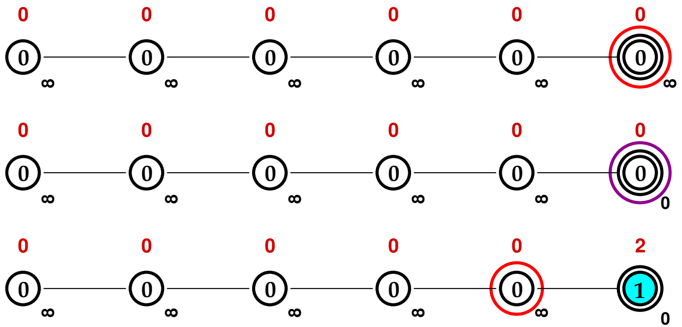
- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Output Bit:** Inside Circle. **Black Numeral:** Level.
- **Red Numeral:** Energy.
- **Red Circle:** Enabled to Reset.



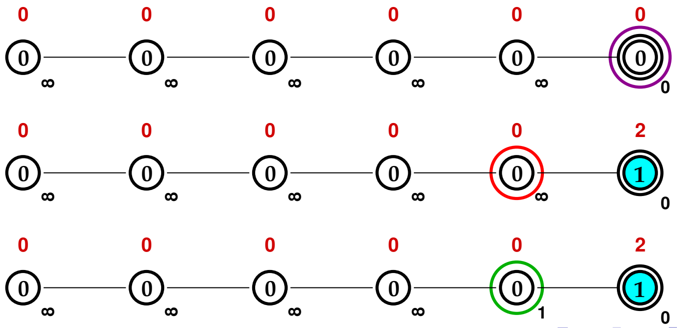
- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Output Bit:** Inside Circle. **Black Numeral:** Level.
- **Red Numeral:** Energy.
- **Red Circle:** Enabled to Reset.
- **Magenta Circle:** Enabled to Initialize.



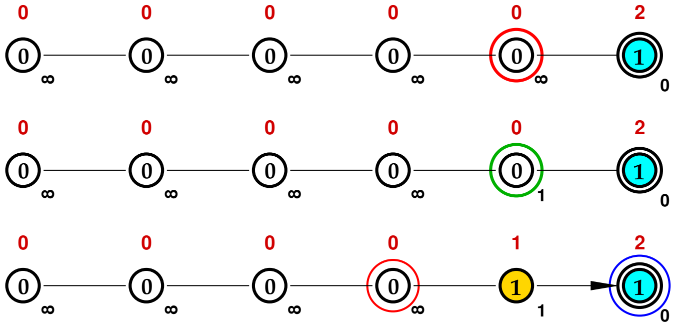
- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Output Bit:** Inside Circle. **Black Numeral:** Level.
- **Red Numeral:** Energy.
- **Red Circle:** Enabled to Reset.
- **Magenta Circle:** Enabled to Initialize.
- **Cyan:** Color = 1.



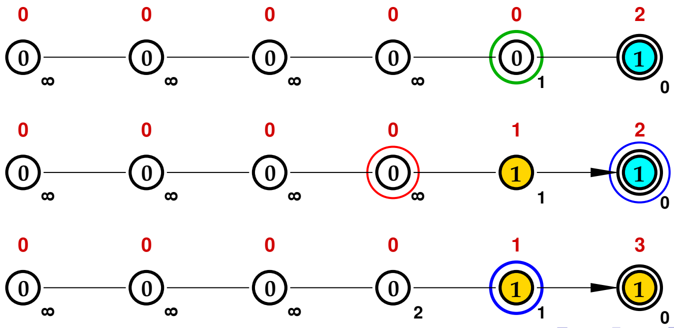
- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Output Bit:** Inside Circle. **Black Numeral:** Level.
- **Red Numeral:** Energy.
- **Red Circle:** Enabled to Reset.
- **Magenta Circle:** Enabled to Initialize.
- **Cyan:** Color = 1.
- **Green Circle:** Enabled to Join. Can only attach to Color 1.



- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Output Bit:** Inside Circle. **Black Numeral:** Level.
- **Red Numeral:** Energy.
- **Red Circle:** Enabled to Reset.
- **Cyan:** Color = 1; **Gold:** Color = 0.
- **Green Circle:** Enabled to Join. Can only attach to Color 1.
- **Blue Circle:** Enabled to Change Color.



- **Chain Example.** **Double Circle:** Input Bit = 1, Otherwise 0.
- **Red Numeral:** Energy.
- **Red Circle:** Enabled to Reset.
- **Cyan:** Color = 1; **Gold:** Color = 0.
- **Green Circle:** Enabled to Join. Can only attach to Color 1.
- **Blue Circle:** Enabled to Change Color.
- **Color Waves** Convergecast. **Absorbed** by Clusterhead.



- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Red Numeral:** Energy.
- **Red Circle:** Enabled to Reset.
- **Cyan:** Color = 1; **Gold:** Color = 0.
- **Green Circle:** Enabled to Join. Can only attach to Color 1.
- **Blue Circle:** Enabled to Change Color.
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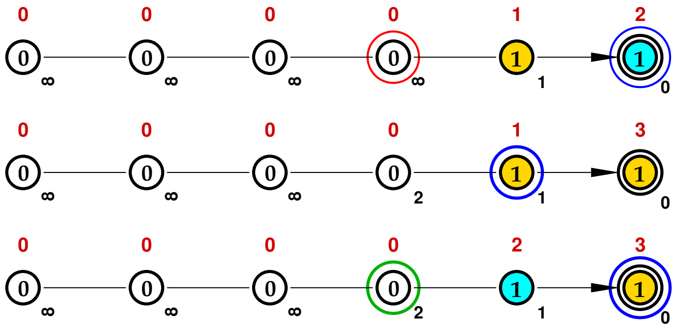

- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Red Numeral:** Energy.
- **Red Circle:** Enabled to Reset.
- **Cyan:** Color = 1; **Gold:** Color = 0.
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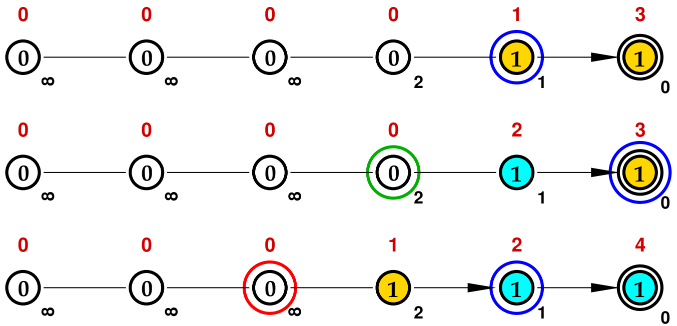
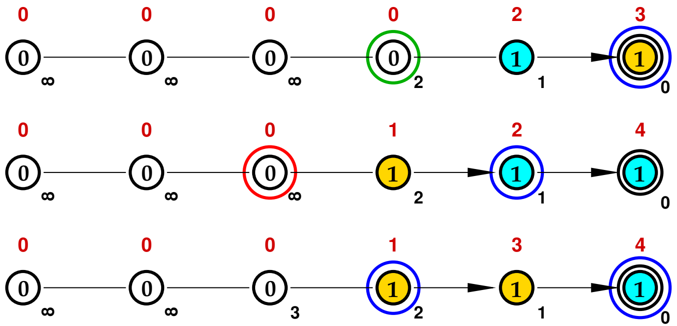


- **Chain Example.** **Double Circle:** Input Bit = 1, Otherwise 0.
- **Red Numeral:** Energy.
- **Red Circle:** Enabled to Reset.
- **Cyan:** Color = 1; **Gold:** Color = 0.
- **Green Circle:** Enabled to Join. Can only attach to Color 1.
- **Blue Circle:** Enabled to Change Color.
- **Color Waves** Convergecast. **Absorbed** by Clusterhead.



- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Red Numeral:** Energy.
- **Red Circle:** Enabled to Reset.
- **Cyan:** Color = 1; **Gold:** Color = 0.
- **Green Circle:** Enabled to Join. Can only attach to Color 1.
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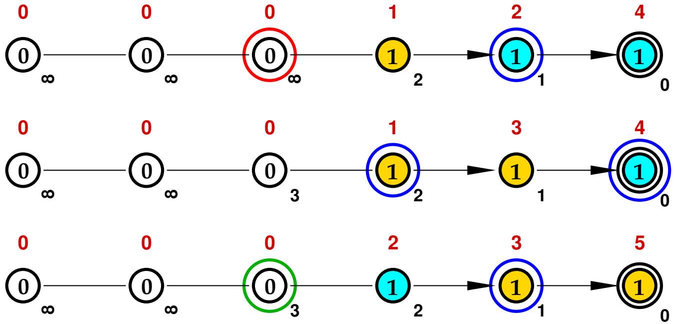

- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Red Numeral:** Energy.
- **Red Circle:** Enabled to Reset.
- **Cyan:** Color = 1; **Gold:** Color = 0.
- **Green Circle:** Enabled to Join. Can only attach to Color 1.
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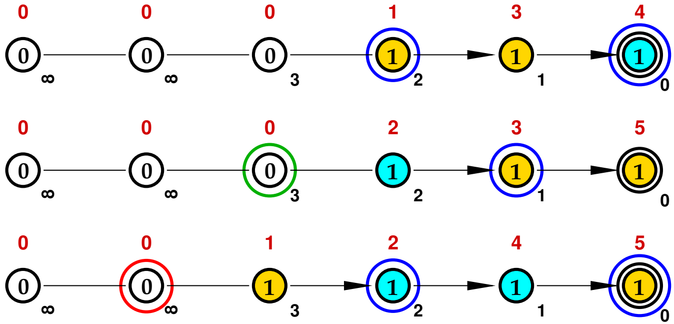

- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Red Numeral:** Energy.
- **Red Circle:** Enabled to Reset.
- **Cyan:** Color = 1; **Gold:** Color = 0.
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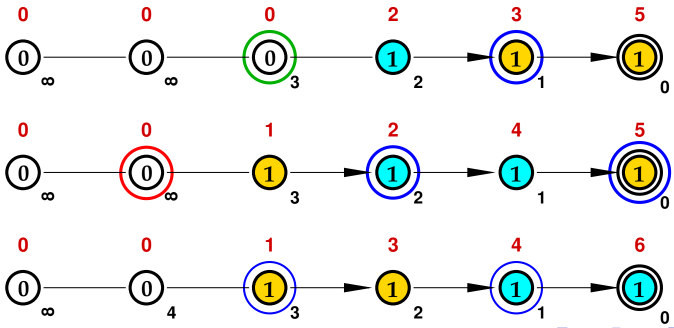

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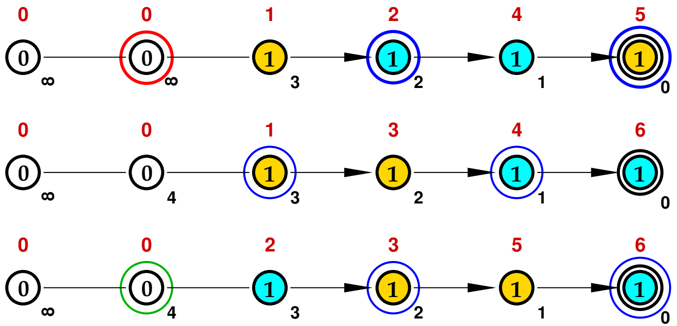

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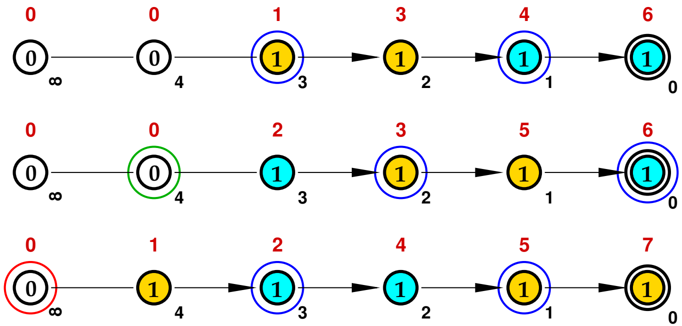
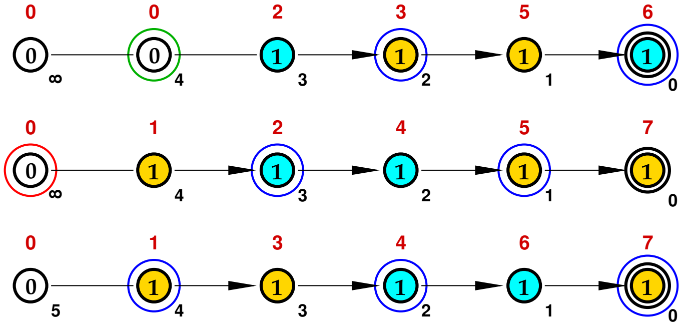
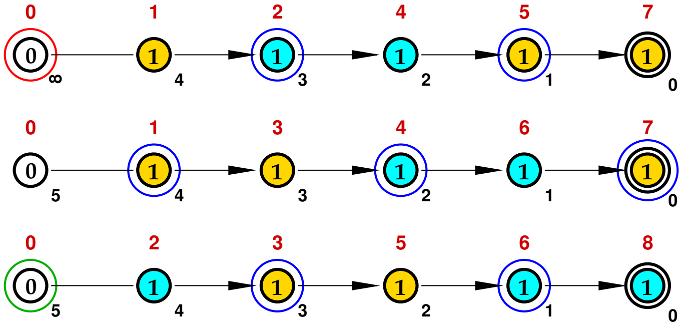
- **Blue Circle:** Enabled to Change Color.
- **Color Waves** Convergecast. **Absorbed** by Clusterhead.



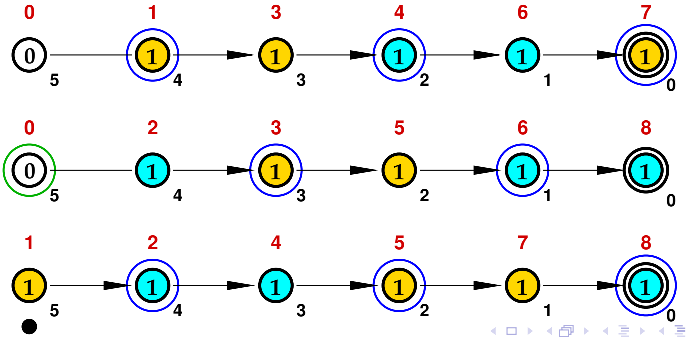
- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Red Numeral:** Energy.
- **Red Circle:** Enabled to Reset.
- **Cyan:** Color = 1; **Gold:** Color = 0.
- **Green Circle:** Enabled to Join. Can only attach to Color 1.
- **Blue Circle:** Enabled to Change Color.
- **Color Waves** Convergecast. **Absorbed** by Clusterhead.



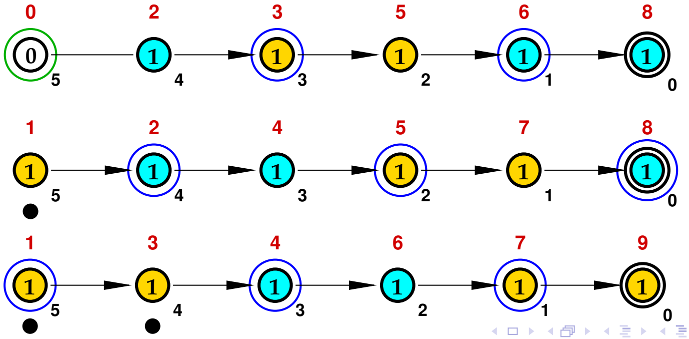
- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Red Numeral:** Energy.
- **Cyan:** Color = 1; **Gold:** Color = 0.
- **Green Circle:** Enabled to Join. Can only attach to Color 1.
- **Blue Circle:** Enabled to Change Color.
- **Color Waves** Convergecast. **Absorbed** by Clusterhead.



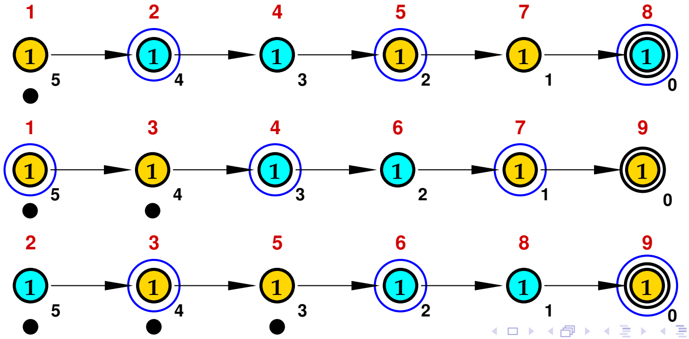
- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Red Numeral:** Energy.
- **Cyan:** Color = 1; **Gold:** Color = 0.
- **Blue Circle:** Enabled to Change Color.
- **Color Waves** Convergecast. **Absorbed** by Clusterhead.
- Configuration is **Legitimate.** All Output Bits = 1.



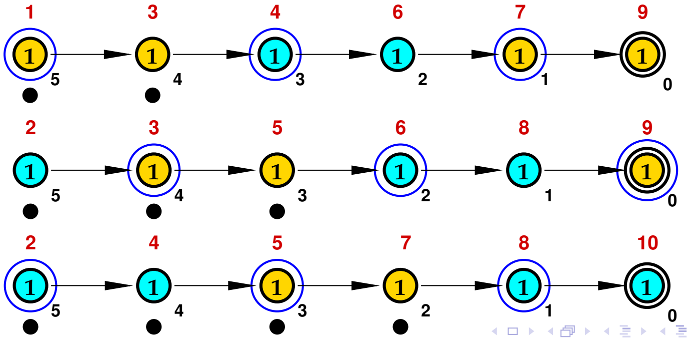
- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Red Numeral:** Energy.
- **Cyan:** Color = 1; **Gold:** Color = 0.
- **Blue Circle:** Enabled to Change Color.
- **Color Waves** Convergecast. **Absorbed** by Clusterhead.
- Configuration is **Legitimate.** All Output Bits = 1.
- **Black Dot:** Done. Convergecast.



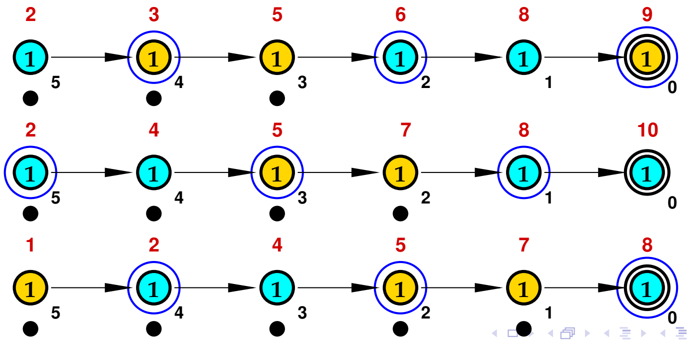
- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Red Numeral:** Energy.
- **Cyan:** Color = 1; **Gold:** Color = 0.
- **Blue Circle:** Enabled to Change Color.
- **Color Waves** Convergecast. **Absorbed** by Clusterhead.
- Configuration is **Legitimate.** All Output Bits = 1.
- **Black Dot:** Done. Convergecast.



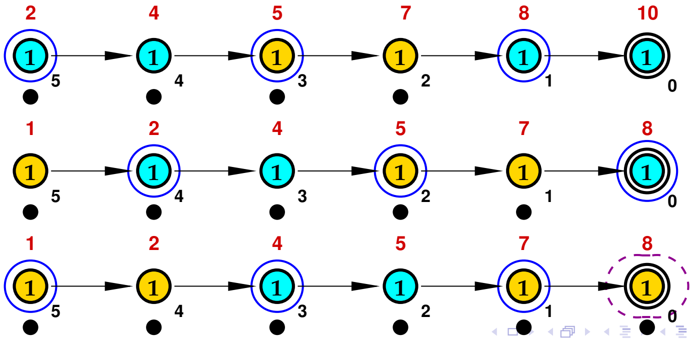
- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Red Numeral:** Energy.
- **Cyan:** Color = 1; **Gold:** Color = 0.
- **Blue Circle:** Enabled to Change Color.
- **Color Waves** Convergecast. **Absorbed** by Clusterhead.
- Configuration is **Legitimate.** All Output Bits = 1.
- **Black Dot:** Done. Convergecast.



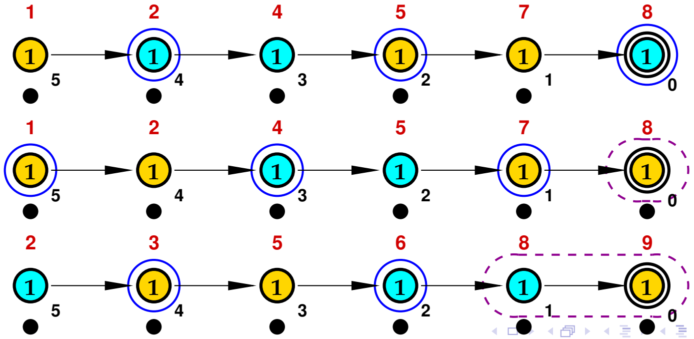
- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Red Numeral:** Energy.
- **Cyan:** Color = 1; **Gold:** Color = 0.
- **Blue Circle:** Enabled to Change Color.
- **Color Waves** Convergecast. **Absorbed** by Clusterhead.
- Configuration is **Legitimate.** All Output Bits = 1.
- **Black Dot:** Done. Convergecast.



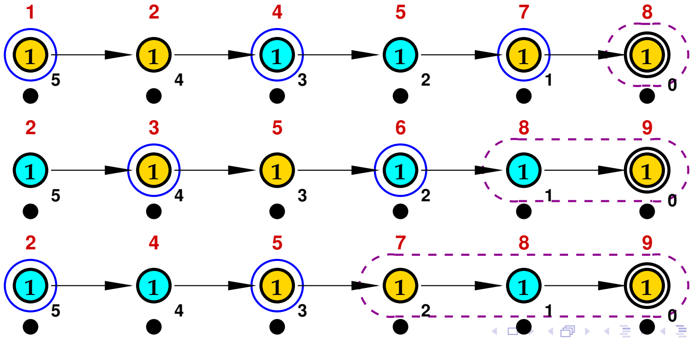
- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Red Numeral:** Energy. No **Increase** if Clusterhead is Frozen.
- **Cyan:** Color = 1; **Gold:** Color = 0.
- Configuration is **Legitimate.** All Output Bits = 1.
- **Black Dot:** Done. Convergecast.
- Clusterhead Done, Hence Color-Frozen.
  Clusterhead will No Longer Absorb Color Waves.



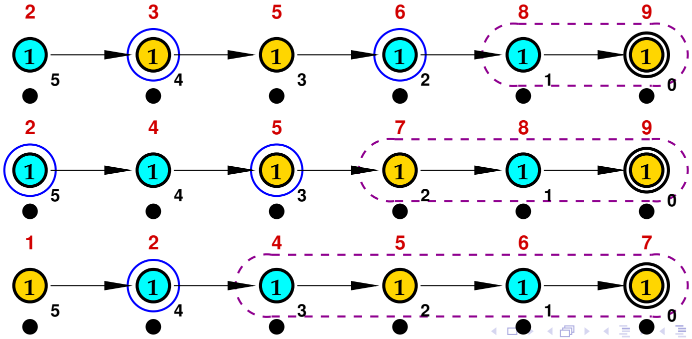
- **Chain Example.** **Double Circle:** Input Bit = 1, Otherwise 0.
- **Red Numeral:** Energy. No **Increase** if Clusterhead is Frozen.
- **Cyan:** Color = 1; **Gold:** Color = 0.
- Configuration is **Legitimate.** All Output Bits = 1.
- Clusterhead Done, Hence Color-Frozen.
  Clusterhead will No Longer Absorb Color Waves.
- **Dashed Oval:** Color-Locked Processes. Alternating Colors.



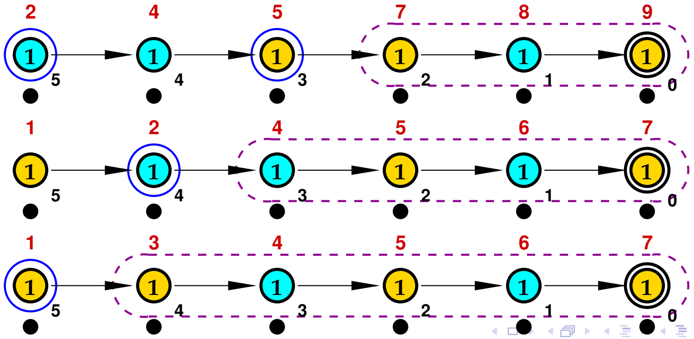
- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Red Numeral:** Energy. No **Increase** if Clusterhead is Frozen.
- **Cyan:** Color = 1; **Gold:** Color = 0.
- Configuration is **Legitimate.** All Output Bits = 1.
- Clusterhead Done, Hence Color-Frozen.
  Clusterhead will No Longer Absorb Color Waves.
- **Dashed Oval:** Color-Locked Processes. Alternating Colors.



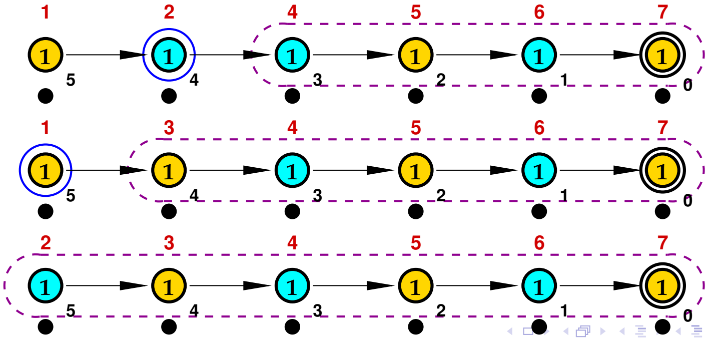
- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Red Numeral:** Energy. No **Increase** if Clusterhead is Frozen.
- **Cyan:** Color = 1; **Gold:** Color = 0.
- Configuration is **Legitimate.** All Output Bits = 1.
- Clusterhead Done, Hence Color-Frozen.
  Clusterhead will No Longer Absorb Color Waves.
- **Dashed Oval:** Color-Locked Processes. Alternating Colors.



- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Red Numeral:** Energy. No **Increase** if Clusterhead is Frozen.
- **Cyan:** Color = 1; **Gold:** Color = 0.
- Configuration is **Legitimate.** All Output Bits = 1.
- Clusterhead Done, Hence Color-Frozen.
  Clusterhead will No Longer Absorb Color Waves.
- **Dashed Oval:** Color-Locked Processes. Alternating Colors.



- **Chain Example. Double Circle:** Input Bit = 1, Otherwise 0.
- **Red Numeral:** Energy. No **Increase** if Clusterhead is Frozen.
- **Cyan:** Color = 1; **Gold:** Color = 0.
- Configuration is **Legitimate.** All Output Bits = 1.
- Clusterhead Done, Hence Color-Frozen.
  Clusterhead will No Longer Absorb Color Waves.
- All Processes **Color-Locked.** Final Configuration.



1 2 4 5 6 7
1 1 1 1 1 1
5 4 3 2 1 0
1 3 4 5 6 7
1 1 1 1 1 1
5 4 3 2 1 0
2 3 4 5 6 7
1 1 1 1 1 1
5 4 3 2 1 0