Homework - Version B.

Exercise 1

We define the syntactic category of bits $B = \{0, 1\}$. By $b$ we denote a meta-variable ranging over $B$. We define inductively a set of bitstrings $BS$ by the following BNF:

$$bs := b \mid 0 \ bs \ 0 \mid 01 \ bs \ 10$$

where $bs$ is a meta-variable ranging over the set of bitstrings $BS$. Of the following two statements, one is wrong and one is right. You get to tell which is which and justify your answers by either a proof (by induction) or a counter-example.

1. In the set of bistrings $BS$, all elements contain at least one 0.
2. Every bitstring in the set $BS$ is a palindrome.

A palindrome is a sequence of symbols that reads the same from right to left or left to right (e.g. ‘Rise to vote sir’ is one, 1001 is one too).

The first statement is false. Indeed, the first rule provides that 1 belongs to $BS$, which is a counter-example.

The second statement is true, let’s prove it by induction.

Basic case: the atoms of this set are bits in $B$. Moreover, 0 and 1 are indeed palindromes, so that the property holds for atoms.

Case of the rule ‘if $bs \in BS$ then $0 \ bs \ 0 \in BS$:
our induction hypothesis is that $bs$ is a palindrome, and we have to show that $0 \ bs \ 0$ is one too, which is true (I skip the details).

Case of the rule ‘if $bs \in BS$ then $01 \ bs \ 10 \in BS$:
our induction hypothesis is that $bs$ is a palindrome, and we have to show that $01 \ bs \ 10$ is one too, which is true (I skip the details).

Conclusion : we have proven by structural induction on the set $BS$ that every element of $BS$ is a palindrome.

Exercise 2

We consider the following program.

begin
\begin{verbatim}
var \hspace{1em} z_1 := 43;
var \hspace{1em} z_2 := 5 \times (z_1 - 1);
proc \hspace{1em} toto \hspace{1em} is \hspace{1em} z_1 := 3 \times (z_2 + z_1);
begin \hspace{1em} var \hspace{1em} z_2 := 1;
proc \hspace{1em} toto \hspace{1em} is \hspace{1em} z_1 := z_2 + 45;
proc \hspace{1em} q \hspace{1em} is \hspace{1em} call \hspace{1em} toto;
call \hspace{1em} q;
end \hspace{1em} call \hspace{1em} toto;
end
\end{verbatim}
end
You have been presented three different semantics for the While language with blocks and procedures: one with dynamic links for variables and procedures, another with dynamic links for variables but static links for procedures, and finally one with static links for variables and procedures.

What values are associated to $z_1$ and $z_2$ at the end of this program according to each of the three semantics you know? Justify your answer (you can either draw the tree or precise the state or the variable environment and the storage function after each ';').

**Dynamic-dynamic semantics:**

\[
\begin{align*}
\sigma, env_P & \\
\text{begin} & \quad \text{var } z_1 := 43; \\
& \quad \text{var } z_2 := 5 * (z_1 - 1); \\
& \quad \text{proc } toto \text{ is } z_1 ::= 3 * (z_2 + z_1); \\
& \quad \text{begin } z_2 := 1; \\
& \quad \text{proc } q \text{ is call } toto; \\
& \quad \text{call } toto; \\
& \quad \text{end}
\end{align*}
\]

Ask yourself why in $\ast$, the procedure environment is $env_P$. It is because we apply the sequence rule to the body of the big block, and in the sequence rule, $S_1$ and $S_2$ are evaluated in the same environment!

**Dynamic-static semantics:**

\[
\begin{align*}
\sigma, env_P & \\
\text{begin} & \quad \text{var } z_1 := 43; \\
& \quad \text{var } z_2 := 5 * (z_1 - 1); \\
& \quad \text{proc } toto \text{ is } z_1 ::= 3 * (z_2 + z_1); \\
& \quad \text{begin } z_2 := 1; \\
& \quad \text{proc } q \text{ is call } toto; \\
& \quad \text{call } toto; \\
& \quad \text{end}
\end{align*}
\]

**Static-static semantics:**

\[
\begin{align*}
env_V, sto, env_P & \\
\text{begin} & \quad \text{var } z_1 := 43; \\
& \quad \text{var } z_2 := 5 * (z_1 - 1); \\
& \quad \text{proc } toto \text{ is } z_1 ::= 3 * (z_2 + z_1); \\
& \quad \text{begin } z_2 := 1; \\
& \quad \text{proc } q \text{ is call } toto; \\
& \quad \text{call } toto; \\
& \quad \text{end}
\end{align*}
\]