Symbolic Methods for the Automatic Search of Attacks Against Some Block Ciphers

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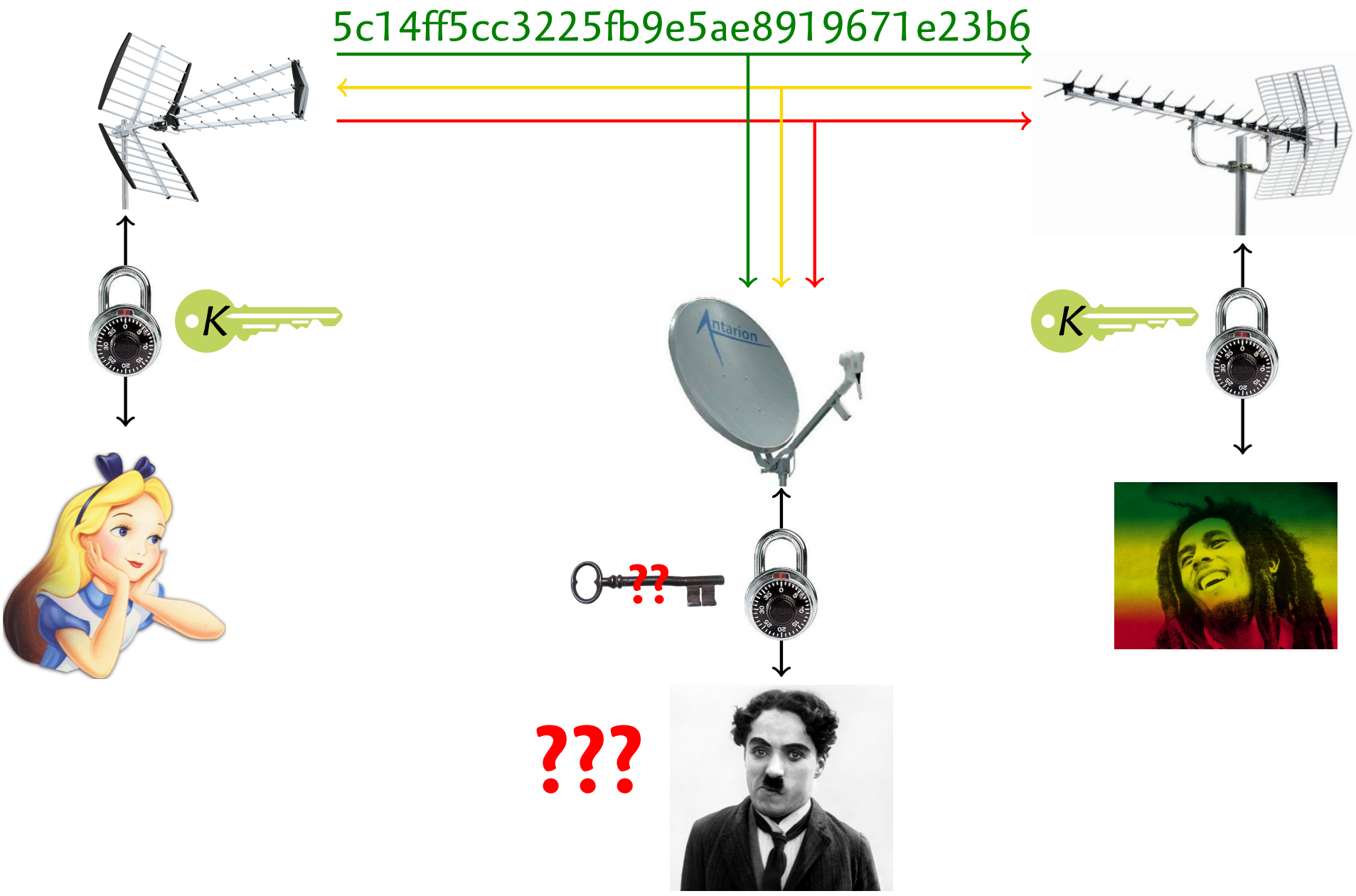
(joint work with Patrick Derbez and Pierre-Alain Fouque)

Université de Versailles St-Quentin en Yvelines

LSV Seminar
November 15, 2011

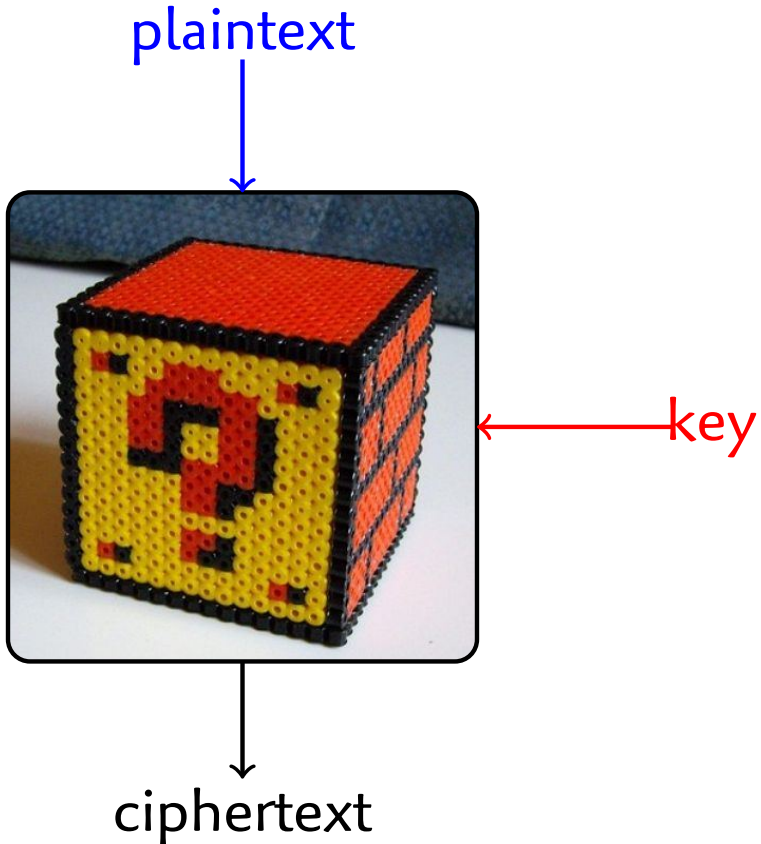
A (Very Brief) Introduction to Cryptography: Encryption



Block-Cipher Cryptanalysis: the Object

a Block Cipher

$$E : \underbrace{\{0,1\}^k}_{\text{key}} \times \underbrace{\{0,1\}^n}_{\textit{plaintext}} \to \underbrace{\{0,1\}^n}_{\textit{ciphertext}}$$



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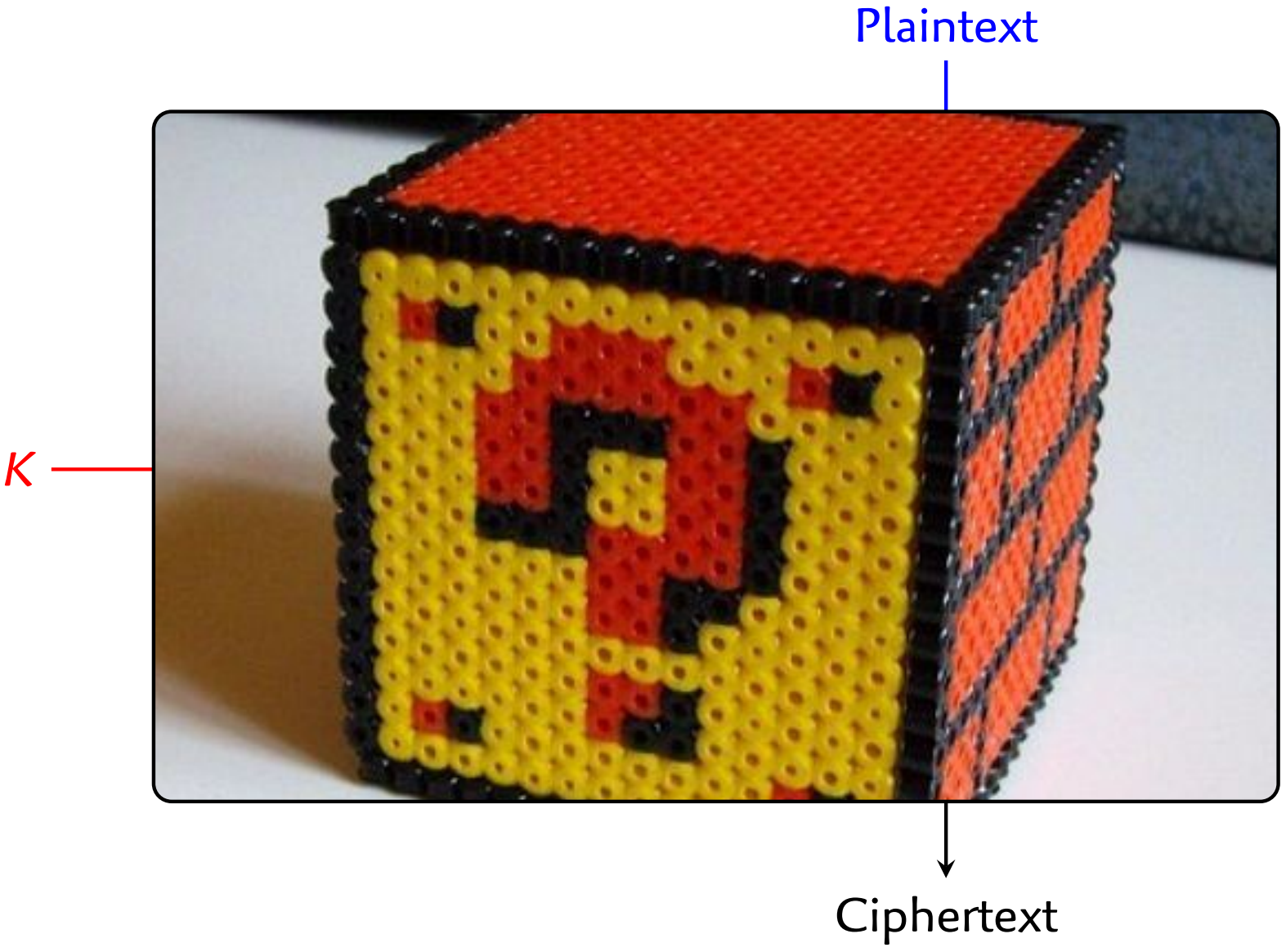

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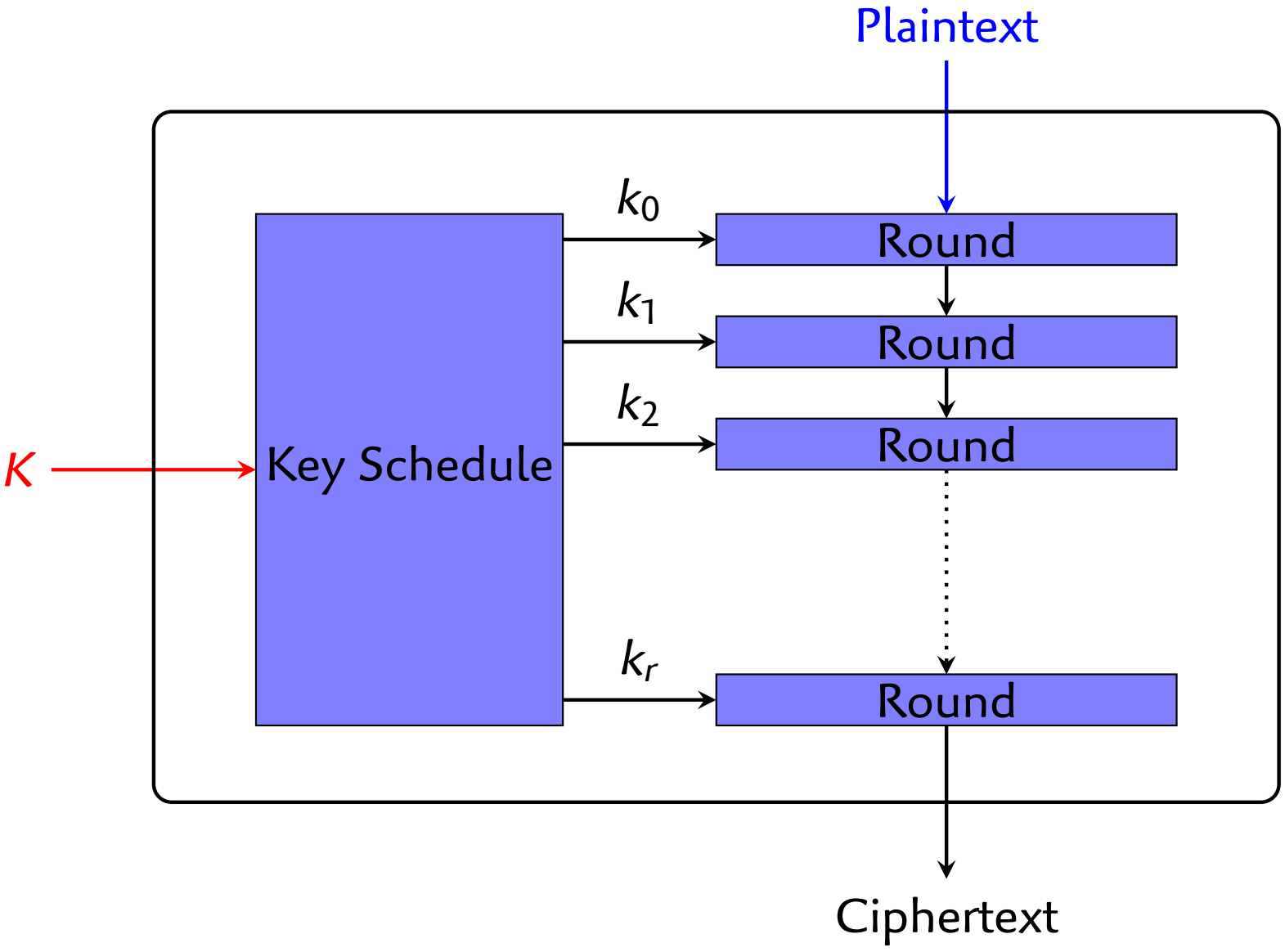
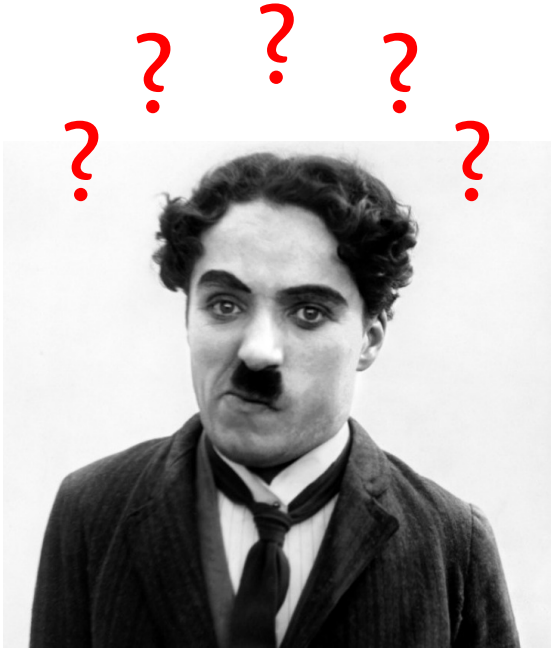


Block-Cipher Cryptanalysis: the Subject

an Attacker

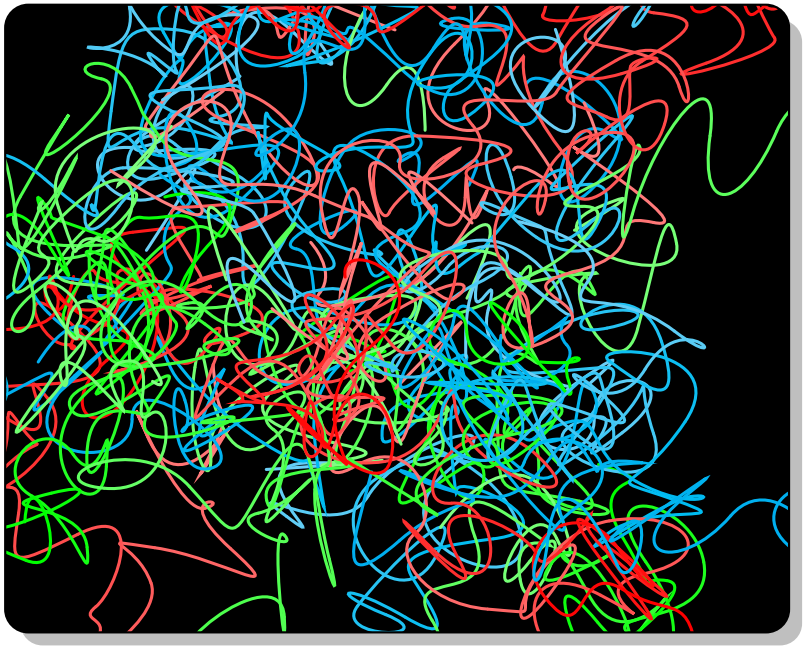
- Goal :
  - In Theory: **distinguish from random permutation**
  - In Practice: **recover the secret key**
- Resources:
  - Time: less than 2^k encryptions
  - Data: less than 2^n plaintext/ciphertext pairs

Block-Cipher Cryptanalysis: the Game



plaintext →

← ciphertext



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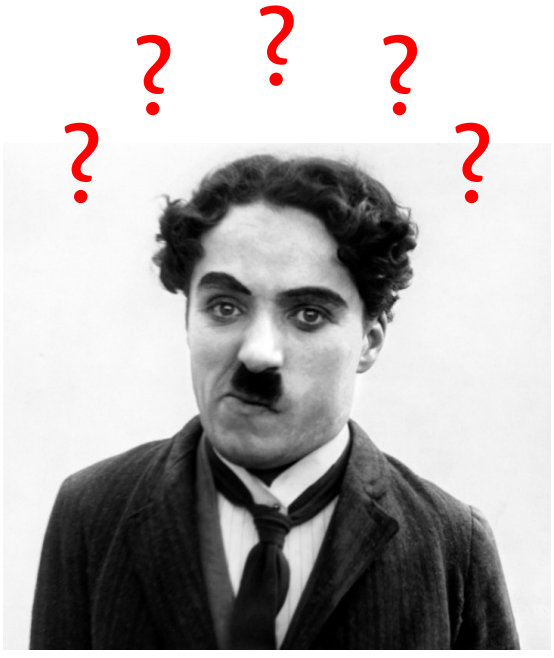

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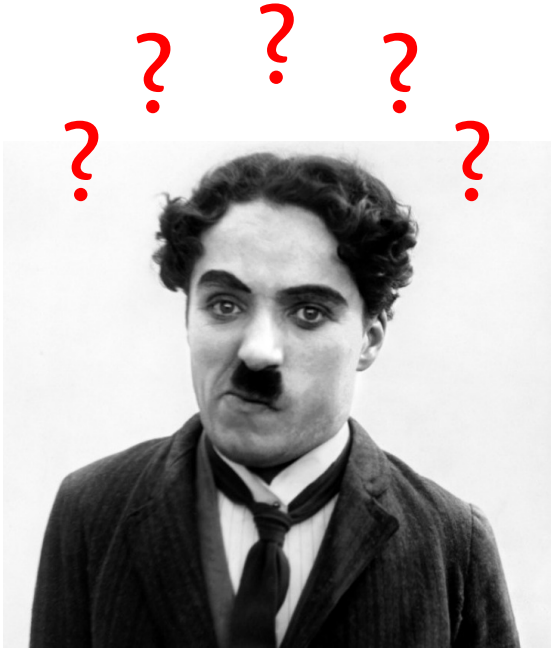

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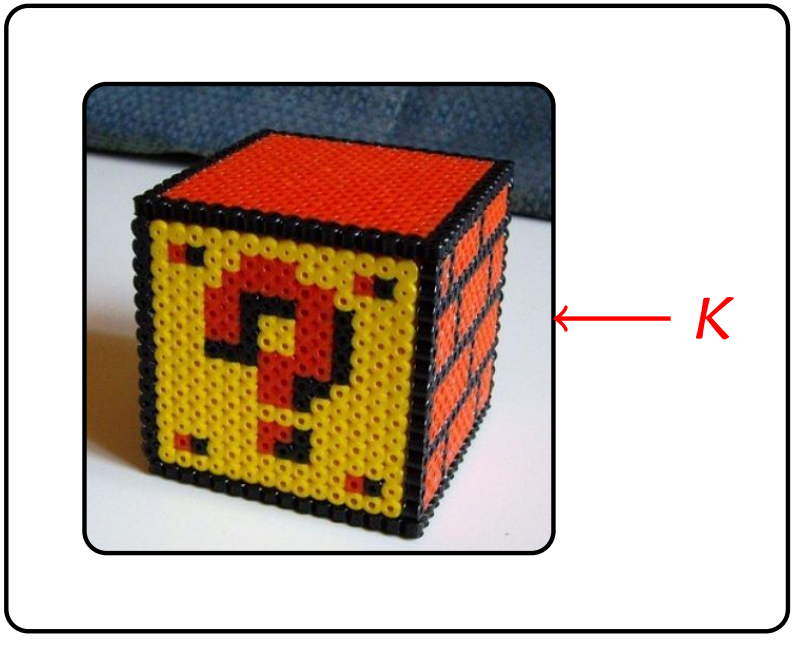
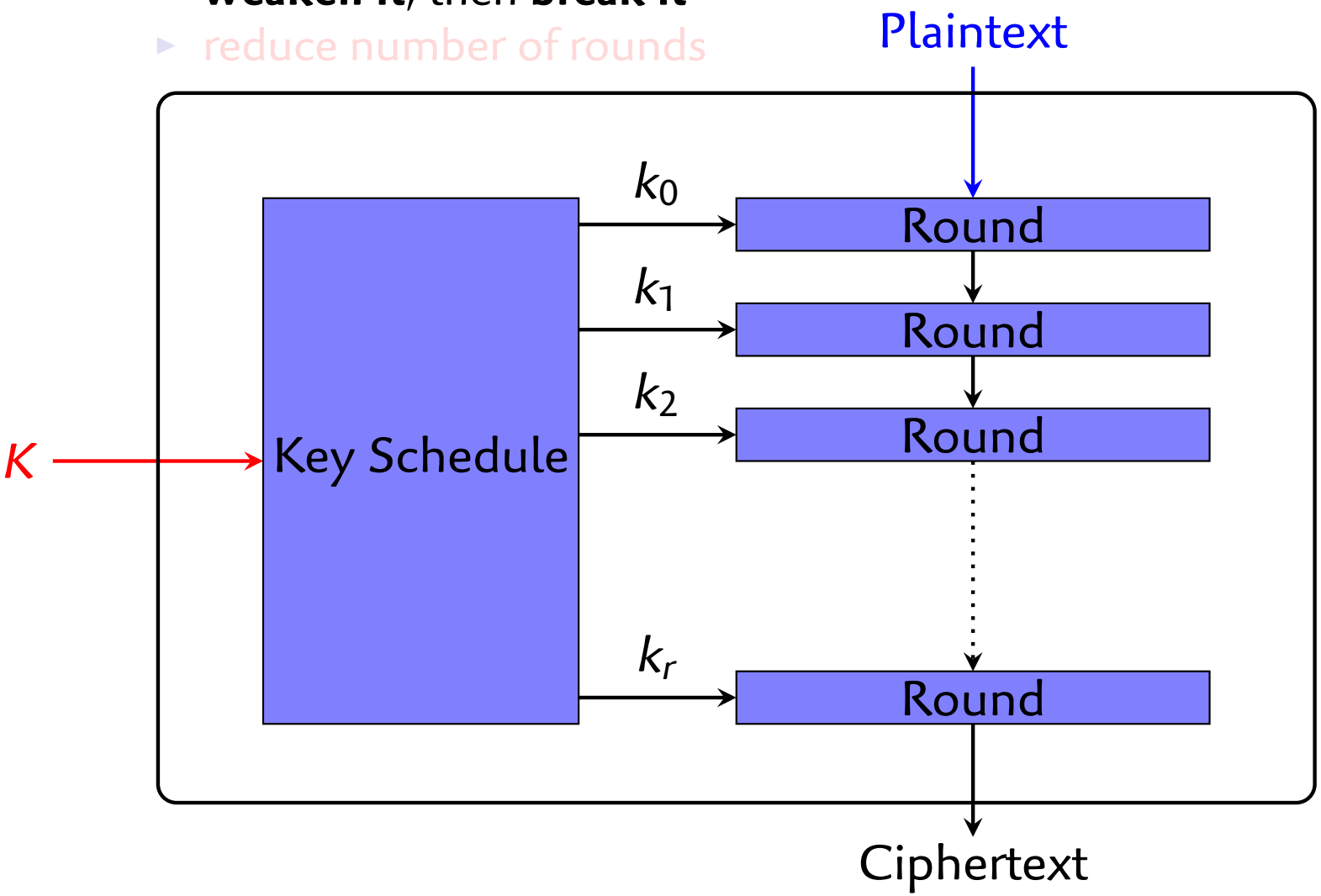


What Can We Do When Block Ciphers Are Too Strong For Us?

- **Solution # 1:**
  - **weaken it**, then **break it**
  - reduce number of rounds



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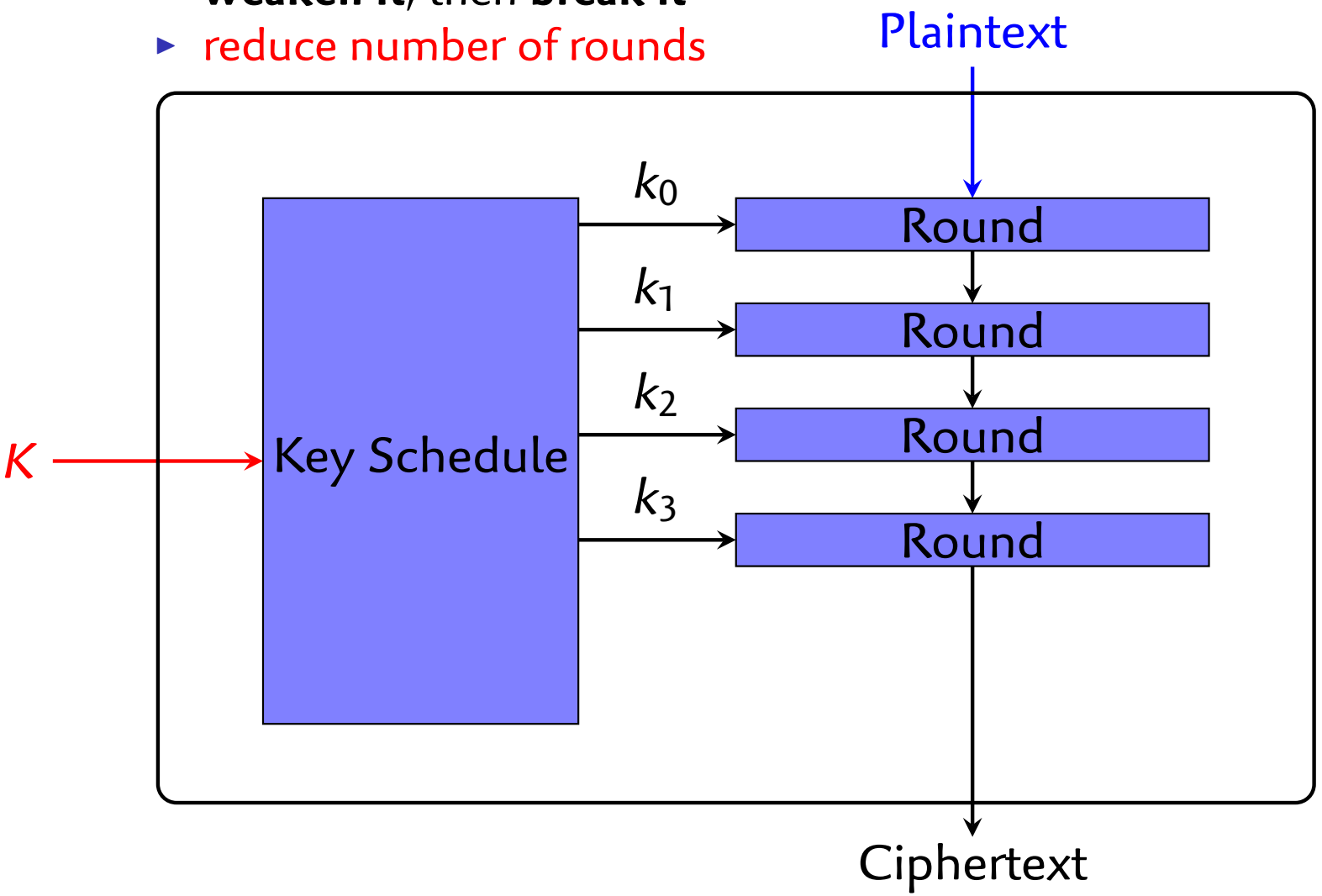

What to do when block ciphers are too strong for us?

- **Solution # 2**:
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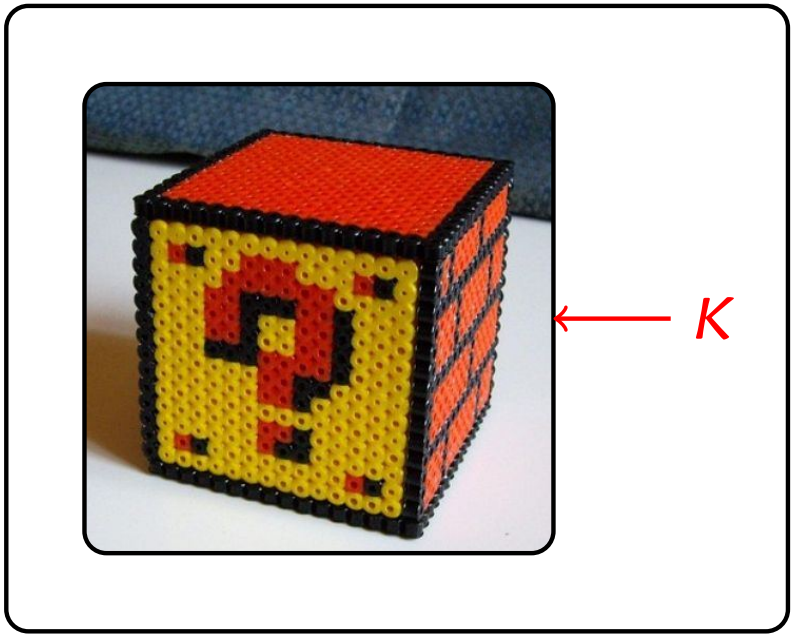
  - (adaptively) chosen ciphertexts, related keys, etc.

plaintext →

← ciphertext



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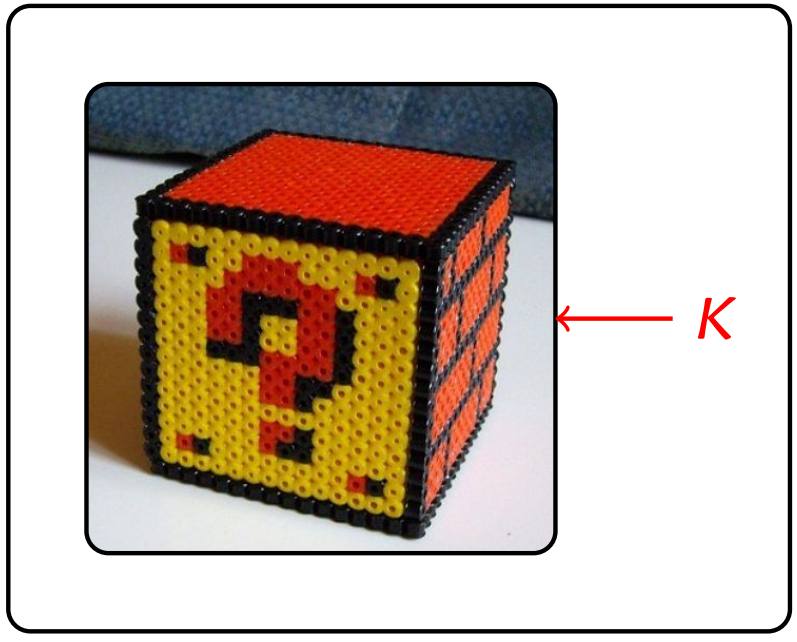

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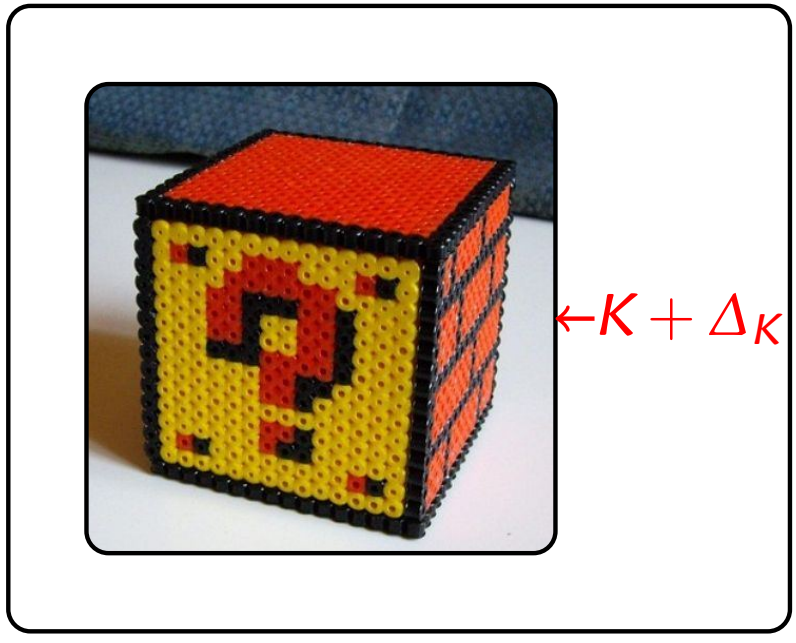

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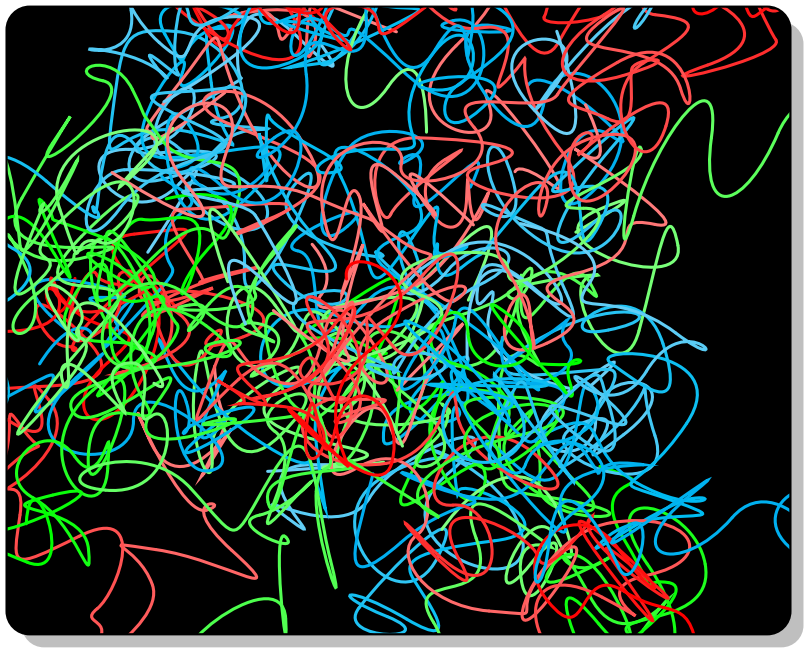
  - side channels, fault injection, different attack games...



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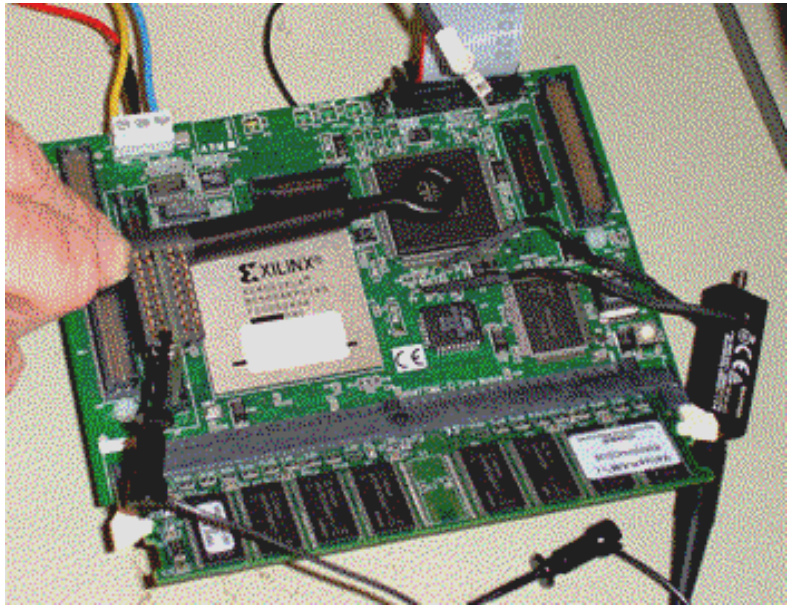

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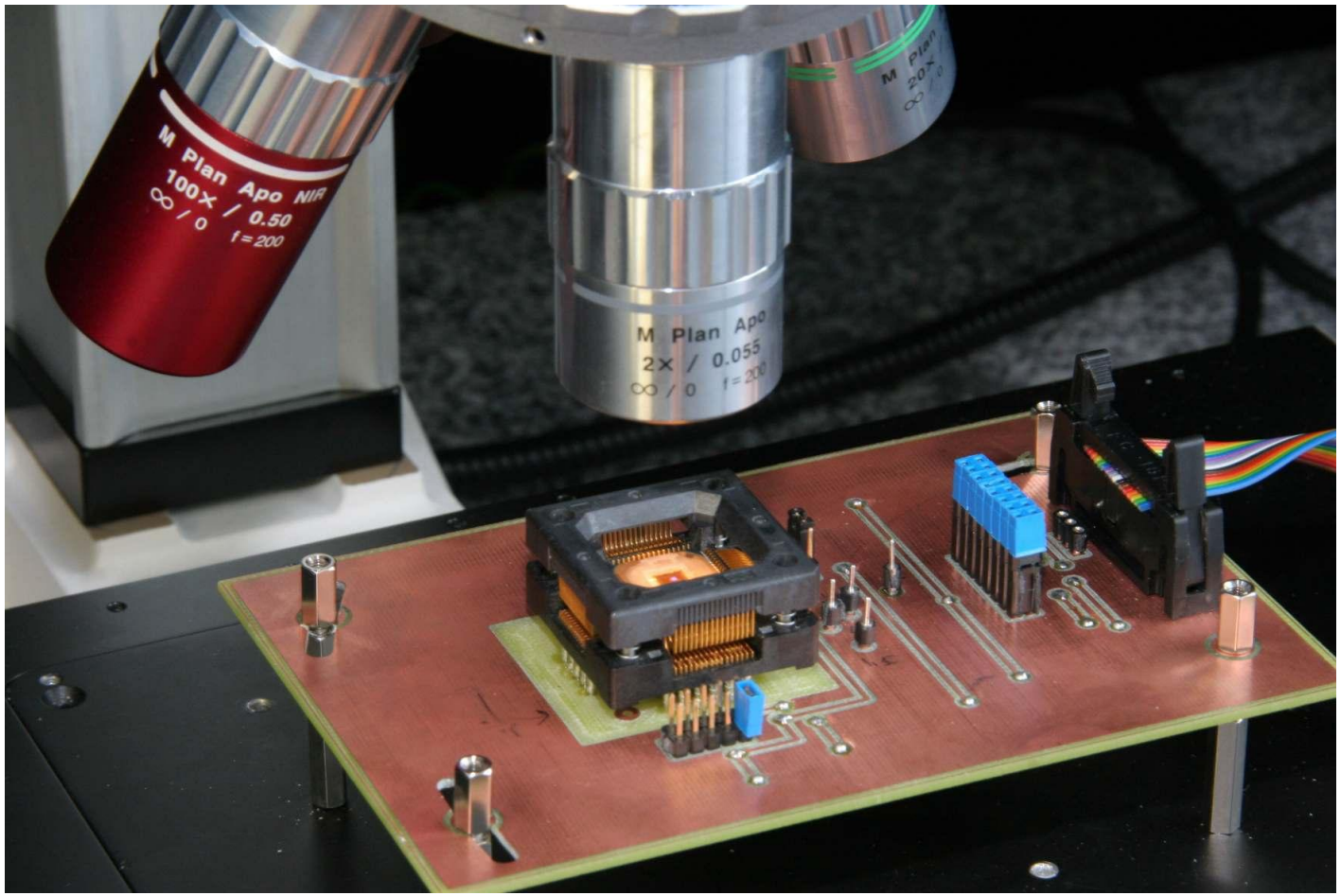

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In this talk :

Low Data Complexity Attacks

- Must be faster than exhaustive search
- Only very few plaintext/ciphertext pairs available

Why ?

- Rather unexplored territory
- What is harder in practice?
  - **performing** 2^{50} elementary operations?
  - or **acquiring** 50 Plaintext/Ciphertext pairs?
- LDC attacks can sometimes be recycled, and used as sub-components in other attacks
  - *e.g.* attack on GOST uses a 2-plaintext attack on 8 rounds

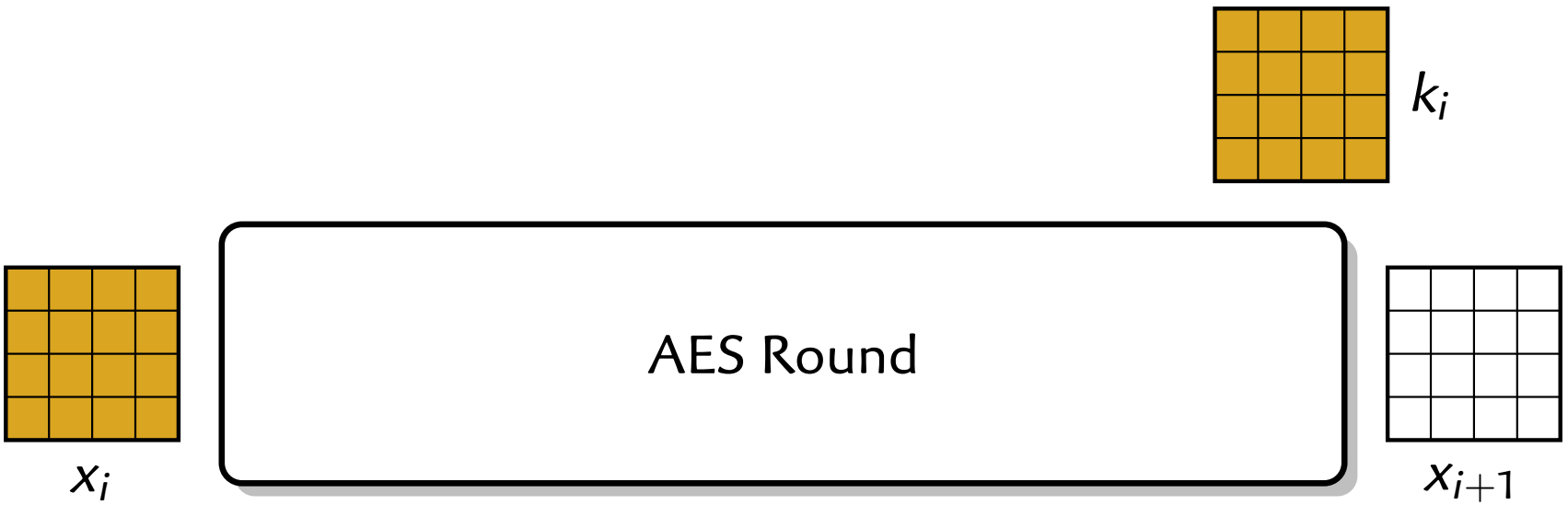
Target Block Cipher: the Advanced Encryption Standard

- Designed by Rijmen and Daemen
- Winner of AES competition in 2001
- One of the most widely used encryption primitive

AES basic structures

- Substitution-Permutation network
- Block size: 128 bits
- key lengths: **128**, 192 or 256 bits
- 10 rounds for the 128-bit version

Description of the AES



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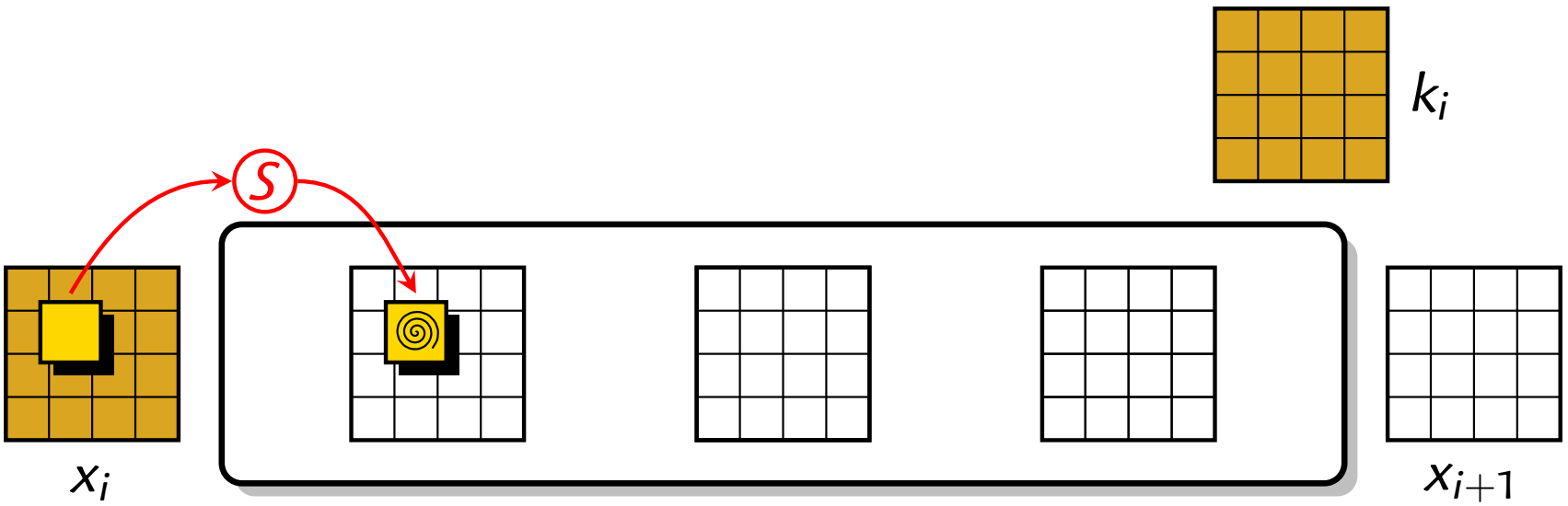

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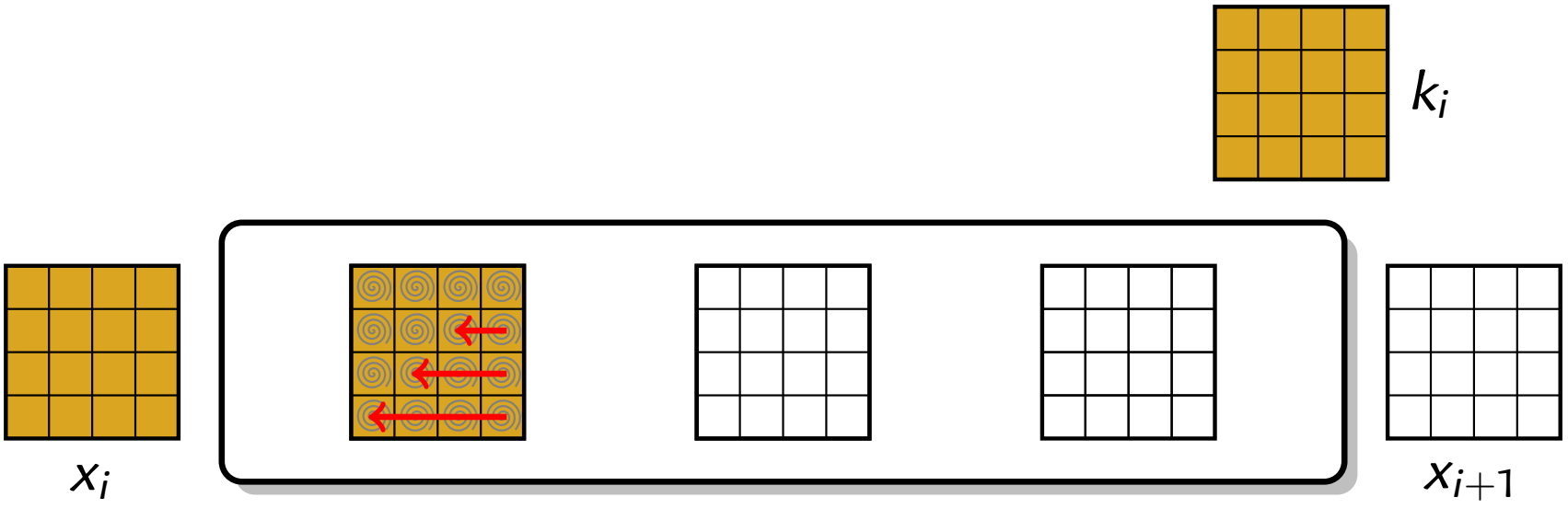

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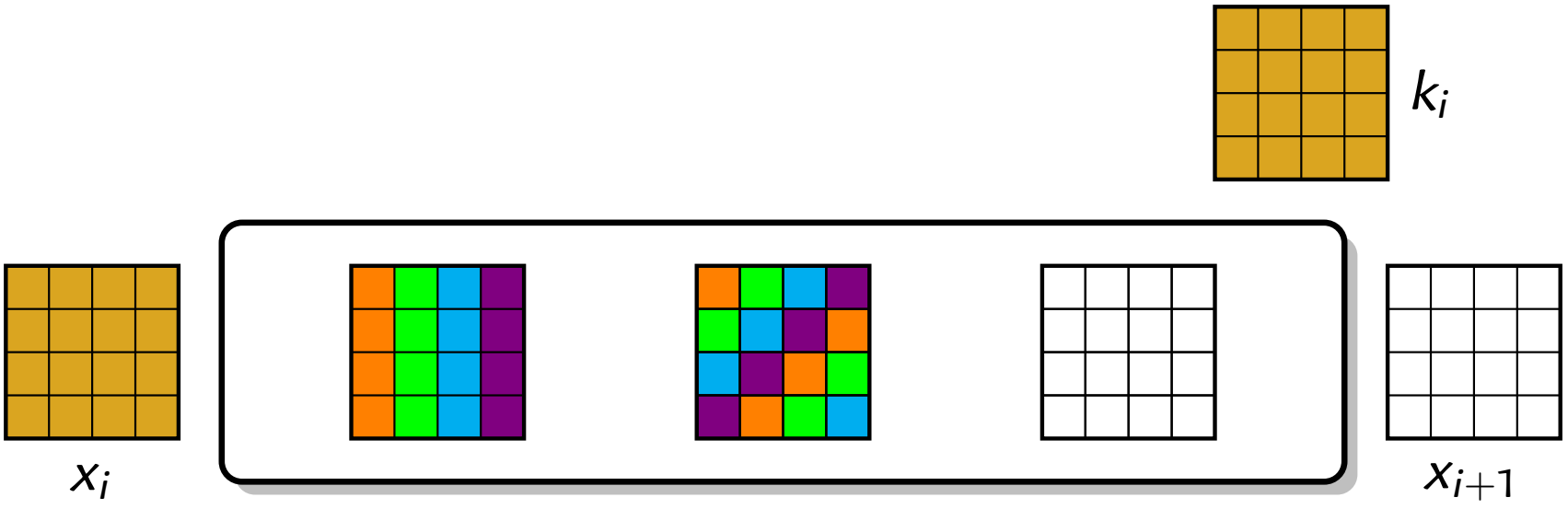

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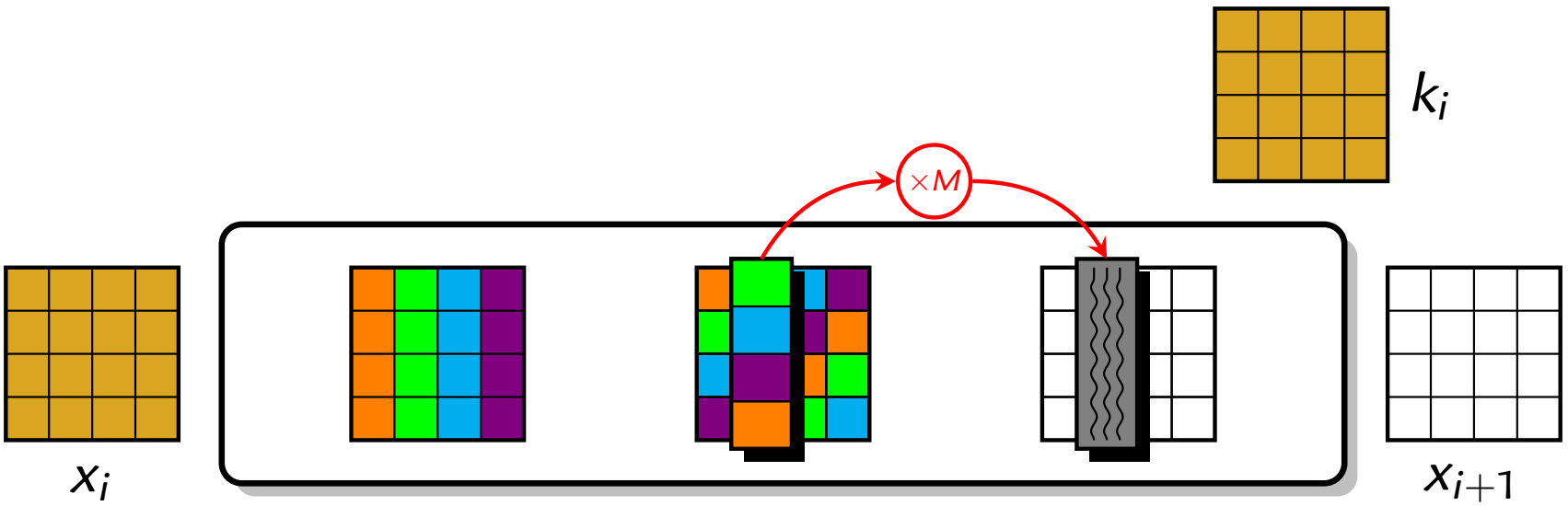

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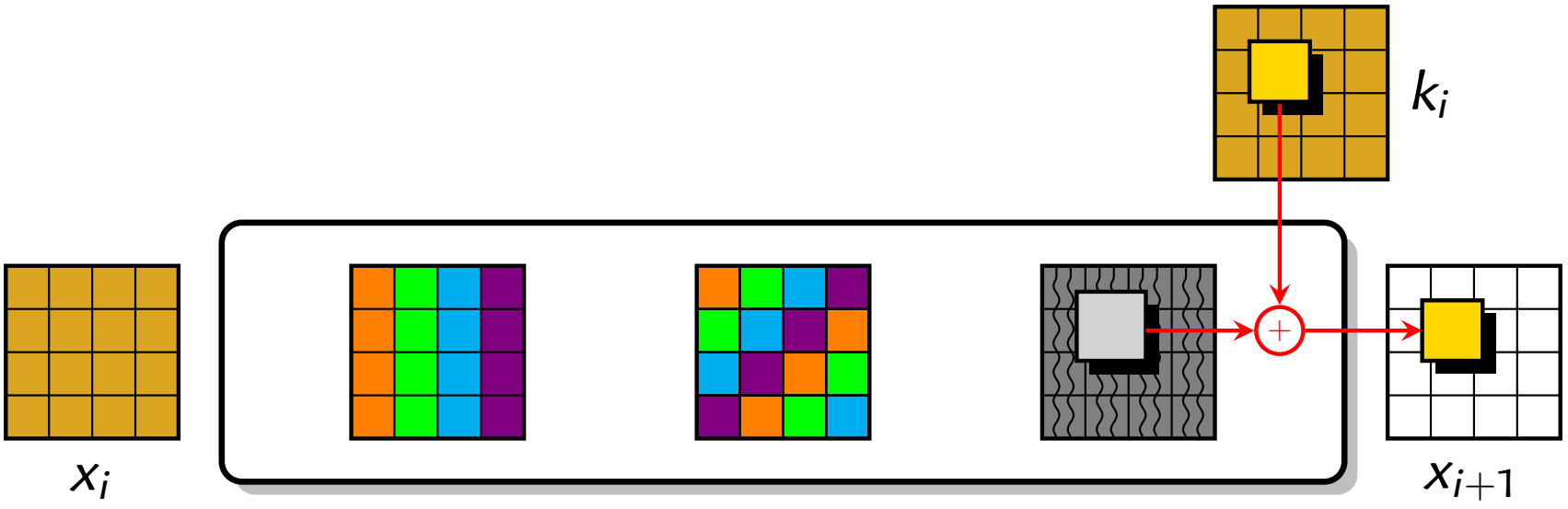

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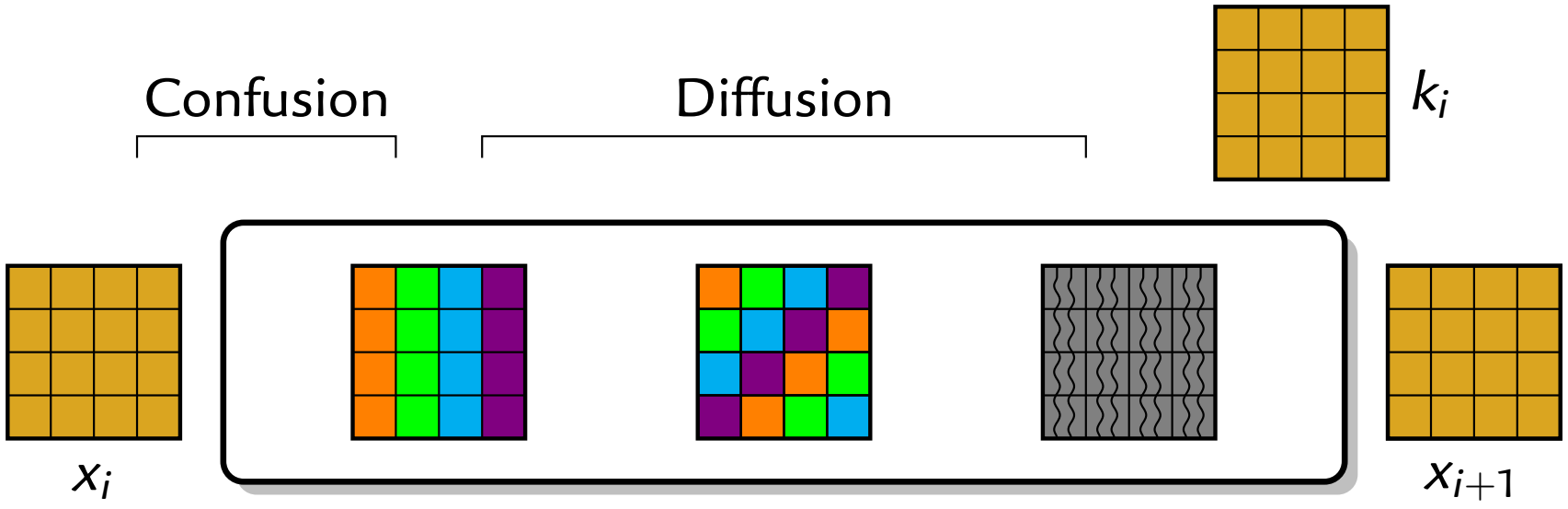
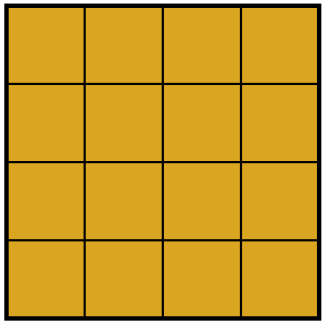
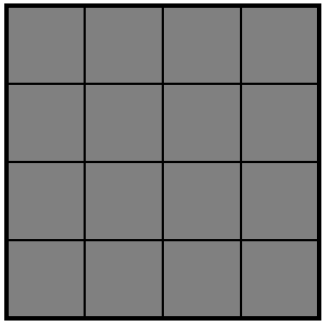


Description of the AES: the Key-Schedule



k_i

k_{i+1}



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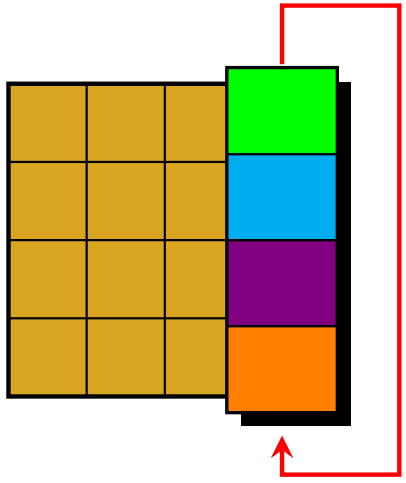

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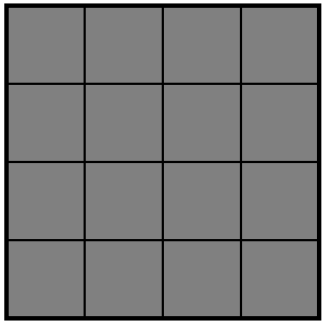

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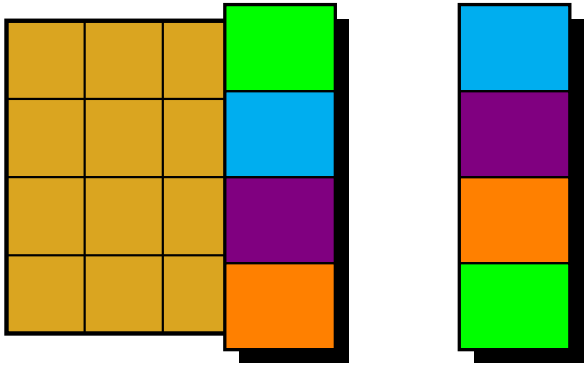

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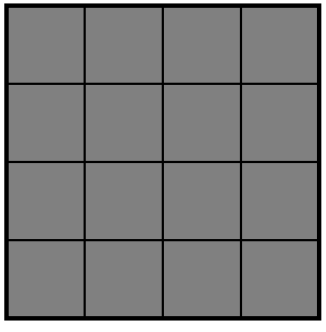

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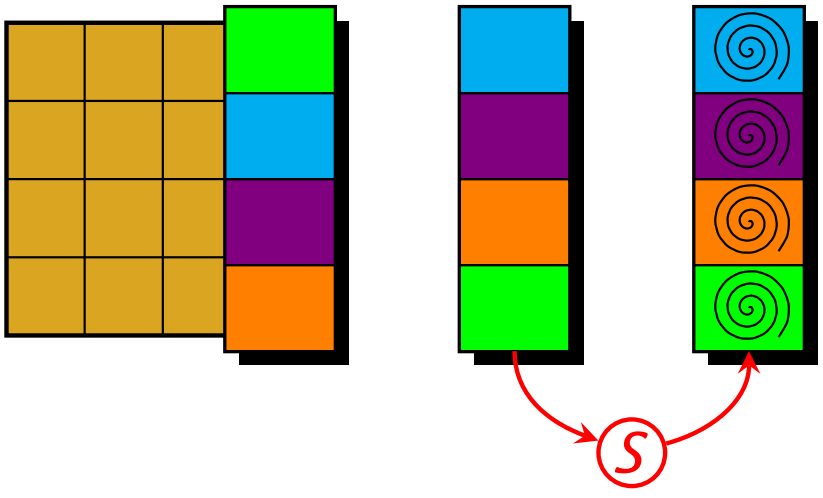

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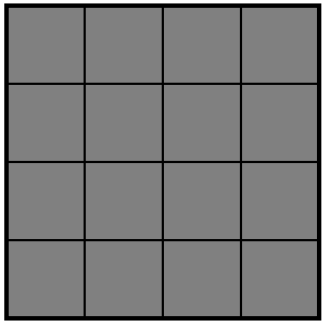

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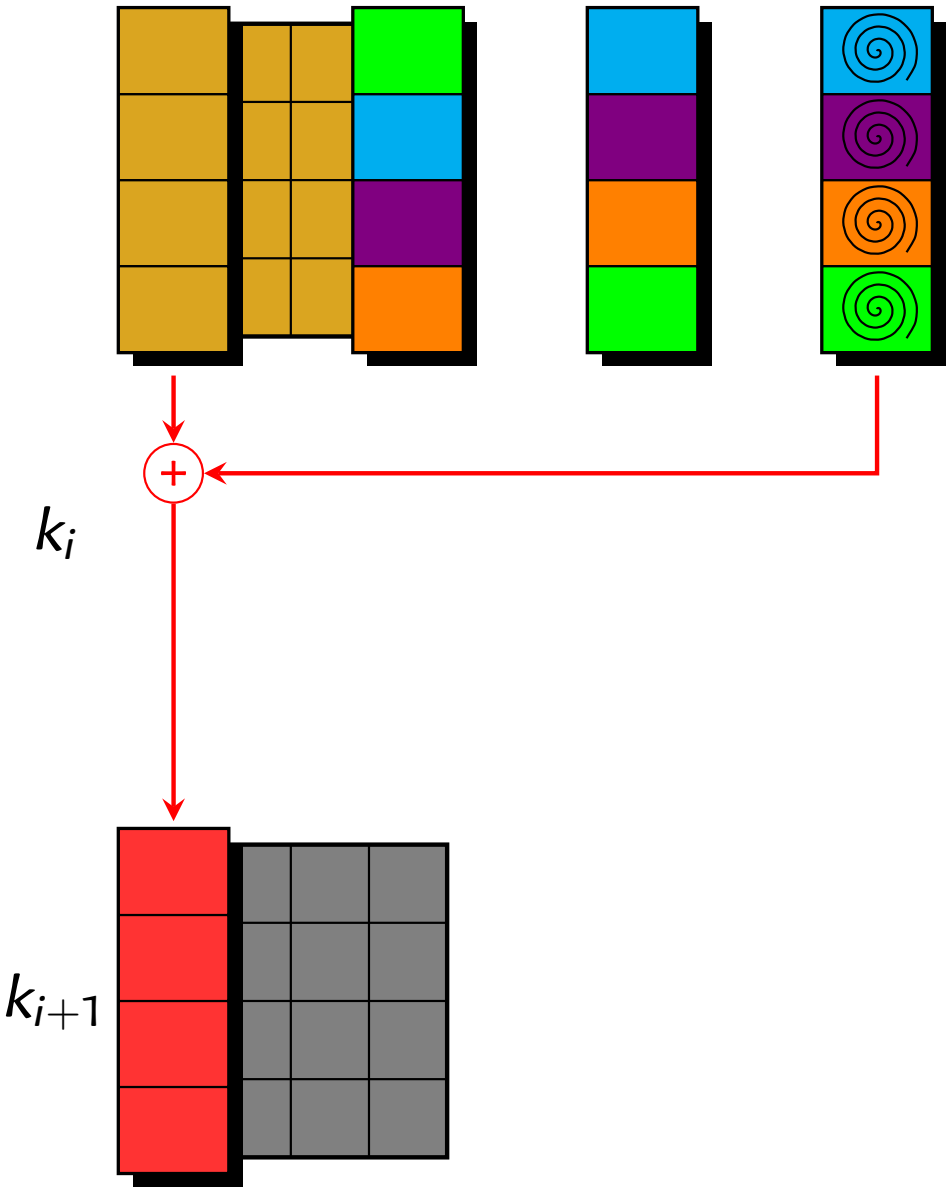

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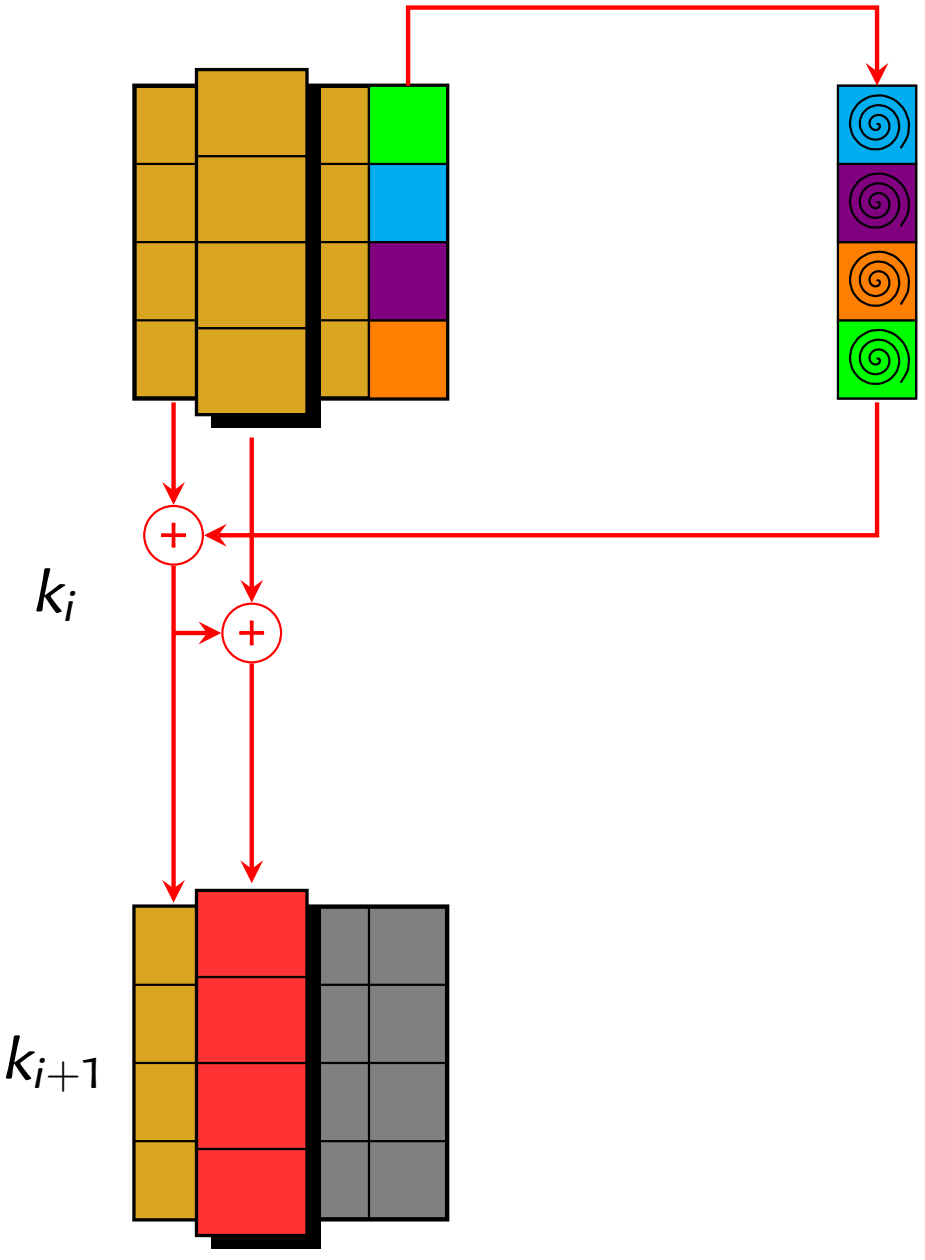

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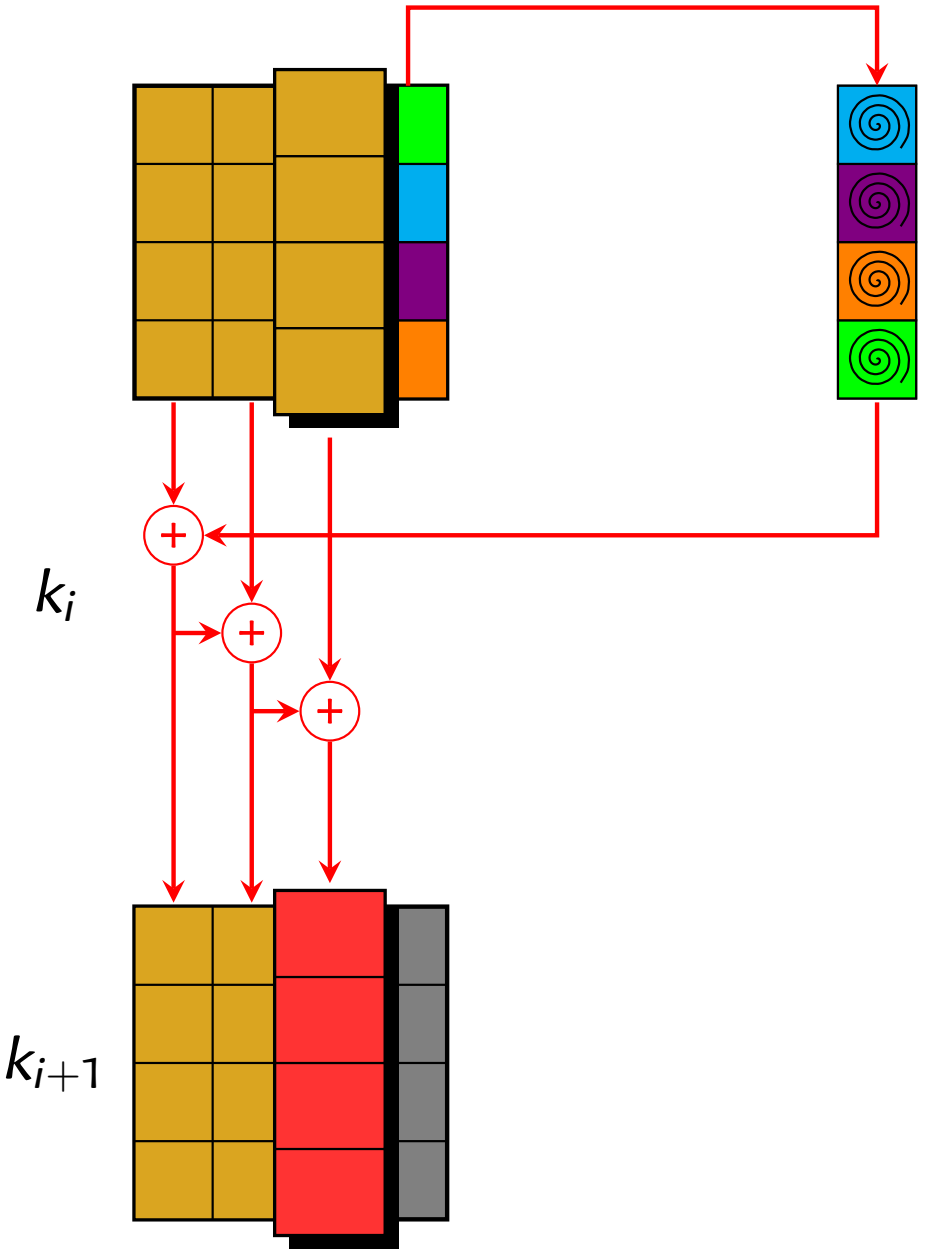

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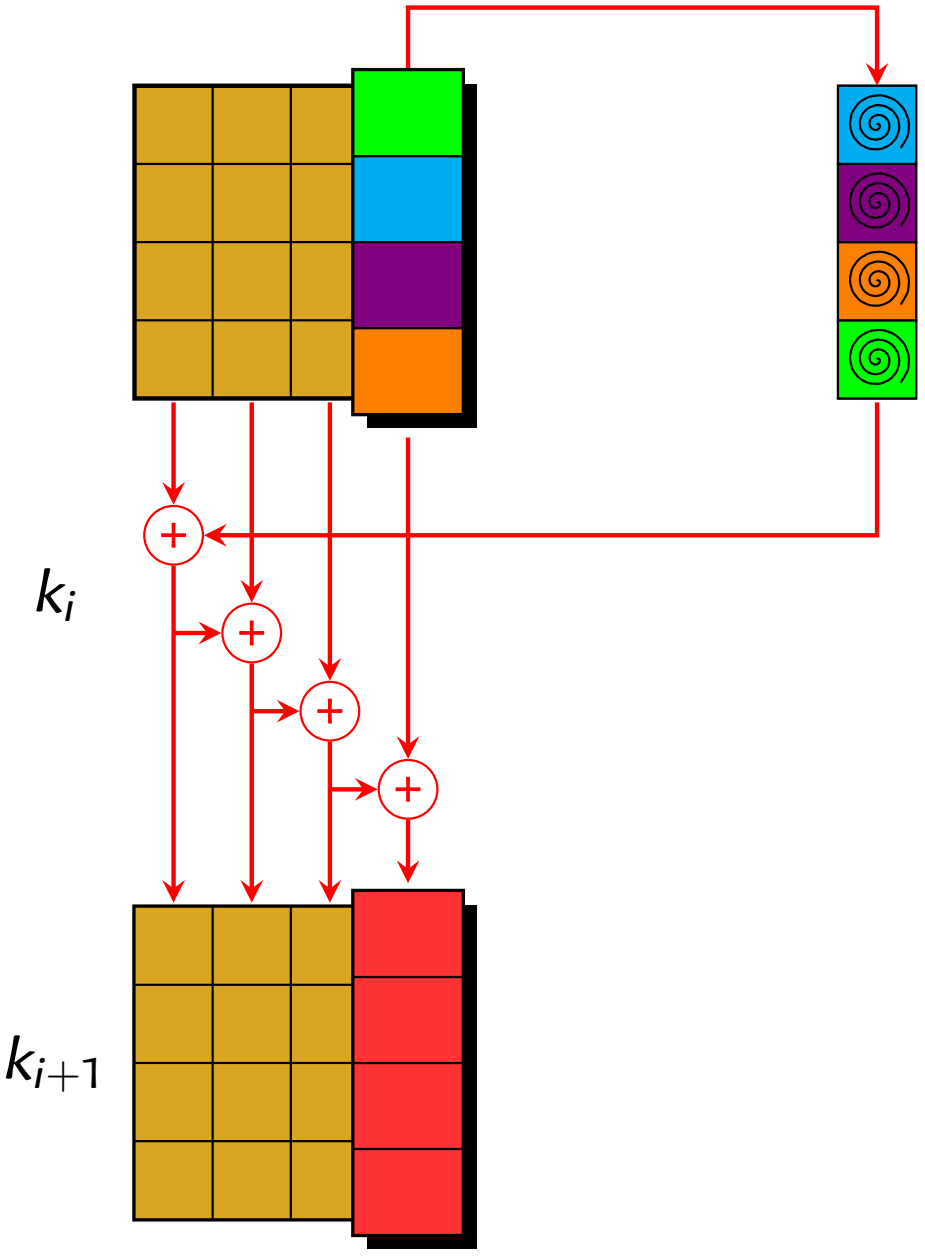

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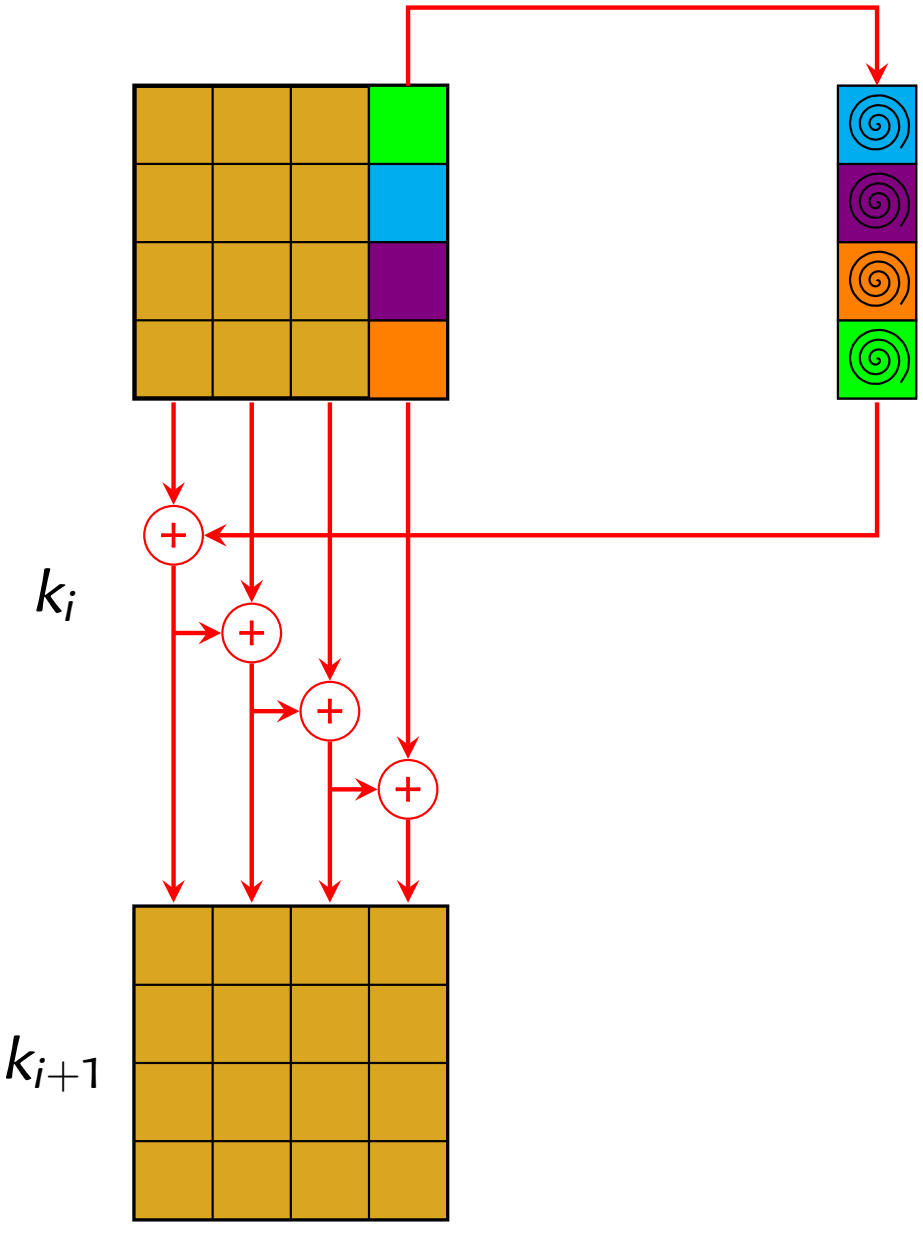

Techniques for Low Data Complexity Attacks

The problem with "Usual" attack techniques

- **Statistical attacks** (*e.g.*, [impossible] differential, linear)
- **"Golden-plaintext" attacks** (*e.g.*, reflexion, slide)

They require **(VERY) LARGE QUANTITY** of data

What's Left?

- Algebraic Attacks
- Meet-in-the-Middle attacks
- Guess-and-Determine attacks

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The AES Has a Clean Description over $\mathbb{F}_{256}$

$$x_0[j] = P[j] + k_0[j]$$
$$y_i[j] = S(x_i[j])$$
$$x_{i+1} = \begin{pmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{pmatrix} \begin{pmatrix} y_i[0] & y_i[4] & y_i[8] & y_i[12] \\ y_i[5] & y_i[9] & y_i[13] & y_i[1] \\ y_i[10] & y_i[14] & y_i[2] & y_i[6] \\ y_i[15] & y_i[3] & y_i[7] & y_i[11] \end{pmatrix} + k_{i+1}$$

- **Equation** = linear combination of **Terms** over $\mathbb{F}_{256}$
- **Term** = X_i or $S(X_i)$

The equations are:

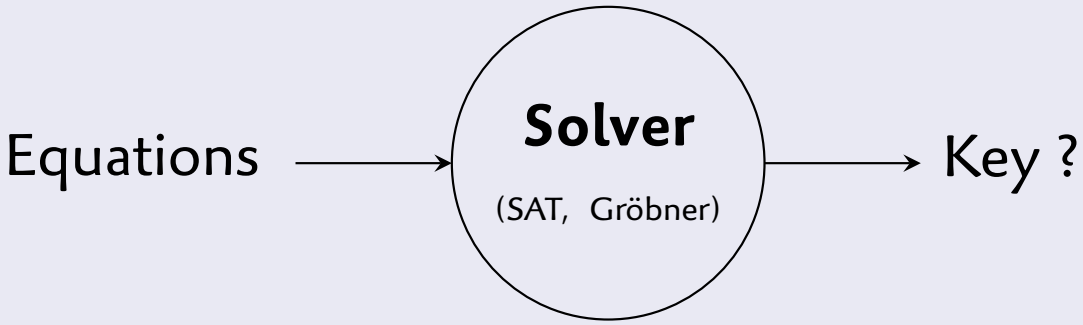
- sparse: each equation relates, at most, five variables
- structured: each variable appears in, at most, four equations
  - A **Gröbner basis** of the equations is **known** (but useless)

Working With the Equations

Is it a Problem?

- **Concerns** about the AES's algebraic simplicity have been expressed several times

Algebraic Cryptanalysis: the Direct Approach



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- 2005 : Previous claim **debunked**
  - No practical results
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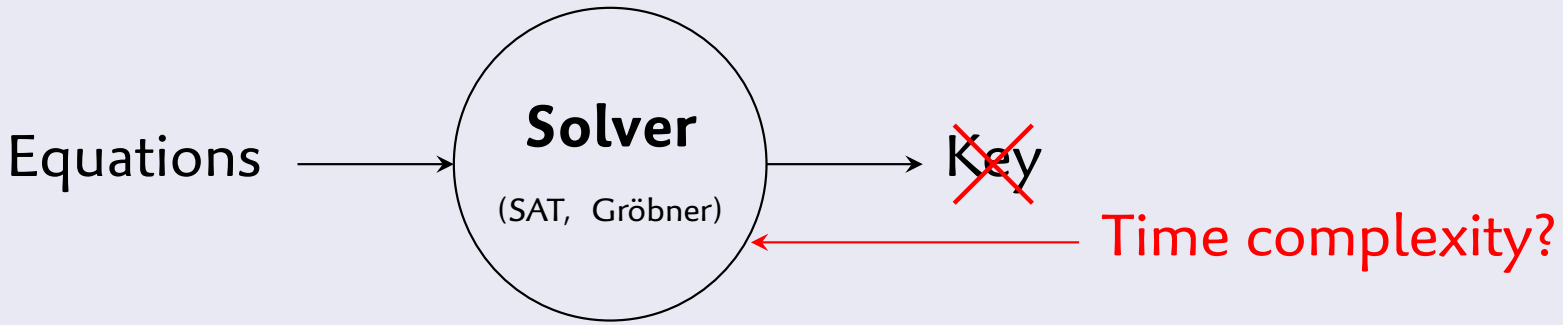

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The Problem With Algebraic Attacks

Why is it failing?

- Main obstacle = S-box
- has only "bad" representations

Pluging the description of the S-box into the equations makes them hard.

$\Longrightarrow$ Could we not do that?

Goal

Find a way to solve the equations **independently of a particular choice of** S

Paradigm Shift

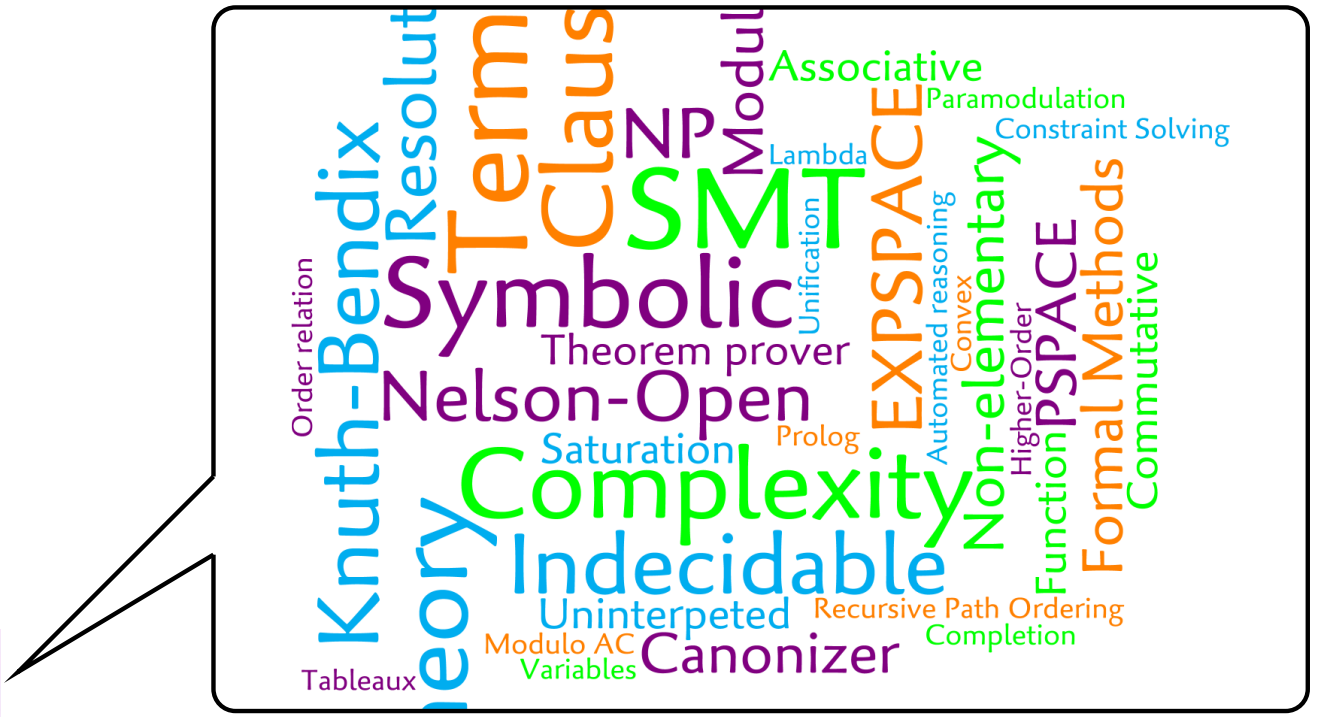
Old Goal

Break Cryptographic Stuff

Paradigm Shift

New Goal

(efficiently) Solve linear equations over $\mathbb{F}_{256}$ with an **uninterpreted** permutation symbol



Paradigm Shift

New Goal

(efficiently) Solve linear equations over $\mathbb{F}_{256}$ with an **uninterpreted** permutation symbol

DISCLAIMER: there is (probably) a lot of existing litterature devoted to this particular problem, or to similar-looking ones, that I am not aware of.

In this talk, I present our own answers, which are based on cryptographic techniques.

They crucially rely on the fact that variables live in a finite domain.

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Meet-in-the-Middle Attacks

A Well-Known Example: Double-DES

$$2DES_{k_1,k_2} = DES_{k_2} \circ DES_{k_1}$$

- Initialize a Hash Table
- For all k_1, store $M = DES_{k_1}(P) \rightarrow k_1$
- For all k_2, look-up $M = DES^{-1}_{k_2}(C)$

Time complexity $\approx 2^k$ encryptions, with $2k$-bit keys!

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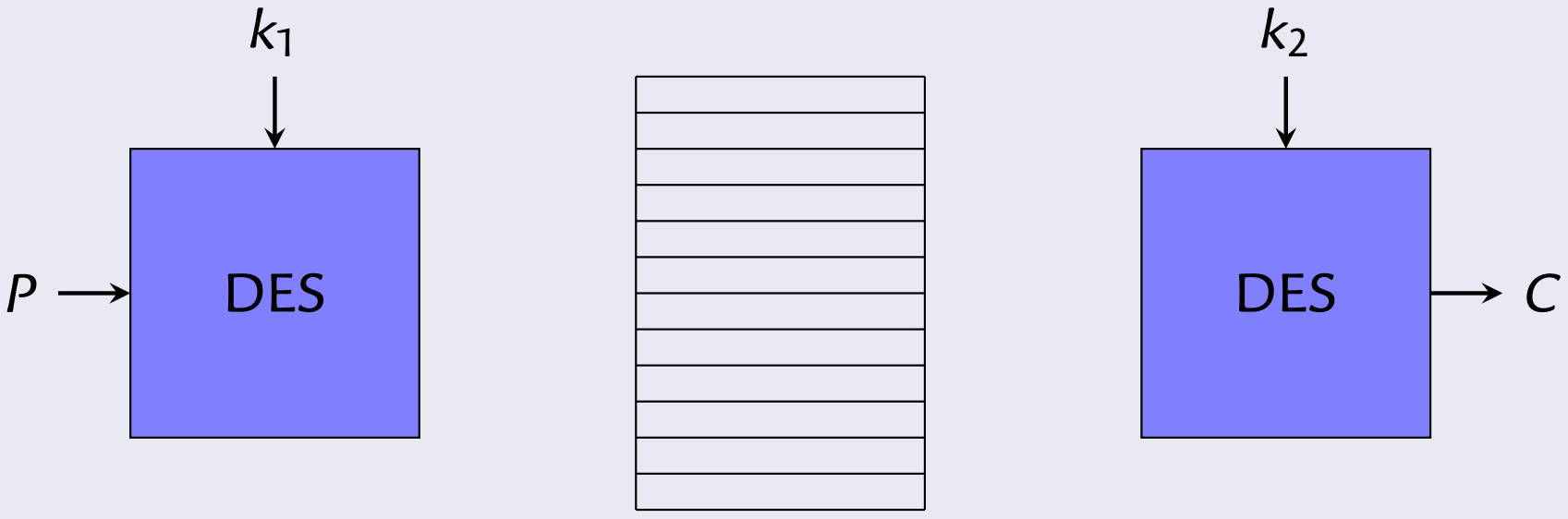

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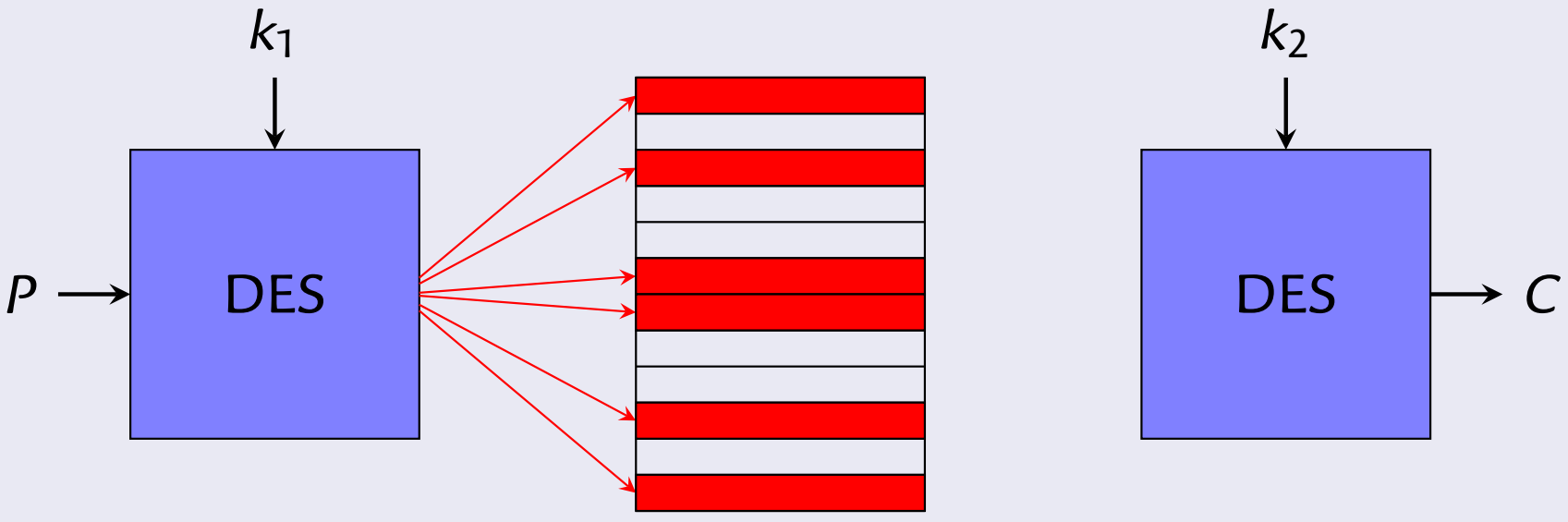

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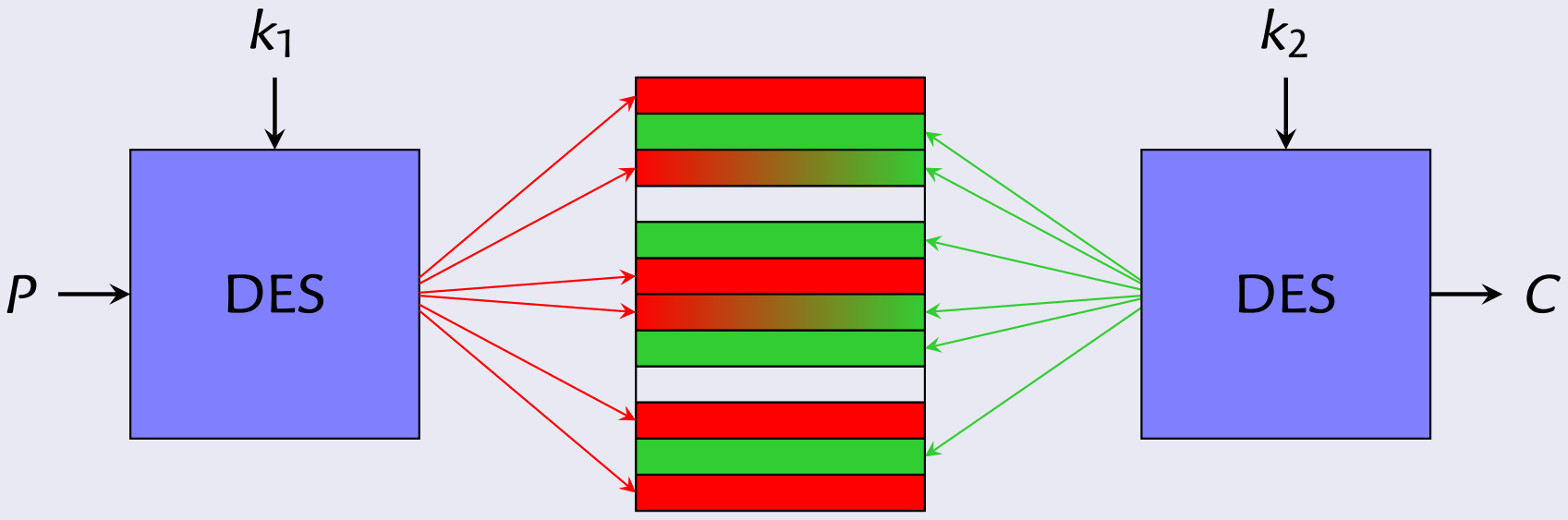
Meet-in-the-Middle Attacks

A Well-Known Example: Double-DES



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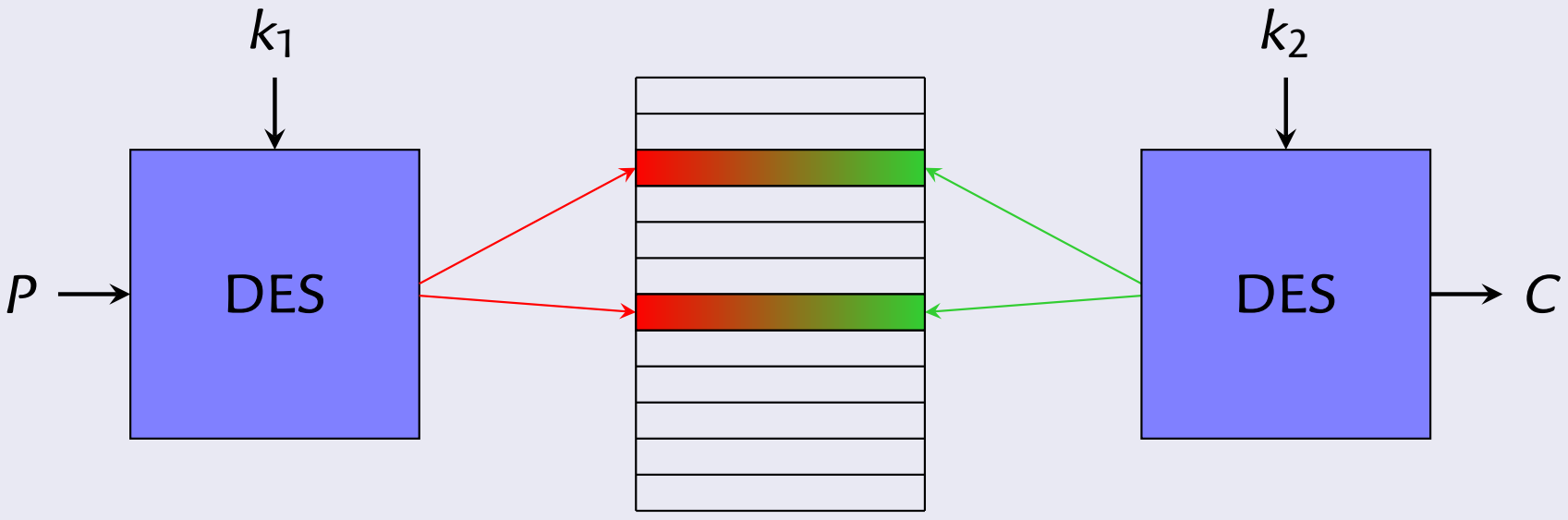

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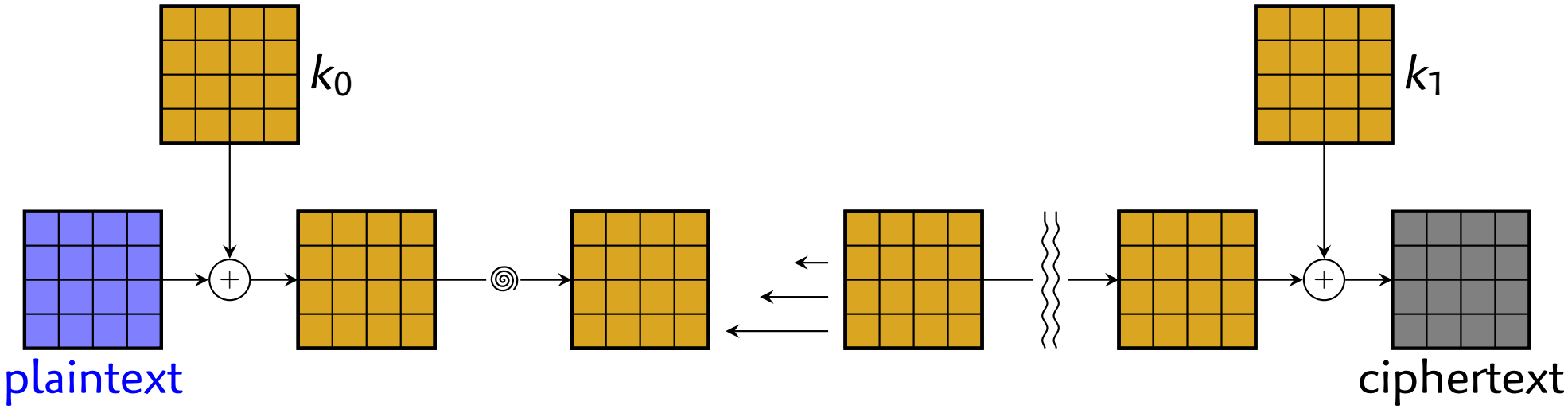

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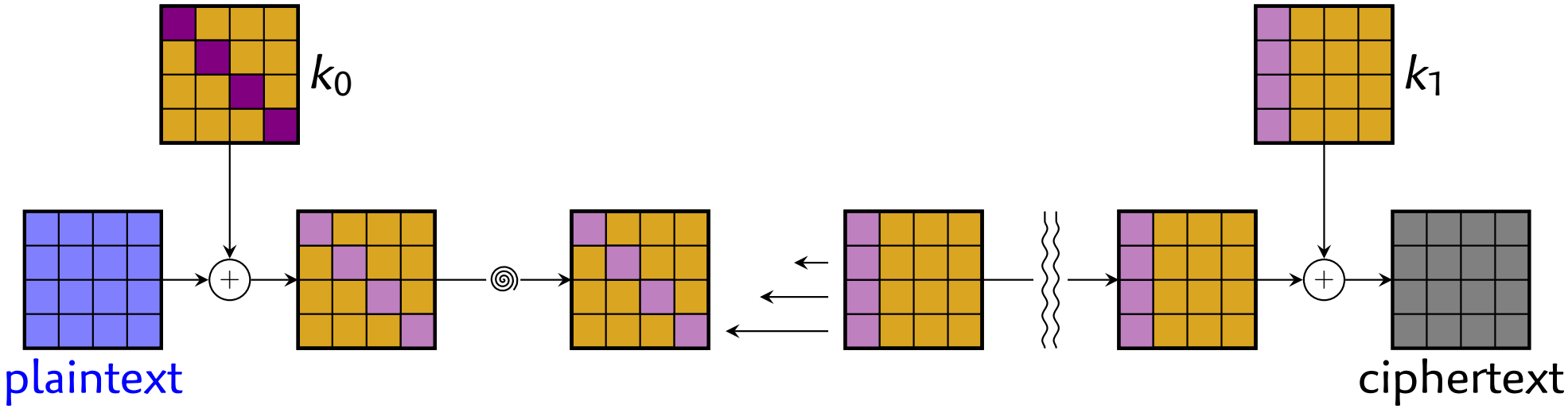

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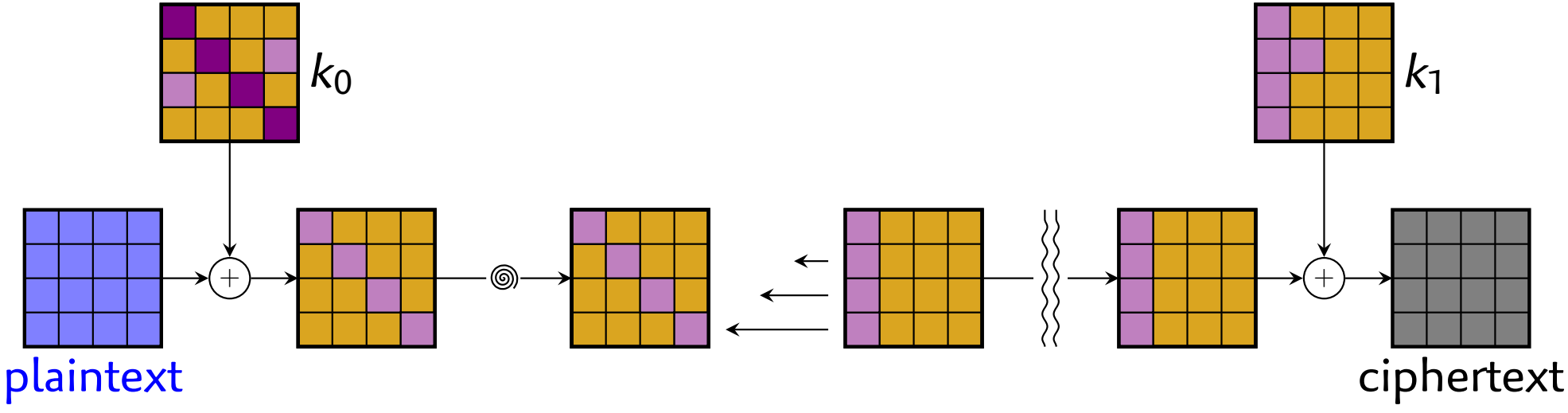

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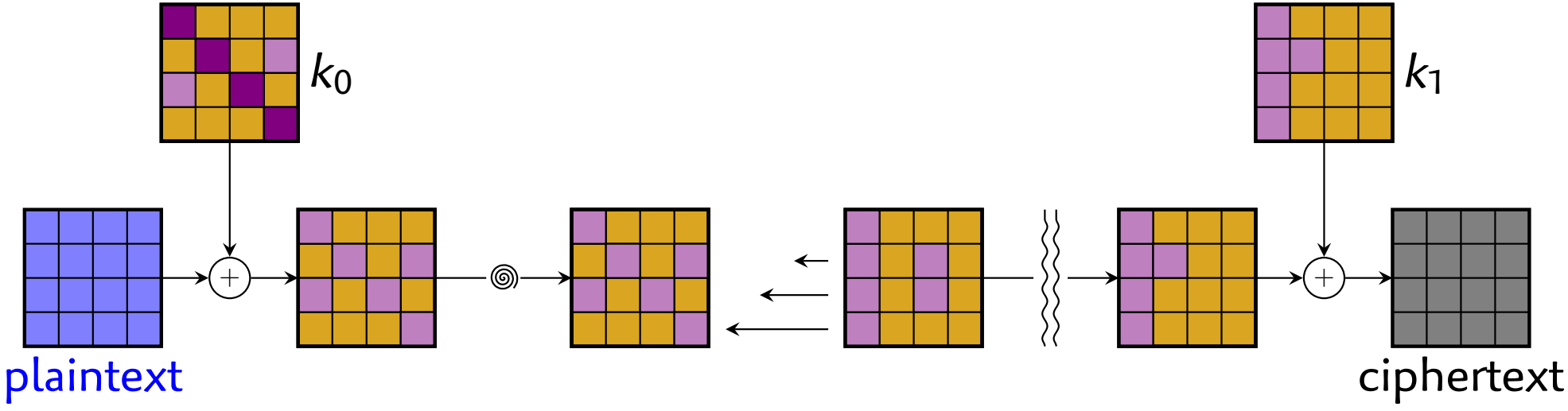

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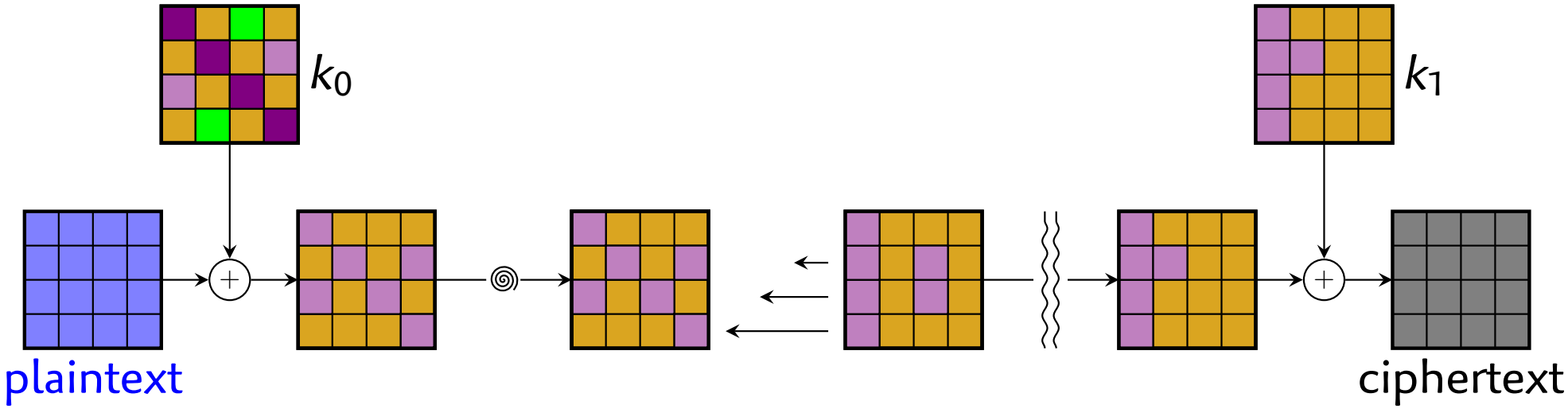

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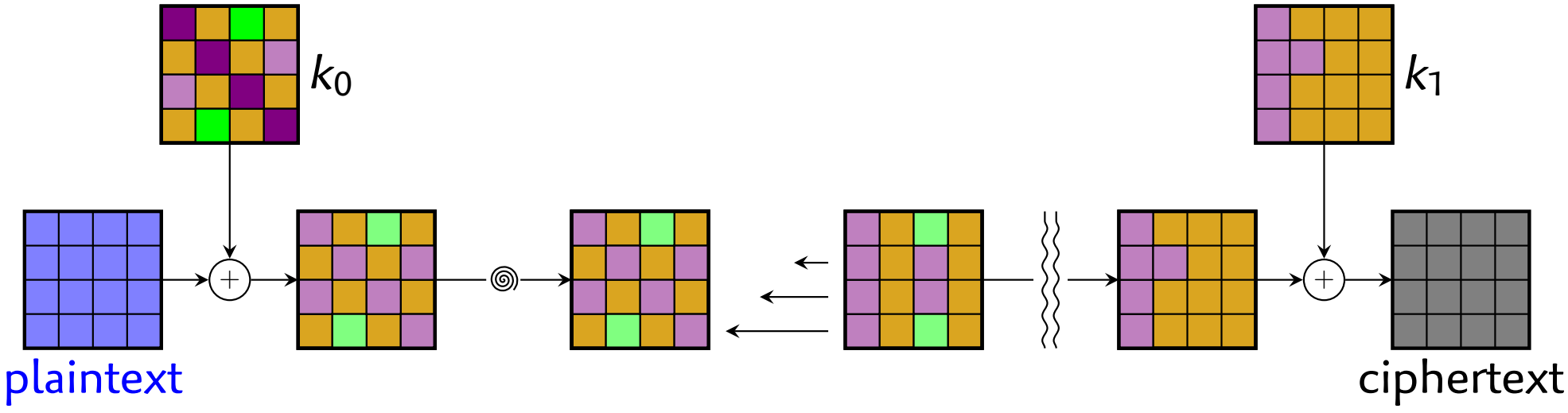

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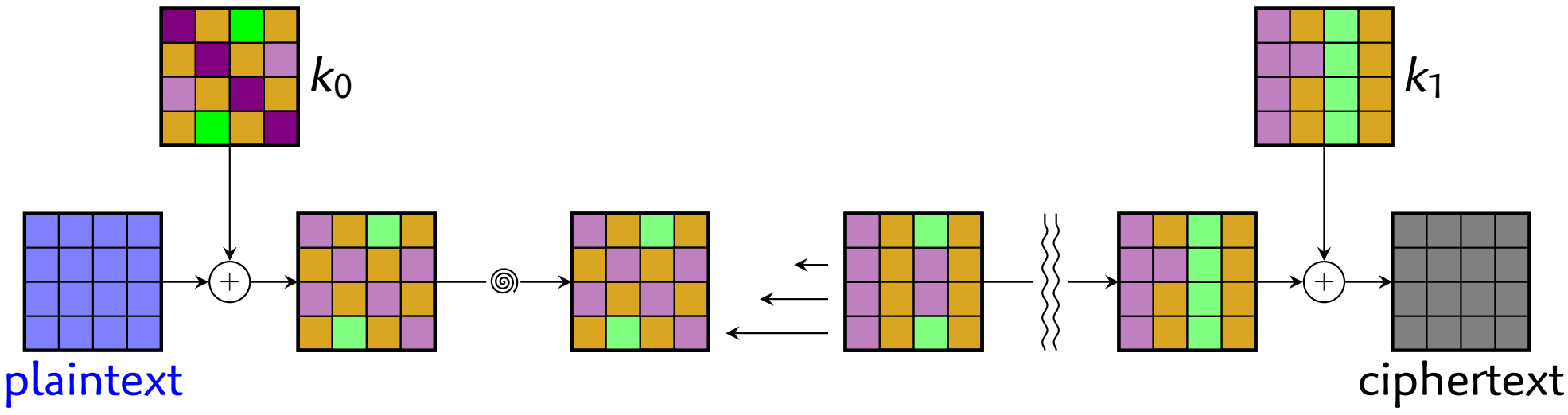

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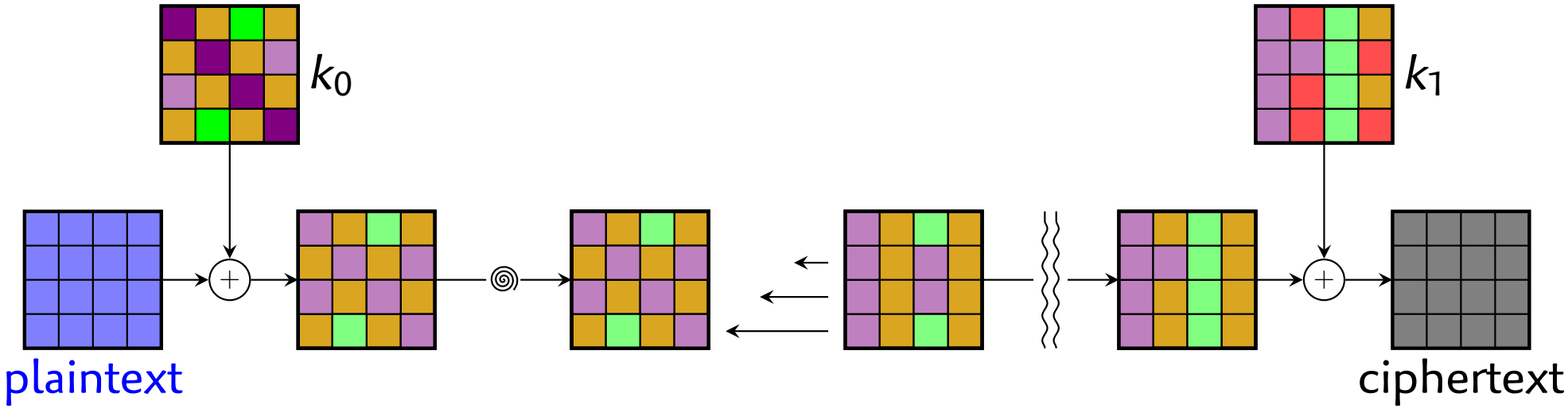

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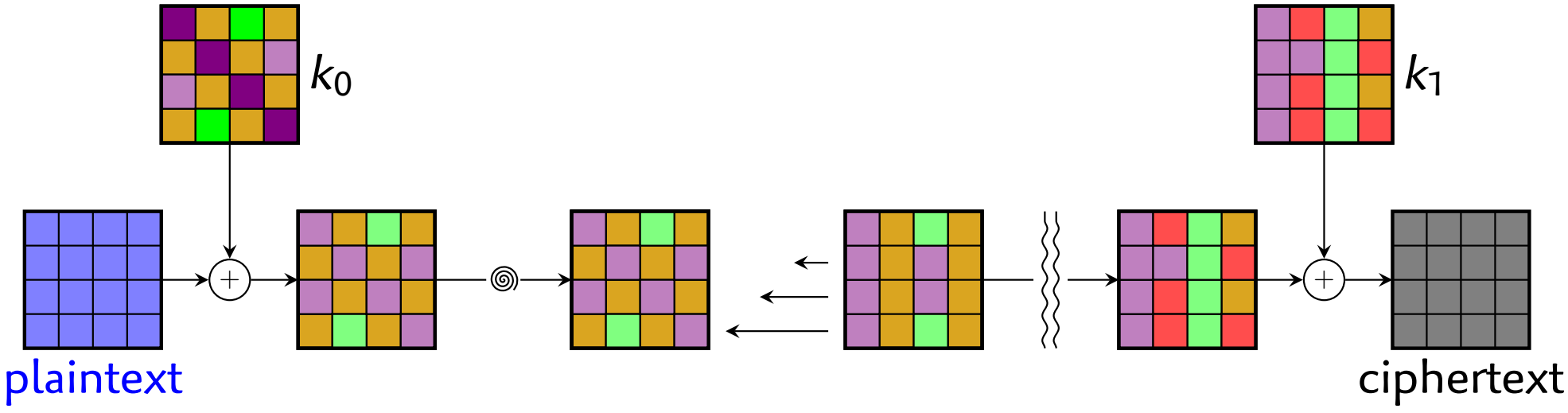

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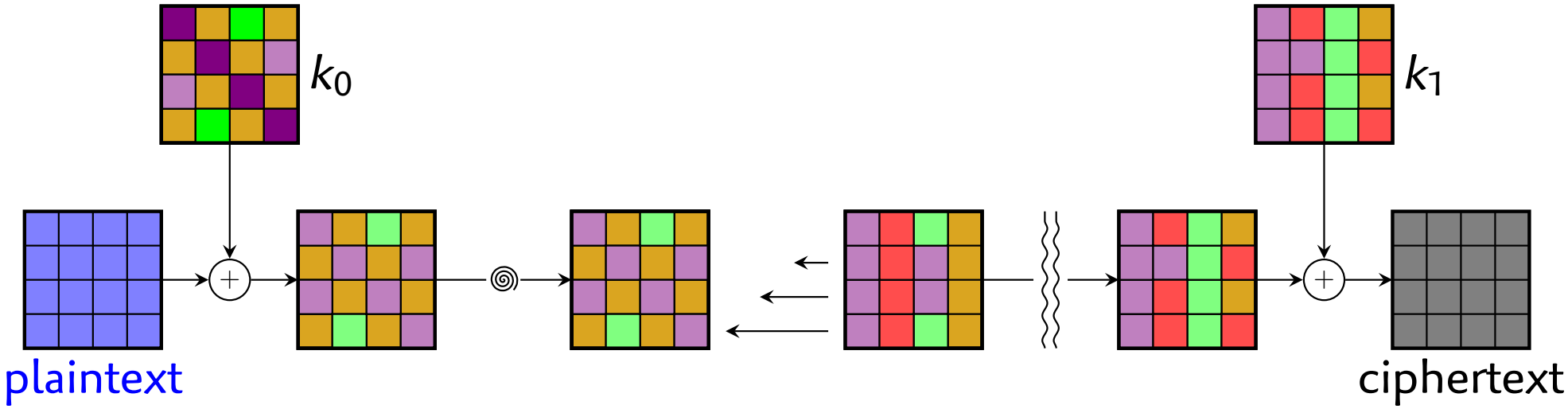

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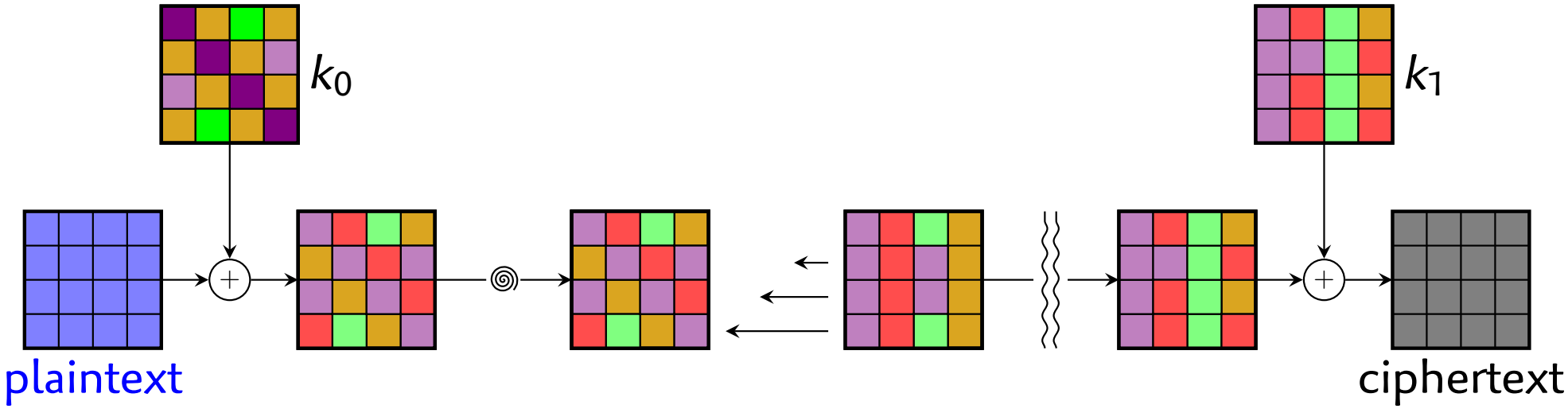

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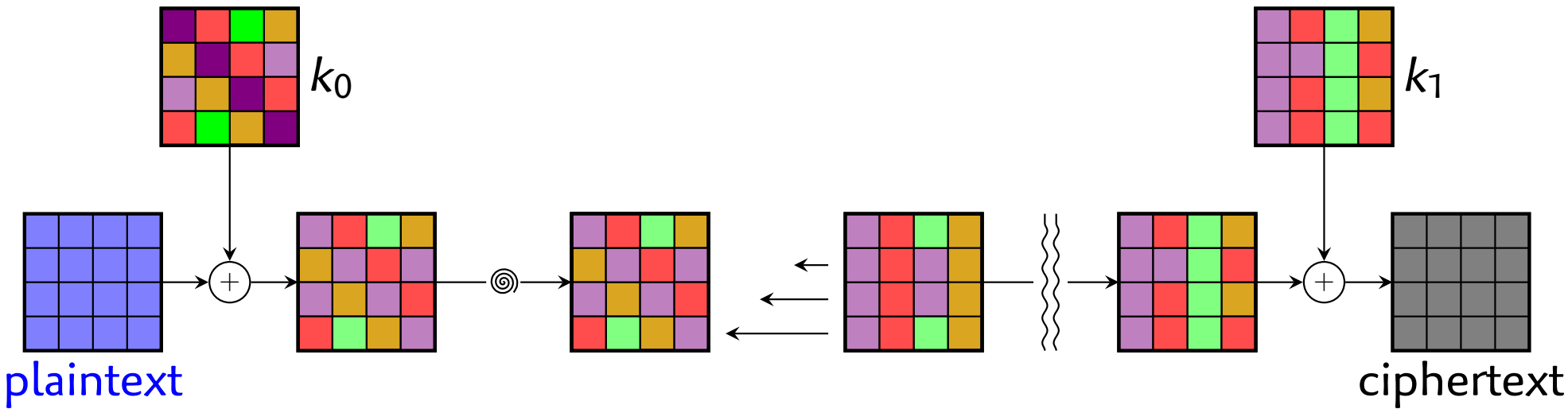

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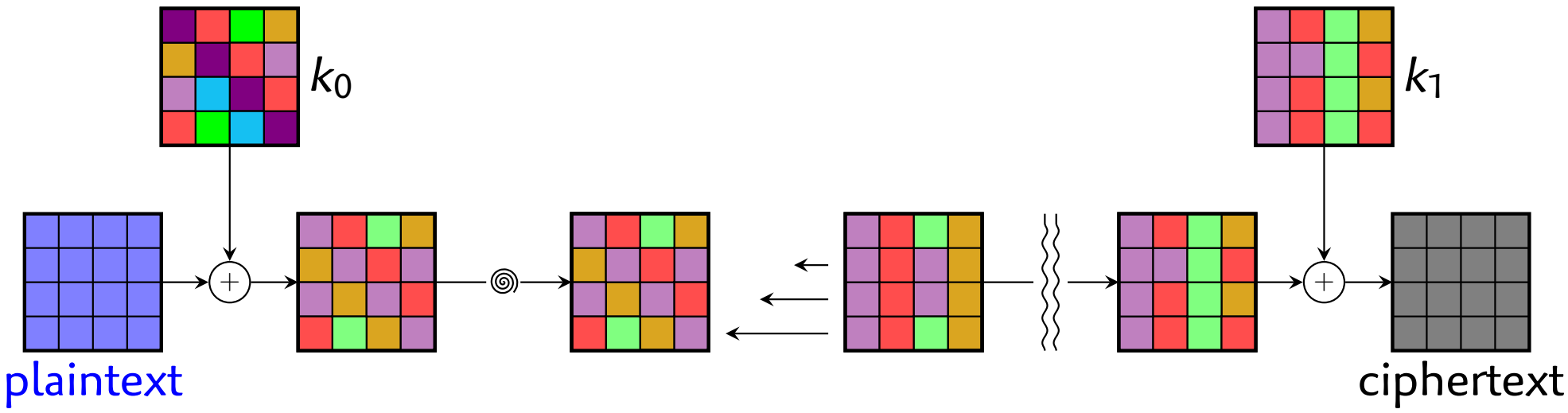

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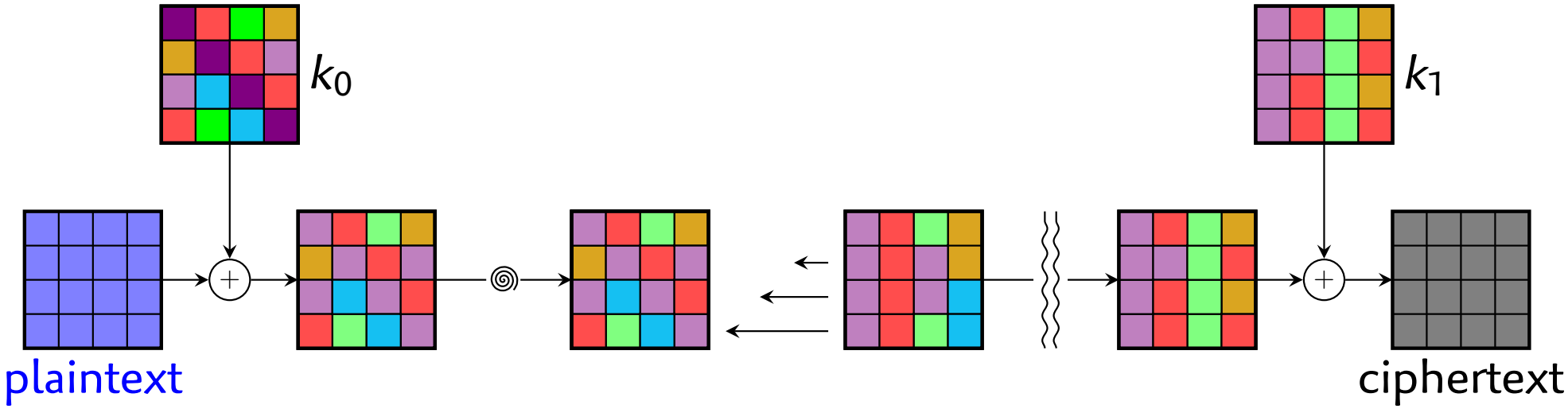

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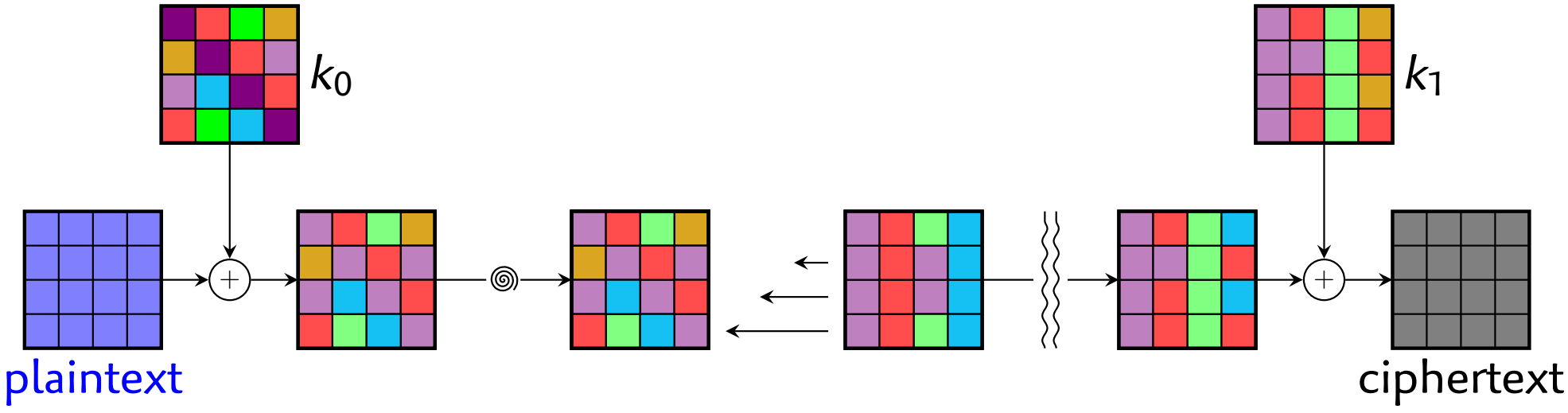

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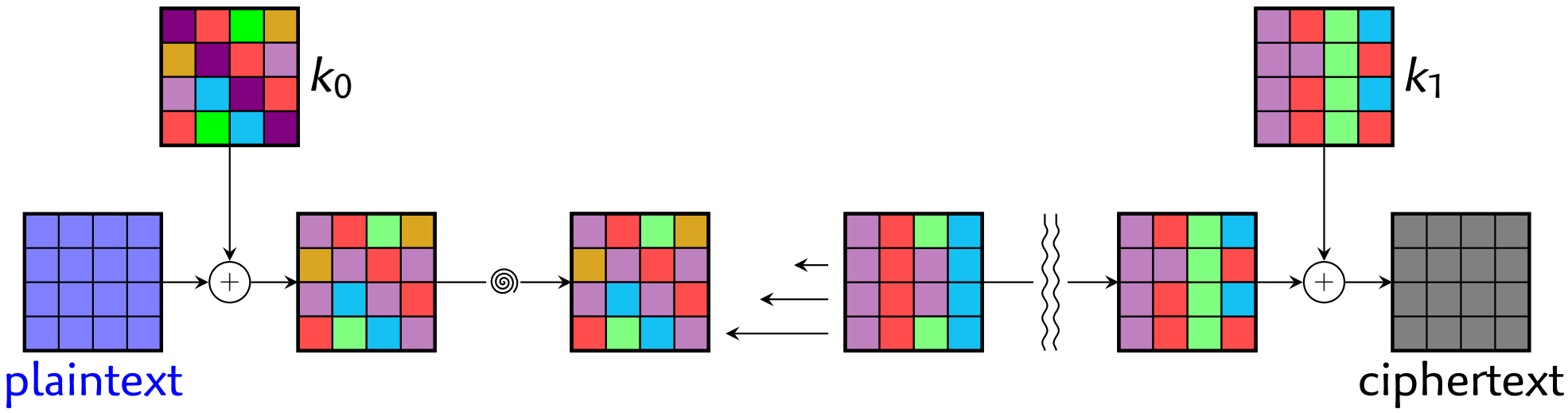

Simple in Theory...

- After a few Hours of doing this by hand
- larger systems (more rounds) → it gets even harder

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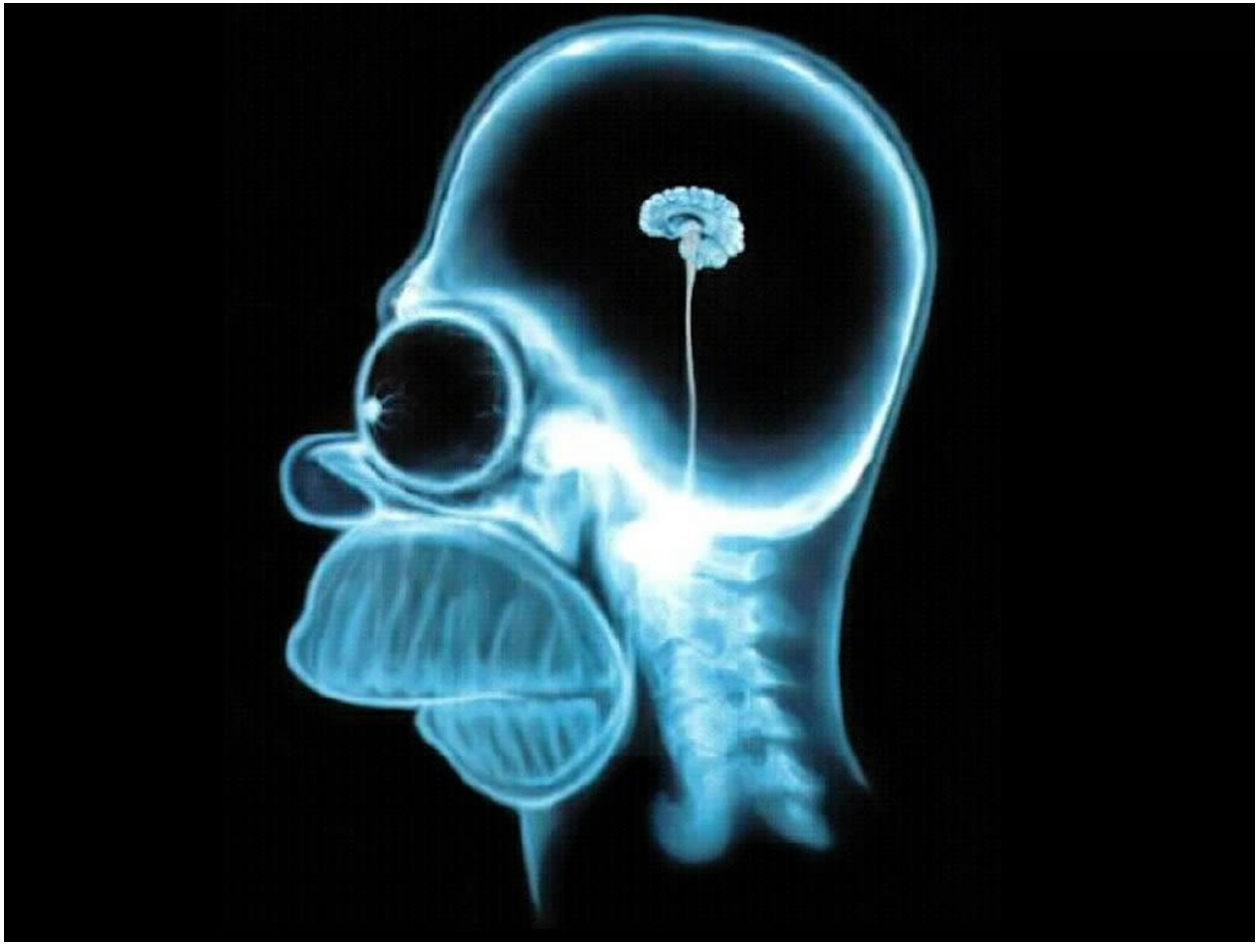

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- less guessed variables $\rightarrow$ it gets harder
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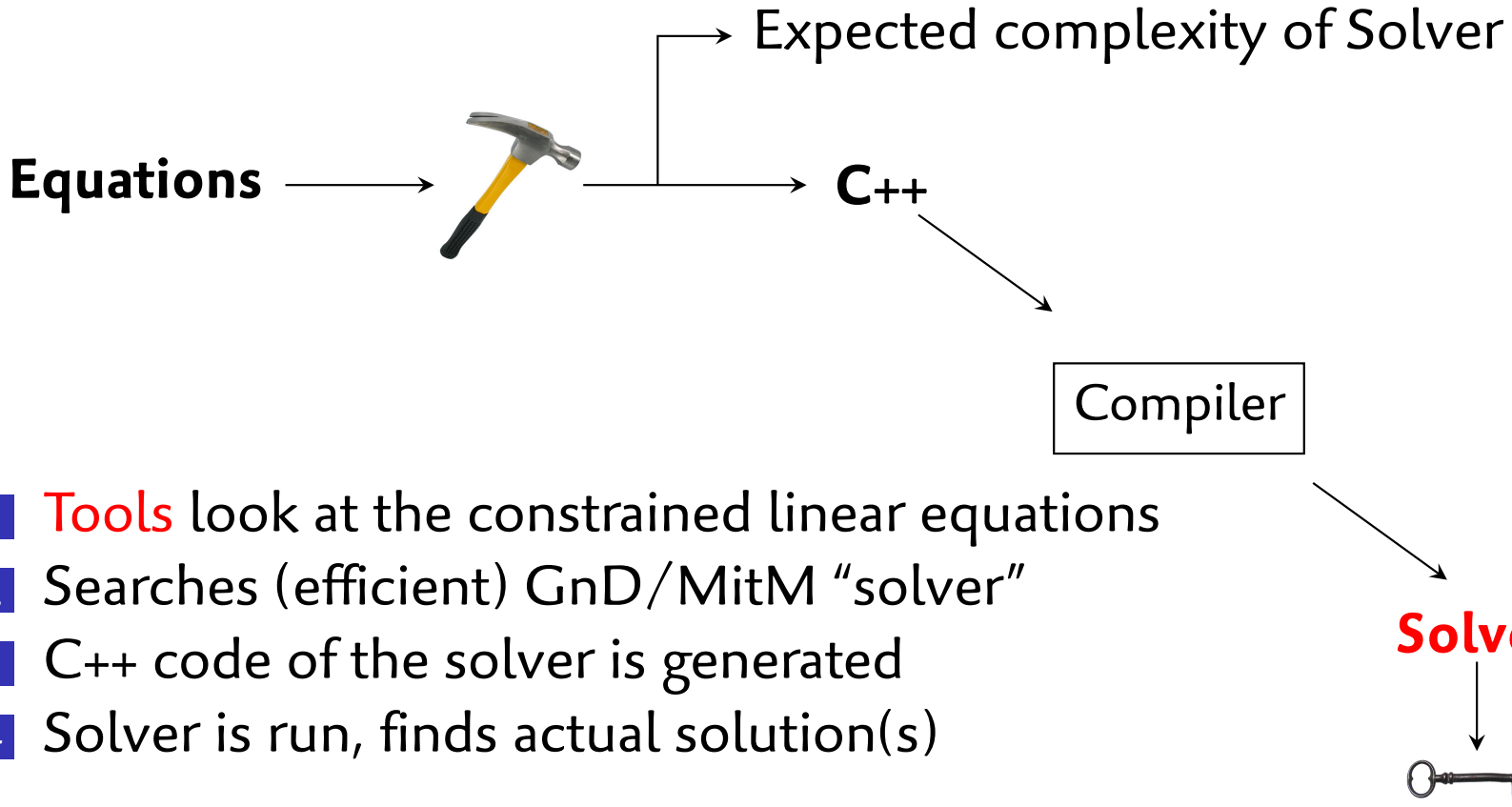
A Two-Phase Approach



1. Tools look at the constrained linear equations
2. Searches (efficient) GnD/MitM "solver"
3. C++ code of the solver is generated
4. Solver is run, finds actual solution(s)

In Cryptographic Terms...

- The Equations describe a cryptographic **Primitive**
- The Solver is an **Attack** against the primitive
- The Tools is a fully-automatic **Attack-Finder**

Finding Guess-and-Determine Attacks Automatically

The equations are **sparse**

All terms known except one $\rightarrow$ knowledge propagation

$$\text{e.g.} \quad x_i + S(z_j) + 03 \cdot z_k = 0$$

The equations are (essentially) **linear**

Gaussian elimination allows more knowledge propagation:

$$\text{e.g.} \quad \begin{cases} x_i + S(z_j) & +03 \cdot z_k & & +7\mathtt{f} \cdot u_\ell & = & 0 \\ 3\mathtt{d} \cdot x_j & +56 \cdot z_k & +S(v_r) & +9\mathtt{a} \cdot u_\ell & = & 0 \\ \mathtt{c2} \cdot y_s & +84 \cdot z_k & +\mathtt{cf} \cdot S(v_r) & & = & 0 \end{cases}$$

All terms known except one in a linear combination

A DPLL-Like Search Procedure

The DPLL procedure for SAT-solvers (big success story!)

- Propagate constraints as much as possible
- SAT $\rightarrow$ Return model
- When stuck, choose a variable and guess it
- UNSAT $\rightarrow$ Backtrack
- Tricks to prune the search-space (backjumping, learning)

The DPLL procedure for... Us?

- Propagate knowledge as much as possible
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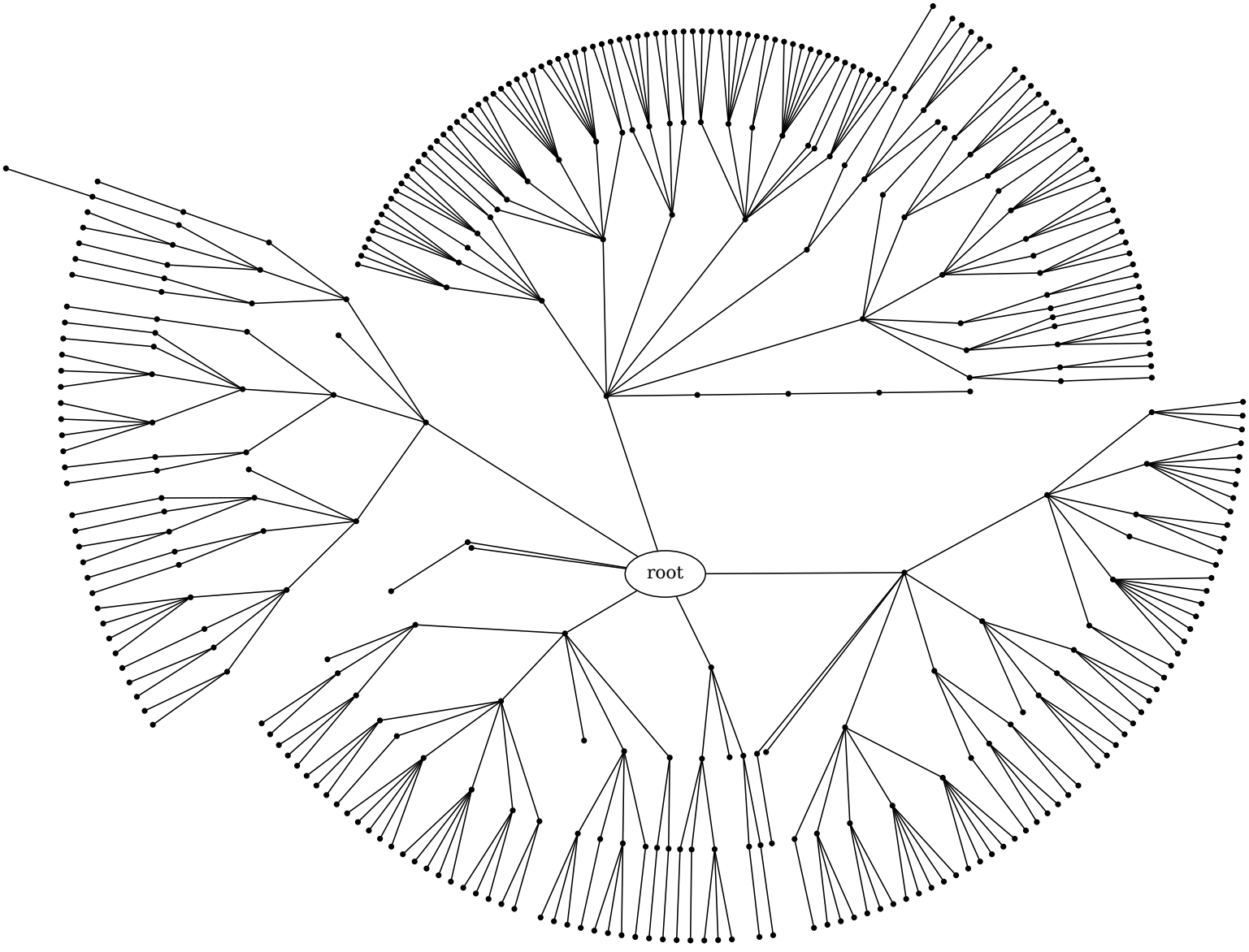
Attacking 1 round: Trace ($\geq$ 20 variables)



Implementation Details

Actual Implementation

- 5000 lines of OCaml
- Efforts put into efficiency
- Non-trivial sparse linear algebra
  - Triangular solver, re-echelonization
- Distributed: Home-made MapReduce on top of OcamlMPI
  - Run on $\approx$ 500 MIPS-like cores for a month without problem
- Generates C code of the solver

The Tool Put to Work

Results on round-reduced version of the AES-128

		Tool-found	**Human-found**
#Rounds	Data	Time	Time
1	1 pair	$\mathbf{2^{40}}$	2^{48}
2	1 pair	2^{80}	2^{96}
3	1 pair	2^{120}	not found

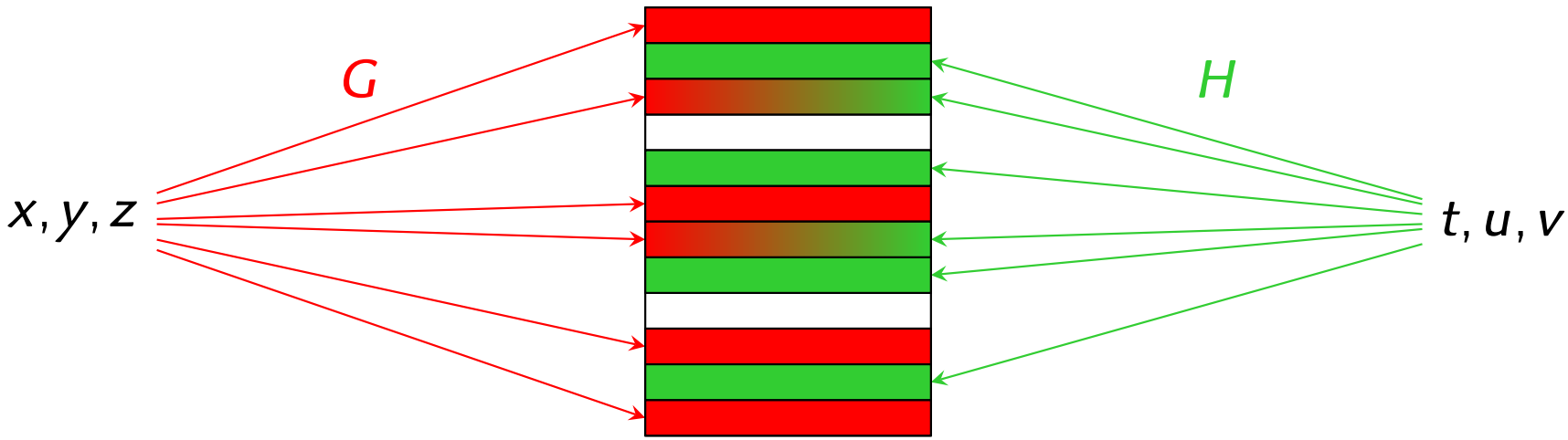
- The 1-round attack generated by the tool does work in 18h
- These attacks are guaranteed to be the best in this category

$\Longrightarrow$ ($\leq$ 3 round)-AES: Exhaustively explored the search space

Finding Meet-in-the-Middle Solvers

Idea: Partition the Set of Variables in Two

$$F(x, y, z, t, u, v) = 0 \iff G(x, y, z) = H(t, u, v)$$



Meet-in-the-Middle Solver

- **for all** x, y, z, **store** $G(x, y, z) \mapsto (x, y, z)$ in a hash table
- **for all** u, v, t, **look-up** $H(u, v, t)$ in the hash table
  - On average one value of (x, y, z) per value of (u, v, t).

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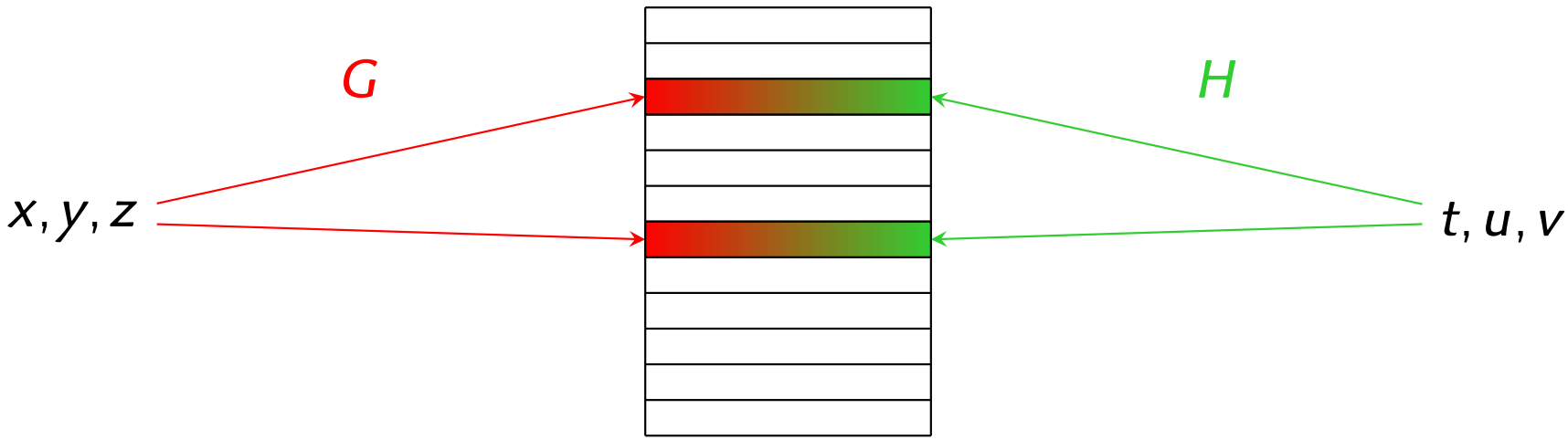

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Variable Elimination

Suppose we have a partition $X \cup Y$ of the variables.

Question: are there equations in only X (resp. Y)?

- Intuition: **yes**, because equations are very sparse
- **How** to find them?
- Generic Problem known as **Elimination**
  - Want to get rid of some variable(s)
- Just selecting a subset of the initial system seems **bad**
- Polynomial systems: computation of Elimination Ideals
  - Gröbner Basis computation with ad hoc term order

A Compromise Solution

- **easy** to find the linear combinations with variables in X
  - Vector-space Intersection, **Polynomial** time complexity

Recursive Top-Down Approach

Objective: Find **Balanced** partition such that

$$F(x,y,z,t,u,v) = 0 \iff \begin{cases} G_1(x,y,z) & = & H_1(t,u,v) \\ G_2(x,y,z) & = & 0 \\ 0 & = & H_2(t,u,v) \end{cases}$$

Improved Solving Algorithm

- **for all** solutions of $G_2(x,y,z) = 0$
  - **Store** $G_1(x,y,z) \rightarrow (x,y,z)$ in a hash table
- **for all** solutions of $H_2(u,v,t) = 0$
  - **Look-up** $H_1(u,v,t)$ in the hash table
- Each match suggests a complete solution
- Problem reduces to solving two **sub-problems recursively**

Same problem
Smaller instan

Higher-Order Point of View

$$F(X,Y)=0 \iff \begin{cases} G_1(X) &= H_1(Y) \\ G_2(X) &= 0 \\ 0 &= H_2(Y) \end{cases}$$

In fact we have described a Combination operation:

- Algorithm $\mathcal{A}_1$ solves $G_2(X)=0$
- Algorithm $\mathcal{A}_2$ solves $H_2(Y)=0$

$\Longrightarrow$ "meet-in-the-middle" algorithm $\mathcal{A}_1 \bowtie \mathcal{A}_2$ for $F(X,Y)=0$

$$\mathcal{A}_1 : \texttt{solver}(X), \qquad \mathcal{A}_2 : \texttt{solver}(Y),$$
$$\bowtie : \texttt{solver}(X) \to \texttt{solver}(Y) \to \texttt{solver}(X \cup Y)$$

- "Properties" (complexity) of $\mathcal{A}_1 \bowtie \mathcal{A}_2$ easy to determine

$$T(\mathcal{A}_1 \bowtie \mathcal{A}_2) = T(\mathcal{A}_1) + T(\mathcal{A}_2) + \#Sols(F)$$

$$M(\mathcal{A}_1 \bowtie \mathcal{A}_2) = \max\left\{M(\mathcal{A}_1), M(\mathcal{A}_2), \min\left[\#Sols(G_2), \#Sols(H_2)\right]\right\}$$

Towards Solving the Whole Problem

The actual difficulty...

... is to find a nice partition $X \cup Y$.

- **Bottom-up** approach
- Start from trivial algorithms that enumerate a single variable
  - Usually no equation in a single variable

  $\Longrightarrow$ Solutions = $\mathbb{F}_{256}$

- Combine them until **saturation**, throw bad/redundant ones

Just like

- Buchberger's algorithm for Gröbner Bases
- Knuth-Bendix completion for equational theories
- Resolution/paramodulation/whatever for 1st-order logic

The Saturation Procedure

Simplified Version

1: **function** BESTSOLVERS($\mathbb{E}$)
2: $G \leftarrow \{\text{BaseSolver}(x) : x \in \mathbb{X}\}$
3: $\mathcal{P} \leftarrow \{(G_i, G_j) : 1 \leq i < j \leq |G|\}$
4: **while** $\mathcal{P} \neq \emptyset$ **do**
5: Pick $(\mathcal{A}_1, \mathcal{A}_2) \in \mathcal{P}$ and remove it from $\mathcal{P}$
6: $\mathcal{C} \leftarrow \mathcal{A}_1 \bowtie \mathcal{A}_2$
7: **if** $\mathcal{C}$ not subsumed by anything in G **then**
8: $\mathcal{P} \leftarrow \mathcal{P} \cup \{(\mathcal{A}, \mathcal{C}) : \mathcal{A} \in G\}$
9: $G \leftarrow G \cup \{\mathcal{C}\}$
10: **end while**
11: **return** G
12: **end function**

- Patrick Derbez : Implementation, Improvements, Tricks, ...
- 10'000 lines of C

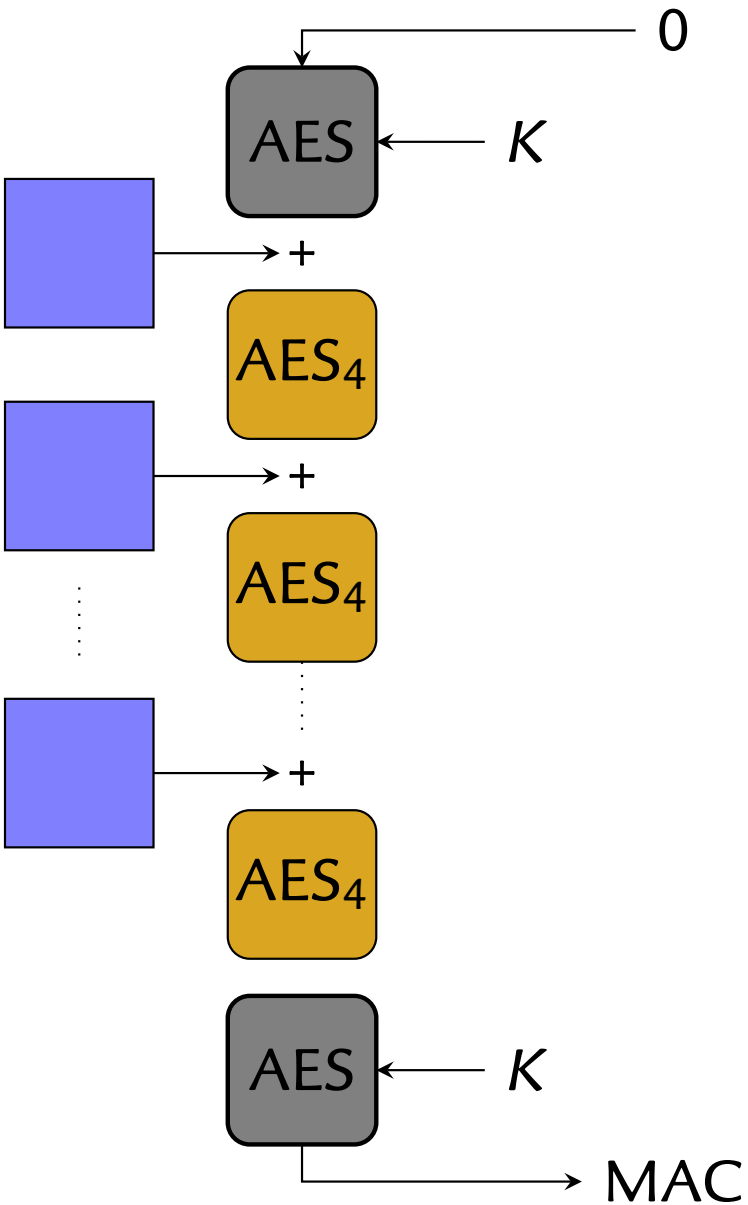
Results (Reduced AES)

Attacks on round reduced version of the AES-128

		Tool-found	Human-found
#Rounds	Data	Time	Time
1	**1 KP**	**2^{32}**	**2^{48}**
2	1 KP	2^{64}	2^{80}
2	**2 KP**	**2^{32}**	**2^{48}**
2	**2 CP**	**2^{8}**	**2^{28}**
3	1 KP	2^{96}	
3	**2 CP**	**2^{16}**	**2^{32}**
4	1 KP	2^{120}	
4	2 CP	2^{80}	2^{104}
4	**4 CP**	**2^{32}**	
4	5 CP		2^{64}
4.5	1 KP	2^{120}	

The attacks that are practical have been actually run and verified

Description of Pelican-MAC



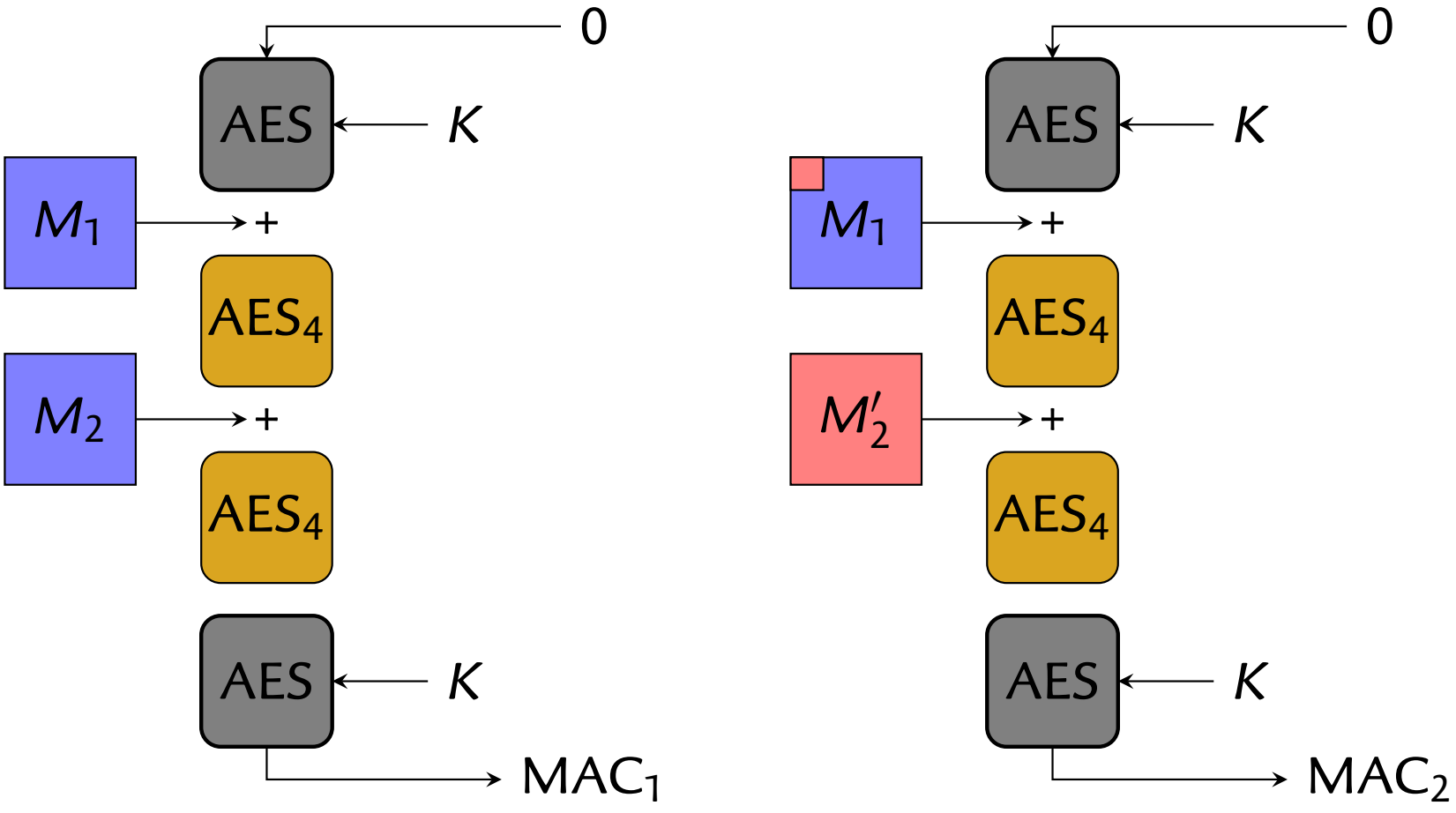
- MAC based on the AES
- Also by Rijmen & Daemen

- "Provably" secure up to 2^{64}
- Best known attack in $2^{85.5}$

- Initial state randomized with K
- 16-byte message block XORed
- 4 keyless AES rounds
- Finalization: full AES

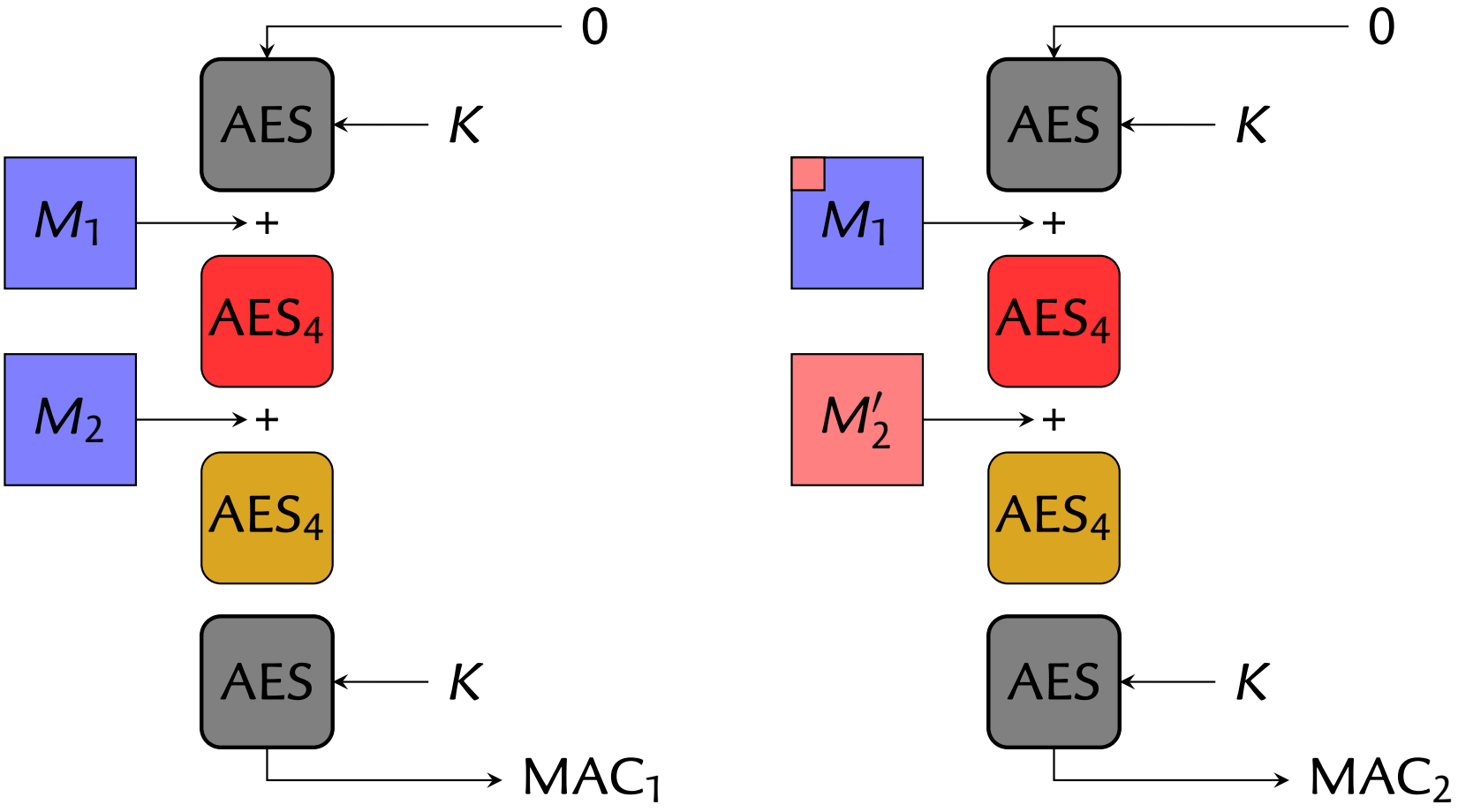
- Knowing the state $\rightarrow$ forgeries

Pelican-MAC: Finding an Internal State Collision



1. Pick random M_1
2. Try 2^{64} random M_2 and 2^{64} random M'_2
3. Look for $\mathrm{MAC}_1 = \mathrm{MAC}_2$

Pelican-MAC: Finding an Internal State Collision



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3. Look for $\mathrm{MAC}_1 = \mathrm{MAC}_2$

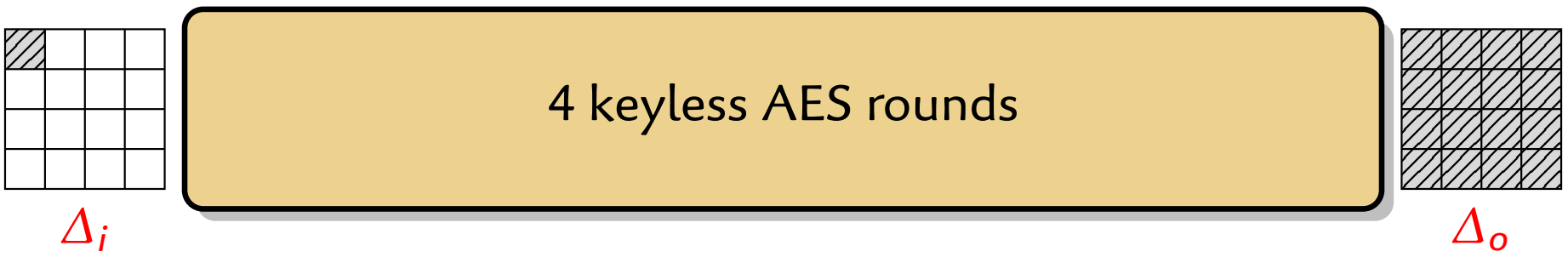
Results: Pelican-MAC

Attack Plan

- Query the MAC, find Internal Collision. Time = 2^{64}
- Solve

$$\mathrm{AES}_4(x \oplus \Delta_i) = \mathrm{AES}_4(x) \oplus \Delta_o$$

and recover x (the internal state). Time = 2^{32}



- 4 Hash Tables to build: 4×2^{32} opérations
- Isolate $\approx 2^{32}$ possible internal states

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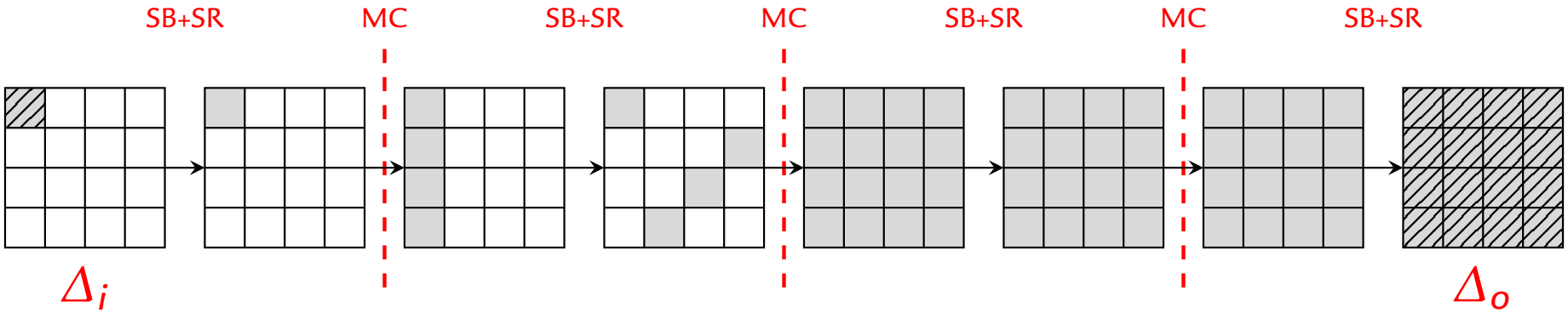

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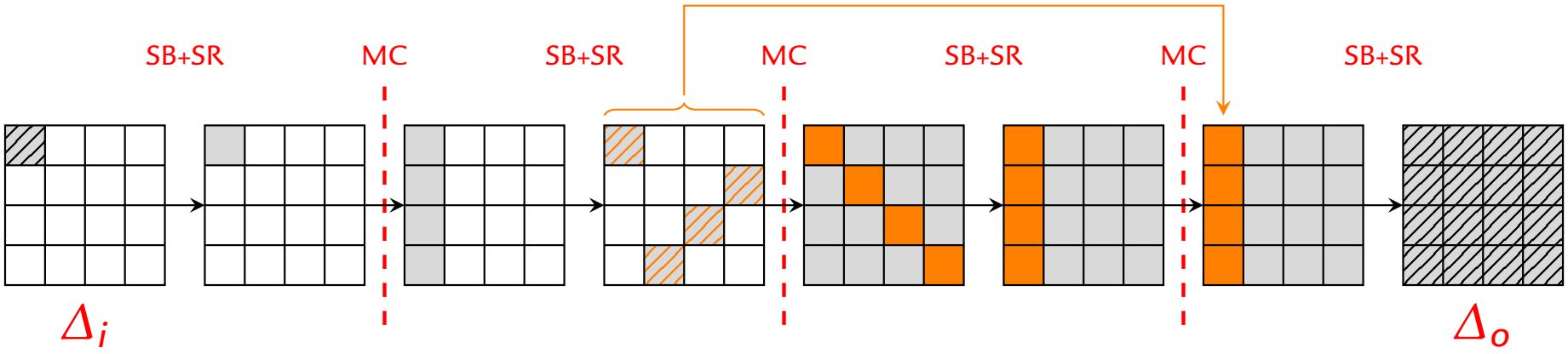

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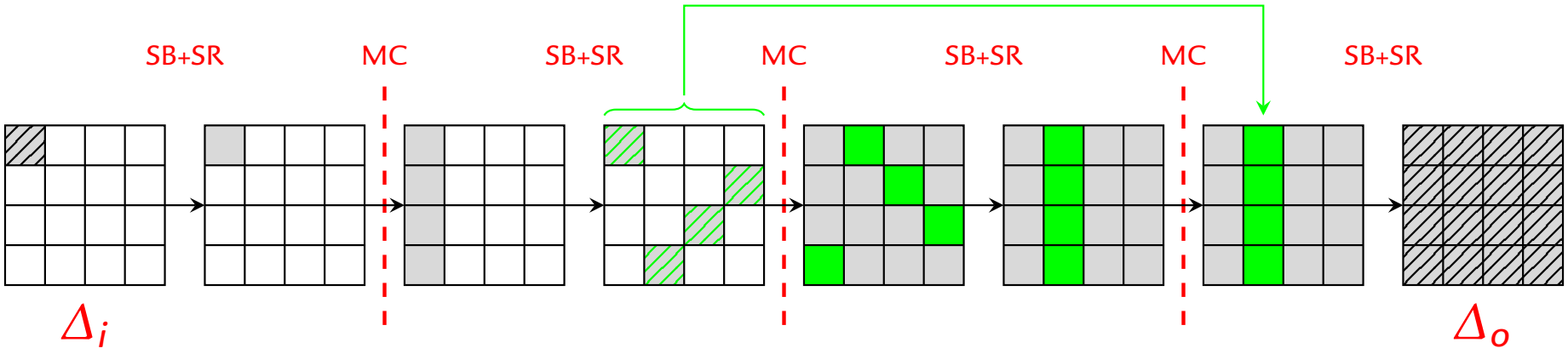

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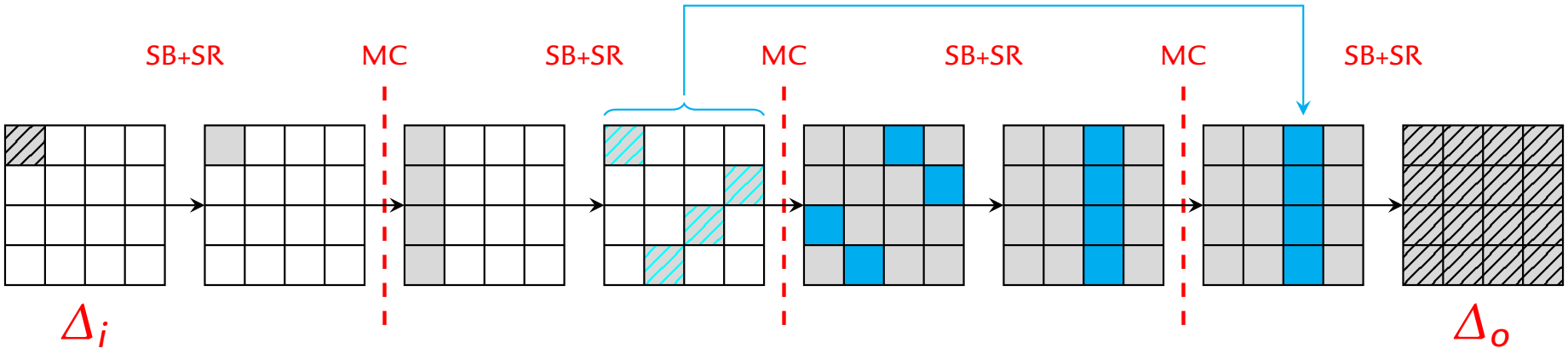

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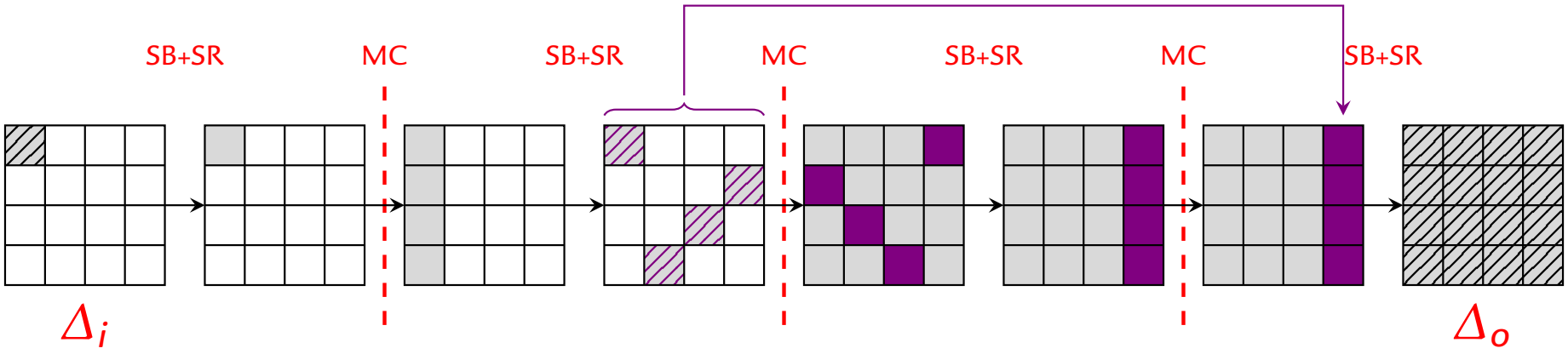

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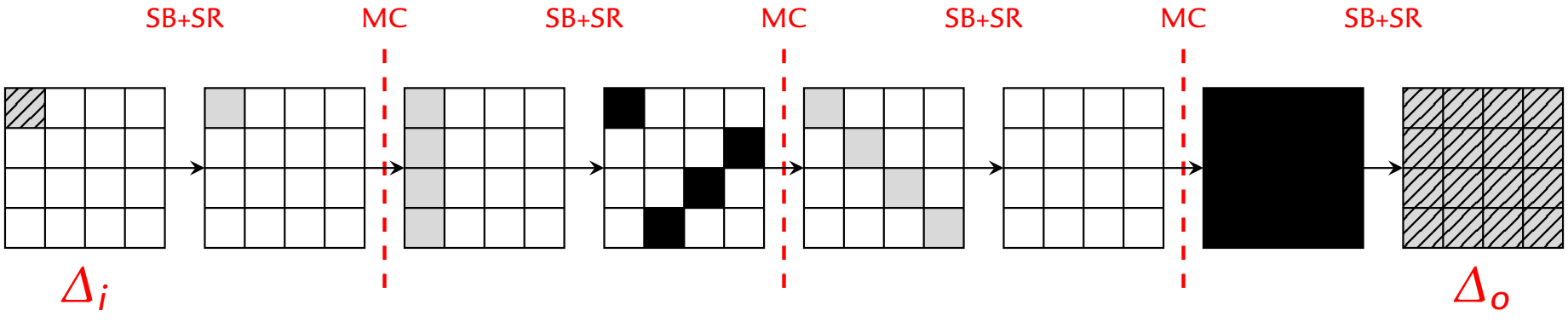

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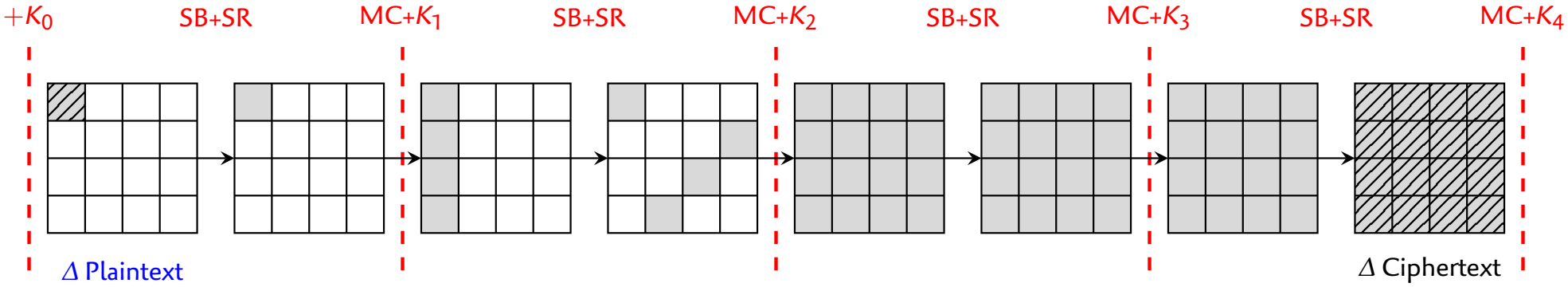
Results: 4 AES Rounds, 4 Chosen Messages (1 active byte)



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3. Obtain differences before `MixColumn` on 4 bytes
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5. $\approx 2^8$ possible column ● pass the test
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7. deduce K_4, check

▸ Skip

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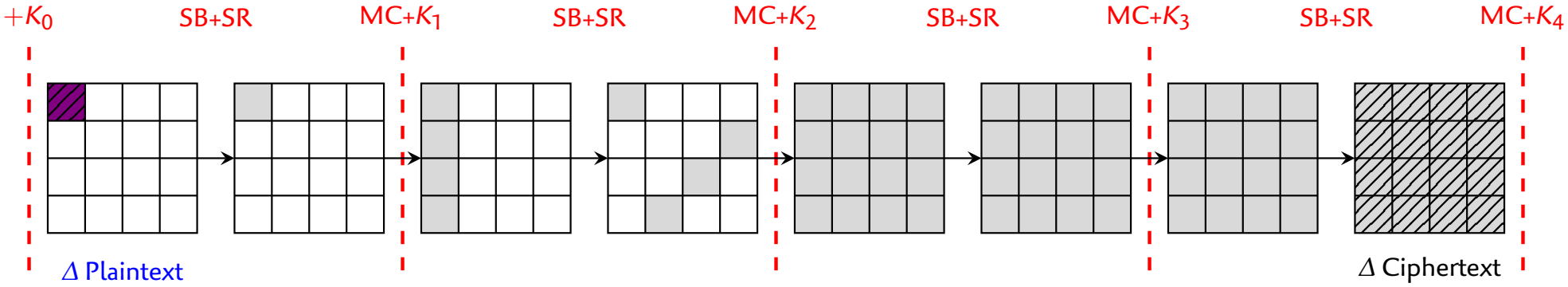

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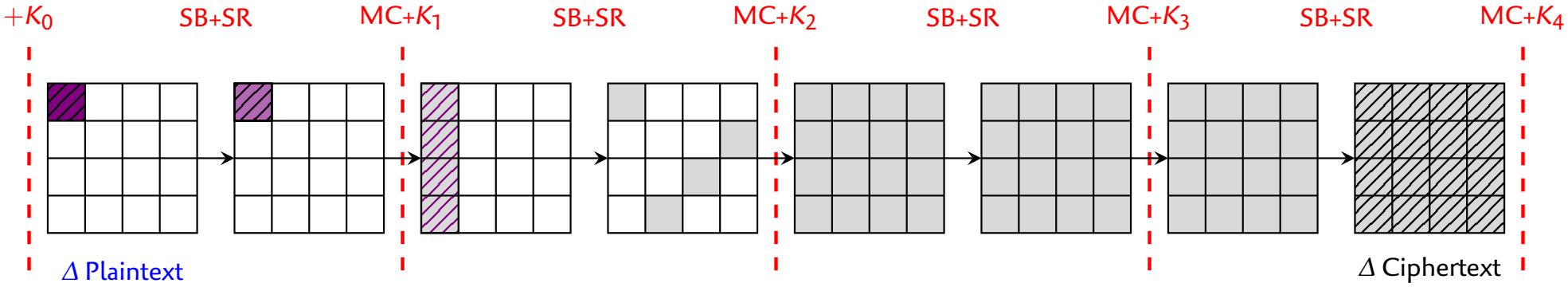

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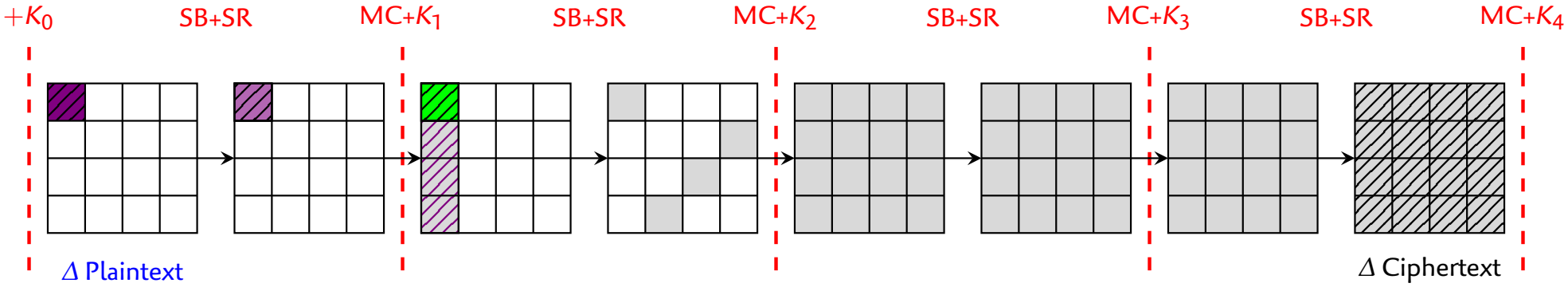

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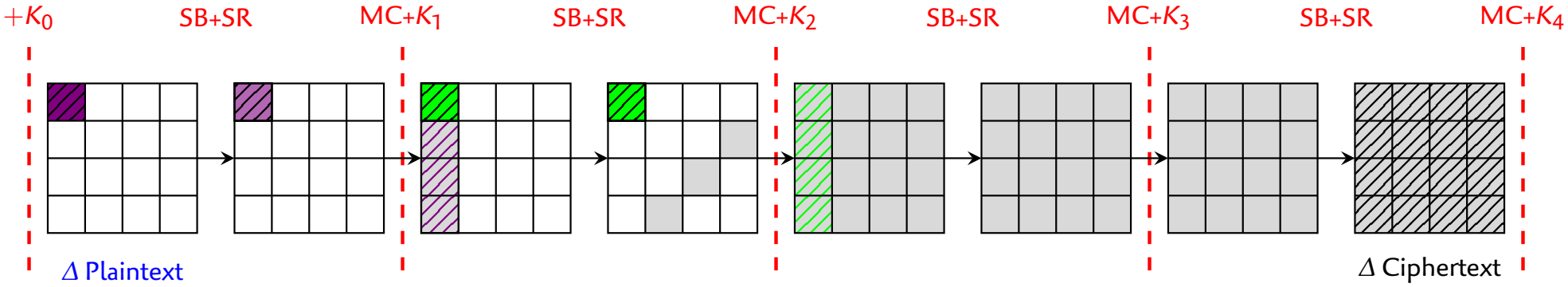

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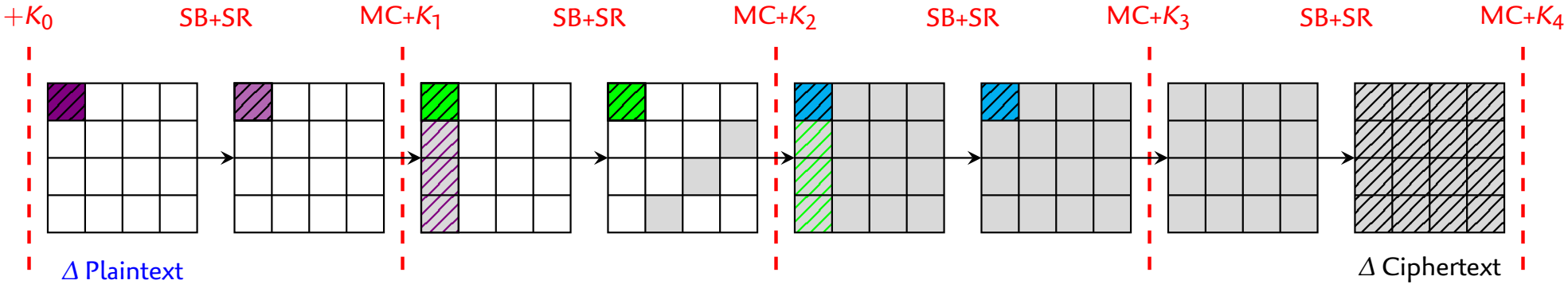

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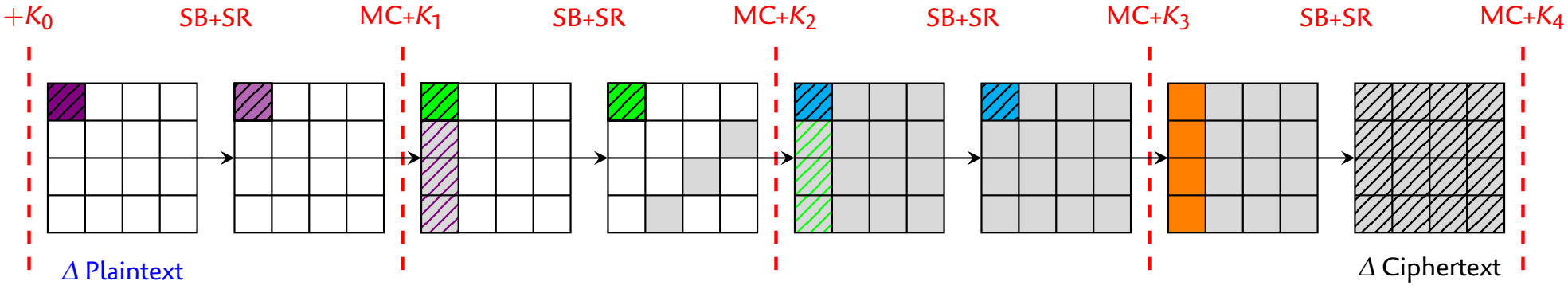

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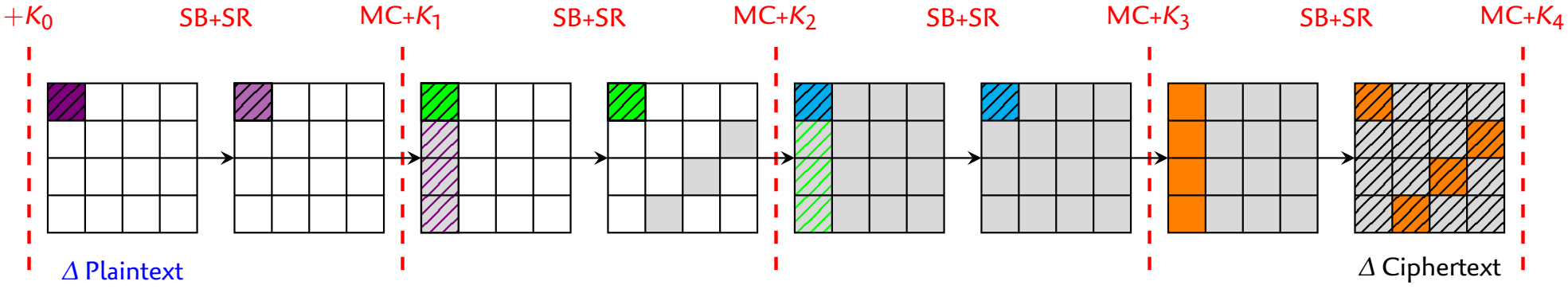

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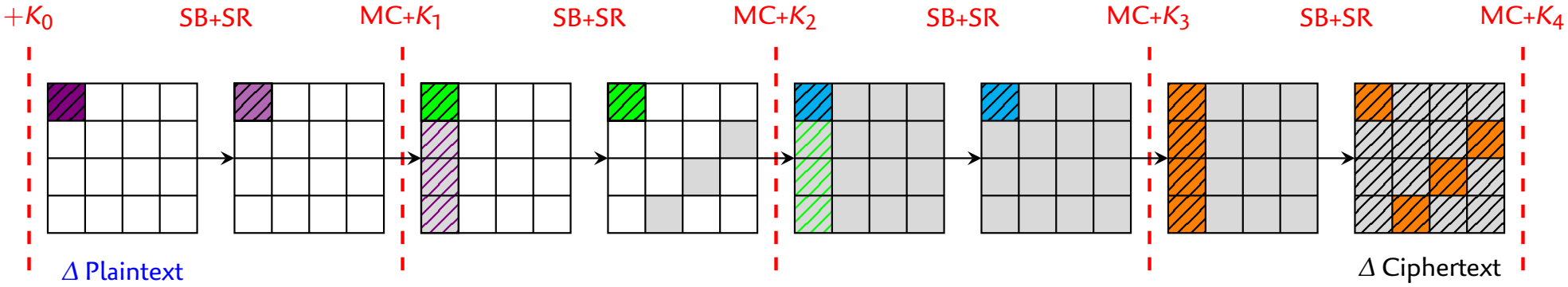

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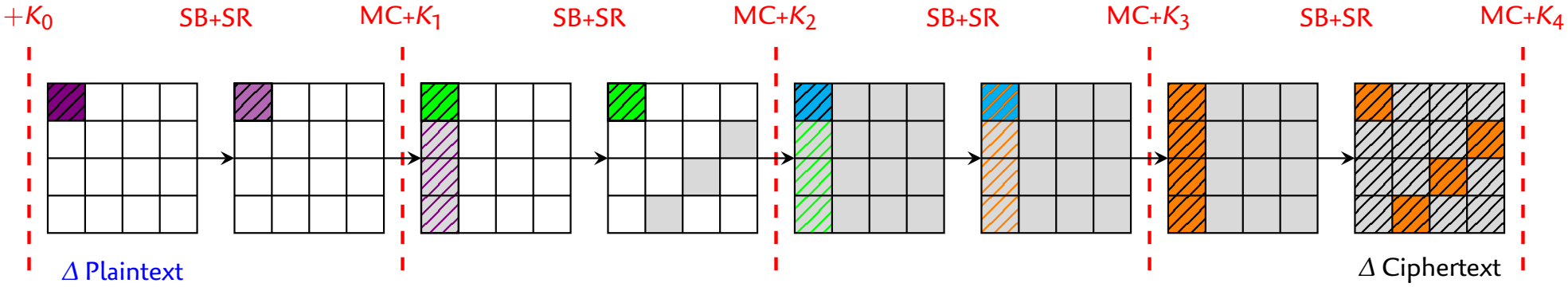

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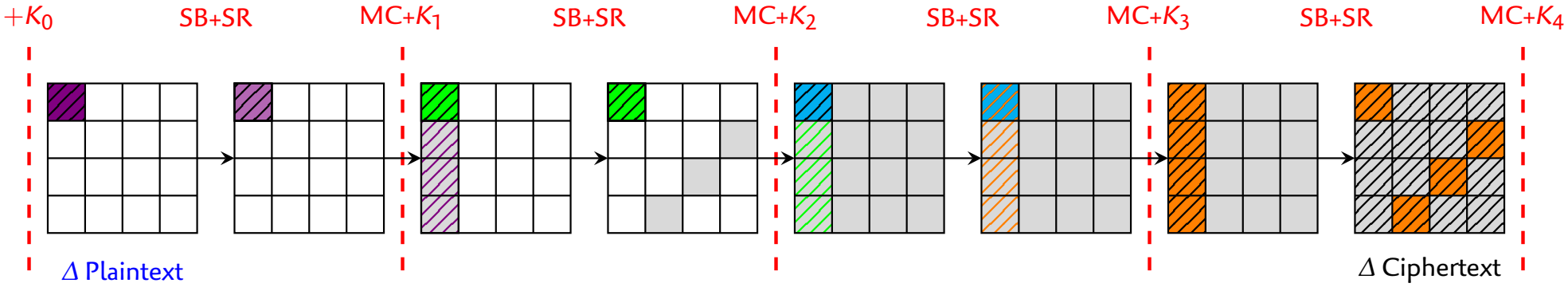

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Other Results

The method is somewhat generic, and applies to AES, SQUARE, PHOTON, SkipJack, LEX, Alpha-MAC, Pelican-MAC, etc.

LEX

- Stream Cipher by Biryukov, eStream Final Round candidate
  - Based on the AES, faster than CTR mode
  - Attacks by Dunkelman and Keller of complexity 2^{100}
- With the tool(s), new attacks found
  - Higher-order differential, complexity 2^{80}
  - Uses 80 tera-bytes of key-stream
  - success probability 2^{-5}

Summary

- Cryptanalysis-inspired methods to solve linear equations with an uninterpreted permutation
- Automatically find the best known (low data complexity) attacks on round-reduced AES, Pelican-MAC, LEX
- Can generate C++ code of the attacks
- Tools publicly available at:

`http://www.di.ens.fr/~bouillaguet/`

Future Work

- We are currently limited by the performance of our tools
  - exponential in every possible parameters
- We need to think on improving the search algorithms
  - have notions of critical/useless pairs
  - better subsumption tests
  - better pair selection heuristics
- We should exploit more thoroughly the input structure
  - similarity between the rounds
  - between concurrent encryption of several messages

And...

Thank You