Self-Stabilizing Leader Election in Polynomial Steps\(^1\)

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Problem

- **Silent Self-stabilizing Leader Election**
- Model:
  - Locally shared memory model
  - Read/write atomicity
  - Distributed unfair daemon
- Network:
  - Any connected topology
  - Bidirectional
  - Identified
- No global knowledge on the network
## State of the Art

<table>
<thead>
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<th>Model</th>
<th>Paper</th>
<th>Knowledge</th>
<th>Daemon</th>
<th>Complexity</th>
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<td>Message Passing</td>
<td>Afek, Bremler, 1998</td>
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<td>Θ(log $n$)</td>
<td>$O(n)$</td>
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<tr>
<td></td>
<td>Awerbuch et al, 1993</td>
<td>×</td>
<td>Θ(log $D \log n$)</td>
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<td></td>
<td>Burman, Kutten, 2007</td>
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<td>Θ(log $D \log n$)</td>
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<td>Locally Shared</td>
<td>Dolev, Herman, 1997</td>
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<td>Θ($N \log N$)</td>
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<td>Kravchik, Kutten, 2013</td>
<td>×</td>
<td>Synchronous</td>
<td>Θ(log $n$)</td>
<td>$O(D)$</td>
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<tr>
<td></td>
<td>Datta et al, 2011</td>
<td>×</td>
<td>Unfair</td>
<td>Θ(log $n$)</td>
<td>$O(n)$</td>
</tr>
</tbody>
</table>

$D$: Diameter  
$D \geq \mathcal{D}$: Upper bound on the diameter  
$n$: Number of nodes  
$N \geq n$: Upper bound on the number of nodes  
$B$: Upper bound on the link-capacity
### Our Contribution

#### Algorithm $\mathcal{LE}$

- **Memory requirement asymptotically optimal**: $\Theta(\log n)$ bits/process
- **Stabilization time (worst case):**
  - $3n + \mathcal{D}$ rounds
  - Lower Bound: $\frac{n^3}{6} + \frac{5}{2}n^2 - \frac{11}{3}n + 2$ steps,
  - Upper Bound: $\frac{n^3}{2} + 2n^2 + \frac{n}{2} + 1$ steps
Our Contribution

Algorithm $\mathcal{LE}$

- Memory requirement asymptotically optimal: $\Theta(\log n)$ bits/process
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Analytical Study of Datta et al, 2011$^2$

- Stabilization time not polynomial in steps:
  - $\forall \alpha \geq 3$, $\exists$ networks and executions in $\Omega(n^{\alpha+1})$ steps.

---

$^2$Datta, Larmore, and Vemula. Self-stabilizing Leader Election in Optimal Space under an Arbitrary Scheduler. 2011
Design of the Leader Election Algorithm
Simplified Algorithm (Non Self-stabilizing)

Join a Tree

3 variables per process $p$

- $p.idR \in \mathbb{N}$: ID of the root
- $p.par \in \mathcal{N}_p \cup \{p\}$: Parent pointer
- $p.level \in \mathbb{N}$: Level

Initial Configuration

$p.idR = p$
$p.par = p$
$p.level = 0$

Key: $\langle idR, level \rangle$
Simplified Algorithm (Non Self-stabilizing)

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Simplified Algorithm (Non Self-Stabilizing)

Self-stabilization $\implies$ Arbitrary initialization

Key: $\langle idR, level \rangle$
Simplified Algorithm (Non Self-Stabilizing)

Self-stabilization $\implies$ Arbitrary initialization $\implies$ Fake ids

Fake id
\[
\langle 1,1 \rangle \downarrow \langle 3,0 \rangle \downarrow \langle 4,0 \rangle \downarrow \langle 1,1 \rangle
\]

Fake id

Key: $\langle idR, level \rangle$
Simplified Algorithm (Non Self-Stabilizing)

Self-stabilization $\implies$ Arbitrary initialization $\implies$ Fake ids

Key: $\langle idR, level \rangle$

Diagram:

- Node 2 with key $\langle 1, 1 \rangle$
- Node 3 with key $\langle 1, 2 \rangle$
- Node 4 with key $\langle 1, 2 \rangle$
- Node 5 with key $\langle 1, 1 \rangle$

Nodes are connected in a line from 2 to 5.
Simplified Algorithm: Removal of Fake Ids

Reset

Inconsistency

\[ \langle 1, 1 \rangle \leftrightarrow \langle 1, 2 \rangle \leftrightarrow \langle 1, 2 \rangle \leftrightarrow \langle 1, 1 \rangle \]

Key: \( \langle idR, level \rangle \)
Reset

- \( p.idR = p \)
- \( p.par = p \)
- \( p.level = 0 \)

Inconsistency

\[
\langle 1, 1 \rangle \quad \langle 1, 2 \rangle \quad \langle 1, 2 \rangle \\
2 \quad 3 \quad 4 \quad 5
\]

Key: \( \langle idR, level \rangle \)
Simplified Algorithm: Removal of Fake Ids

Reset

- $p.idR = p$
- $p.par = p$
- $p.level = 0$

Key: $\langle idR, level \rangle$

![Diagram of nodes and edges with keys $\langle 2, 0 \rangle$, $\langle 1, 2 \rangle$, $\langle 1, 2 \rangle$, $\langle 5, 0 \rangle$]
Simplified Algorithm: Removal of Fake Ids

Reset

- $p.idR = p$
- $p.par = p$
- $p.level = 0$

Inconsistency

```
⟨2, 0⟩  ⟨1, 2⟩  ⟨1, 2⟩  ⟨5, 0⟩
2        3        4        5
```

Key: $\langle idR, level \rangle$
Simplified Algorithm: Removal of Fake Ids

Reset

- \( p.idR = p \)
- \( p.par = p \)
- \( p.level = 0 \)

Key: \( \langle idR, level \rangle \)
Simplified Algorithm: Removal of Fake Ids

Reset

Key: \( \langle idR, level \rangle \)
Simplified Algorithm: Removal of Fake Ids

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Key: \( \langle idR, level \rangle \)
Simplified Algorithm: Removal of Fake Ids

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Key: \(\langle idR, level \rangle\)
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Reset

Key: $\langle idR, level \rangle$
Simplified Algorithm: Removal of Fake Ids

Reset

Key: \( \langle idR, level \rangle \)
Simplified Algorithm: Removal of Fake Ids

Reset

Key: \langle idR, level \rangle
Abnormal Trees

Key: \(\langle idR, level \rangle\)
Abnormal Trees

Key: \( \langle idR, level \rangle \)
Abnormal Trees

Key: \( \langle idR, level \rangle \)
Abnormal Trees

Key: \( \langle idR, level \rangle \)

Abnormal root

Anaïs Durand (VERIMAG)
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Abnormal Trees

Key: \langle idR, level \rangle

Abnormal root
Abnormal Trees

\[ (3, 0), (3, 1), (3, 1), (3, 1), (3, 1), (3, 1), (3, 1), (3, 1) \]

\[ T_1, T_2, T_3 \]

\[ \langle 3, 1 \rangle, \langle 3, 2 \rangle, \langle 1, 2 \rangle, \langle 1, 0 \rangle \]

Key: \( \langle idR, level \rangle \)
Abnormal Trees

Key: \( \langle \text{idR}, \text{level} \rangle \)
Cleaning
Cleaning

$EB$-action

Key: $\langle idR, level \rangle$

Clean  $EBroadcast$  $EFeedback$

$\langle 1, 0 \rangle$

$\langle 1, 1 \rangle$

$\langle 1, 1 \rangle$
Cleaning

$EB$-action

Key: $\langle idR, level \rangle$

$Clean$ $EBroadcast$ $EFeeback$

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Cleaning

Key: $\langle idR, \text{level} \rangle$  Clean  EBroadcast  EFeedback
Cleaning

EF-action

EB

Key: \( (idR, \text{level}) \)

- Clean
- EBroadcast
- EFeedback
Cleaning

Key: $\langle \text{idR}, \text{level} \rangle$

- Clean
- EBroadcast
- EFeedback
Cleaning

Key: \((idR, level)\)
- Green: Clean
- Blue: EBroadcast
- Red: EFeedback

Diagram:
- R-action
- EF
- Nodes: 6, 2, 8
- Edges:
  - 6 to 2: \((1, 0)\) \((1, 1)\)
  - 6 to 8: \((1, 0)\) \((1, 1)\)
Cleaning

Key: \(\langle idR, level \rangle\)

- Green: Clean
- Blue: EBroadcast
- Red: EFedback

Diagram with nodes and arrows indicating R-actions, EF, and key notation.
Stabilization Time in Rounds

- No alive abnormal tree created
- Height of an abnormal tree: at most $n$
Stabilization Time in Rounds

- No alive abnormal tree created
- Height of an abnormal tree: at most $n$
- **Cleaning:**
  - EB-wave: $n$
  - EF-wave: $n$
  - R-wave: $n$
Stabilization Time in Rounds

- No alive abnormal tree created
- Height of an abnormal tree: at most $n$
- **Cleaning:**
  - EB-wave: $n$
  - EF-wave: $n$
  - R-wave: $n$
- **Building of the Spanning Tree:** $D$
Stabilization Time in Rounds

- No alive abnormal tree created
- Height of an abnormal tree: at most $n$
- **Cleaning:**
  - EB-wave: $n$
  - EF-wave: $n$
  - R-wave: $n$
- **Building of the Spanning Tree:** $D$

$$O(3n + D) \text{ rounds}$$
Lower Bound on the Worst Case Stabilization Time in Rounds

- $k$ links
- $j = k + 3$
- $D = n - k$

Key:

- $\langle idR, level \rangle$
- Clean
- EBroadcast
- EFeedback

Diagram:

- Nodes labeled with $\langle 0, n-1 \rangle$, $\langle 0, n-2 \rangle$, $\langle 0, 0 \rangle$, $\langle 0, j-2 \rangle$, $\langle 0, j-3 \rangle$, $\langle 0, 3 \rangle$, $\langle 0, 2 \rangle$, $\langle 0, 1 \rangle$
- Arrows indicating communication or state transitions
Lower Bound on the Worst Case Stabilization Time in Rounds

- $k$ links
- $j = k + 3$
- $D = n - k$

Key: $\langle idR, level \rangle$

- $Clean$
- $EBroadcast$
- $EFeedback$
Lower Bound on the Worst Case Stabilization Time in Rounds

- $k$ links
- $j = k + 3$
- $\mathcal{D} = n - k$

![Diagram](image)

Key: $\langle idR, level \rangle$

- Green: Clean
- Blue: EBroadcast
- Red: EFeeback

$n$
Lower Bound on the Worst Case Stabilization Time in Rounds

- $k$ links
- $j = k + 3$
- $D = n - k$

Key:

- $\langle idR, level \rangle$
- $\text{Clean}$
- $\text{EBroadcast}$
- $\text{EF eedback}$

$n$
Lower Bound on the Worst Case Stabilization Time in Rounds

- $k$ links
- $j = k + 3$
- $D = n - k$

$n + n$
Lower Bound on the Worst Case Stabilization Time in Rounds

- $k$ links
- $j = k + 3$
- $D = n - k$

Key:

$\langle idR, level \rangle$
- $\text{Clean}$
- $\text{EBroadcast}$
- $\text{EFeedback}$

$n + n$
Lower Bound on the Worst Case Stabilization Time in Rounds

- $k$ links
- $j = k + 3$
- $D = n - k$

$n + n + n$
Lower Bound on the Worst Case Stabilization Time in Rounds

- $k$ links
- $j = k + 3$
- $D = n - k$

$$j = k + 3$$

$D = n - k$

$\langle 2, n-k \rangle$

$\langle 2, 0 \rangle$

$\langle 2, 1 \rangle$

$\langle 2, 1 \rangle$

$\langle 2, 1 \rangle$

$\langle 2, 1 \rangle$

$\langle 1, 0 \rangle$

Building

Key: $\langle idR, level \rangle$

$n + n + n$
Lower Bound on the Worst Case Stabilization Time in Rounds

- $k$ links
- $j = k + 3$
- $D = n - k$

$n + n + n + (n - k)$
Lower Bound on the Worst Case Stabilization Time in Rounds

- $k$ links
- $j = k + 3$
- $\mathcal{D} = n - k$

$n + n + n + (n - k) = \text{exactly } 3n + \mathcal{D} \text{ rounds}$
Stabilization Time in Steps

- A segment
- Another segment

Death of an abnormal tree

At most $n$ alive abnormal trees + No alive abnormal tree created

$\rightarrow$ At most $n + 1$ segments

In a segment $idR$:

- $J$-action
- $J$-action
- $J$-action
- $EB$-action
- $EF$-action
- $R$-action

$\Rightarrow O(n)$ actions per process

$O(n^3)$ steps

Lower Bound: $n^3 + 5n^2 + 2n - 11$ steps

Upper Bound: $n^3 + 2n^2 + n + 1$ steps
Stabilization Time in Steps

At most $n$ alive abnormal trees + No alive abnormal tree created
Stabilization Time in Steps

At most $n$ alive abnormal trees + No alive abnormal tree created

$\rightarrow$ At most $n + 1$ segments
**Stabilization Time in Steps**

At most $n$ alive abnormal trees $+$ No alive abnormal tree created $\rightarrow$ At most $n + 1$ segments

In a segment

$\text{idR} : 7 \xrightarrow{\text{J-action}} 5 \xrightarrow{\text{J-action}} 3 \xrightarrow{\text{J-action}} 2 \xrightarrow{\text{EB-action} \rightarrow \text{EF-action} \rightarrow \text{R-action}} 7 \xrightarrow{\text{J-action}} 3$

Death of an abnormal tree $=$ End of the segment

Death of an abnormal tree
Stabilization Time in Steps

A segment  Another segment

Death of an abnormal tree

At most $n$ alive abnormal trees $+$ No alive abnormal tree created
$\longrightarrow$ At most $n + 1$ segments

In a segment

$idR : 7 \xrightarrow{\text{J-action}} 5 \xrightarrow{\text{J-action}} 3 \xrightarrow{\text{J-action}} 2 \xrightarrow{\text{EB-action \hspace{1cm} EF-action \hspace{1cm} R-action}} 7 \xrightarrow{\text{J-action}} 3$

Death of an abnormal tree $=$ End of the segment

- $n - 1$ J-actions
- 1 EB-action
- 1 EF-action
- 1 R-action
Stabilization Time in Steps

At most $n$ alive abnormal trees + No alive abnormal tree created $\rightarrow$ At most $n + 1$ segments

In a segment

$\text{idR} : 7 \xrightarrow{J\text{-action}} 5 \xrightarrow{J\text{-action}} 3 \xrightarrow{J\text{-action}} 2 \xrightarrow{EB\text{-action}} \xrightarrow{EF\text{-action}} \xrightarrow{R\text{-action}} 7 \xrightarrow{J\text{-action}} 3$

Death of an abnormal tree $= \text{End of the segment}$

- $n - 1$ $J$-actions
- 1 $EB$-action
- 1 $EF$-action
- 1 $R$-action

$\Rightarrow O(n)$ actions per process
Stabilization Time in Steps

Death of an abnormal tree

At most $n$ alive abnormal trees + No alive abnormal tree created

$\rightarrow$ At most $n + 1$ segments

In a segment

$idR: 7 \xrightarrow{J\text{-action}} 5 \xrightarrow{J\text{-action}} 3 \xrightarrow{J\text{-action}} 2 \xrightarrow{EB\text{-action} \rightarrow EF\text{-action} \rightarrow R\text{-action}} 7 \xrightarrow{J\text{-action}} 3$

Death of an abnormal tree = End of the segment

$\bullet$ $n - 1$ $J$-actions

$\bullet$ 1 $EB$-action

$\bullet$ 1 $EF$-action

$\bullet$ 1 $R$-action

$\Rightarrow O(n)$ actions per process

$O(n^3)$ steps

Lower Bound: $\frac{n^3}{6} + \frac{5}{2}n^2 - \frac{11}{3}n + 2$ steps

Upper Bound: $\frac{n^3}{2} + 2n^2 + \frac{n}{2} + 1$ steps
Lower Bound on the Worst Case Stabilization Time in Steps

Key: \(\langle idR, level \rangle\)
- \(\text{Clean}\)
- \(\text{EBroadcast}\)
- \(\text{EFeedback}\)

\[
\begin{align*}
\langle n-4, 0 \rangle & \rightarrow \langle n-3, 0 \rangle \\
\langle n-3, 0 \rangle & \rightarrow \langle n-2, 0 \rangle \\
\langle n-2, 0 \rangle & \rightarrow \langle n-1, 0 \rangle \\
\langle n-1, 0 \rangle & \rightarrow \langle n, 0 \rangle \\
\langle n, 0 \rangle & \rightarrow \langle 2n, 0 \rangle \\
\langle 2n, 0 \rangle & \rightarrow \langle 2n-1, 0 \rangle \\
\langle 2n-1, 0 \rangle & \rightarrow \langle 2n-2, 0 \rangle \\
\langle 2n-2, 0 \rangle & \rightarrow \langle n+1, 1 \rangle
\end{align*}
\]
Lower Bound on the Worst Case Stabilization Time in Steps

Key: $\langle idR, level \rangle$
- Clean
- EBroadcast
- EFeedback

Case of the reset of $2n-4$ processes:

$$\sum_{j=1}^{n-1} j - 1 \sum_{i=1}^{2n} i \Rightarrow \Theta(n^3)$$

steps

Reset

Build

$\langle n-1, 0 \rangle$

$\langle n-2, 0 \rangle$

$\langle n-3, 0 \rangle$

$\langle 2n-2, 0 \rangle$

$\langle 2n-3, 0 \rangle$

$\langle 2n-4, 0 \rangle$

$\langle 2n, 0 \rangle$

$\langle n+1, 1 \rangle$

$\langle 1, 0 \rangle$
Lower Bound on the Worst Case Stabilization Time in Steps

Key: \( \langle idR, level \rangle \)
- Clean
- EBroadcast
- EFeedback

Case of the reset of \( 2n - 4 \) processes:

\[
2n - 1 \quad 2n - 2 \quad 2n \quad 2n - 3 \quad 2n - 4 \quad \ldots
\]

\[\langle n - 1, 0 \rangle \quad \langle n - 2, 0 \rangle \quad \langle n - 3, 0 \rangle \quad \langle n - 4, 0 \rangle \]

\[\langle 1, 0 \rangle \quad \langle 2, 0 \rangle \quad \langle 2n, 0 \rangle \]

\[\sum_{j=1}^{n-1} j - \sum_{i=1}^{2n-4} i \Rightarrow \Theta(n^3) \text{ steps} \]
Lower Bound on the Worst Case Stabilization Time in Steps

\[
\begin{align*}
2n - 4 & \quad \langle n - 4, 3 \rangle \\
2n - 2 & \quad \langle n - 4, 2 \rangle \\
2n & \quad \langle n - 4, 1 \rangle \\
2n - 4 & \quad \langle n - 4, 0 \rangle \\
n + 1 & \quad \langle 1, 0 \rangle
\end{align*}
\]

Key: \( \langle idR, level \rangle \)
- Clean
- EBroadcast
- EFeedback

\[\Omega(n) \text{ reset} \Rightarrow n \sum_{j=1}^{n-j} j - 1 \sum_{i=1}^{n-j} i \Rightarrow \Omega(n^3) \text{ steps}\]
Lower Bound on the Worst Case Stabilization Time in Steps

Key: \( \langle idR, level \rangle \)
- Clean
- EBroadcast
- EFeedback

\[ \langle n-4, 3 \rangle \]
\[ 2n-1 \]
\[ \langle n-4, 2 \rangle \]
\[ 2n-2 \]
\[ \langle n-4, 1 \rangle \]
\[ 2n-3 \]
\[ \langle n-4, 0 \rangle \]
\[ 2n-4 \]
\[ n+1 \]
\[ \langle 2n, 0 \rangle \]
\[ \langle 2n, 0 \rangle \]
\[ \cdots \]

\[ \Theta(n^2) \] reset \( \Rightarrow \)
\[ n \sum_{j=1}^{n} j - 1 \]
\[ \sum_{i=1}^{n} i \]
\[ \Rightarrow \]
\[ \Theta(n^3) \] steps
Lower Bound on the Worst Case Stabilization Time in Steps

\[\sum_{j=1}^{n} j - 1 \leq \sum_{i=1}^{n} i \Rightarrow \Theta(n^3) \text{ steps}\]

Key: \(\langle idR, level\rangle\)
- Clean
- EBroadcast
- EFeedback
Lower Bound on the Worst Case Stabilization Time in Steps

\[ \Theta(n^3) \text{ steps} \]
Lower Bound on the Worst Case Stabilization Time in Steps

$$2^n, 0$$

$$n + 1, 0$$

$$2n, 0$$

$$\langle n-4, 3 \rangle$$

$$\langle 2n-2, 0 \rangle$$

$$\langle 2n-3, 0 \rangle$$

$$\langle 2n-4, 0 \rangle$$

Key: $$\langle idR, level \rangle$$

Green: Clean

Blue: EBroadcast

Red: EFeedback

$$\Theta(n)$$ reset $$\Rightarrow n \sum_{j=1}^{j-1} \sum_{i=1}^{2^n-4} \Rightarrow \Theta(n^3)$$ steps
Lower Bound on the Worst Case Stabilization Time in Steps

Case of the reset of $2n - 4$

processes: $2n - 1$  $2n - 2$  $2n - 3$  $2n - 4$  ...
Lower Bound on the Worst Case Stabilization Time in Steps

\[
\begin{align*}
2n-1 & \Rightarrow \langle 2n-2, 1 \rangle \\
2n-2 & \Rightarrow \langle 2n-2, 0 \rangle \\
n+1 & \Rightarrow \langle 2n, 0 \rangle \\
n & \Rightarrow \langle 2n, 0 \rangle \\
2n-3 & \Rightarrow \langle 2n-3, 0 \rangle \\
2n-4 & \Rightarrow \langle 2n-4, 0 \rangle \\
\vdots & \\
\end{align*}
\]

Key: \(\langle idR, level \rangle\)

\(\langle idR, level \rangle\) 
- \(Clean\) 
- \(EBroadcast\) 
- \(EFeedback\)

Case of the reset of \(2n - 4\)

Processes: 
- \(2n - 1\) 
- \(2n - 2\) 
- \(2n - 3\) 
- \(2n - 4\) 
- \(\ldots\)

\(\text{idR} = 2n - 2\)
Lower Bound on the Worst Case Stabilization Time in Steps

Case of the reset of $2n - 4$

processes: $2n - 1$  $2n - 2$  $2n - 3$  $2n - 4$  ... 

idR = $2n-2$  idR = $2n-3$  

...
Lower Bound on the Worst Case Stabilization Time in Steps

Case of the reset of $2n - 4$

Processes: $2n - 1$, $2n - 2$, $2n - 3$, $2n - 4$, \ldots

Key: $\langle \text{idR}, \text{level} \rangle$

- **Clean**
- **EBroadcast**
- **EFeeback**
Lower Bound on the Worst Case Stabilization Time in Steps

Key: \langle idR, level \rangle

Clean EBroadcast EFeedback

Case of the reset of $2n - 4$

processes: $2n - 1$ $2n - 2$ $2n - 3$ $2n - 4$ $\ldots$
Lower Bound on the Worst Case Stabilization Time in Steps

Case of the reset of $2n - 4$

Processes: $2n - 1$  $2n - 2$  $2n - 3$  $2n - 4$  $\ldots$

$\langle idR = 2n-2, \ldots \rangle$

$\langle idR = 2n-3, \ldots \rangle$

$\langle idR = 2n-4, \ldots \rangle$

Key: $\langle idR, level \rangle$

- Clean
- EBroadcast
- EFeedback
Lower Bound on the Worst Case Stabilization Time in Steps

Case of the reset of $2n - 4$

processes: $2n - 1$  $2n - 2$  $2n - 3$  $2n - 4$  ...
Lower Bound on the Worst Case Stabilization Time in Steps

Case of the reset of $2n - 4$

$\sum_{i=1}^{j-1} i = \frac{j(j-1)}{2}$

$\Theta(n)$ reset $\Rightarrow \sum_{j=1}^{n} \sum_{i=1}^{j-1} i \Rightarrow \Theta(n^3)$ steps
Analytical Study of Datta et al, 2011

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3 Datta, Larmore, and Vemula. Self-stabilizing Leader Election in Optimal Space under an Arbitrary Scheduler. 2011
Principles

Join a tree

Key: $\langle idR, level \rangle$

- Can be joined
- Cannot be joined
Principles

Join a tree

Key: \( \langle idR, level \rangle \)

- Can be joined
- Cannot be joined
Principles

Change of color

Key: \langle idR, level \rangle

- Can be joined
- Cannot be joined
Principles
Change of color

Key: \(\langle idR, level \rangle\)

- Can be joined
- Cannot be joined
Principles

Change of color

Key: \(\langle idR, level \rangle\)  
- Can be joined
- Cannot be joined
Principles

Color Waves Absorption

Normal tree

Abnormal tree

Key: $\langle idR, level \rangle$  
- Can be joined
- Cannot be joined
Principles

Color Waves Absorption

Key: \(\langle idR, level \rangle\)  
- Can be joined  
- Cannot be joined

Normal tree

Abnormal tree
Principles
Color Waves Absorption

Key: \( \langle idR, level \rangle \)  
- Can be joined
- Cannot be joined

Normal tree

Abnormal tree
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

- $(i, j).ID = (i - 1)\beta + j$
- $(i, j).idR = 0$
- $\star$ Can be joined
- $\bigstar$ Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$\star (i, j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$

- Can be joined
- Cannot be joined
Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
$(i, j).ID = (i - 1)\beta + j$
$(i, j).idR = 0$
- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$\star (i, j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

- 

$(i,j).ID = (i-1)\beta + j$
- 

$(i,j).idR = 0$
- 

- Blue circle: Can be joined
- Red circle: Cannot be joined

\begin{itemize}
  \item $1,1$
  \item $2,1$
  \item $3,1$
  \item $4,1$
  \item $5,1$
  \item $6,1$
  \item $7,1$
  \item $8,1$
  \item $1,2$
  \item $2,2$
  \item $3,2$
  \item $4,2$
  \item $5,2$
  \item $6,2$
  \item $7,2$
  \item $8,2$
  \item $1,3$
  \item $2,3$
  \item $3,3$
  \item $4,3$
  \item $5,3$
  \item $6,3$
  \item $7,3$
  \item $8,3$
\end{itemize}
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

- $(i, j).ID = (i - 1)\beta + j$
- $(i, j).idR = 0$
- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i-1)\beta + j$

- $\star (i, j).idR = 0$
- Blue can be joined
- Red cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

- Can be joined
- Cannot be joined

\(\beta\)
**Datta et al, 2011**

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

\[ \beta \]

---

**Key:**

$$(i, j).ID = (i - 1)\beta + j$$

$$(i, j).idR = 0$$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i,j).ID = (i - 1)\beta + j$

$(i,j).idR = 0$

- Can be joined
- Cannot be joined

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Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

- $(i,j).ID = (i - 1)\beta + j$
- $(i,j).idR = 0$
- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$\star (i, j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

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- $(i, j).idR = 0$
- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

- $(i, j).ID = (i - 1)\beta + j$
- $(i, j).idR = 0$
- Can be joined
- Cannot be joined

$\beta^2$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$\star (i, j).idR = 0$

- Can be joined
- Cannot be joined
Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

- Can be joined
- Cannot be joined

$\beta^2$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i,j).ID = (i - 1)\beta + j$

$(i,j).idR = 0$

- Blue: Can be joined
- Red: Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$\beta^2$

Key:

(i, j).ID = $(i - 1)\beta + j$

(i, j).idR = 0

Can be joined

Cannot be joined
Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$

- Can be joined
- Cannot be joined

\[
\beta^2
\]
Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
(i, j).ID = (i − 1)$\beta + j$

$\star (i, j).idR = 0$

- Can be joined
- Cannot be joined

\[ \beta^2 \]
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$\beta^2$

Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

![Diagram showing self-stabilizing leader election algorithm]

Key:

- $(i,j).ID = (i-1)\beta + j$
- $(i,j).idR = 0$
- Blue nodes: Can be joined
- Red nodes: Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$\beta^2$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$\star (i, j).idR = 0$

- Can be joined
- Cannot be joined

$\beta^2$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$\star (i, j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n^8}{8}$

$\beta^2$

Key:

- $(i, j).ID = (i - 1)\beta + j$
- $(i, j).idR = 0$

- Can be joined
- Cannot be joined
Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

(i, j).ID = (i - 1)$\beta + j

$\star (i, j).idR = 0$

Can be joined

Cannot be joined

$\beta^2$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

- Can be joined
- Cannot be joined

$\beta^2$
Datta et al, 2011
Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
(i, j).ID = (i − 1)$\beta + j
\((i, j).idR = 0

- Can be joined
- Cannot be joined

$\beta^2$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i,j).ID = (i - 1)\beta + j$

$\star (i,j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
- $(i, j).ID = (i - 1)\beta + j$
- $(i, j).idR = 0$
- Blue circle: Can be joined
- Red circle: Cannot be joined

$\beta^2$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$\star (i, j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$\beta^2$

Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

\[ \beta^2 \]

Key:
(i, j).ID = (i - 1)\beta + j
(i, j).idR = 0

Can be joined
Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$\beta^2$

Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

(i, j).ID = (i - 1)$\beta + j$

(i, j).idR = 0

* Can be joined

Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$\beta^2$

Key:

$(i,j).ID = (i-1)\beta + j$

$(i,j).idR = 0$

- Can be joined
- Cannot be joined

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Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$\star (i, j).idR = 0$

Can be joined

Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

- $(i, j).ID = (i - 1)\beta + j$
- $(i, j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$\beta^2$

Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

- Can be joined
- Cannot be joined

Can be joined

Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
- $(i, j).ID = (i - 1)\beta + j$
- $(i, j).idR = 0$
- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$\star (i, j).idR = 0$

- Can be joined
- Cannot be joined

$\beta^2$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i,j).ID = (i - 1)\beta + j$

$(i,j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

Can be joined

Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
$(i, j).ID = (i - 1)\beta + j$

$$(i, j).idR = 0$$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$*(i, j).idR = 0$

- Can be joined
- Cannot be joined

$\beta^2$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

\[ \beta^2 \]

Key:

- $(i, j).ID = (i - 1)\beta + j$
- $\star (i, j).idR = 0$

Circle: Can be joined

Circle with star: Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

- Can be joined
- Cannot be joined

$\beta^2$
Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

- Can be joined
- Cannot be joined

$\beta^2$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$$(i, j).ID = (i - 1)\beta + j$$

$$(i, j).idR = 0$$

- Can be joined
- Cannot be joined
Datta et al, 2011
Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
$$(i, j).ID = (i - 1)\beta + j$$
$$\star (i, j).idR = 0$$
- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$\beta^3$

Key:
- $(i, j).id = (i - 1)\beta + j$
- $(i, j).idR = 0$

- Can be joined
- Cannot be joined

[(Diagram showing the self-stabilizing leader election process)]
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n^4}{8}$

Key:
$(i,j).ID = (i-1)\beta + j$
$(i,j).idR = 0$
- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$\beta^3$

Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

Can be joined

Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$\bigstar (i, j).idR = 0$

- Can be joined
- Cannot be joined
Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$$(i, j).ID = (i - 1)\beta + j$$

$(i, j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i,j).ID = (i - 1)\beta + j$

$\star (i,j).idR = 0$

- Gray: Can be joined
- Red: Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$\star (i, j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

- $(i, j).ID = (i - 1)\beta + j$
- $(i, j).idR = 0$
- Can be joined
- Cannot be joined

$\beta^3$
Datta et al, 2011
Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
- $(i, j).ID = (i - 1)\beta + j$
- $(i, j).idR = 0$
- Can be joined
- Cannot be joined

$\beta^3$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$\beta^3$

**Key:**

$(i, j).ID = (i - 1)\beta + j$

$\star (i, j).\text{idR} = 0$

- Can be joined
- Cannot be joined

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Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$\blackstar (i, j).idR = 0$

- Can be joined
- Cannot be joined

\[ \beta^3 \]
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$$(i, j).IID = (i - 1)\beta + j$$

$$(i, j).idR = 0$$

- Can be joined
- Cannot be joined

$\beta^3$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1) \beta + j$

$\star (i, j).idR = 0$

- Can be joined
- Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
$(i, j).ID = (i - 1)\beta + j$

*(i, j).idR = 0*

- Can be joined
- Cannot be joined

$\beta^3$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

- Blue: Can be joined
- Red: Cannot be joined

$\beta^3$
Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:
- $(i,j).ID = (i - 1)\beta + j$
- $(i,j).idR = 0$

- Can be joined
- Cannot be joined

$\beta^3$
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

Key:

$(i, j).ID = (i - 1)\beta + j$

$(i, j).idR = 0$

- [Blue] Can be joined
- [Red] Cannot be joined
Datta et al, 2011

Execution in $\Omega(n^4)$ steps: $\beta = \frac{n}{8}$

$\beta = \Omega(n) \Rightarrow \Omega(n^4)$

Key:

- $(i, j).ID = (i - 1)\beta + j$
- $(i, j).idR = 0$
- Can be joined
- Cannot be joined

Diagram showing nodes with labels $(i, j)$ and connections indicating whether they can or cannot be joined.

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\(\forall \alpha \geq 3, \exists\) networks and executions in \(\Omega(n^{\alpha+1})\) steps.
Goal

Design a self-stabilizing leader election algorithm that stabilizes in $O(D)$ rounds.
Goal
Design a self-stabilizing leader election algorithm that stabilizes in $O(D)$ rounds.

Hypotheses
- Unfair daemon
- Memory requirement of $\Theta(\log n)$ bits/process
Perspectives

Goal
Design a self-stabilizing leader election algorithm that stabilizes in $O(D)$ rounds.

Hypotheses
- Unfair daemon
- Memory requirement of $\Theta(\log n)$ bits/process

- With the knowledge of $D \geq D$, ($D = O(D)$) : $\checkmark$
Perspectives

Goal
Design a self-stabilizing leader election algorithm that stabilizes in $O(D)$ rounds.

Hypotheses
- Unfair daemon
- Memory requirement of $\Theta(\log n)$ bits/process

- With the knowledge of $D \geq D$, ($D = O(D)$) : √
- Without any global knowledge : ??
Thank you for your attention.

Do you have any questions?
Rounds

1\textsuperscript{st} round

2\textsuperscript{nd} round

Processes

Key:

Enabled ⭐️

Activated ⭐

Neutralized ⭐️

Time

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Self-Stabilizing Leader Election
September 29, 2014
Experimental Results

Average stabilization time in rounds in UDGs ($n = 1000$)
Experimental Results

Average stabilization time in steps in UDGs ($D = 15$)