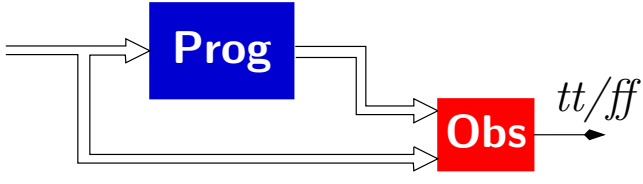
From Discrete Duration Calculus to Synchronous Observers

Laure Gonnord
Nicolas Halbwachs
Pascal Raymond

Vérimag, Grenoble

Specifying (safety) properties with synchronous observers



Well-known advantages:

- **maximal power**
- **same language for programs and properties**
- **specifications are "executable"**

Higher-level formalisms sometimes needed

Specification formalisms (e.g., temporal logics)

- **simulation needed**
- **is decidability an issue?**

Decision procedures for TL

$$\varphi \longrightarrow \mathcal{A}_\varphi$$

$$\sigma \in \mathcal{L}(\mathcal{A}_\varphi) \iff \sigma \models \varphi$$

$$\varphi \text{ satisfiable} \iff \mathcal{L}(\mathcal{A}_\varphi) \neq \emptyset$$

Easy if $\mathcal{A}_\varphi$ rational

Model-checking of TL

$$\boldsymbol{P} \models \varphi \Longleftrightarrow \boldsymbol{\mathcal{L}}(\boldsymbol{P} \times \boldsymbol{\mathcal{A}}_{\neg\varphi}) = \emptyset$$

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$\longrightarrow$ symbolic automata, possibly extended with variables (counters)

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Translating high level specification languages into symbolic automata

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$\sigma \models \varphi$ **iff**

- **executable**
- **use in verification, runtime verification, testing...**
- **no explosion**
- **not necessarily finite state (counters...)**

A well-known case: REGLO [Raymond96]
Translation of regular expressions into symbolic acceptors (Lustre observers)
Strictly linear size

A new experience: QDDC [Pandya 2000-03]

$\longrightarrow$ one conclusion: there are common (safety) properties which are surprizingly hard to express with observers, i.e., hard to check "on the fly"

QDDC: Quantified Discrete Duration Calculus

Examples

"Whenever p has been true continuously during at least n steps, q holds"

$$\Box\left((\lceil\!\lceil p \rceil\!\rceil \wedge \eta \geq n) \Rightarrow true^{\frown}\lceil q \rceil^{0}\right)$$

or $p \xrightarrow{n} q$

"In any interval of duration d, p holds at least k times"

$$\Box(\eta \geq d \Rightarrow \Sigma p \geq k)$$

QDDC: Syntax

Propositions (on states)

$$P ::= 0 \mid 1 \mid p \mid \neg P \mid P \wedge P \mid P \vee P$$

Formulas (on traces and windows)

$$\varphi ::= \lceil P \rceil^0 \mid \lceil\lceil P \rceil\rceil \mid \eta \; op \; c \mid \Sigma P \; op \; c \mid$$

$$\varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \exists p \; \varphi \mid \varphi_1 \frown \varphi_2$$

$$(op \in \{\leq, <, =, \neq, >, \geq\})$$

QDDC: Semantics

States: sets of basic propositions (or Boolean valuations of propositional symbols) — models of propositions: $s \models P$

Traces: finite sequences of states

$$\sigma = \sigma_1\sigma_2 \ldots \sigma_n \qquad |\sigma| = n$$

Windows: intervals in a trace

$$\sigma[b, e] = \sigma_b\sigma_{b+1} \ldots \sigma_e \quad 1 \leq b \leq e \leq |\sigma|$$

QDDC: Semantics (cont.)

$$
\begin{array}{lll}
\boldsymbol{\sigma}[b,e] \models \lceil P \rceil^0 & \textbf{iff} & b = e \textbf{ and } \boldsymbol{\sigma}_b \models P \\
\boldsymbol{\sigma}[b,e] \models \lceil\lceil P \rceil & \textbf{iff} & b < e \textbf{ and } \forall i, b \leq i < e, \boldsymbol{\sigma}_i \models P \\
\boldsymbol{\sigma}[b,e] \models \eta \; op \; c & \textbf{iff} & (e-b) \; op \; c \\
\boldsymbol{\sigma}[b,e] \models \Sigma P \; op \; c & \textbf{iff} & Card\,\{i | b \leq i < e, \boldsymbol{\sigma}_i \models P\} \; op \; c \\
\boldsymbol{\sigma}[b,e] \models \exists p \, \varphi & \textbf{iff} & \exists \boldsymbol{\sigma}', \; \boldsymbol{\sigma}'[b,e] \models \varphi, \\
 & & \textbf{and } \forall i, \forall q \neq p, \boldsymbol{\sigma}_i(q) = \boldsymbol{\sigma}'_i(q) \\
\boldsymbol{\sigma}[b,e] \models \varphi_1 \frown \varphi_2 & \textbf{iff} & \exists i, b \leq i \leq e, \\
 & & \boldsymbol{\sigma}[b,i] \models \varphi_1 \textbf{ and } \boldsymbol{\sigma}[i,e] \models \varphi_2
\end{array}
$$

$\boldsymbol{\sigma} \models \varphi$ **iff** $\boldsymbol{\sigma}[1, |\boldsymbol{\sigma}|] \models \varphi$

Decidable, as long as constants are known

(no parameters)

QDDC: Derived operators

$\lceil\!\lceil P \rceil\!\rceil = \lceil\!\lceil P \rceil\!\rceil^\frown \lceil P \rceil^0$ **(right-closed interval)**

$\Diamond\varphi = true^\frown \varphi^\frown true$ **(eventually φ)**

$\Box\varphi = \neg\Diamond\neg\varphi$ **(always φ)**

$P_1 \xrightarrow{c} P_2 = \neg\Diamond\left((\lceil\!\lceil P_1 \rceil\!\rceil \wedge \eta \geq c)^\frown \lceil \neg P_2 \rceil^0\right)$

(whenever P_1 has been continuously true during c steps, P_2 is true)

Our extensions: **(forget decidability)**

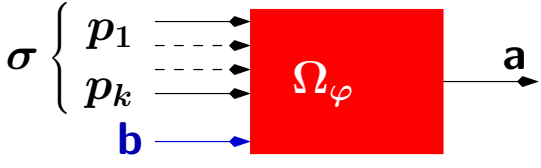
- **Constants c (in $\eta \; op \; c$, $\Sigma P \; op \; c$, $P_1 \xrightarrow{c} P_2$) can be symbolic (parameters)**
- **The atomic propositions can be conditions on parameters (e.g., $c_1 \geq c_2$)**

Observers of formulas (in Lustre)

inputs:

- σ **(sequence of values for the propositions)**
- **a "starter"** b **(a Boolean, true only once)**

output a=true at t **iff** $\sigma, [t_b, t] \models \varphi$



Example 1: $\lceil\lceil p \rceil\rceil$

True of each interval $[t_b, t]$ where p always holds

The observer output should

- be true before the (unique) occurrence of b

- take the value of p when b

- remain true as long as p is true

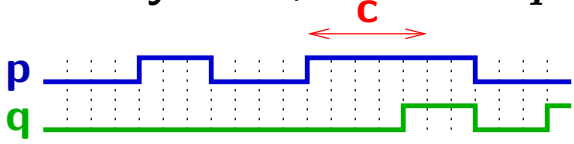
a = before(b) or (if b then p else p and pre(a))

where

before(b) = not b -> (not b and pre(before(b)))

Example 2: $p \xrightarrow{c} q$

True on $[t_b, t]$ iff all subinterval of length c where p is always true, end with q true



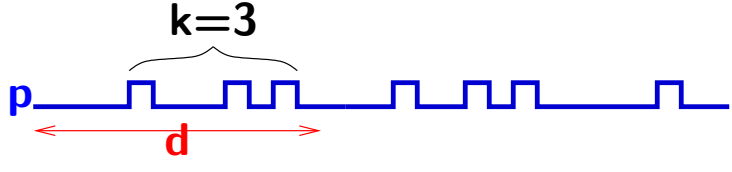

```
a = before(b) or
        ((true -> pre(a)) and (age(p)>=c => q))
```

where

```
age(p) = if p then (0-> pre(age(p))+1 else 0
```

Example 3: $\Box(\eta \geq d \Rightarrow \Sigma p \geq k)$

Each interval longer than d contains at least k occurrences of p



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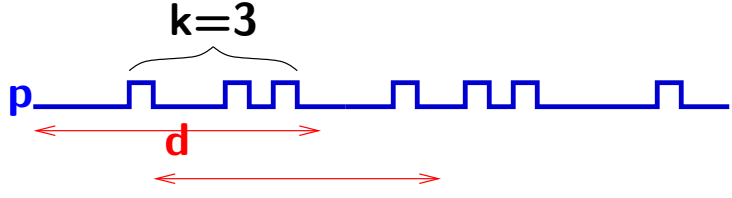

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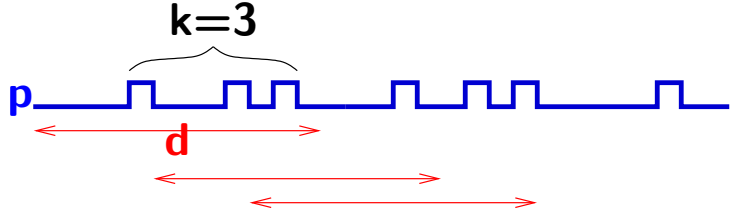

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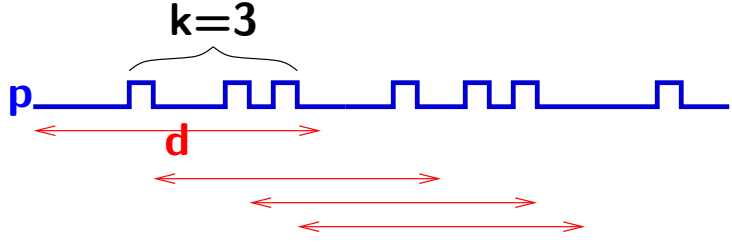

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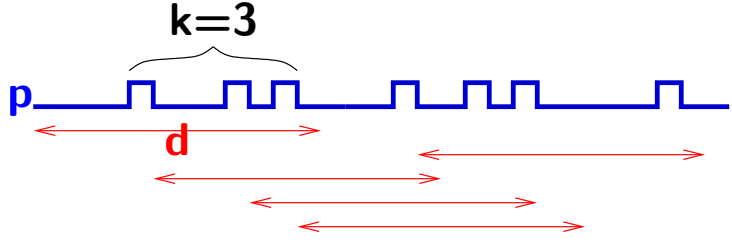

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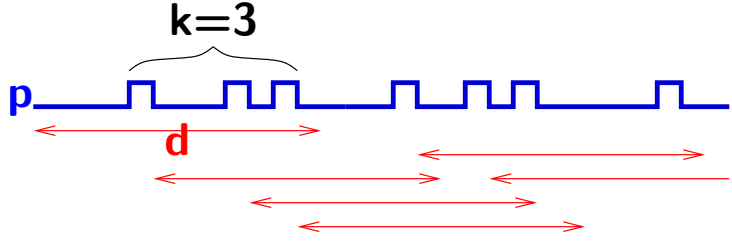

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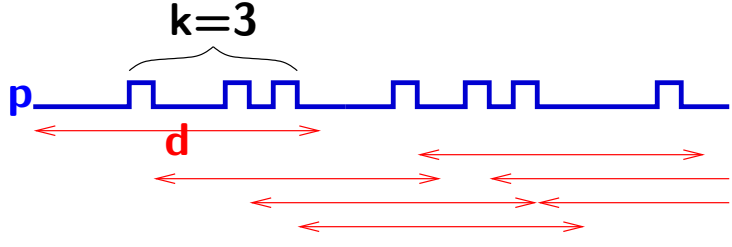

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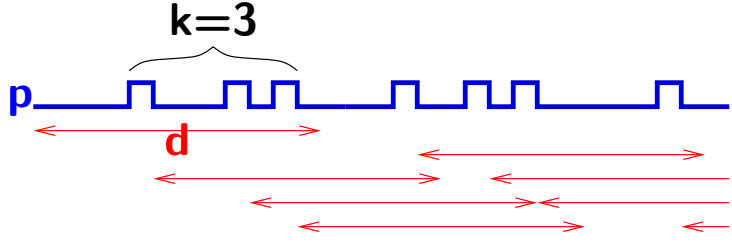

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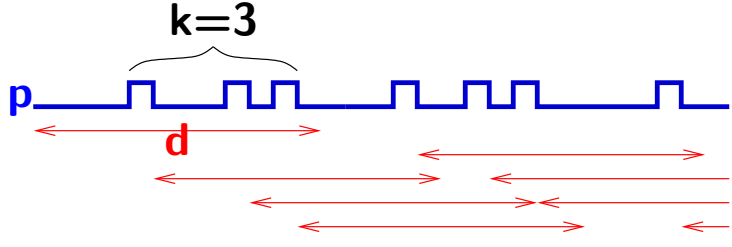

k (or d) counters needed!

Problem if d and k are parameters...

(for testing, one could use generic arrays. Out of reach of verification tools...)

A solution: non-deterministic observers

Idea: non deterministically start a counter at each instant — by means of an additional input, say x (oracle).

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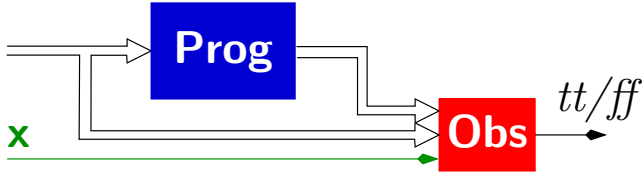
Since a model-checker verifies that the output is always true whatever be the inputs, it will verify whatever be x, so, for all intervals.



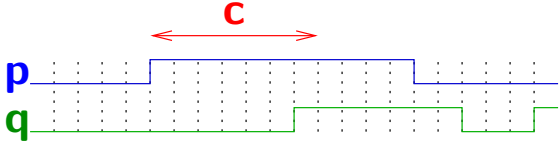
Example 3 (revisited): $\Box(\eta \geq d \Rightarrow \Sigma p \geq k)$

```
a = before(b) or
      ((true -> pre(a)) and length<d or nb_p ≥ k)
length = nb_since(true,x)
nb_p = nb_since(p,x)

where
 nb_since(b1,b2) =
      if before(b2) then 0
      else if b2 then if b1 then 1 else 0
      else pre(nb_since(b1,b2)) +
           if b1 then 1 else 0
```

An example of verification

Prove that $\left(p \xrightarrow{c} q \wedge d \geq c\right) \Longrightarrow (p \xrightarrow{d} q)$



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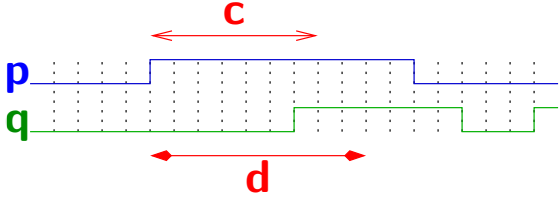

An example of verification (cont.)

Prove that, whatever be p, q, c, d and x, the output a is always true

```
a1 = before(x) or                    -- p -c-> q
     ((true -> pre(a1)) and (nb_since(p,x)<c or q)
a2 = before(x) or                    -- p -d-> q
     ((true -> pre(a2)) and (nb_since(p,x)<d or q)
-- (p -c-> q / d ≥ c) => p -d-> q
a = (a1 and d ≥ c) => a2;
```

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Instantly proved, both by Lesar and Nbac

A fragment of QDDC
recognizable by non-deterministic acceptors

Why only a fragment?

- **Liveness properties expressible in QDDC**
- **Oracles can only be universally quantified**
- **Formulas $\exists p\, \varphi$ and $\varphi_1 \frown \varphi_2$ need existentially quantified oracles**

3-levels syntax

$$\varphi ::= \neg\psi$$

$$\psi ::= \xi \mid \psi_1 \vee \psi_2 \mid \psi_1 \wedge \psi_2 \mid \exists \boldsymbol{p}\ \psi \mid \psi_1 \frown \psi_2$$

$$\xi ::= \lceil \boldsymbol{P} \rceil^0 \mid \lceil\lceil \boldsymbol{P} \rceil \mid \boldsymbol{\eta}\ op\ \boldsymbol{c} \mid \boldsymbol{\Sigma P}\ op\ \boldsymbol{c} \mid \neg\xi$$

Example: $\Box(\eta > c \Rightarrow \Sigma p \geq d)$

Translation into basic QDDC:

$$\neg\,(true^\frown(\eta > c \wedge \Sigma p < d)^\frown true)$$

Translation into Lustre:

```
a = not (   nb_since(true,x) > c and
            nb_since(p,x) < d )
```

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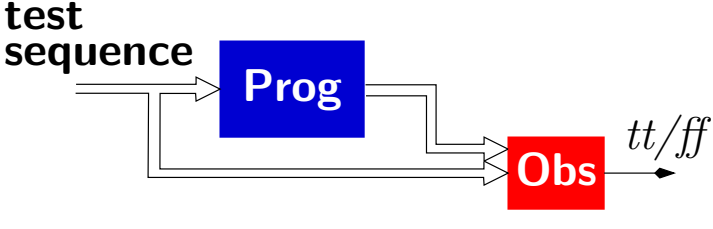

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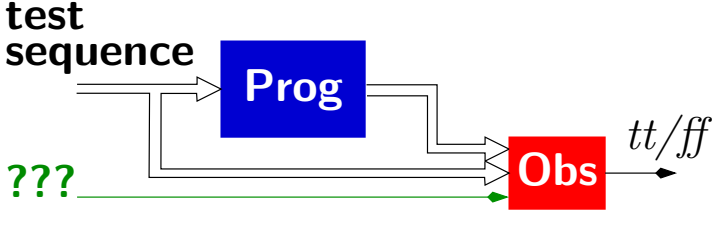

A deterministic fragment

Sources of non-determinism:

- $\exists p\ \varphi$: **completely non-deterministic, hopeless**
- $\varphi_1 \frown \varphi_2$: **often used in a deterministic way**
  ex. $\lceil\!\lceil P \rceil\!\rceil \frown \lceil\!\lceil Q \rceil\!\rceil^0$

$\longrightarrow$ **replace** $\varphi_1 \frown \varphi_2$ **by** $\varphi_1 \; then \; \varphi_2$**:**

the interval consists of a maximal interval satisfying φ_1, followed by an interval satisfying φ_2

φ_1 restricted to formulas that can only change from $true$ to $false$.

Two-levels syntax

$$\varphi \ ::= \ \psi \mid \mathit{end}(\boldsymbol{P}) \mid \psi \ \mathit{then} \ \varphi \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi$$

$$\psi \ ::= \ \mathit{begin}(\boldsymbol{P}) \mid \lceil\!\lceil \boldsymbol{P} \rceil\!\rceil \mid \boldsymbol{\eta} \leq \boldsymbol{c} \mid \boldsymbol{\Sigma P} \leq \boldsymbol{c} \mid \mathit{age}(\boldsymbol{P}) \leq \boldsymbol{c} \mid \psi_1 \wedge \psi_2 \mid \psi_1 \vee \psi_2$$

Examples

$$\lceil\!\lceil \boldsymbol{P} \rceil\!\rceil^\frown \lceil \boldsymbol{Q} \rceil^0 \ \equiv \ \lceil\!\lceil \boldsymbol{P} \rceil\!\rceil \wedge \mathit{end}(\boldsymbol{Q})$$

$$\boldsymbol{p} \xrightarrow{\boldsymbol{c}} \boldsymbol{q} \ \equiv \ \mathit{age}(\boldsymbol{p}) \leq \boldsymbol{c} \vee \boldsymbol{q}$$

$$\lceil\!\lceil \boldsymbol{P} \rceil\!\rceil^\frown \lceil\!\lceil \mathit{begin}(\neg \boldsymbol{P}) \wedge \boldsymbol{Q} \rceil\!\rceil \ \equiv \ \lceil\!\lceil \boldsymbol{P} \rceil\!\rceil \ \mathit{then} \ \lceil\!\lceil \boldsymbol{Q} \rceil\!\rceil$$

Example of translation

$$\lceil\!\lceil P \rceil\!\rceil^\frown \lceil Q \rceil^0 \equiv \lceil\!\lceil P \rceil\!\rceil \wedge end(Q)$$

Obs$_{\lceil\!\lceil P \rceil\!\rceil}$ **= after(b) and pre(always_from(Obs**$_P$**,b))**

Obs $_{end(Q)}$ **= after(b) and Obs**$_Q$

Obs$_{\lceil\!\lceil P \rceil\!\rceil^\frown \lceil Q \rceil^0}$ **= Obs**$_{\lceil\!\lceil P \rceil\!\rceil}$ **and Obs** $_{end(Q)}$

Conclusion

- **There are common safety properties which are not obvious to translate into symbolic acceptors**
- **Non-deterministic acceptors increase the descriptive power**
- **What about really complex formalisms (SUGAR...)?**