Verifying lossy channel systems

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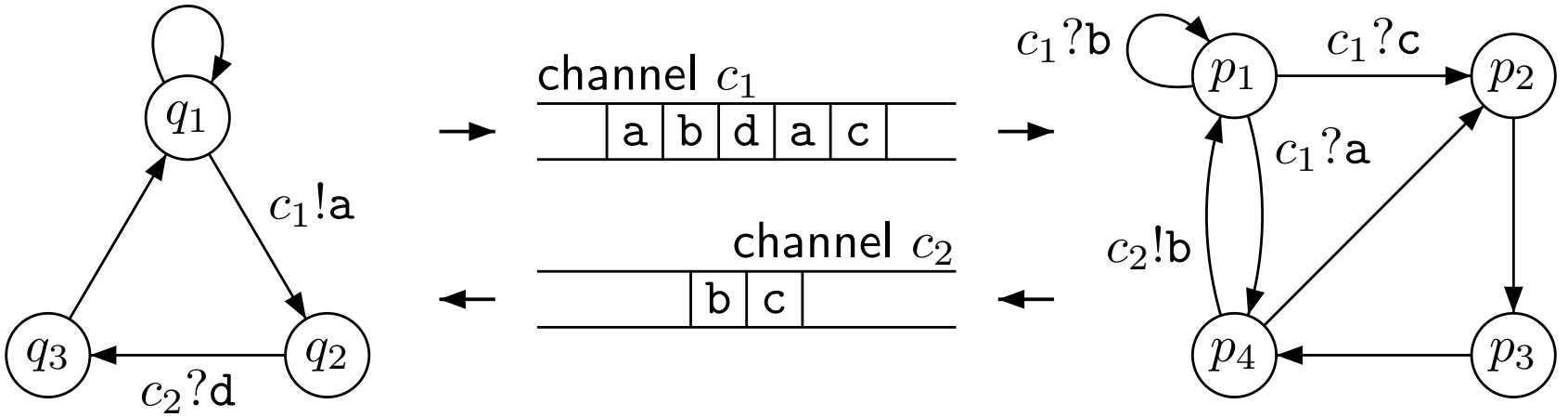
Channel Systems

A.k.a. "communicating finite-state machines"



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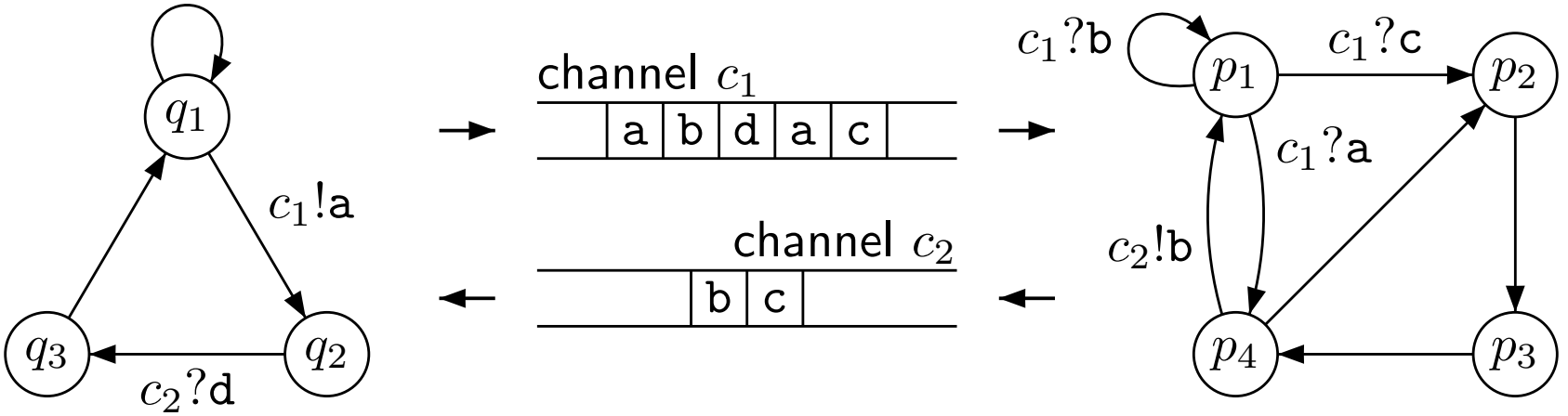

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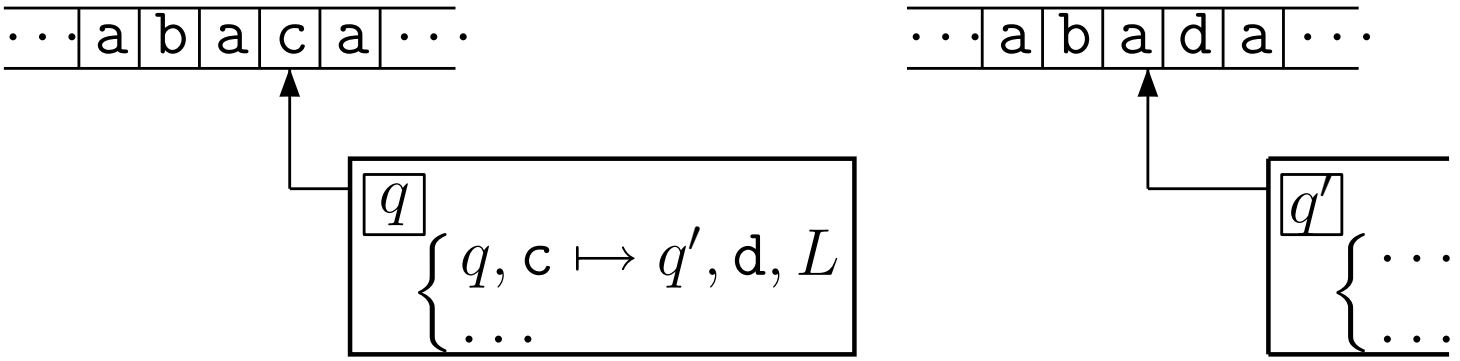
Turing powerful!

Simulating Turing Machines



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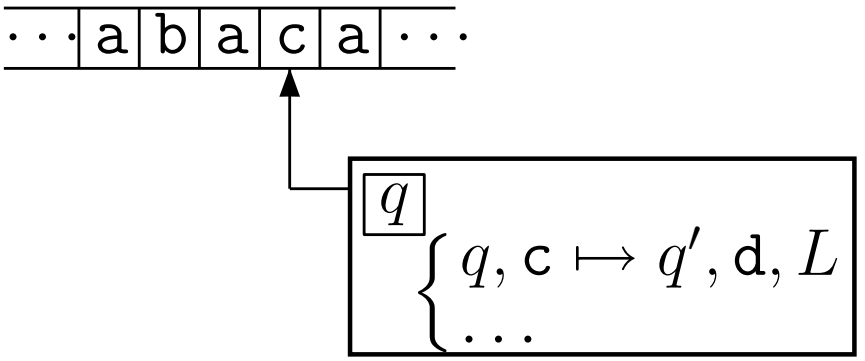

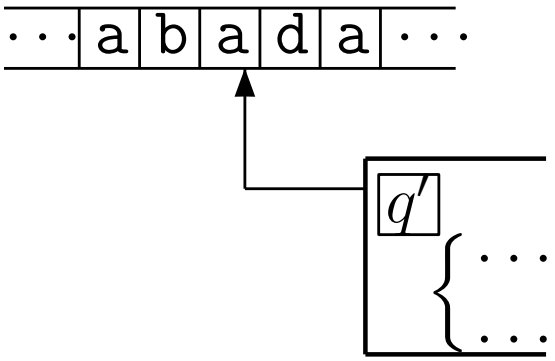

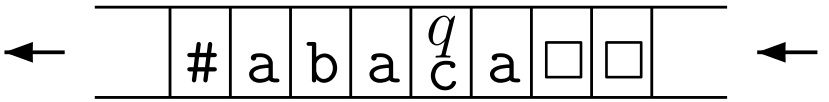

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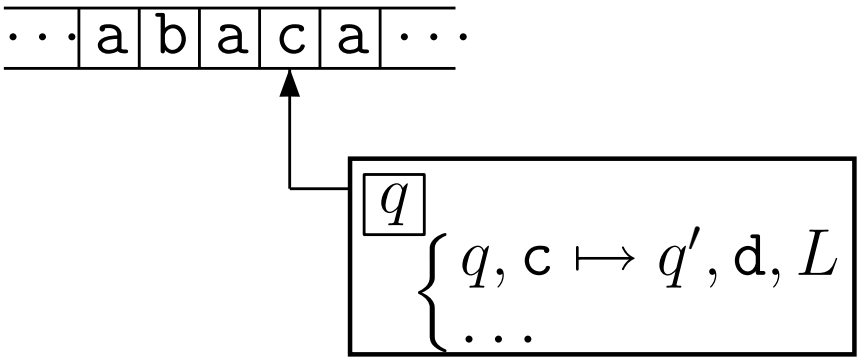

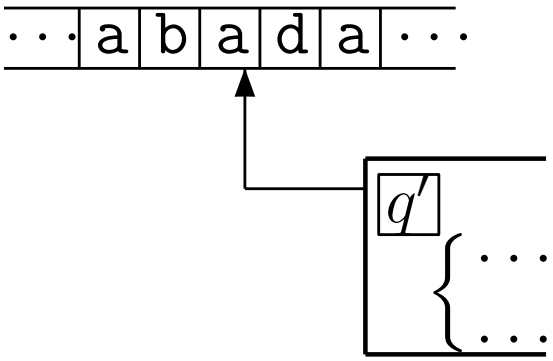

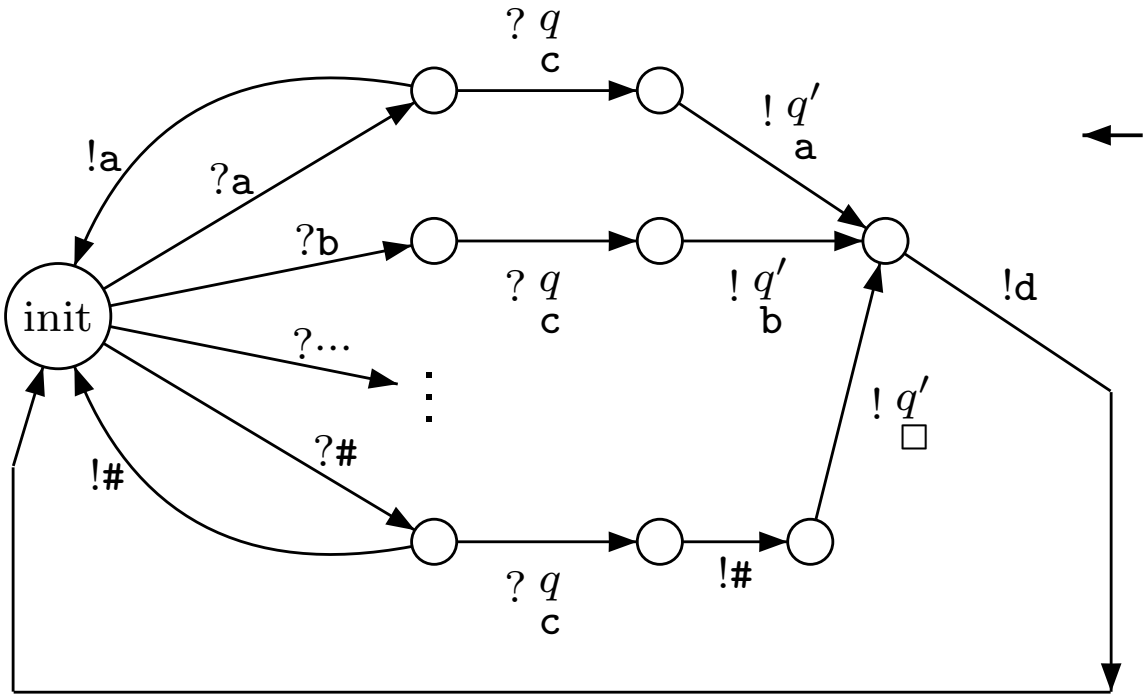

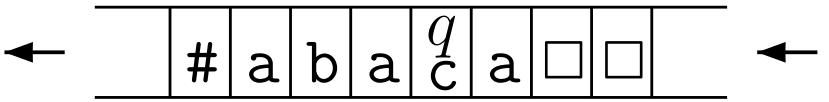

Model Checking Channel Systems

Rice Theorem for Channel Systems: All nontrivial behavioural properties of channel systems are undecidable.

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Side question: Such protocols are everywhere!! Why weren't lossy channel systems identified earlier???

Outline of the Talk

Basics and Definitions

Algorithm for Inevitability

Algorithm for Reachability

Hard Problems With a Recurring idea

Why Study Probabilistic Systems?

Channel Systems: Perfect

W.l.o.g., we only consider systems made of one component and several channels.

$S = \langle Q, \Sigma, C, \Delta \rangle$ with
– $Q = \{q, \ldots\}$, the *control states*
– $\Sigma = \{a, b, \ldots\}$, the *messages*
– $C = \{c_1, c_2, \ldots, c_n\}$, the *channels*
– $\Delta \subseteq Q \times C \times \{?, !\} \times \Sigma \times Q$, the *rules*

Rules in Δ written e.g. as "$(q, c!a, q')$"

A *configuration* of S: $\sigma = \langle q, u_1, \ldots, u_n \rangle$

Perfect steps: $\langle q, u_1, \ldots, u_n \rangle \rightarrow \langle r, v_1, \ldots, v_n \rangle$ if
— $(q, c_i?a, r)$ is a rule, $u_i = a.v_i$ and $v_j = u_j$ for $j \neq i$, or
— $(q, c_i!a, r)$ is a rule, $v_i = u_i.a$ and $v_j = u_j$ for $j \neq i$.
NB: no test for emptiness

Channel Systems: Lossy

Subword ordering: $abba \sqsubseteq abracadabra$

Subword relation for configurations: $\sigma \sqsubseteq \sigma'$

Lossy steps: $\sigma \rightarrow_{\text{lossy}} \sigma' \stackrel{\text{def}}{\Leftrightarrow} \sigma \sqsupseteq \delta \rightarrow_{\text{perf}} \delta' \sqsupseteq \sigma'$

Corollary: If $\sigma_1 \rightarrow \sigma_2$ then $\sigma_1' \rightarrow \sigma_2'$ for any $\sigma_1' \sqsupseteq \sigma_1$ and $\sigma_2' \sqsubseteq \sigma_2$.

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Lemma [Higman 1952]: the subword ordering is a well-quasi-order (wqo), i.e. any infinite sequence $u_0, u_1, u_2, \ldots$ of words has an infinite increasing subsequence $u_{i_0} \sqsubseteq u_{i_1} \sqsubseteq u_{i_2} \sqsubseteq \cdots$

$\Rightarrow$ Applies equivalently to configurations of S ordered by $\sqsubseteq$.

Corollary. Any set of configurations has a finite number of minimal elements.

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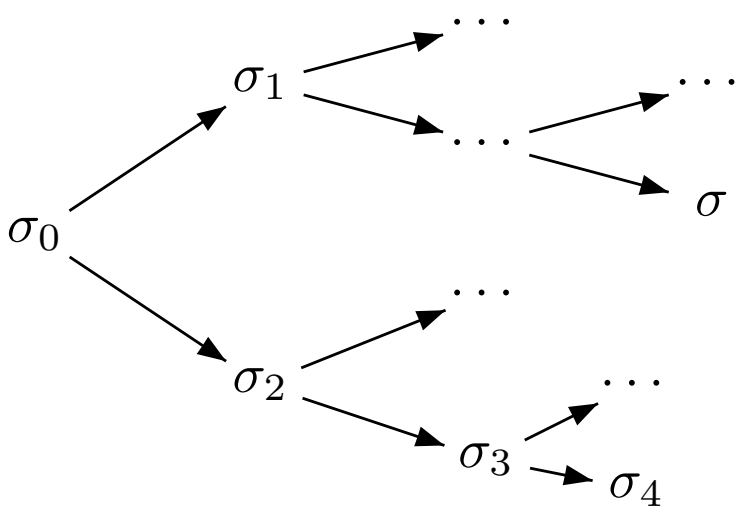

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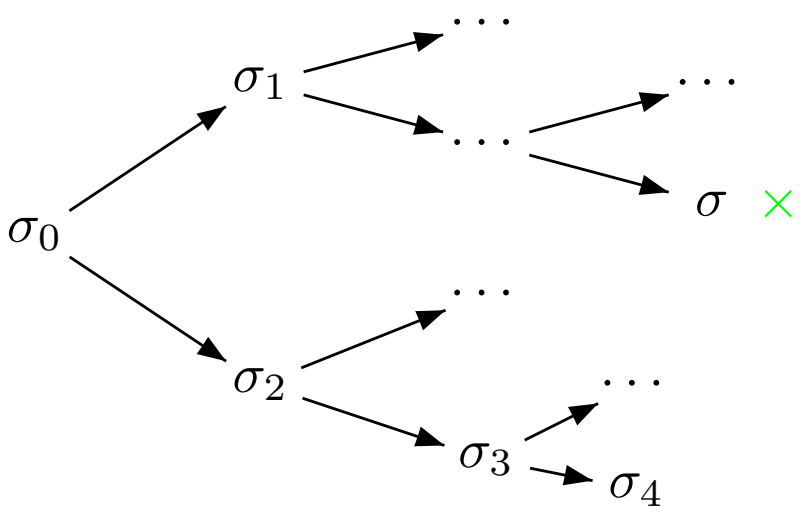

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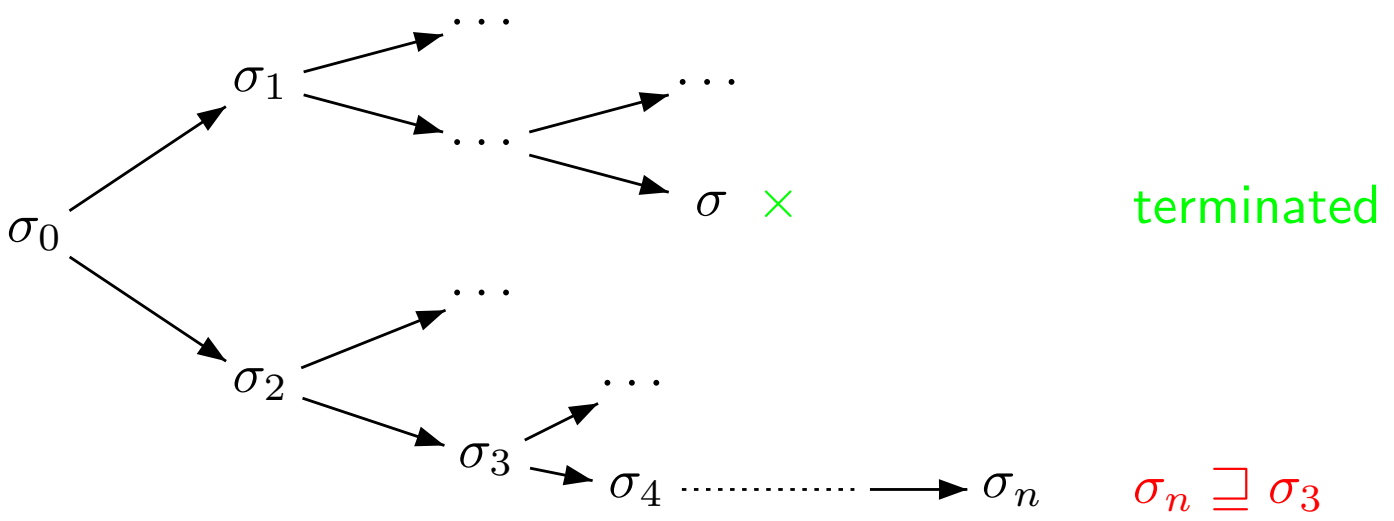

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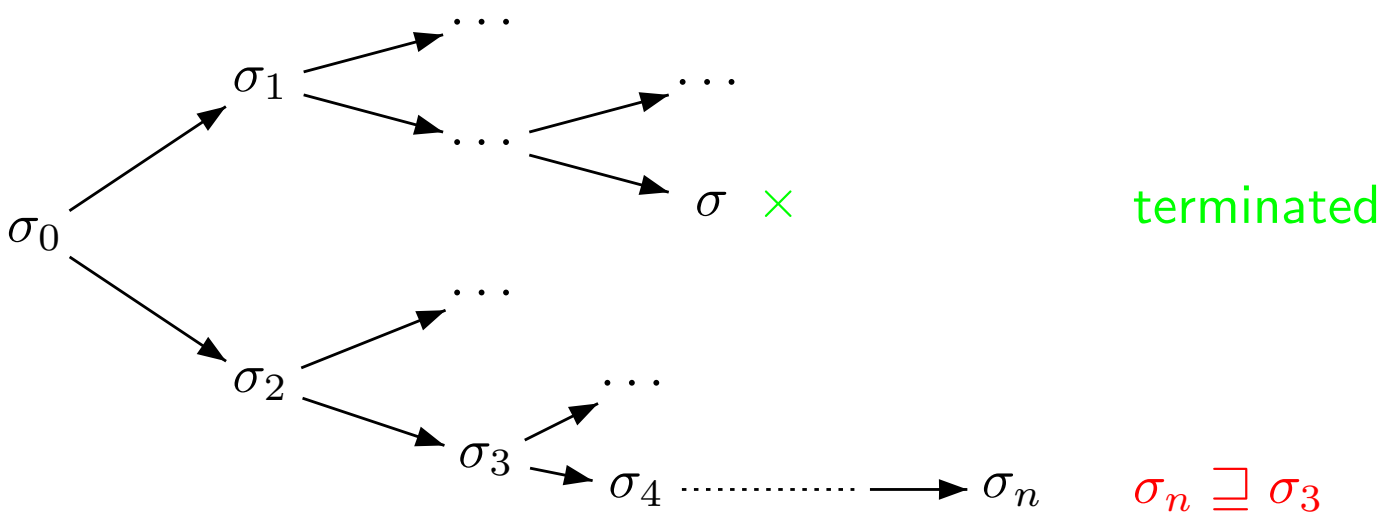

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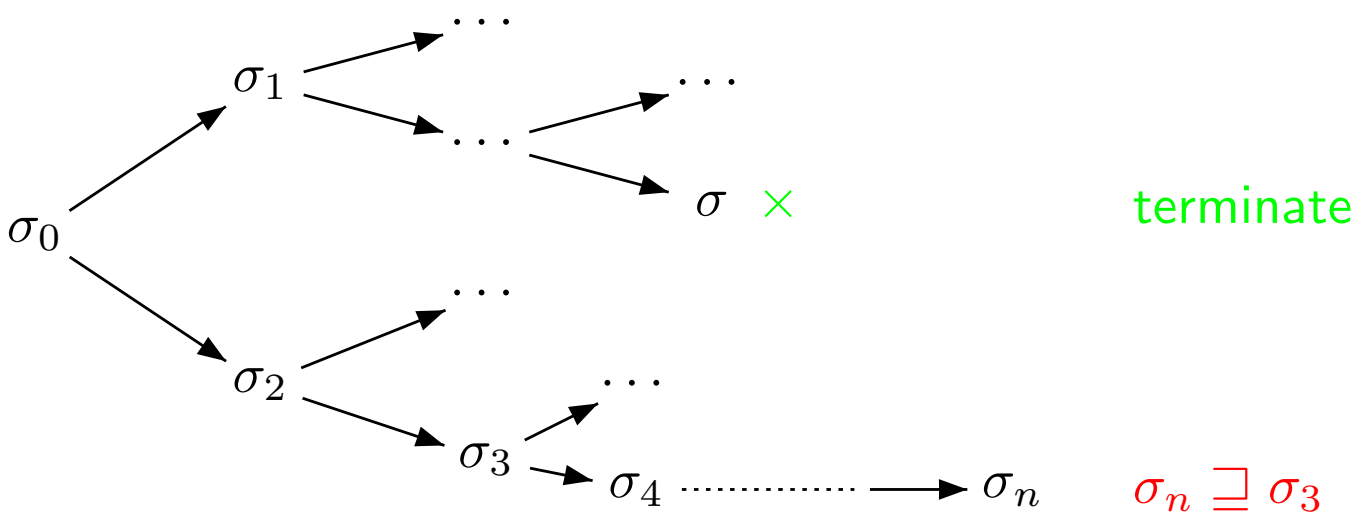

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By Higman's & König's Lemmas, the algorithm must eventually conclude.
(NB: works more generally for $\mathsf{A}\varphi\mathsf{U}\psi$ properties.)

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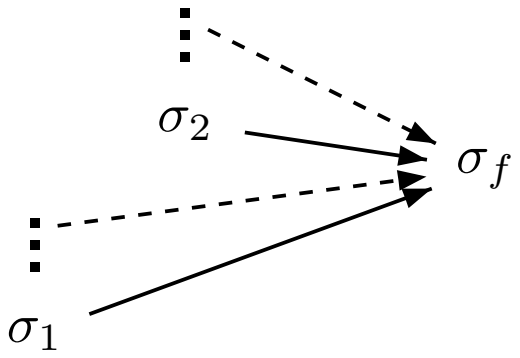

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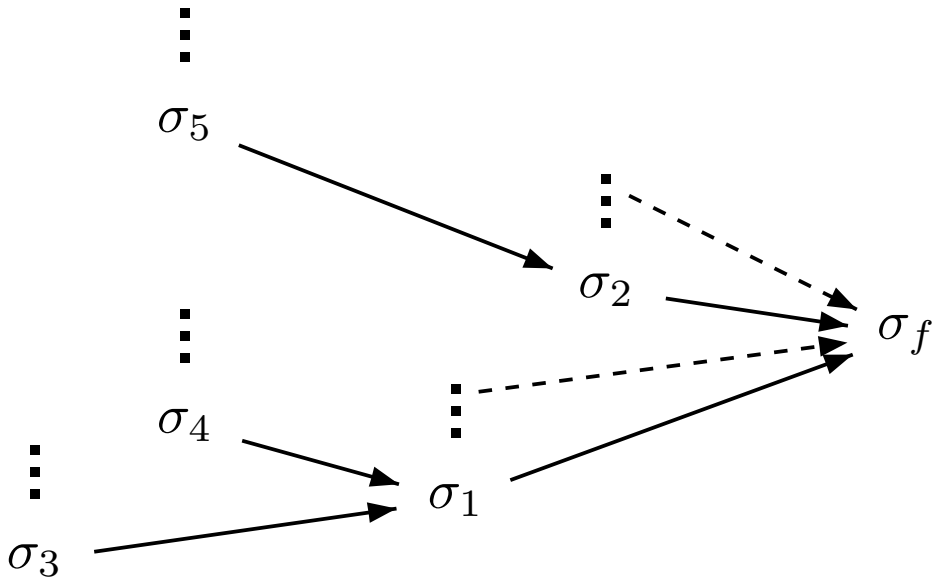

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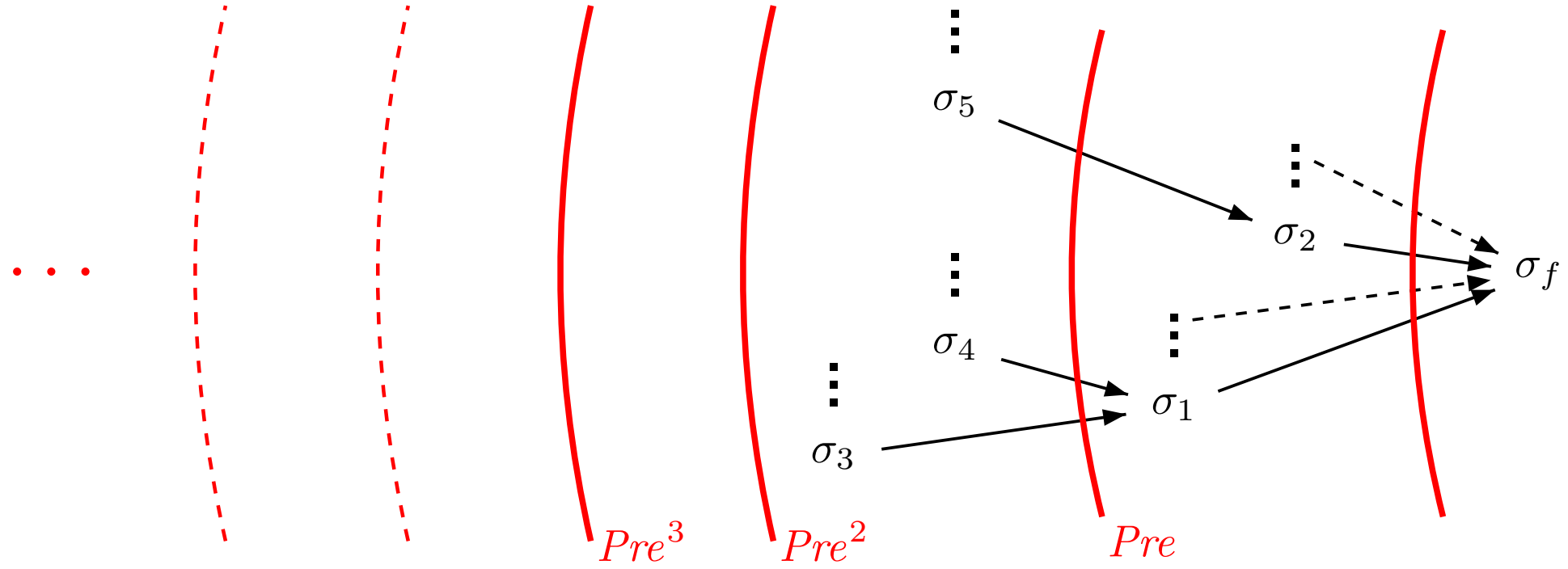

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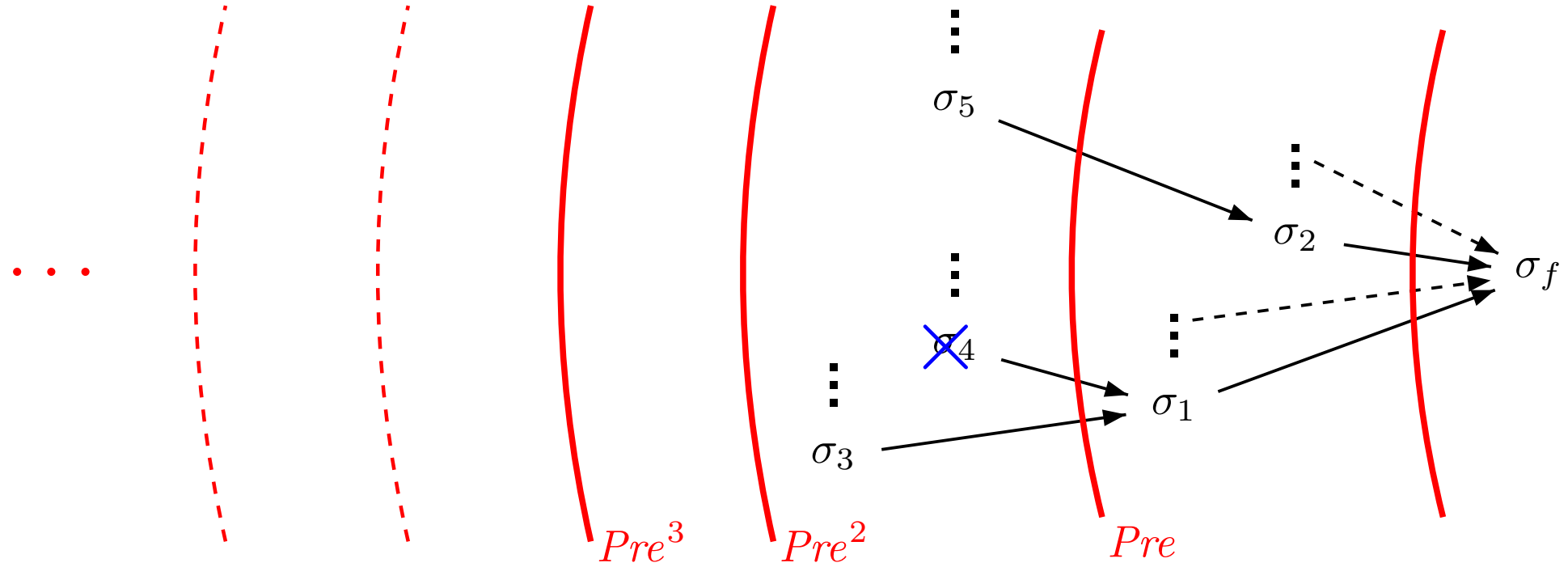

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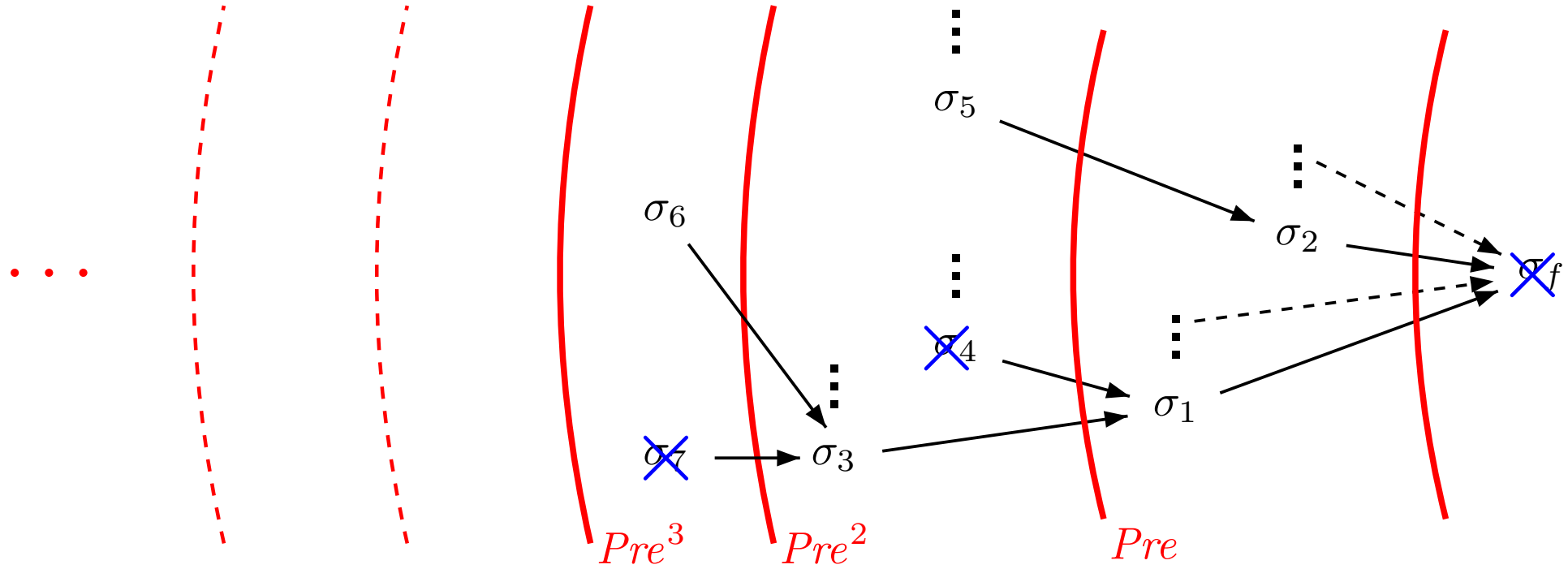

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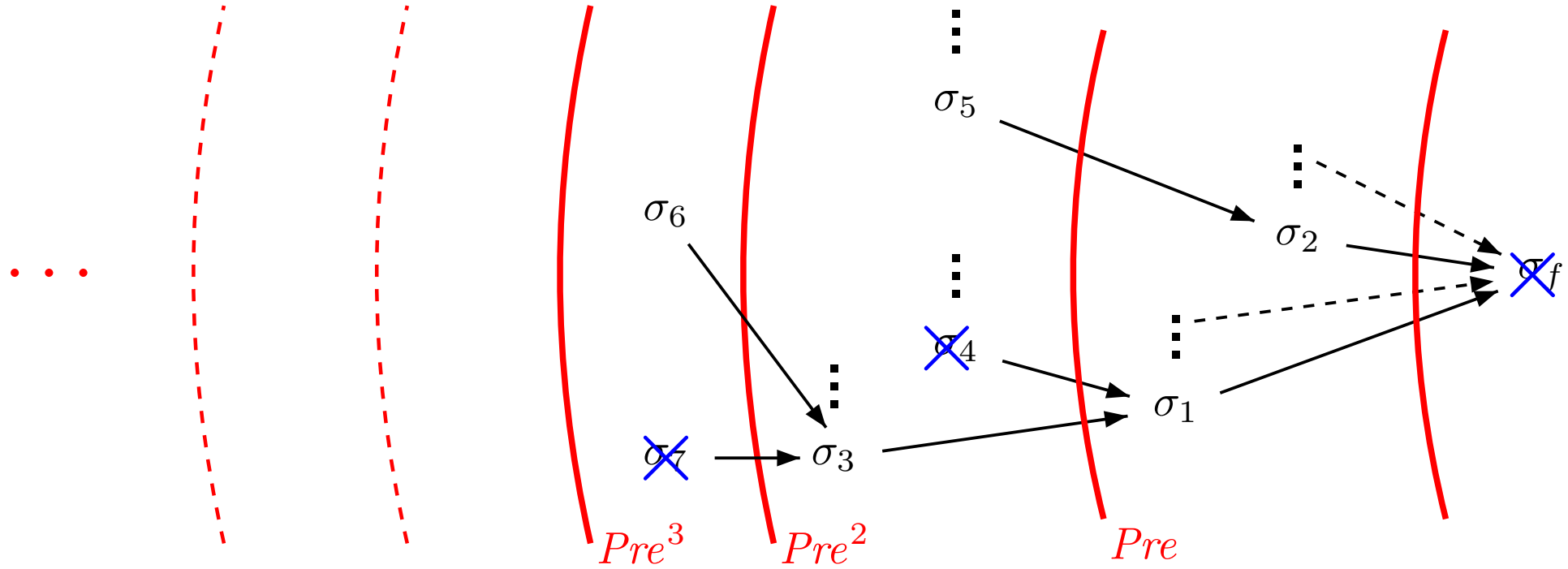

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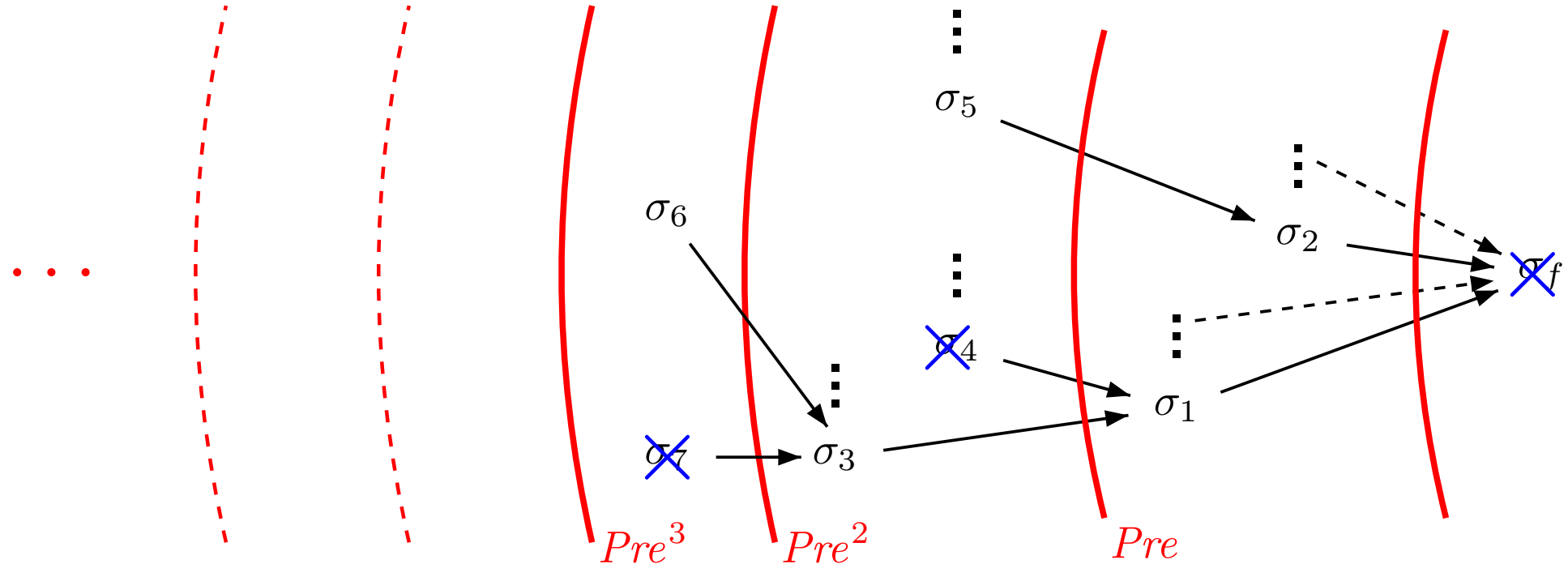

$\lim_i Pre^i = Pre^*$ assuming $\sigma_2 \sqsubseteq \sigma_4$ (say)

At step k we know the minimal elements of Pre^k.
Higman's Lemma ensures $Pre^k = Pre^{k+1}$ for some k.
$\Rightarrow Pre^*$ is effectively computable.

Other Decidable Problems?

Safety properties are decidable.

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Simulation and bisimulation with a finite state system are decidable.

Question: Are lossy channel systems a *trivial* model?

Answer: No! Many problems are undecidable, all other are hard.

We illustrate this in a uniform way...

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Problem is to decide whether there is a run from some σ_0 that visits some control state r infinitely often.

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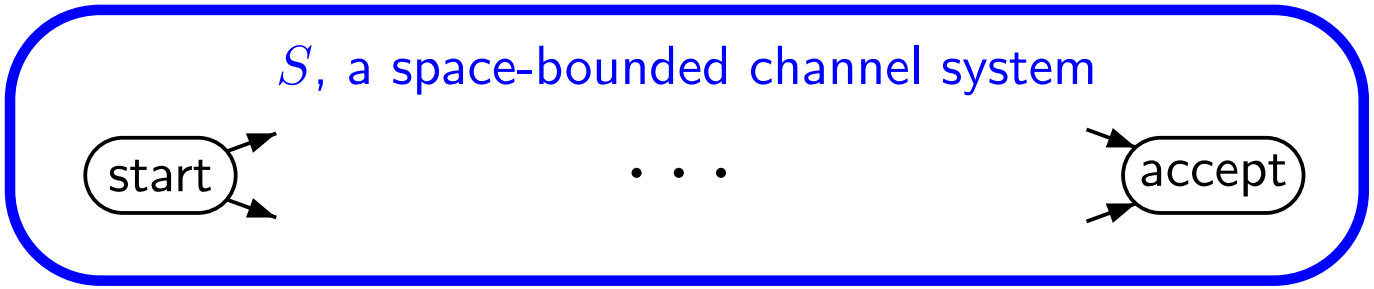

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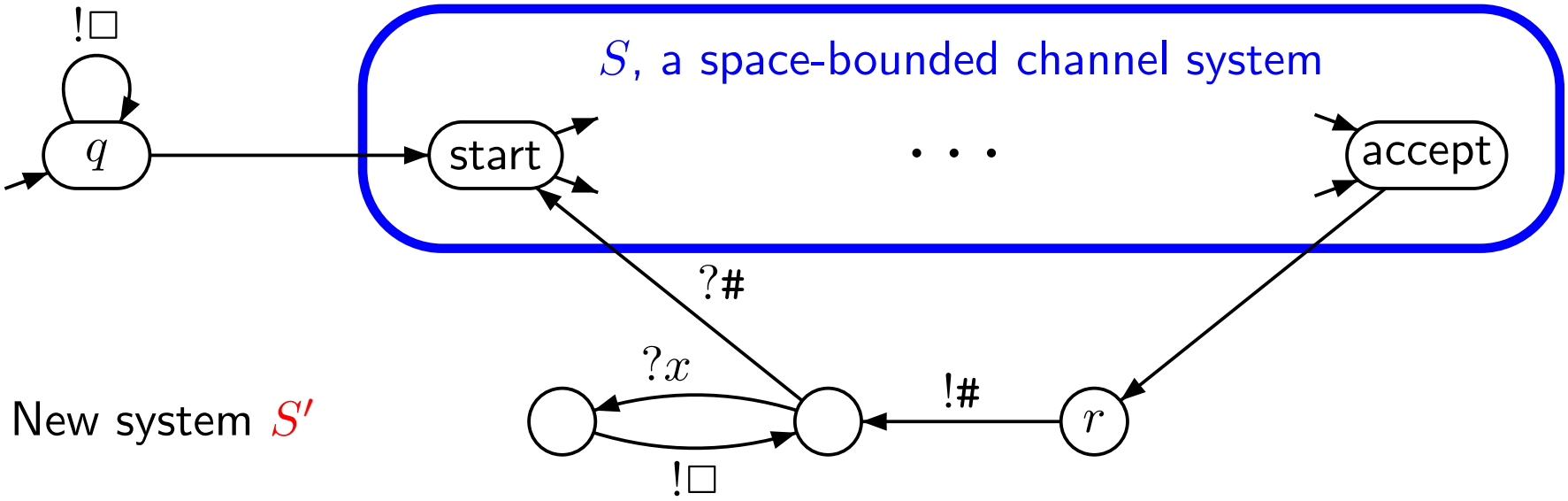

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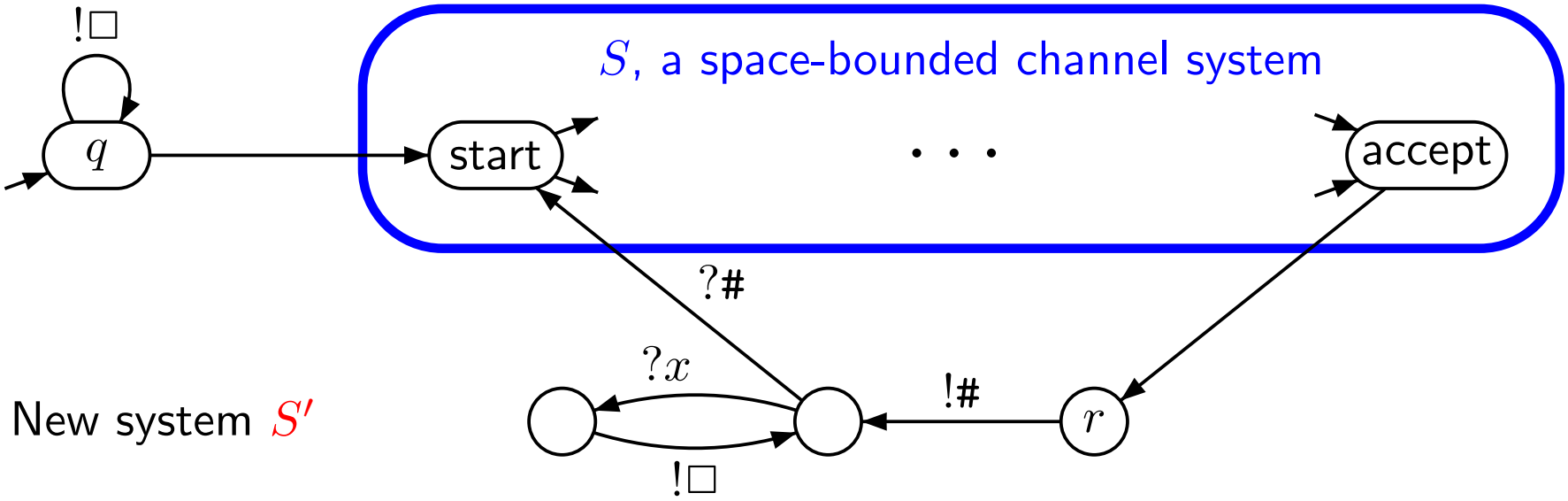

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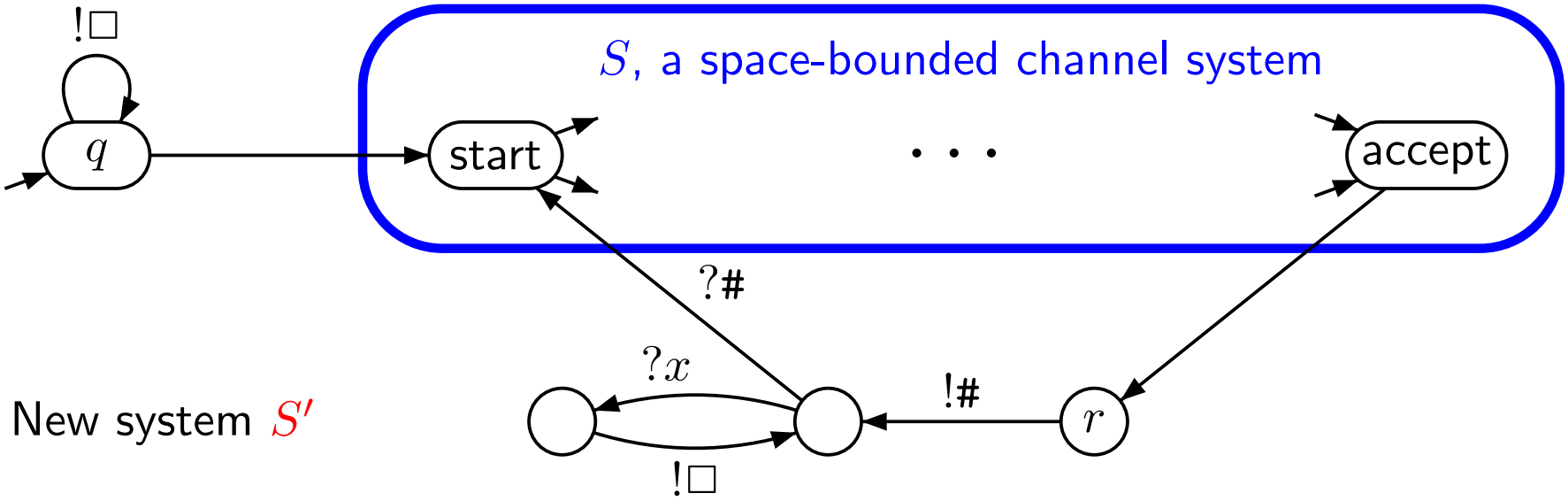

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Corollary: Liveness properties of lossy channel systems are undecidable.

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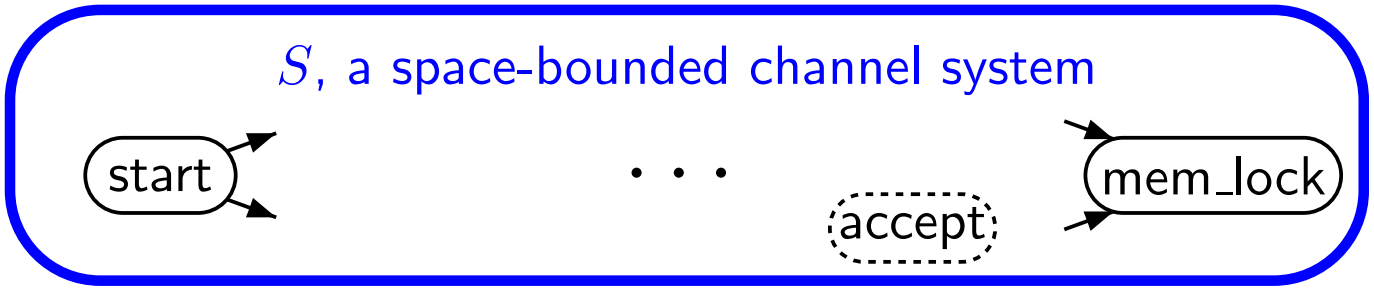

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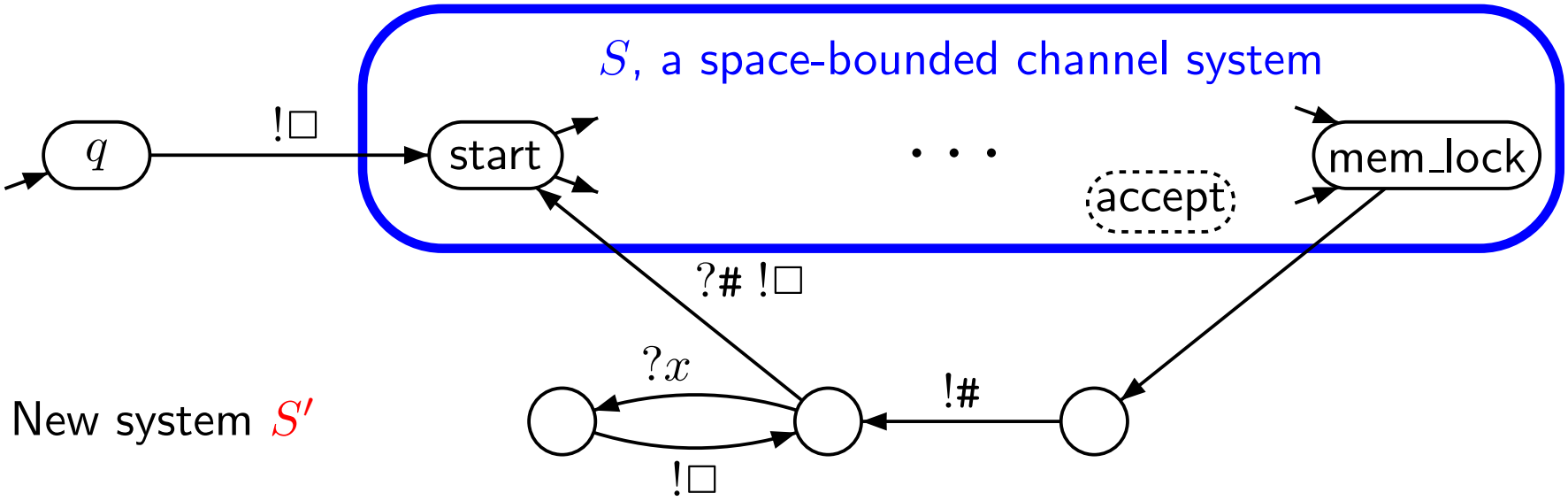

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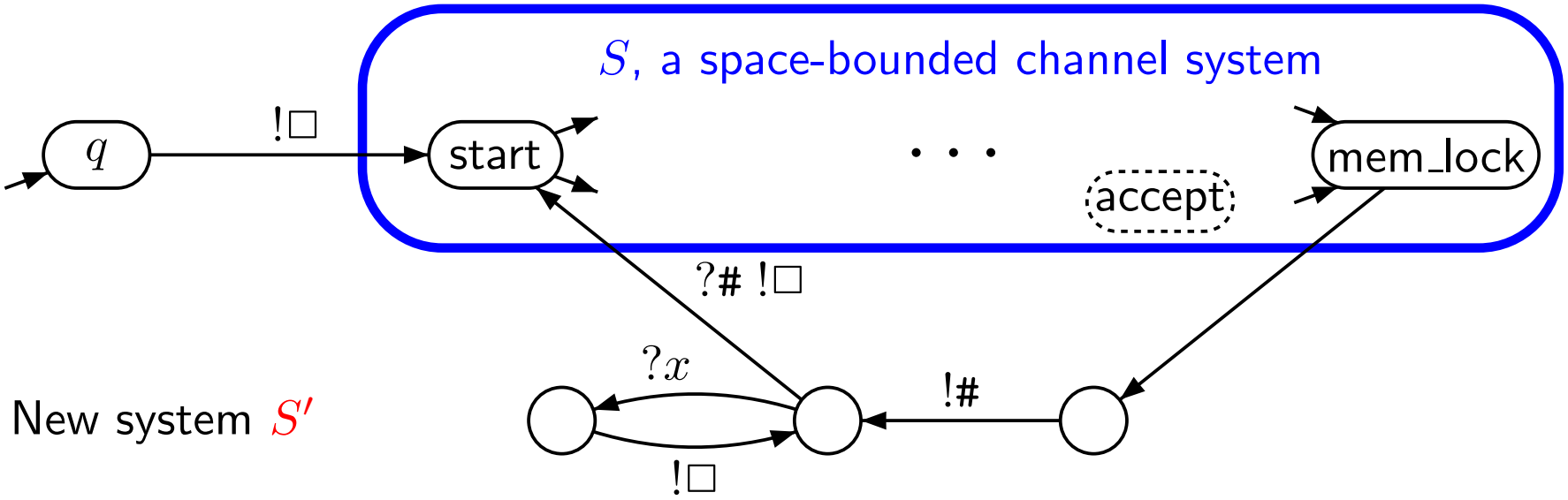

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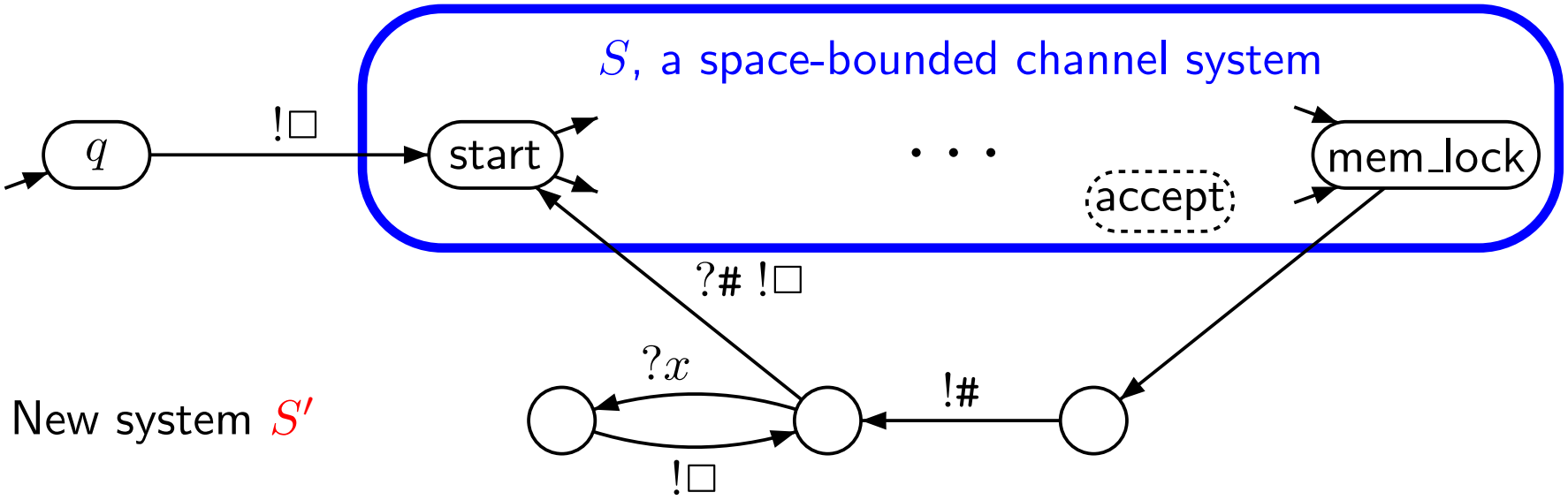

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Corollary: $Post^*$ is not effectively computable (though it is finitely recognizable).

Some Other Results

Theorem (Cécé): Reachability is decidable for ring networks with one lossy channel.

Open Problem: Characterize network topologies that provide decidability results.

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Theorem (S. 2001): All behavioral equivalences are undecidable between lossy channel systems.

Theorem (Masson & S. 2002): Termination under fair scheduling is decidable (for systems without multiplexing).

Inevitability under fair scheduling is undecidable.

Verification Is Hard

Let f_k be the usual k-nested primitive recursive map:

$$f_2(n) \stackrel{\text{def}}{=} 2n$$

$$f_k(n) \stackrel{\text{def}}{=} \underbrace{f_{k-1} \circ \cdots \circ f_{k-1}}_{n \text{ times}}(1) \qquad \text{if } k > 2.$$

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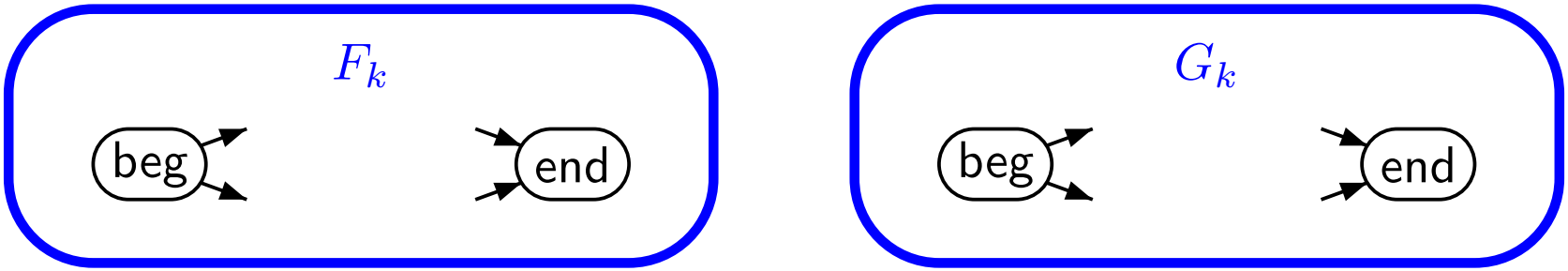
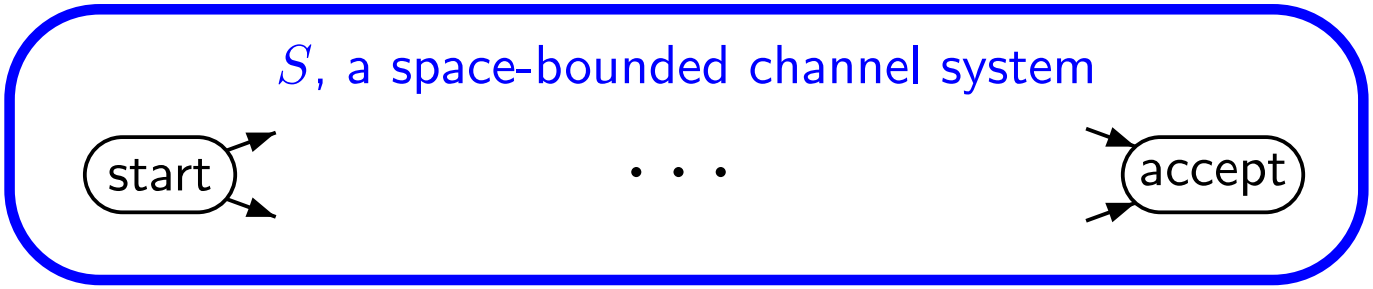


Facts: F_n and G_k have size $O(k)$.

In F_k, $\langle \text{beg}, \text{a}\Box^n\text{b} \rangle \xrightarrow{*} \langle \text{end}, \text{a}\Box^m\text{b} \rangle$ **iff** $m \leq f_k(n)$.

In G_k, $\langle \text{beg}, \text{a}\Box^n\text{b} \rangle \xrightarrow{*} \langle \text{end}, \text{a}\Box^m\text{b} \rangle$ **iff** $f_k(m) \leq n$.

Verification Is Hard – cont'd



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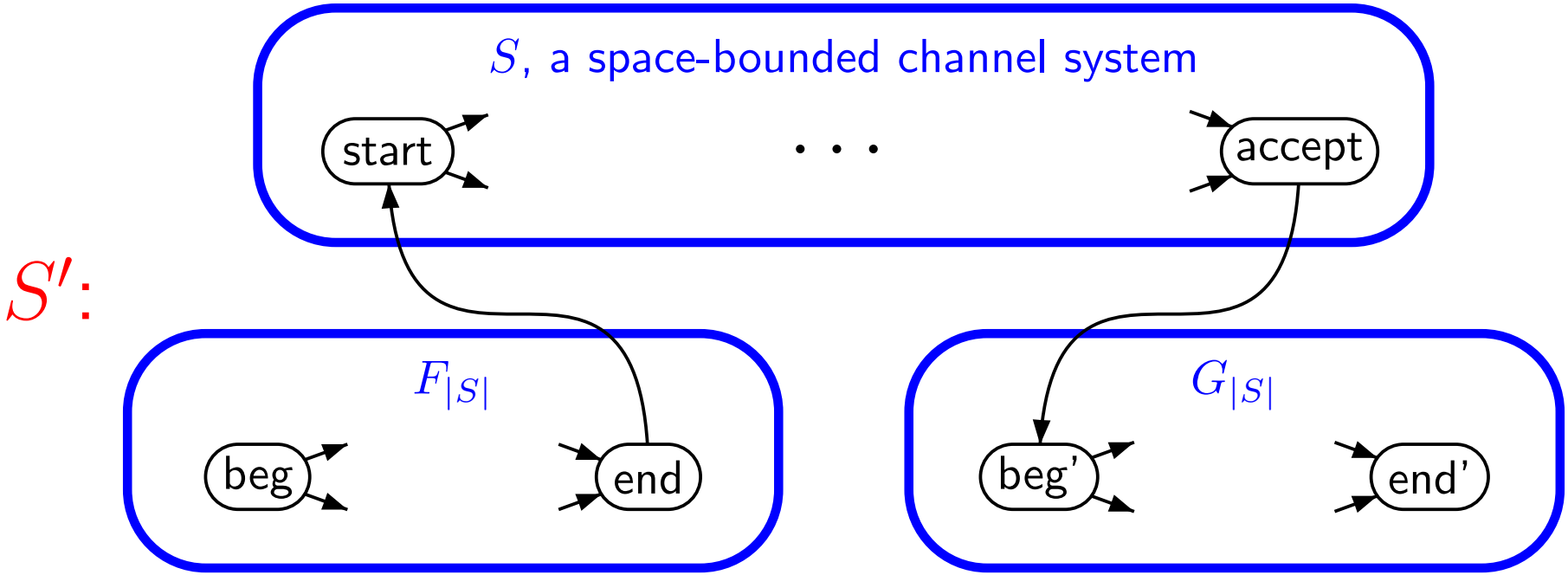

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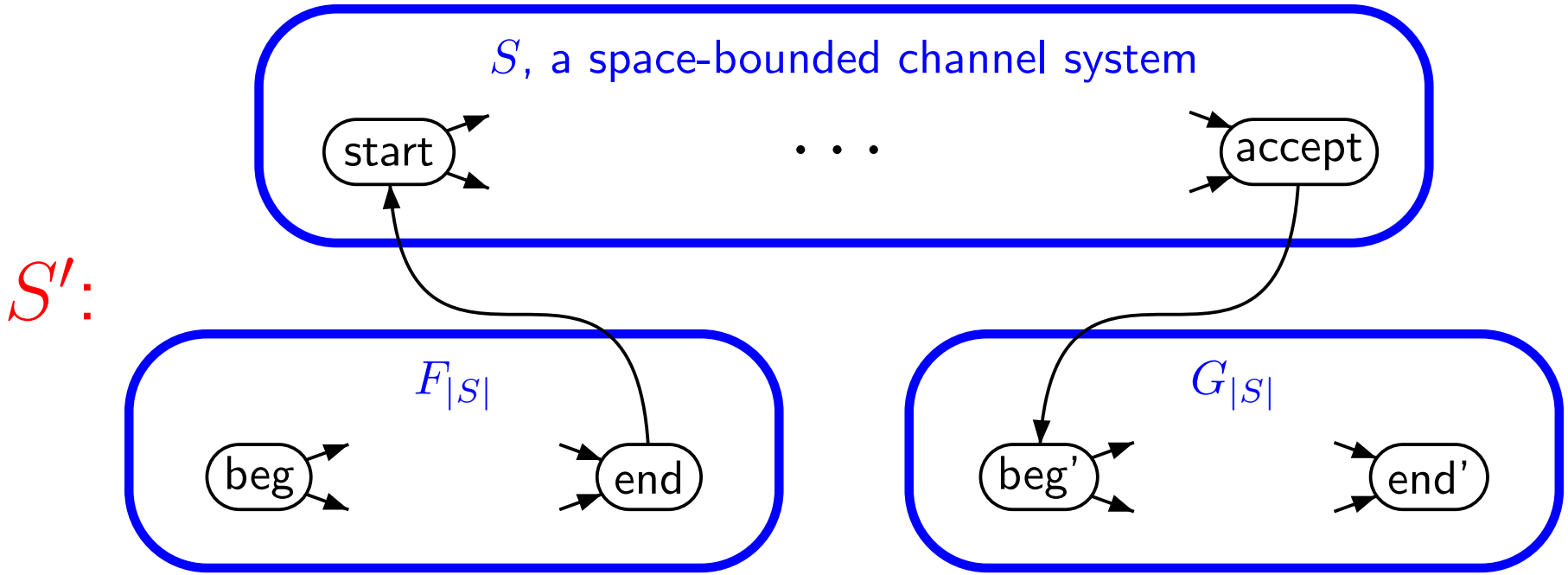

Observe: $\langle \text{beg}, \text{a}\Box^{|S|}\text{b} \rangle \xrightarrow{*} \langle \text{end'}, \text{a}\Box^{|S|}\text{b} \rangle$ in lossy S' **iff** perfect S accepts in $Ack(|S|)$ space.

Corollary (S. 2002): Reachability in lossy channel systems is not primitive recursive.

(NB: Applies to all other known decidable problems.)

Probabilistic Lossy Channel Systems

Definition: A Probabilistic LCS is a LCS equipped with
– positive weights on rules, and
– a constant probability $p_{\text{loss}} \in (0, 1)$.

Semantics in form of a countable Markov chain.

Two interpretations of p_{loss}: global-fault vs. local-fault model.

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Semantics in form of a countable Markov chain.

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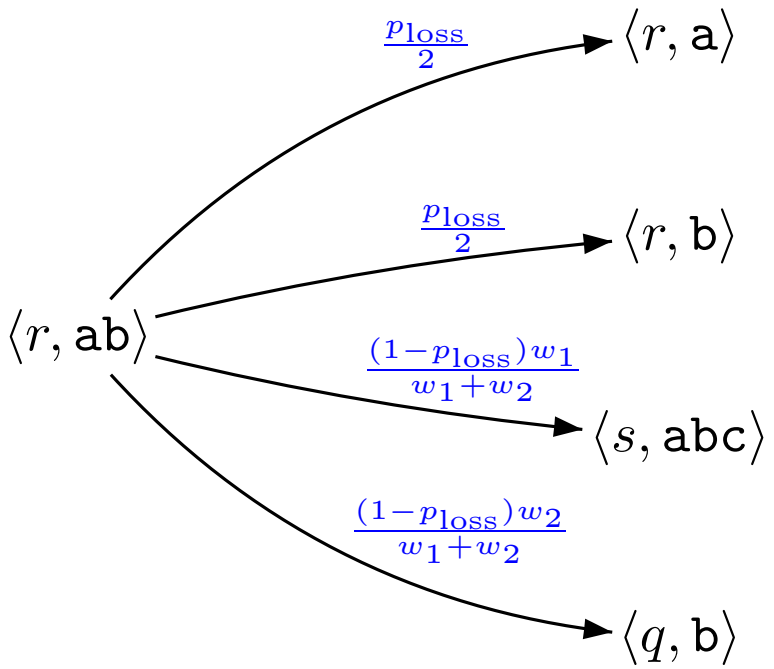

Probabilistic Lossy Channel Systems

Definition: A Probabilistic LCS is a LCS equipped with
– positive weights on rules, and
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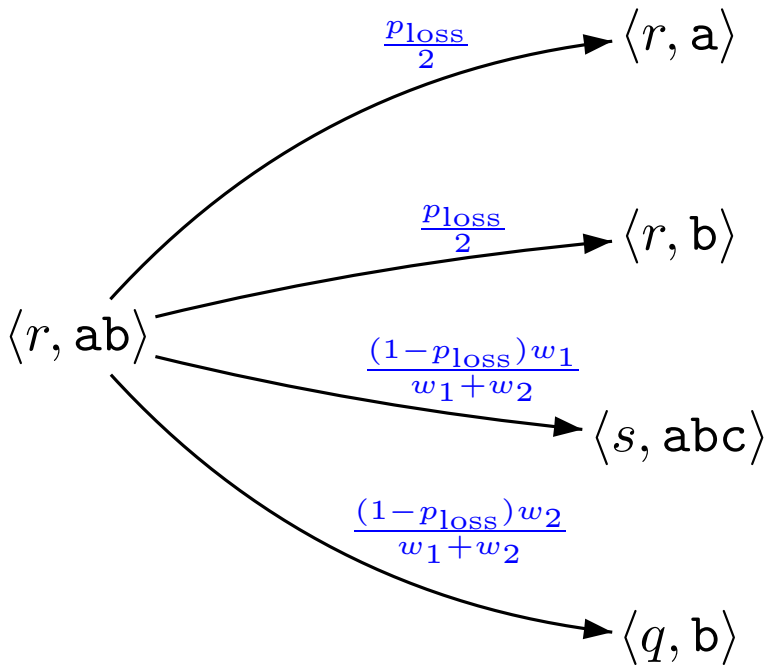

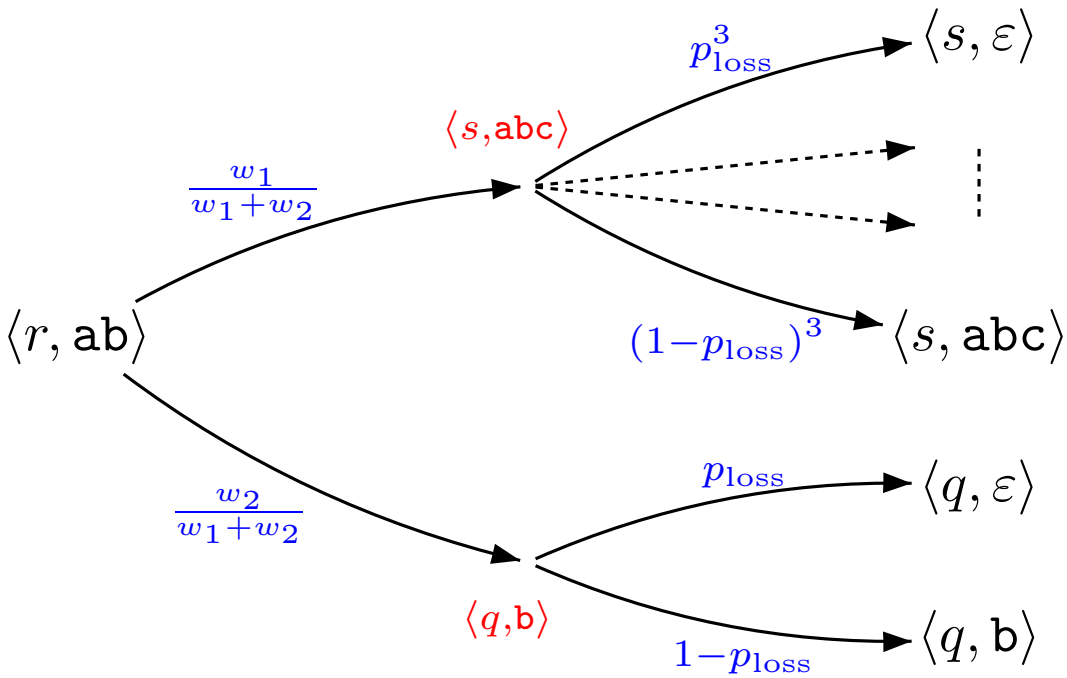

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More realist than just non-deterministic losses (protocols are designed with the idea that losses are not that likely).

Randomization rules out unrealistically nasty behaviors

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All these positive results rely on finite attractors.

Theorem (Rabinovich 2003): Approximability of $\mathbb{P}(\varphi)$ when a finite attractor exists.

Randomization helps!!

Beyond Markov Chains

The problem with PLCS's is that you have to view rules as probabilistic instead of nondeterministic.

Classically, nondeterminism in rules comes from:

– arbitrary interleaving of asynchronous components

– abstraction of real-life programs

– open systems

– early designs

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You want a *Markov decision process* model, where rules are nondeterministic and losses are probabilistic!! [Bertrand & S. 2003].

Then we can ask questions such as "*does* $\mathbb{P}(\varphi) = 1$ *under all scheduling policies?*" (This is the adversarial qualitative viewpoint)

Unrealistically nasty scheduling policies

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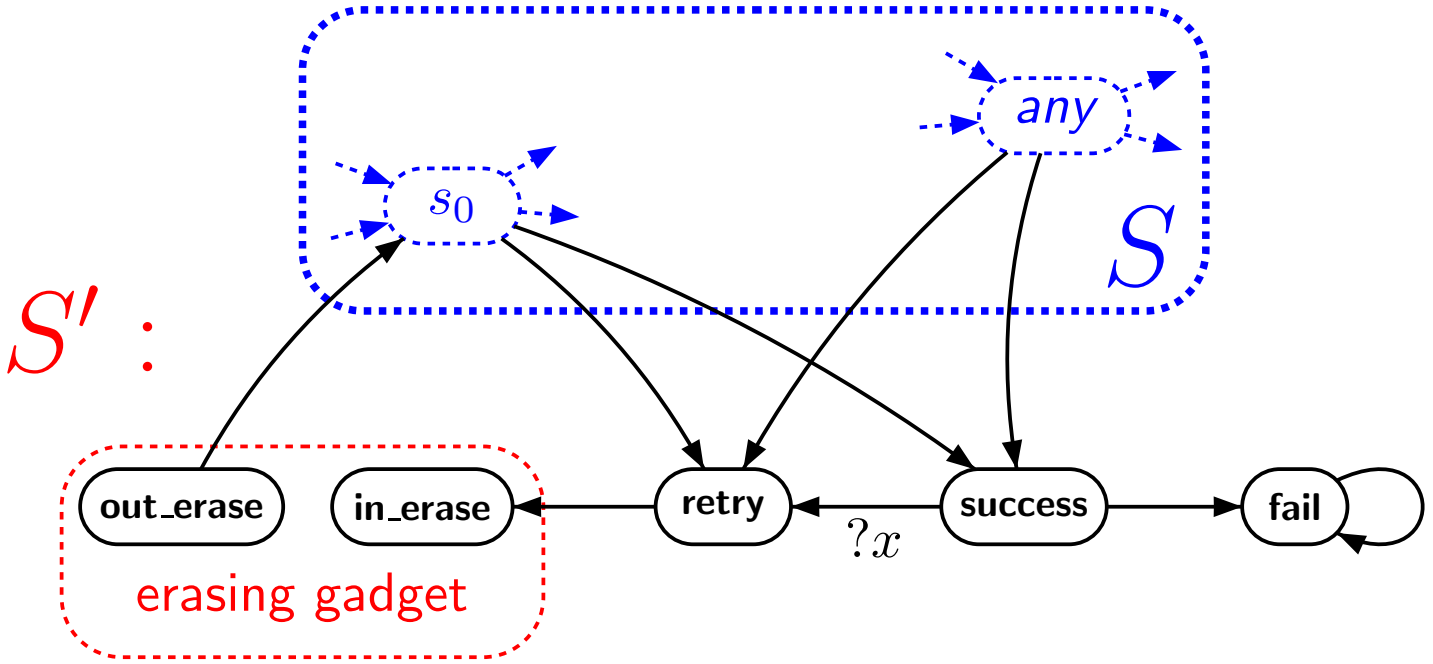

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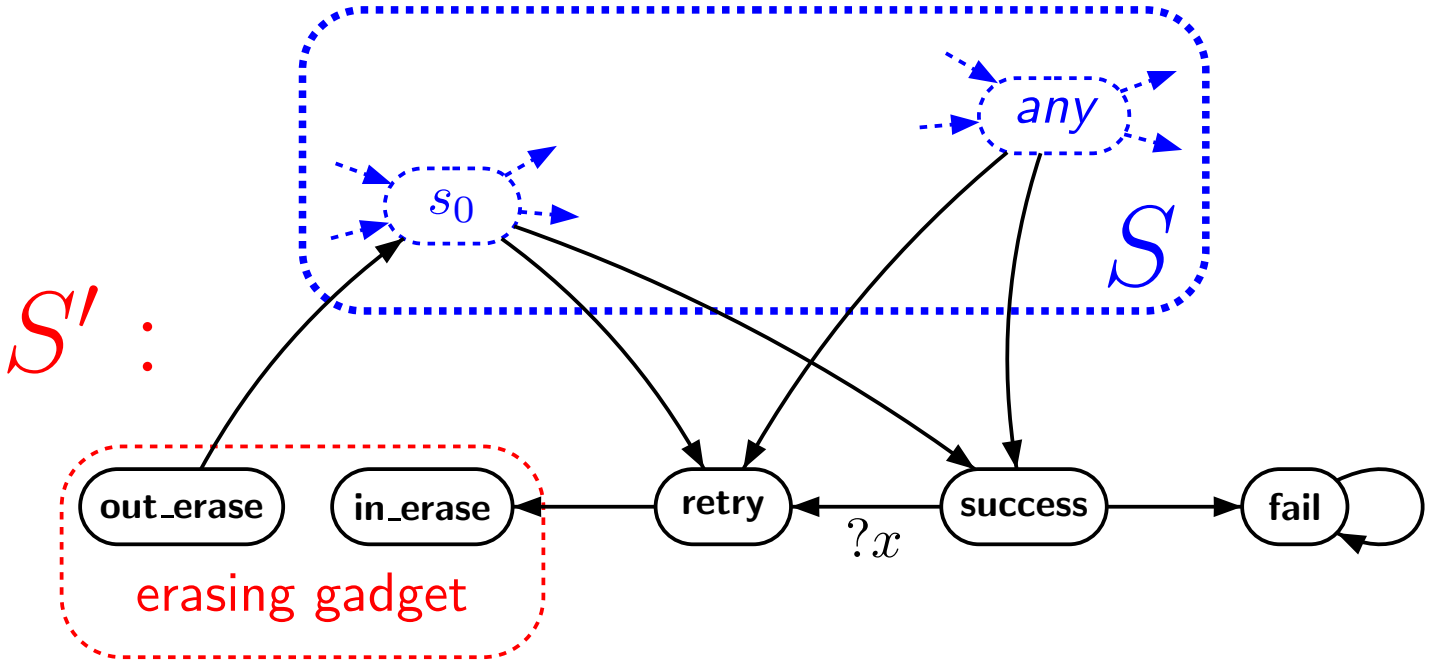

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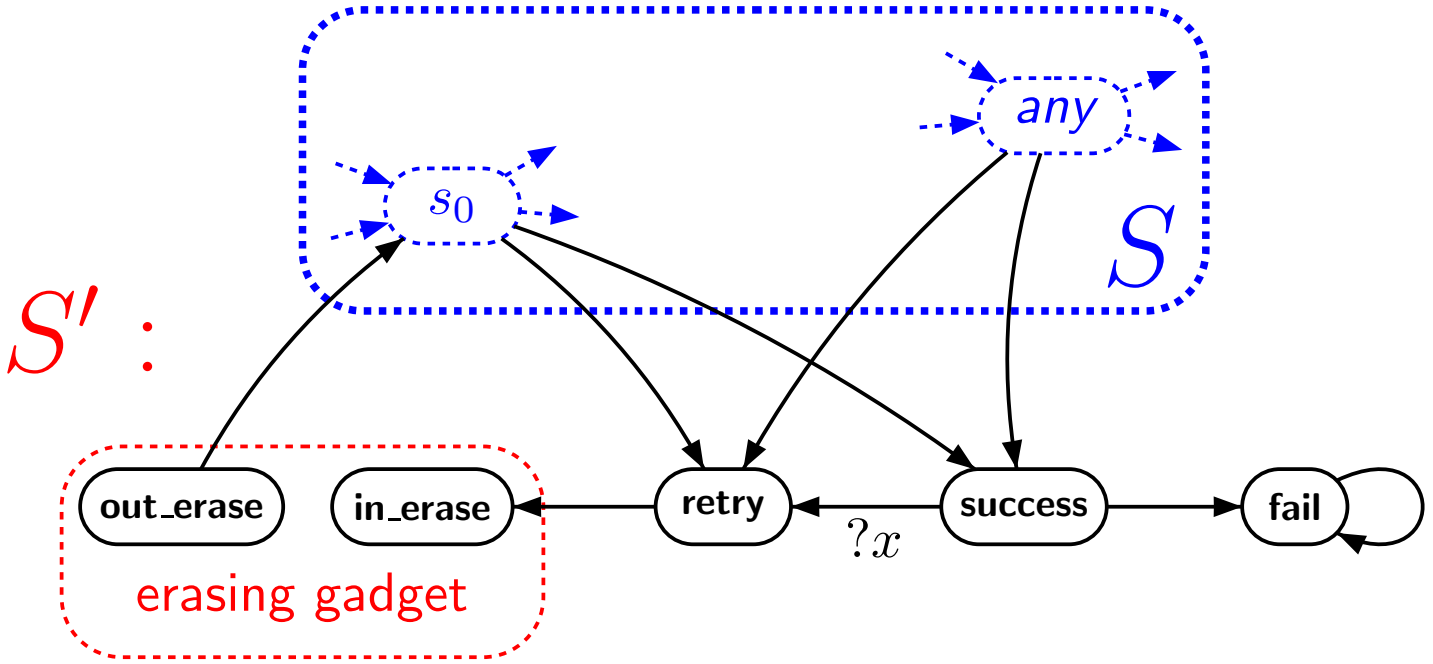
Unrealistically nasty scheduling policies



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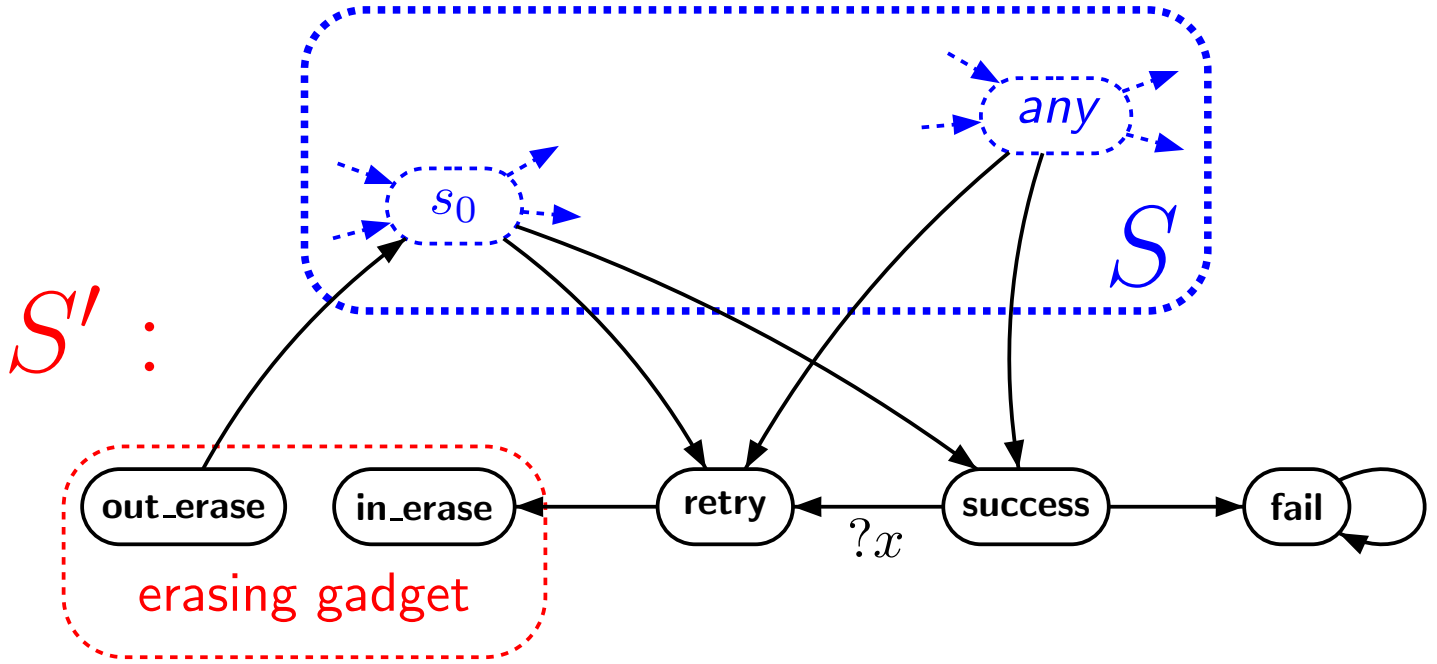

Question: is there a scheduling policy that makes S' visit **success** infinitely often with > 0 probability?

Answer: yes iff (nondeterministic) S is unbounded!

Corollary: model checking qualitative properties under all scheduling policies is undecidable.

All Is Not Lost!

In previous proof, the nasty scheduling policy is unrealistic.

E.g. it needs remember infinitely many things.

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Theorem (Bertrand & S. 2003): model checking qualitative properties under all *finite-memory policies* is decidable.

Some remaining open problems:

– What about cooperative qualitative model checking?

– What about computing minimal and maximal probabilities?

Concluding remarks

A fascinating model, bordering on undecidability.

Many open questions remain. Join us!!

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Algorithms for reachability and safety properties do work in practice.

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Lossy channel systems are useful too!

Algorithms for reachability and safety properties do work in practice.

Today, the Markovian decision process model with decidable adversarial qualitative properties is the best approximation to decidable model checking.

Will it work in practice?

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