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Compositional Minimisation of Finite State Systems Using Interface Specifications

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Abstract. We present a method for the *compositional construction* of the *minimal transition system* that represents the semantics of a given distributed system. Our aim is to control the *state explosion* caused by the interleavings of actions of communicating parallel components by *reduction steps* that exploit *global* communication constraints given in terms of *interface specifications*. The *effect* of the method, which is developed for *bisimulation semantics* here, depends on the structure of the distributed system under consideration, and the *accuracy* of the interface specifications. However, its *correctness* is independent of the correctness of the interface specifications provided by the program designer.

1. Introduction

Many tools for the *automatic analysis or verification of finite state distributed systems* are based on the construction of the *global state graph* of the system under consideration (cf. [CES83, CPS93, FSS83, Ste94]). Thus, they often fail because of the *state explosion problem*: the state space of a distributed system potentially increases *exponentially* in the number of its parallel components. To overcome this problem techniques have been developed in order to avoid the construction of the complete state graph (cf. [BFH90, CLM89, CR94, CS90b, DGG93, Fer88, GL93, Jos87, KM89, Kru89, LSW94, LT88, LX90, Pel93, Pnu90, SG89, SG90, Vaa90, Val93, Wal88, Win90, WL89]). In this paper we present

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a *method for the compositional minimisation of finite state distributed systems*, which is practically motivated by the following observation. For the verification of a system it is usually sufficient to consider an *abstraction* of its global state graph, because numerous computations are irrelevant from the observer's point of view. Such abstractions often allow us to reduce the state graph drastically by collapsing semantically equivalent states to a single state without affecting the observable behaviour. For example, the so obtained minimisation of a complex communication protocol may be a simple buffer.

Let us refer to the size of the original state space of a system S as its *apparent complexity*, and to the size of the minimised state space as its *reduced complexity*. The intention of our method is to avoid the apparent complexity by constructing the minimal system representation taking context information into account. Unfortunately, the straightforward idea to just successively combine and minimise the components of the system is not satisfactory, because such a "local" minimisation does not take context constraints into account and, therefore, may even lead to subsystems with a higher reduced complexity than the apparent complexity of the overall system. This is mainly due to the fact that parts need to be considered that can never be reached in the global context. *Partial* or *loose* specifications allow us to "cut off" these unreachable parts. As in [CS90b, Kru89, LT88, SG90, Wal88] we exploit this feature to take advantage of context information. Furthermore, we refer to the size of the maximal transition system that is encountered by our method as the *algorithmic complexity*.

Our method, called $\mathcal{RM}$*-Method*,[1] is tailored for establishing $\mathrm{P} \models \mathit{Spec}$, i.e. whether P satisfies the specification or property *Spec*, when P is a system in standard concurrent form, i.e. $\mathrm{P} = (p_1 \|_{I_1} \ldots \|_{I_{n-1}} p_n)\langle L\rangle$, which is annotated by interface specifications, and *Spec* is consistent with the semantical equivalence under consideration, i.e. $P \models \mathit{Spec} \iff Q \models \mathit{Spec}$ if P and Q are semantically equivalent. To simplify the development of our theory, we assume that the processes p_i are already given as *transition systems* and that $\|$ represents the *parallel composition operator*, $\langle L\rangle$ is a *window* or *hiding operator* that abstracts from the activities considered as internal by transforming them into the unobservable action τ, and I_i are *interface specifications* between $R_i =_{df} (p_1 \| \ldots \| p_i)$ and $Q_i =_{df} (p_{i+1} \| \ldots \| p_n)$. Interface specifications are intended to describe supersets of the set of sequences that can be observed at the associated interfaces. We represent interface specifications as processes with only observable behaviour and that cannot perform any internal step. A central result of the paper is that the branching structure of an interface specification is unimportant in our framework. Only its associated language has an impact.

The point of our method is the successive construction of *partially defined* transition systems P_i (for $1 \leq i \leq n$) with the following properties:

1. P_i is less specified than R_i, i.e. P_i is smaller than R_i with respect to the *specification preorder* $\preceq$. This guarantees the *correctness* of the $\mathcal{RM}$-Method, which states that $\mathcal{P}_n \models \mathit{Spec}$ implies $\mathrm{P} \models \mathit{Spec}$, if *Spec* is consistent with the kernel of $\preceq$ (i.e. $\preceq \cap \succeq$).
2. P_n is semantically equivalent to the full system P, whenever the interface specifications are *correct*.[2] This guarantees the *completeness* of the $\mathcal{RM}$-Method.

[1] $\mathcal{RM}$-Method stands for Reduction-Minimisation-Method.

[2] This does *not* mean that, in general, P_i and R_i are semantically equivalent for $1 \leq i \leq n-1$.

3. P_i has the least number of states and transitions in its semantic equivalence class.

In this paper, we are dealing with a refinement $\approx^{\mathrm{d}}$ of *observational equivalence* [Mil80, Mil89]. However, the method also adapts to other equivalences. Technically, we use a new operator, called *reduction operator*, in each step P_i where $1 \leq i < n$. The purpose of this operator is to cut off all states and transitions of the 'intermediate' transition systems that are not reachable according to the corresponding interface specification I_i.

An important factor in this approach are the interface specifications, which should be provided by the program designer. However, the *correctness* of the $\mathcal{RM}$-Method does not depend on the correctness of these interface specifications. They are only used to "guide" the reduction. Thus, wrong interface specifications never lead to wrong results, i.e. if $\mathcal{P}_n \models \mathit{Spec}$ is valid, then P satisfies *Spec*, too, provided that *Spec* is consistent with the kernel of $\preceq$. Otherwise, if $\mathcal{P}_n \models \mathit{Spec}$ is *not* provable, then P may or may not satisfy *Spec*. Therefore, wrong interface specifications may only prevent a successful verification of a valid statement. However, the $\mathcal{RM}$-Method is *complete* in the following sense: if all the considered interface specifications are correct and if *Spec* is a $\approx^{\mathrm{d}}$-consistent property, then $\mathcal{P}_n \models \mathit{Spec} \iff \mathrm{P} \models \mathit{Spec}$. It should be noted that the *total definedness* of $\mathcal{P}_n$ already implies the semantical equivalence of $\mathcal{P}_n$ and P and, therefore, the completeness for all $\approx^{\mathrm{d}}$-consistent properties. This criterion is sufficient for most practical applications.

1.1. Related Work

A great effort has already been made in order to avoid the construction of the complete state graph, and therefore to avoid the state explosion problem. Roughly, the proposed methods can be split into two categories, *compositional verification* and *compositional minimisation*. Characteristic for the former category is that the global system need not be considered at all during the verification process, and for the latter that a minimal semantically equivalent representation of the global system is constructed. This minimal representation can subsequently be used for all kinds of verification.

A pure approach to *compositional verification* has been proposed by Winskel in [Win90], where rules are given to decompose assertions of the form $\mathrm{P} \models \Phi$ depending on the syntax of the program P and the formula Φ. Unfortunately, the decomposition rules for processes involving the parallel operator are very restricted. Larsen and Xinxin [LX90] follow a similar line, however, their decomposition rules are based on an operational semantics of contexts rather than the syntax. In order to deal with the problems that arise from parallel compositions, Pnueli [Pnu90] proposed a "conditional" inference system where assertions of the form $\phi \mathrm{P} \psi$ can be derived, meaning that the program P satisfies the property ψ under the condition that its environment satisfies ϕ. This inference system has been used by Shurek and Grumberg in [SG90], where a semi-automatic modular verification method is presented which, like ours, is based on "guesses" for context specifications. However, in contrast to our method it requires a separate proof of the correctness of these guesses. Another method based on interface specifications which must be proved correct separately is given in [Kru89]. Josko [Jos87] also presented a method, where the assumptions on the environment of a com-

ponent are expressed by a formula, which must be proved in a separate step. The main disadvantage of his method is that the algorithm is exponential in the size of the assumptions about the environment. Other methods try to avoid the state explosion problem using preorders for verification [GW91, Pel93, Val93] where unnecessary interleavings of actions are suppressed. In [LSW94] a constraint-oriented state-based proof methodology for concurrent software systems is presented which exploits compositionality and abstraction for the reduction of the (possibly infinite) verification problem under consideration. There, Modal Transition Systems are used for fine-granular, loose state-based specifications of constraints.

Halbwachs et al. [BFH90] proposed a method of the second category. It constructs directly a transition system minimised with respect to bisimulations by successive refinement of a single state. In this method symbolic computation is needed in order to keep the expressions small which in general may grow exponentially. Another approach of this category was presented by Clarke et al. [CLM89]. They exploit the knowledge about the alphabet of interest in order to abstract and minimise the system's components. By using $\langle L \rangle$ operators together with an elementary rule for distributing them over the parallel operator (see Proposition 2.7) our method covers this approach. Larsen and Thomsen [LT88], and Walker [Wal88] use partial specifications in order to take context constraints into account. Our method is an elaboration of theirs. It uses a more appropriate preorder and defines a concrete strategy for (semi-)automatic proofs where the required user support is kept to a minimum. Also Vaandrager [Vaa90] observes that in most situations partial information about the traces of processes is sufficient to prove that part of a specification is redundant and can be omitted.

The methods proposed in [BCG86, KM89, SG89, WL89] are tailored to verify properties of classes of systems that are systematically built from large numbers of identical processes. These methods are somewhat orthogonal to ours. This suggests to consider a combination of both types of methods.

In practice, Binary Decision Diagrams are used to code state graphs for an interesting class of systems [Bry86]. These codings do not explode directly, but they may explode during verification. All mentioned techniques can be accompanied by *abstraction*. Parallel systems may be dramatically reduced by suppressing constraints that are irrelevant for the verification of the particular property under consideration [CC77, CGL92, CR94, DGG93, LGS+95].

1.2. Structure of the Paper

The remainder of the paper is structured as follows. Section 2 presents the basic notions, and Section 3 the reduction operators on which our method, the $\mathcal{RM}$-Method, is based. Subsequently, Section 4 develops the $\mathcal{RM}$-Method for the compositional minimisation of finite state distributed systems, proves its correctness and completeness, and illustrates its power by means of an example, where the apparent exponential complexity is reduced to a linear algorithmic complexity. Finally, Section 5 draws our conclusions. A version of this paper including detailed proofs is available as technical report [GSL95].

2. General Notions

Our framework is based on processes (systems) as labelled transition systems extended by an undefinedness predicate on states. Processes can be structured by means of parallel composition and hiding, thus allowing a hierarchical treatment. The introduction of undefinedness predicates naturally leads to a specification-implementation preorder between processes, which induces a slightly finer semantics on processes than observational equivalence [Mil80, Mil89]. This equivalence is captured by our technique, which is based on the notion of *interface specification* introduced subsequently.

2.1. Representation of Processes

We model distributed systems by *extended transition systems*, i.e. a transition system which is extended by an *undefinedness predicate* that plays an important role in the correctness proof of our $\mathcal{RM}$-Method.

Definition 2.1. (Extended Transition Systems)
An *extended (finite state) transition system* is a quadruple $(S, \mathcal{A} \cup \{\tau\}, \longrightarrow, \uparrow)$ where

1. S is a finite set of *processes* or *states*,
2. $\mathcal{A}$ is a finite *alphabet* of *observable actions*, and τ represents an internal or unobservable action not in $\mathcal{A}$,
3. $\longrightarrow \subseteq S \times \mathcal{A} \cup \{\tau\} \times S$ is a *transition relation*, and
4. $\uparrow \subseteq S \times 2^{\mathcal{A} \cup \{\tau\}}$ is a predicate expressing *guarded undefinedness*.[3]

Typically, S is a set of program states, and the relationship $p \xrightarrow{a} q$ indicates that p can evolve to q under the observation of a. We write $p \xrightarrow{a}$ for $\exists q \in S.\, p \xrightarrow{a} q$. Moreover, we use for convenience the convention $p \xrightarrow{\epsilon} p$ for all $p \in Proc$, i.e. ϵ denotes an 'idling' or 'trivial' step. Finally, $p \uparrow a$ expresses that an a-transition would allow p to enter an undefined state. We say that p is *a-undefined* in this case. Thus, transition systems involving the undefinedness predicate are only *partially defined* or *specified*. It is this notion of partial specification together with its induced preorder which provides the framework for proving our method correct.

Processes are rooted extended transition systems, i.e. they consist of an extended transition system and a designated start state.

Definition 2.2. (Processes)
Let $T = (S, \mathcal{A} \cup \{\tau\}, \longrightarrow, \uparrow)$ be an extended transition system. A *process* is a tuple $(S_p, \mathcal{A}_p \cup \{\tau\}, \longrightarrow_p, \uparrow_p, p)$ for a state $p \in S$ where

- S_p is the set of states that are reachable from p in T,
- $\mathcal{A}_p =_{df} \mathcal{A}$, and
- $\longrightarrow_p$ and $\uparrow_p$ are $\longrightarrow$ and $\uparrow$ restricted to S_p, respectively.

p is called *start state* of the process. The set of all processes is denoted by $\mathcal{P}$.

[3] 2^M denotes the power set of the set M.

In future, obvious indices are dropped, and we write p for $(S_p, \mathcal{A}_p \cup \{\tau\}, \longrightarrow_p, \uparrow_p, p)$. The following property characterises the subset of "standard" transition systems: a process p is *totally defined* if its undefinedness predicate $\uparrow_p$ is empty. Otherwise it is called *partial*. Moreover, if $p, q \in \mathcal{P}$ are identical up to renamings of states, we call p and q *isomorphic*, in signs $p \cong q$. If no confusion arises with syntactic equality, we simply write $p = q$. A process p is called *deterministic* if $\forall q \in S_p, a \in \mathcal{A}_p \cup \{\tau\}.\ |\{q' | q \xrightarrow{a}_p q'\}| \leq 1$. Otherwise p is called *nondeterministic*.

As usual, processes can be assigned a language which we need when dealing with interface specifications. Since we want the language of a process only to contain observable actions and not the internal, invisible action τ, we define the following *weak* transition relation and *weak* undefinedness predicate.

Definition 2.3. (Weak Transition Relation and Undefinedness)
Let $(S, \mathcal{A} \cup \{\tau\}, \longrightarrow, \uparrow)$ be an extended transition system. The *weak transition relation* $\Longrightarrow \subseteq S \times \mathcal{A} \cup \{\epsilon\} \times S$ and the *weak undefinedness predicate* $\Uparrow \subseteq S \times 2^{\mathcal{A} \cup \{\epsilon\}}$ are defined as the least relations satisfying for all $p, q \in S$ and all $a \in \mathcal{A}$.

1. $p \xrightarrow{\tau}{}^{*} \xrightarrow{a} \xrightarrow{\tau}{}^{*} q$ implies $p \overset{a}{\Longrightarrow} q$,
2. $p \xrightarrow{\tau}{}^{*} q$ implies $p \overset{\epsilon}{\Longrightarrow} q$,
3. $q \uparrow a$ and $p \overset{\epsilon}{\Longrightarrow} q$ implies $p \Uparrow a$,
4. $q \uparrow \tau$ and $p \overset{\epsilon}{\Longrightarrow} q$ implies $p \Uparrow \epsilon$,
5. $q \Uparrow \epsilon$ and $p \overset{a}{\Longrightarrow} q$ implies $p \Uparrow a$, and
6. $p \Uparrow \epsilon$ implies $p \Uparrow a$.

Now, we are able to define the *language of processes*.

Definition 2.4. (Language of Processes)
The *language*, $\mathcal{L}(p)$, of a partially defined process p is defined as the least fixed point of the following equation system.

$$\mathcal{L}(p) = \begin{cases} \mathcal{A}_p^* & \text{if } p \Uparrow \varepsilon \\ \bigcup \{a \cdot \mathcal{L}_a(p) \mid \mathcal{L}_a(p) \neq \emptyset\} \cup \{\varepsilon\} & \text{otherwise} \end{cases}$$

and

$$\mathcal{L}_a(p) = \begin{cases} \mathcal{A}_p^* & \text{if } p \Uparrow a \\ \bigcup \{\mathcal{L}(p') \mid p \overset{a}{\Longrightarrow} p'\} & \text{otherwise} \end{cases}$$

for any action $a \in \mathcal{A} \cup \{\epsilon\}$. Furthermore, given a language $\mathcal{L}$, we denote the language of its a-suffixes, $\{w \mid a \cdot w \in \mathcal{L}\}$, by $\mathcal{L}_a$.

The well-definedness of the above definition follows from elementary fixed point theory. Note that this definition is standard for totally defined processes. The language of an a-undefined process includes any sequence of actions starting with a and the language of an ϵ-undefined process p is $\mathcal{A}_p^*$ which reflects our intuition that the language of an undefined state is unconstraint. Therefore, we make the worst case assumption that the language of an undefined process contains *all* possible sequences of actions.

2.2. Parallel Composition and Hiding

We now introduce a binary *parallel operator* $\|$ and unary *hiding* or *window operators* $\langle L\rangle$ on processes, where L is the set of actions remaining visible. Intuitively, $p\|q$ is the parallel composition of the processes p and q with synchronisation of the actions common to both of their alphabets and interleaving of the others (like in CSP [Hoa85]), and $p\langle L\rangle$ is the process in which only the actions in L are observable.

Definition 2.5. (Operational Semantics)
Let $p = (S_p, \mathcal{A}_p \cup \{\tau\}, \longrightarrow_p, \uparrow_p, p)$, $q = (S_q, \mathcal{A}_q \cup \{\tau\}, \longrightarrow_q, \uparrow_q, q) \in \mathcal{P}$, let $p', p'' \in S_p$, $q', q'' \in S_q$, and let L be a set of visible actions. We define the alphabets of the processes $p\langle L\rangle$ and $p\|q$ by $\mathcal{A}_{p\langle L\rangle} =_{df} \mathcal{A}_p \cap L$ and $\mathcal{A}_{p\|q} =_{df} \mathcal{A}_p \cup \mathcal{A}_q$, respectively. Their state sets are defined as the subsets of states of $\{p'\langle L\rangle \mid p' \in S_p\}$ and $\{p'\|q' \mid p' \in S_p, q' \in S_q\}$ which are reachable from the initial states $p\langle L\rangle$ and $p\|q$, respectively, according to the following transition relations defined in *Plotkin style notation.*

1. $\dfrac{p' \xrightarrow{a}_p p''}{p'\langle L\rangle \xrightarrow{a}_{p\langle L\rangle} p''\langle L\rangle} \quad a \in L$

2. $\dfrac{p' \xrightarrow{a}_p p''}{p'\langle L\rangle \xrightarrow{\tau}_{p\langle L\rangle} p''\langle L\rangle} \quad a \notin L$

3. $\dfrac{p' \xrightarrow{a}_p p''}{p'\|q' \xrightarrow{a}_{p\|q} p''\|q'} \quad a \notin \mathcal{A}_q$

4. $\dfrac{q' \xrightarrow{a}_q q''}{p'\|q' \xrightarrow{a}_{p\|q} p'\|q''} \quad a \notin \mathcal{A}_p$

5. $\dfrac{p' \xrightarrow{a}_p p'' \quad q' \xrightarrow{a}_q q''}{p'\|q' \xrightarrow{a}_{p\|q} p''\|q''} \quad a \neq \tau\,.$

The undefinedness predicates of $p\langle L\rangle$ and $p\|q$ are defined by:

6. $\dfrac{p' \uparrow_p a}{p'\langle L\rangle \uparrow_{p\langle L\rangle} a} \quad a \in L$

7. $\dfrac{p' \uparrow_p a}{p'\langle L\rangle \uparrow_{p\langle L\rangle} \tau} \quad a \notin L$

8. $\dfrac{p' \uparrow_p a}{(p'\|q') \uparrow_{p\|q} a} \quad a \notin \mathcal{A}_q$

9. $\dfrac{p' \uparrow_p a}{(p'\|q') \uparrow_{p\|q} a} \quad q' \xrightarrow{a}_q$

10. $\dfrac{q' \uparrow_q a}{(p'\|q') \uparrow_{p\|q} a} \quad a \notin \mathcal{A}_p$

11. $\dfrac{q' \uparrow_q a}{(p'\|q') \uparrow_{p\|q} a} \quad p' \xrightarrow{a}_p$

12. $\dfrac{p' \uparrow_p a \quad q' \uparrow_q a}{(p'\|q') \uparrow_{p\|q} a}\,.$

Thus, $p' \uparrow_p a$ ($q' \uparrow_q a$) implies $(p'\|q') \uparrow_{p\|q} a$, whenever q' (p') does not preempt the execution of a, i.e. whenever $a \notin \mathcal{A}_q$ or $q' \xrightarrow{a}_q$ ($a \notin \mathcal{A}_p$ or $p' \xrightarrow{a}_p$). Remember that $\tau \notin \mathcal{A}_p$ for any p. The exact meaning of this definition becomes clear in Section 3, where we introduce *reduction operators*. We may immediately conclude from Definition 2.5 the following properties.

Proposition 2.6. (Associativity & Commutativity)
The parallel operator $\|$ is *associative* and *commutative* in the following sense.

1. $\forall p, q, r \in \mathcal{P}.\ (p\|q)\|r \cong p\|(q\|r)$, and
2. $\forall p, q \in \mathcal{P}.\ p\|q \cong q\|p$.

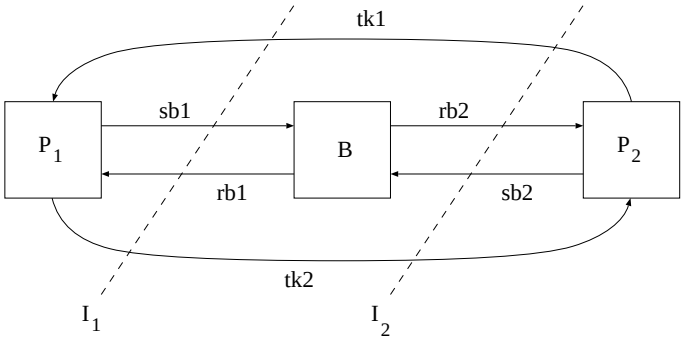


Fig. 1. Communication diagram of the example system

Thus, processes of the form $(p_1 \| \ldots \| p_n)\langle L\rangle$ are well-defined. Our method concentrates on this form which is called *standard concurrent form* in CCS [Mil80, Mil89].

Usually, the following correspondence between the parallel operator and the window operators is exploited in compositional minimisation techniques.

Proposition 2.7. (Window Operator Law)
Let $p, q \in \mathcal{P}$ and let L, L' be sets of visible actions satisfying $L' \supseteq L \cup (\mathcal{A}_p \cap \mathcal{A}_q)$. Then $(p\|q)\langle L\rangle \cong (p\langle L'\rangle \| q)\langle L\rangle$.

This proposition allows us to localise global hiding informations. In fact, this localisation is the essence of the construction of *interface processes* in [CLM89]. The proof of the proposition is done by induction similar to the proof of Theorem 3.9 including a case analysis according to Definition 2.5 in the induction step.

We finish this section by presenting a simple example, which accompanies the development of our method.

Example 2.8. Our example system $System =_{df} (P_1 \| B \| P_2)\langle\{\texttt{tk1}, \texttt{tk2}\}\rangle$, presented in Figure 1, consists of three processes P_1, B, and P_2 with alphabets $\mathcal{A}_{P_1} = \{\texttt{tk1}, \texttt{tk2}, \texttt{rb1}, \texttt{sb1}\}$, $\mathcal{A}_B = \{\texttt{rb1}, \texttt{sb1}, \texttt{rb2}, \texttt{sb2}\}$, and $\mathcal{A}_{P_2} = \{\texttt{tk1}, \texttt{tk2}, \texttt{rb2}, \texttt{sb2}\}$, respectively. I_1 and I_2 indicate interface specifications which are presented and explained in Section 2.4. Process B models a buffer which is used by the processes P_1 and P_2 to exchange data, i.e. P_1 reads data from and sends data to P_2 via B and vice versa. To guarantee mutual exclusion of the "shared" buffer a token is passed through the channels `tk1` and `tk2` between P_1 and P_2. If P_i possesses the token, it may read some data from B via `rb`i and write some data to B via `sb`i. The exact definitions of P_1, B, and P_2 are given in Figure 2. The 'incoming' arrows point to the start states of the processes.

2.3. Semantical Equivalence and Preorder

In this section we define a semantical equivalence of extended labelled transition systems in terms of *observational equivalence* [Mil80, Mil89] and establish a specification-implementation relation in terms of a preorder, which is compatible with this semantics. This preorder plays a key role in the correctness proof of our $\mathcal{RM}$-Method.

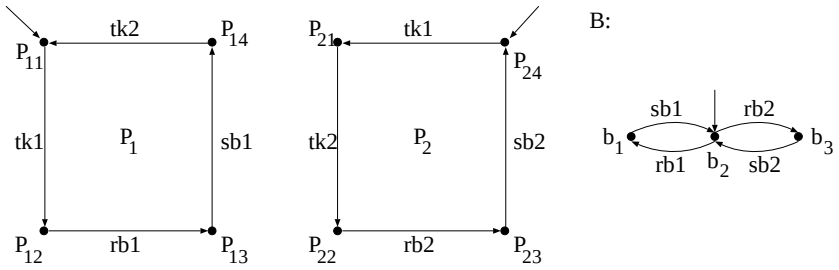


Fig. 2. Definition of P_1, P_2, and B

As already mentioned in Section 1, the minimisation of transition systems is based on the fact that many computations are irrelevant from the observer's point of view. Our notion of semantics, which is defined by means of the following equivalence relation, reflects this intuition by using the weak transition relation and the weak undefinedness predicate as defined in the previous section.[4]

Definition 2.9. (Semantical Equivalence)
Let $(S, \mathcal{A} \cup \{\tau\}, \longrightarrow, \uparrow)$ be an extended transition system. Then $\approx^{\mathrm{d}}$ is the union of all relations $R \subseteq S \times S$ satisfying that $(p,q) \in R$ implies for all $a \in \mathcal{A} \cup \{\epsilon\}$.

1. $p \Uparrow a$ if and only if $q \Uparrow a$,
2. $p \stackrel{a}{\Longrightarrow} p'$ implies $\exists q'.\, q \stackrel{a}{\Longrightarrow} q' \wedge (p',q') \in R$, and
3. $q \stackrel{a}{\Longrightarrow} q'$ implies $\exists p'.\, p \stackrel{a}{\Longrightarrow} p' \wedge (p',q') \in R$.

Two processes $p, q \in \mathcal{P}$ with the same alphabet are equivalent if and only if their extended transition systems can be combined into one extended transition system and the states p and q are equivalent according to the above definition. Moreover, $\approx^{\mathrm{d}}$ coincides with the well-known *observational equivalence* $\approx$ [Mil80, Mil89] if the first of the three defining requirements is dropped. Especially, isomorphic processes are $\approx^{\mathrm{d}}$-equivalent.

The following preorder which intuitively defines a "less defined than" relation between processes is the basis of the framework in which we establish the correctness of our $\mathcal{RM}$-Method (cf. [CS90a]).

Definition 2.10. (Specification Preorder)
Let $(S, \mathcal{A} \cup \{\tau\}, \longrightarrow, \uparrow)$ be an extended transition system. The specification preorder $\preceq$ is the union of all relations $R \subseteq S \times S$ satisfying $(p,q) \in R$ implies for all $a \in \mathcal{A} \cup \{\epsilon\}$ with $\neg(p \Uparrow a)$.

1. $\neg(q \Uparrow a)$,
2. $p \stackrel{a}{\Longrightarrow} p'$ implies $\exists q'.\, q \stackrel{a}{\Longrightarrow} q' \wedge (p',q') \in R$, and
3. $q \stackrel{a}{\Longrightarrow} q'$ implies $\exists p'.\, p \stackrel{a}{\Longrightarrow} p' \wedge (p',q') \in R$.

$\preceq$ is a variant of the divergence preorder [Wal88] in which a-divergence does not require the potential of an a-move. Our modification serves for a different intend. We do not want to cover divergence, i.e. the potential of an infinite internal

[4] This definition of semantical equivalence is adapted from [CS90b] where it is presented for processes expressed in a CCS-based algebra.

computation, but (guarded) undefinedness. This establishes $\preceq$ as a specification-implementation relation: a partial specification p is met by an implementation q if and only if $p \preceq q$; in contrast to [CS90a, Wal88] we do not require an implementation of an a-undefined process to possess any a-transition. This modification enhances the practicality of the preorder as specification-implementation relation. A more detailed discussion can be found in [CS90a].

Observational equivalence $\approx$ and our specification-preorder $\preceq$ induce slightly different semantics on processes. However, by definitions of $\approx^{\mathrm{d}}$, $\approx$, and $\preceq$ we have that $\approx^{\mathrm{d}}$ is a refinement of both.

Proposition 2.11. For all processes $p, q \in \mathcal{P}$ we have $p \approx^{\mathrm{d}} q$ implies $p \approx q$ and $p \preceq q$, and for totally defined processes $\approx^{\mathrm{d}}$, $\preceq$, and $\approx$ coincide.

Moreover, it can be proved in the usual way that both $\|$ and $\langle L \rangle$ preserve $\preceq$ and $\approx^{\mathrm{d}}$ which is of particular importance for our minimisation method.

Proposition 2.12. (Compositionality)
For all processes $p, q, r \in \mathcal{P}$ and all sets L of visible actions we have:

1. $p \preceq q$ implies $p \| r \preceq q \| r$,
2. $p \approx^{\mathrm{d}} q$ implies $p \| r \approx^{\mathrm{d}} q \| r$,
3. $p \preceq q$ implies $p\langle L \rangle \preceq q\langle L \rangle$, and
4. $p \approx^{\mathrm{d}} q$ implies $p\langle L \rangle \approx^{\mathrm{d}} q\langle L \rangle$.

The relationship between the notions *preorder*, *semantic equivalence*, and *languages* is characterised by the following lemma.

Lemma 2.13. For all processes $p, q \in \mathcal{P}$ we have:

1. $p \preceq q$ implies $\mathcal{L}(p) \supseteq \mathcal{L}(q)$, and
2. $p \approx^{\mathrm{d}} q$ implies $\mathcal{L}(p) = \mathcal{L}(q)$.

The proof of the first part is a consequence of the Definitions 2.10 and 2.4, whereas the second part is an immediate consequence of the first one and Proposition 2.11.

The $\mathcal{RM}$-Method presented in Section 4 works for every equivalence relation `er` and every preorder `po` satisfying Propositions 2.12 and Lemma 2.13, whenever $\texttt{er} \subseteq \texttt{po} \cap \texttt{po}^{-1}$ and `er` and `or` coincide on totally defined processes (cf. Proposition 2.11).

2.4. Interface Specifications

In this section we introduce our notion of *interface specification* together with a notion of *correctness*, which guarantees the success of the $\mathcal{RM}$-Method. These notions concentrate on the set of observable sequences that may pass the interface. Thus, the *exact* specification of the interface between processes p and q is the *language* of $(p\|q)\langle \mathcal{A}_p \cap \mathcal{A}_q \rangle$, i.e. its set of observable sequences.

We are going to use interface specifications in order to express context constraints. Therefore, interface specifications are correct or safe if the corresponding exact interface specification is more constraint. This motivates the following definition.

Definition 2.14. (Interface Specifications)
Given two processes $p, q \in \mathcal{P}$ we define:

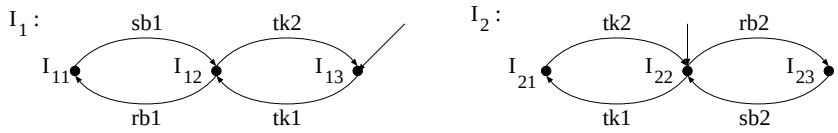


Fig. 3. Interface specifications I_1 and I_2

1. A totally defined process $I \in \mathcal{P}$ without τ-transitions is an *interface specification for* p iff $\mathcal{A}_I \subseteq \mathcal{A}_p$. It is an *interface specification for* p *and* q iff $\mathcal{A}_I = \mathcal{A}_p \cap \mathcal{A}_q$.
2. An interface specification I for p and q is called *correct* for p and q iff $\mathcal{L}((p\|q)\langle\mathcal{A}_p \cap \mathcal{A}_q\rangle) \subseteq \mathcal{L}(I)$.

The set of all interface specifications *for* p is denoted by $\mathcal{I}(p)$, and the set of all *correct* interface specifications for p and q by $\mathcal{I}(p,q)$. Finally, we write $\mathcal{I}$ for $\bigcup\{\mathcal{I}(p) \mid p \in \mathcal{P}\}$, i.e. the set of all totally defined processes without τ-transitions.

Theorem 3.5 shows that these language-based definitions are adequate for our purpose. The following example illustrates the intuition-guided way of deriving interface specifications.

Example 2.15. (Interface Specifications for the Example System)
An interface specification I_1 for the system of Example 2.8 can be constructed according to the following intuition: process P_1 waits for the token passed via `tk1` before it reads data from and writes data to B via `rb1` and `sb1`, respectively. Subsequently, P_1 passes the token to P_2 via `tk2` and waits until it receives the token again.

This intuition would already result in an exact interface specification for P_1 and $B\|P_2$, which is identical to the process P_1 itself. Note that in more complicated examples it is hardly possible to give an exact interface specification. Thus, unprecise interface specifications are often used. For example, the definition of I_1 given on the left in Figure 3 is not exact but still correct since it describes a superset of the exact interface language (cf. Definition 2.14). A similar argument shows the correctness of the definition of I_2 (cf. Figure 3, right) for P_2 and $B\|P_1$.

The languages of I_1 and I_2 result from the following equation systems (cf. Definition 2.4).

$$\begin{array}{lcl} \mathcal{L}(I_1) & = & \mathcal{L}(I_{13}) = \mathtt{tk1} \cdot \mathcal{L}(I_{12}) \cup \{\epsilon\} \\ \mathcal{L}(I_{12}) & = & \mathtt{rb1} \cdot \mathcal{L}(I_{11}) \cup \mathtt{tk2} \cdot \mathcal{L}(I_{13}) \cup \{\epsilon\} \\ \mathcal{L}(I_{11}) & = & \mathtt{sb1} \cdot \mathcal{L}(I_{12}) \cup \{\epsilon\} \end{array}$$

$$\begin{array}{lcl} \mathcal{L}(I_2) & = & \mathcal{L}(I_{22}) = \mathtt{tk1} \cdot \mathtt{tk2} \cdot \mathcal{L}(I_{22}) \cup \mathtt{rb2} \cdot \mathtt{sb2} \cdot \mathcal{L}(I_{22}) \cup \\ & & \{\mathtt{tk1}\} \cup \{\mathtt{rb2}\} \cup \{\epsilon\}. \end{array}$$

Note, however, that our method does not require to compute $\mathcal{L}(I_1)$ and $\mathcal{L}(I_2)$.

Applying Lemma 2.13, Definition 2.14, and Propositions 2.11 and 2.12, we obtain the following lemma.

Lemma 2.16. (Properties of Interface Specifications)
For all processes $p, p', q \in \mathcal{P}$ we have:

1. $p \preceq p'$ implies $\mathcal{I}(p,q) \subseteq \mathcal{I}(p',q)$, and

2. $p \approx^{\mathrm{d}} p'$ implies $\mathcal{I}(p,q) = \mathcal{I}(p',q)$.

The following proposition, which is particularly important for the completeness proof of the $\mathcal{RM}$-Method, is a consequence of Definition 2.14, Proposition 2.7, and Lemma 2.13(2.).

Proposition 2.17. For all processes $p, q \in \mathcal{P}$ and all sets L of visible actions we have: $\mathcal{I}(p,q) = \mathcal{I}(p\langle(\mathcal{A}_p \cap \mathcal{A}_q) \cup L\rangle, q)$.

3. Reduction Operators

Here, we propose a general notion of *reduction operators*, and a special instance of it, $\overline{\Pi}$, which is suitable for our purposes (cf. Section 3.1). $\overline{\Pi}$ is analysed from two different views, the theoretical view (cf. Section 3.2) and the algorithmic view (cf. Section 3.3).

3.1. General Definitions and Properties

Reduction operators are characterised by three properties.

Definition 3.1. (Reduction Operators)
A partial mapping $\Pi : \mathcal{I} \times \mathcal{P} \longrightarrow \mathcal{P}$ is called *reduction operator* if

(i) $\forall p \in \mathcal{P}, I \in \mathcal{I}(p).\ \Pi(I,p) \preceq p$ *(Correctness for arbitrary interfaces)*

(ii) $\forall p, q \in \mathcal{P}, I \in \mathcal{I}(p,q).\ \Pi(I,p) \| q \approx^{\mathrm{d}} p \| q$ *(Context preservation for correct interfaces)*

(iii) $\forall p \in \mathcal{P}, I \in \mathcal{I}(p).\ \mid S_{\Pi(I,p)} \mid \leq \mid S_p \mid$ and $\mid \longrightarrow_{\Pi(I,p)} \mid \leq \mid \longrightarrow_p \mid$ *(Reduction)*[5]

In the following we often write $\Pi_I(p)$ instead of $\Pi(I,p)$.

The intuition behind this definition is the following: a reduction operator Π should eliminate those states and transitions of a process $p \in \mathcal{P}$ which are not reachable in each global context satisfying the interface specification $I \in \mathcal{I}(p)$. This 'algorithmic' intuition guarantees the first two conditions, which are essential for a sensible notion of reduction operator: the first condition is a correctness requirement. The reduction always yields a process which behaves on its defined part as p. The second condition guarantees that the reduction does not affect the behaviour of p in a context satisfying the (correct) interface specification. Finally, the third condition reflects the primary intuition of reduction: the number of states and transitions shall be reduced. This is by no means guaranteed by a decrease in the preorder!

The following technical proposition follows easily from Definition 3.1, Proposition 2.12 and Lemma 2.16 (2.).

Proposition 3.2. Let Π be a reduction operator. Then we have for all $p, p', q \in \mathcal{P}$ and $I \in \mathcal{I}(p,q)$: $p \approx^{\mathrm{d}} p'$ implies $\Pi_I(p) \| q \approx^{\mathrm{d}} \Pi_I(p') \| q$.

As we show in Section 3.2, the following operator $\overline{\Pi}$ satisfies the conditions of Definition 3.1.

[5] $|M|$ denotes the cardinality of the set M.

Definition 3.3. (The Reduction Operator $\overline{\Pi}$)
The reduction operator $\overline{\Pi}$ is defined by $(I,p) \longmapsto \overline{\Pi}(I,p) =_{df} (S, \mathcal{A} \cup \{\tau\}, \longrightarrow, \uparrow, p)$ for $I = (S_I, \mathcal{A}_I, \longrightarrow_I, \emptyset, I) \in \mathcal{I}(p)$ and $p = (S_p, \mathcal{A}_p \cup \{\tau\}, \longrightarrow_p, \uparrow_p, p) \in \mathcal{P}$ where

1. $S = \{q \in S_p | \exists i \in S_I . q \| i \in S_{p\|I}\}$,
2. $\mathcal{A} = \mathcal{A}_p$,
3. $\forall q, q' \in S, a \in \mathcal{A} \cup \{\tau\}. \; q \xrightarrow{a} q'$ iff $\exists i, i' \in S_I . \; q\|i \xrightarrow{a}_{p\|I} q'\|i'$,[6]
4. $\forall q \in S. \, q \uparrow \tau$ iff $q \uparrow_p \tau$, and
5. $\forall q \in S, a \in \mathcal{A}. \, q \uparrow a$ iff
   (a) $q \uparrow_p a$ or
   (b) $\exists q' \in S_p . \, q \xrightarrow{a}_p q'$ and $\nexists q' \in S. \; q \xrightarrow{a} q'$.

In conformance with Definition 3.1 we also write $\overline{\Pi}_I(p)$ for $\overline{\Pi}(I,p)$.

The only difference between $\overline{\Pi}(I,p)$ and the projection of $p\|I$ onto p concerns the undefinedness predicates: $\Pi(I,p)$ inherits all undefinedness predicates from p, and new ones are introduced where transitions of p have been cut off by I. The point of the reduction operator is that for correct interface specifications this second kind of undefinedness disappears again in the full context $\overline{\Pi}(I,p)\|q$. This holds, because if an a-transition of p has been replaced by $\uparrow a$, this predicate disappears in $\overline{\Pi}(I,p)\|q$ exactly if q, in its corresponding state, preempts the execution of an a-transition. Thus, the presence of an $\uparrow a$ in $\overline{\Pi}(I,p)\|q$ indicates a fault in the interface specification, whenever p and q are totally defined processes. Note that it is possible that $\overline{\Pi}(I,p)\|q$ is totally defined, although I is *not* correct for p and q. This is the case if the incorrect parts of I need not be considered for the reduction.

Example 3.4. Consider the process p presented on the left in Figure 4 having the alphabet $\mathcal{A}_p = \{\texttt{tk1}, \texttt{tk2}, \texttt{rb2}, \texttt{sb2}\}$ and the (shaded) start state $(p11\|b2)$, and consider the interface specification I_2 defined in Example 2.15. In order to determine $\overline{\Pi}_{I_2}(p)$ we first consider the projection of $p\|I_2$ onto p (Figure 4, right). Following Definition 3.3, $\overline{\Pi}_{I_2}(p)$ can now be derived by inserting some additional undefinednesses indicating a transition of p which is preempted by the interface. The result of the computation, which we have obtained running the METAFrame environment [SMC96], can be investigated using the *Graph Inspector* of our tool: the field *node syntax* in Figure 4 shows that the highlighted state $(p12\|b2)$ has an `rb2`-undefinedess. A further investigation would reveal the `tk1`-undefinedness of $(p11\|b3)$ and the `rb2`-undefinedness of $(p13\|b1)$.

3.2. Theoretical View

In this section we establish that $\overline{\Pi}$ is indeed a reduction operator. Therefore, the following result is helpful which is established as a byproduct in the next section. It is called *representation independence* and states that not the branching

[6] This implies by the definitions of $S_{p\|I}$ and $\longrightarrow_{p\|I}$ according to Definition 2.2 that $q\|i$ is reachable in $p\|I$.

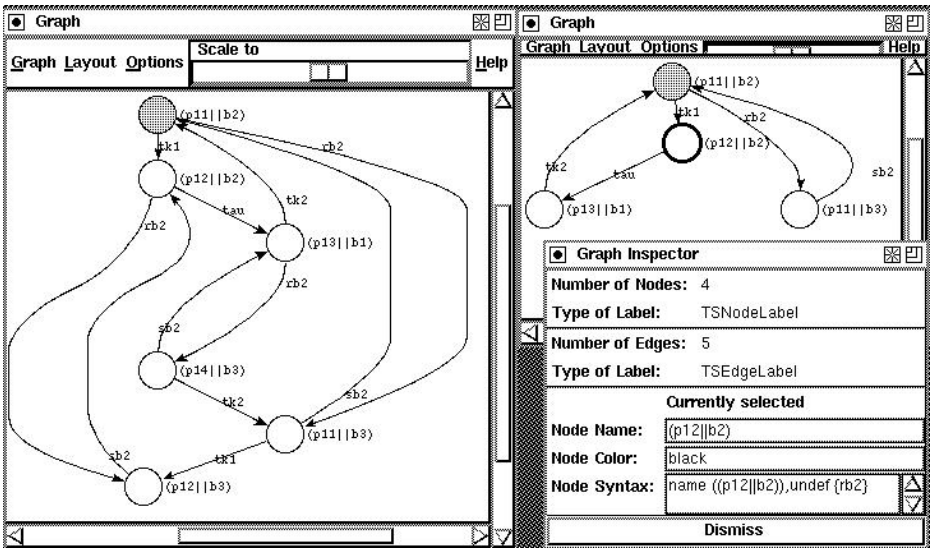


Fig. 4. Example reduction

structure of an interface specification is important but only its language. Therefore, it allows us to assume w.l.o.g. deterministic interface specifications for a proof in the remainder of this section, and it allows researchers to concentrate on interface specifications as languages.

Theorem 3.5. (Representation Independence)
For all $p \in \mathcal{P}$ and for all $I, I' \in \mathcal{I}(p)$ we have:

$$\mathcal{L}(I) = \mathcal{L}(I') \text{ implies } \overline{\Pi}_I(p) = \overline{\Pi}_{I'}(p)\,.$$

The correctness property (cf. Definition 3.1(i)) can be established straightforwardly.

Lemma 3.6. $\forall p \in \mathcal{P}, I \in \mathcal{I}(p).\ \overline{\Pi}_I(p) \preceq p\,.$

The remaining properties require two lemmata. The first lemma, which intuitively states that no "new" states and transitions are inserted, is a consequence of Definition 3.3 (iii) and Definition 2.5 (3) and (5).

Lemma 3.7. Let $p \in \mathcal{P}$ and $I \in \mathcal{I}(p)$ be arbitrary. Then we have:

$$\forall p', p'' \in S_{\overline{\Pi}(I,p)}, a \in \mathcal{A}_p \cup \{\tau\}.\ p' \xrightarrow{a}_{\overline{\Pi}(I,p)} p'' \text{ implies } p' \xrightarrow{a}_p p''\ .$$

The second lemma guarantees that $\overline{\Pi}$ does not cut off too many states or transitions. It requires a more involved argument.

Lemma 3.8. Let $p, q \in \mathcal{P}$, $I \in \mathcal{I}(p,q)$, $a \in \mathcal{A}_p \cup \{\tau, \epsilon\}$, and $p \| q \longrightarrow^*_{p\|q} p' \| q' \xrightarrow{a}_{p\|q} p'' \| q''$. Then we have:

1. $\exists I'' \in S_I.\ p'' \| I''$ is reachable in $p \| I$.
2. $p' \xrightarrow{a}_{\overline{\Pi}(I,p)} p''$.

Proof. Assume w.l.o.g. that I is a deterministic interface specification. We prove Lemma 3.8 by induction on n where n is the length of the path $p\|q \longrightarrow^n_{p\|q} p''\|q''$.

Base Case: $(n = 0)$
Here we have $a = \epsilon$ and $p\|q = p'\|q' = p''\|q''$, i.e. $p = p' = p''$ and $q = q' = q''$. Choose $I'' =_{df} I$ and hence that $p''\|I'' = p\|I$ is reachable in $p\|I$, such that (1.) holds. Statement (2.) is trivial because $p' \xrightarrow{\epsilon}_{\overline{\Pi}(I,p)} p''$.

Induction step: $(n \longrightarrow n+1)$
Here, we have $p\|q \longrightarrow^n_{p\|q} p'\|q' \xrightarrow{a}_{p\|q} p''\|q''$. By induction hypothesis it exists $I' \in S_I$ satisfying:

$(*)$ $\quad p'\|I'$ is reachable in $p\|I$.

The application of $\langle \mathcal{A}_p \cap \mathcal{A}_q \rangle$ yields:

$$(p\|q)\langle \mathcal{A}_p \cap \mathcal{A}_q \rangle \longrightarrow^n_{p\|q} (p'\|q')\langle \mathcal{A}_p \cap \mathcal{A}_q \rangle \xrightarrow{b}_{p\|q} (p''\|q'')\langle \mathcal{A}_p \cap \mathcal{A}_q \rangle$$

where $b = \begin{cases} a & \text{if } a \in \mathcal{A}_q \\ \tau & \text{otherwise} \end{cases}$. Let $b' = \begin{cases} b & \text{if } b \neq \tau \\ \epsilon & \text{otherwise} \end{cases}$. By the induction hypothesis, the premise $\mathcal{L}((p\|q)\langle \mathcal{A}_p \cap \mathcal{A}_q \rangle) \subseteq \mathcal{L}(I)$ (cf. Definition 2.14), the deterministic interface specification I, and Definition 2.4 we conclude the existence of some $I'' \in S_I$ satisfying $I' \stackrel{b'}{\Longrightarrow}_I I''$, i.e.

$$\exists i', i''.\, I' \xrightarrow{\tau}_I \ldots \xrightarrow{\tau}_I i' \xrightarrow{b'}_I i'' \xrightarrow{\tau}_I \ldots \xrightarrow{\tau}_I I''.$$

Hence by Definition 2.5 Rule (3) (and (5) if $b' = a$):

$$p'\|I' \xrightarrow{\tau}_{p\|I} \ldots \xrightarrow{\tau}_{p\|I} p'\|i' \xrightarrow{a}_{p\|I} p''\|i'' \xrightarrow{\tau}_{p\|I} \ldots \xrightarrow{\tau}_{p\|I} p''\|I''.$$

This shows together with $(*)$ that $p''\|I''$ is reachable in $p\|I$, i.e. (1.) holds. Statement (2.) is a consequence of Definition 3.3 (3), because of the existence of i' and i'', the reachability of $p''\|i''$ in $p\|I$, and $p'\|i' \xrightarrow{a}_{p\|I} p''\|i''$. □

Now, we are able to prove the key property for the *completeness proof* of the $\mathcal{RM}$-Method, which implies *context preservation for correct interface specifications* in the sense of Definition 3.1(ii), as we show even *isomorphy*, $=$, instead of *semantical equivalence*.

Proposition 3.9. (Context Preservation)
$\forall p, q \in \mathcal{P}, I \in \mathcal{I}(p,q).\ p\|q = \overline{\Pi}_I(p)\|q$.

Proof. Let $p = (S_p, \mathcal{A}_p \cup \{\tau\}, \longrightarrow_p, \uparrow_p, p), q = (S_q, \mathcal{A}_q \cup \{\tau\}, \longrightarrow_q, \uparrow_q, q) \in \mathcal{P}$, $I \in \mathcal{I}(p,q)$, and $\mathcal{A} =_{df} \mathcal{A}_I = \mathcal{A}_p \cap \mathcal{A}_q$. For this proof we define the following processes:

$$\begin{array}{rcl} p\|q & = & (S_1, \mathcal{A}_p \cup \mathcal{A}_q \cup \{\tau\}, \longrightarrow_1, \uparrow_1, p\|q), \\ \Pi_I(p) & = & (S_{p_I}, \mathcal{A}_p \cup \{\tau\}, \longrightarrow_{p_I}, \uparrow_{p_I}, p), \text{ and} \\ \Pi_I(p)\|q & = & (S_2, \mathcal{A}_p \cup \mathcal{A}_q \cup \{\tau\}, \longrightarrow_2, \uparrow_2, p\|q). \end{array}$$

Then both S_1 and S_2 are subsets of $\{(p'\|q') | p' \in S_p, q' \in S_q\}$. Thus, it remains to show that $S_1 = S_2$, $\longrightarrow_1 = \longrightarrow_2$, and $\uparrow_1 = \uparrow_2$ holds. For this purpose we define for $i = 1, 2$:

- S_i^n, the subset of states reachable in n steps from the initial state,

- $\longrightarrow_i^n$, the set of transitions leaving the states of S_i^n, and
- $\uparrow_i^n$, the undefinedness predicate of states of S_i^n.

Because of $S_1^0 = S_2^0 = \{p \| q\}$, it is enough[7] to verify the following simultaneous induction step for $n \geq 1$ under the induction hypothesis $S_1^{n-1} = S_2^{n-1}$, in order to complete the proof: (1) $\longrightarrow_1^{n-1} = \longrightarrow_2^{n-1}$, (2) $S_1^n = S_2^n$, and (3) $\uparrow_1^{n-1} = \uparrow_2^{n-1}$.

First, we verify Point (1) of the induction step according to the operational rules (cf. Definition 2.5), i.e. we show the equivalence "$p' \| q' \xrightarrow{a}_1 p'' \| q'' \iff p' \| q' \xrightarrow{a}_2 p'' \| q''$". This requires the investigation of Rules 3, 4, and 5 of Definition 2.5. The most complicated case is the following.

Rule 5: Here, we have $a \in \mathcal{A}$, $p' \xrightarrow{a}_p p''$, and $q' \xrightarrow{a}_q q''$:

$$
\begin{array}{lll}
 & & p' \| q' \xrightarrow{a}_1 p'' \| q'' \\
(\text{Rule 5}) & \iff & p' \xrightarrow{a}_p p'' \wedge q' \xrightarrow{a}_q q'' \\
(\text{Lemma 3.8, 3.7, resp.}^8) & \iff & p' \xrightarrow{a}_{p_I} p'' \wedge q' \xrightarrow{a}_q q'' \\
(\text{Rule 5}) & \iff & p' \| q' \xrightarrow{a}_2 p'' \| q''
\end{array}
$$

Point (2) of the induction step is an immediate consequence of Point (1). Thus, it remains to verify Point (3). The τ-undefinedness of a process is not affected by the reduction operator (see Clause (4) of Definition 3.3), which leaves us with the case of an a-undefinedness. In order to prove the equivalence "$(p' \| q') \uparrow_1 a \iff (p' \| q') \uparrow_2 a$" we must deal with the five applicable rules given in Definition 2.5. Most interesting is Rule 12.

Rule 12: Here, we have $p' \uparrow_p a$ and $q' \uparrow_q a$, and therefore:

"$\Rightarrow$":

$$
\begin{array}{lll}
(p' \| q') \uparrow_1 a & \Rightarrow & p' \uparrow_p a \wedge q' \uparrow_q a \\
(\text{Definition 3.3 (5) (a)}) & \Rightarrow & p' \uparrow_{p_I} a \wedge q' \uparrow_q a \\
(\text{Rule 12}) & \Rightarrow & (p' \| q') \uparrow_2 a
\end{array}
$$

"$\Leftarrow$":

$$
\begin{array}{lll}
(p' \| q') \uparrow_2 a & \Rightarrow & p' \uparrow_{p_I} a \wedge q' \uparrow_q a \\
(\text{Definition 3.3 (5)}) & \Rightarrow & (5a)\quad p' \uparrow_p a \wedge q' \uparrow_q a \Rightarrow (\text{Rule 12})\ (p' \| q') \uparrow_1 a \\
 & & \text{or}\quad (5b)\quad p' \xrightarrow{a}_p \wedge q' \uparrow_q a \Rightarrow (\text{Rule 11})\ (p' \| q') \uparrow_1 a
\end{array}
$$

□

[7] Remember that we are dealing with *finite* state systems.
[8] More precisely: for "$\Rightarrow$" we apply Lemma 3.8 and for "$\Leftarrow$" Lemma 3.7.

Together with Lemma 3.6 and the obvious fact that $\overline{\Pi}$ is reducing, this proposition yields the desired result.

Theorem 3.10. The partial mapping $\overline{\Pi} : \mathcal{I} \times \mathcal{P} \longrightarrow \mathcal{P}$ is a *reduction operator*.

3.3. Algorithmic View

In this section we present an algorithmic characterisation of $\overline{\Pi}$ on the basis of a data flow analysis algorithm. As an important byproduct we obtain the proof of Theorem 3.5.

In order to prepare an algorithmic characterisation of the reduction operator $\overline{\Pi}_I(p)$ we define two sets of actions.

Definition 3.11. Let $p = (S_p, \mathcal{A}_p \cup \{\tau\}, \longrightarrow_p, \uparrow_p, p) \in \mathcal{P}$, $I = (S_I, \mathcal{A}_I, \longrightarrow_I, \emptyset, I) \in \mathcal{I}(p)$, and $q \in S_p$. We define the following sets of actions:

- $\mathrm{IA}_{p,I}(q) =_{df} \{a \in \mathcal{A}_I \cup \{\epsilon\} | \exists q \| i \in S_{p\|I} . i \xrightarrow{a}_I\}$.
- $\mathrm{IA}^{\mathrm{all}}_{p,I}(q) =_{df} \mathrm{IA}_{p,I}(q) \cup (\mathcal{A}_p \setminus \mathcal{A}_I) \cup \{\tau\}$.

The intuition for this definition is that the sets $\mathrm{IA}^{\mathrm{all}}_{p,I}(q)$ contain exactly those actions which q can perform within the global context described by I. In particular, as every state can engage in ϵ, this implies that a state q is reachable in the process $\overline{\Pi}_I(p)$ if and only if $\epsilon \in \mathrm{IA}^{\mathrm{all}}_{p,I}(q)$. The following proposition makes this observation more precise. It shows how to compute $\overline{\Pi}_I(p)$ by eliminating states and transitions of p and adding undefinednesses to states of p by means of the sets $\mathrm{IA}^{\mathrm{all}}_{p,I}(q)$.

Proposition 3.12. (Construction of $\overline{\Pi}_I(p)$)
Let $p = (S_p, \mathcal{A}_p \cup \{\tau\}, \longrightarrow_p, \uparrow_p, p) \in \mathcal{P}$, $I \in \mathcal{I}(p)$, and $q \in S_p$. Then $\overline{\Pi}_I(p) = (S, \mathcal{A} \cup \{\tau\}, \longrightarrow, \uparrow, p)$ is determined by the sets $\mathrm{IA}^{\mathrm{all}}_{p,I}(q)$ and a finite sequence of reduction steps for all $q \in S_p$:

1. If $\epsilon \notin \mathrm{IA}^{\mathrm{all}}_{p,I}(q)$, then eliminate q and all transitions ending or starting at q.
2. If $q \xrightarrow{a}_p$ but $a \notin \mathrm{IA}^{\mathrm{all}}_{p,I}(q)$, then eliminate all a-transitions starting at q and add $q \uparrow_p a$ to the undefinedness predicate.

Proof. It suffices to show that

1. $\forall q \in S_p.\ q \in S \Longleftrightarrow \epsilon \in \mathrm{IA}^{\mathrm{all}}_{p,I}(q)$ and
2. $\forall q \in S, a \in \mathcal{A} \cup \{\tau\}.\ q \xrightarrow{a} \Longleftrightarrow\ q \xrightarrow{a}_p \wedge a \in \mathrm{IA}^{\mathrm{all}}_{p,I}(q)$.

The proof of (1.) is straightforward by using the definition of $\mathrm{IA}^{\mathrm{all}}_{p,I}(q)$ and Definition 3.3 (1). A case distinction is needed for (2.): for the case $a \in (\mathcal{A} \setminus \mathcal{A}_I) \cup \{\tau\}$ Definitions 3.3 (3), 2.5 (3), and the definition of $\mathrm{IA}^{\mathrm{all}}_{p,I}(q)$ must be considered. The case $a \in \mathcal{A}_I \cup \{\epsilon\}$ requires to look at Definitions 3.3 (3), 2.5 (5), 2.14, and the definition of $\mathrm{IA}^{\mathrm{all}}_{p,I}(q)$. □

The key to a complete algorithm for $\overline{\Pi}_I(p)$ is the following easy to prove characterisation for the sets $\mathrm{IA}_{p,I}(q)$, which is the basis for the algorithm developed in the next section.

Lemma 3.13. (Characterisation of $\mathrm{IA}_{p,I}(q)$)
Let $q \in S_p$ and $\mathcal{L}_{p,I}(q) =_{df} \bigcup\{\mathcal{L}(i)|q\|i \in S_{p\|I}\}$. Then $\mathrm{IA}_{p,I}(q) = \{a \in \mathcal{A}_I \cup \{\epsilon\}|\exists v \in \mathcal{A}_I^*.\, av \in \mathcal{L}_{p,I}(q)\}$ holds.

This characterisation considering the sets $\mathcal{L}_{p,I}(q)$ leads to an efficient (data flow analysis) algorithm for computing the sets $\mathrm{IA}_{p,I}(q)$ and, therefore, to an efficient implementation of the reduction operator $\overline{\Pi}$.

3.4. Determining $\mathcal{L}_{p,I}(\cdot)$

The development of the data flow analysis algorithm presented in Procedure 3.20 requires a brief review of the relevant data flow analysis scenario.

Given a complete partial order $\langle C; \sqsubseteq\rangle$, whose elements are intended to express the relevant data flow information, the *local abstract semantics* of a process $(S_p, \mathcal{A}_p \cup \{\tau\}, \longrightarrow_p, \uparrow_p, p)$ is defined by the semantic functional $[\![\cdot]\!] : (\longrightarrow_p) \to (C \to C)$ which maps each transition $t \in \longrightarrow_p$ to a transformation on C. The functional $[\![\cdot]\!]$ extends to paths $pth = (t_1, \ldots, t_q)$, $q \geq 0$, in p in the usual way: $[\![pth]\!] =_{df} [\![t_q]\!] \circ \cdots \circ [\![t_1]\!]$.

Let us fix an arbitrary process $p = (S_p, \mathcal{A}_p \cup \{\tau\}, \longrightarrow_p, \uparrow_p, p) \in \mathcal{P}$ and an interface specification $I = (S_I, \mathcal{A}_I, \longrightarrow_I, \emptyset, I) \in \mathcal{I}$. For technical reasons we assume w.l.o.g. that p does not possess any transitions to its start state.

For our application we need the following local abstract semantics over the complete partial order $\langle 2^{\mathcal{L}(I)}; \subseteq\rangle$.

Definition 3.14. (Local Abstract Semantics)
For $a \in \mathcal{A}$ consider the functions $\mathcal{E}_a^I : 2^{\mathcal{L}(I)} \to 2^{\mathcal{L}(I)}$ with

$$\mathcal{E}_a^I(\mathcal{L}) =_{df} \begin{cases} \mathcal{L}_a & \text{if } a \in \mathcal{A}_I \\ \mathcal{L} & \text{otherwise} \end{cases} .$$

Then the local abstract semantic functions for $(q, a, q') \in \longrightarrow_p$ with respect to I are defined by $[\![(q, a, q')]\!] =_{df} \mathcal{E}_a^I$.

The following property of these local abstract semantic functions is important.

Lemma 3.15. (Additivity)
The functions $\mathcal{E}_a^I$ are additive, i.e. for all $\{\mathcal{L}_k|k \geq 0\} \subseteq 2^{\mathcal{L}(I)}$ we have

$$\mathcal{E}_a^I(\bigcup\{\mathcal{L}_k|k \geq 0\}) = \bigcup\{\mathcal{E}_a^I(\mathcal{L}_k)|k \geq 0\}.$$

As a consequence, the local abstract semantic functions $[\![t]\!]$ are additive for all $t \in \longrightarrow_p$.

The local abstract semantics can be globalised according to two strategies: the "operational" *join over all paths (JOP) strategy*, which directly reflects the intuition behind the analysis problem, and the "denotational" *minimal fixed point (MFP) strategy*, which is algorithmic (cf. [Kil73, KU77]).[9] In the following $P[q, q']$ denotes the set of all finite paths from q to q' and c_0 the initial information valid at the start state of p.

[9] Originally, a dual setup was proposed, considering *meet over all paths* and *maximal fixed point* strategies.

Definition 3.16. (*JOP*-Solution)
$\forall q \in S_p, c_0 \in C.\ JOP_{c_0}(q) =_{df} \bigsqcup\{[\![pth]\!](c_0) | pth \in P[p,q]\}$.

For our application we have the following characterisation of the *JOP*-Solution.

Proposition 3.17. (Characterisation of the *JOP*-Solution)
$\forall q \in S_p.\ JOP_{\mathcal{L}(I)}(q) = \mathcal{L}_{p,I}(q)$.

Proof. For "$\supseteq$" one easily establishes

$$\forall q \| i \in S_{p\|I} \exists pth \in P[p,q].\ \mathcal{L}(i) \subseteq [\![pth]\!](\mathcal{L}(I))$$

by induction on the length of a path from $p\|I$ to $q\|i$, and for "$\subseteq$" it is sufficient to prove for all paths $pth \in P[p,q]$ that $w \in [\![pth]\!](\mathcal{L}(I))$ implies $\exists q\|i \in S_{p\|I}.\, w \in \mathcal{L}(i)$ by induction on the length of pth. □

The *MFP*-solution iteratively approximates the smallest solution of a set of simultaneous equations that express consistency between data flow informations with respect to the start information c_0 that is valid in the start state p.

Definition 3.18. (*MFP*-Solution)
The least solution l_{c_0} of the equation system consisting of the equation

$$l(q) = \begin{cases} c_0 & \text{if } q = p \\ l(q) \sqcup \bigsqcup\{[\![(q', a, q)]\!](l(q')) | (q', a, q) \in \longrightarrow_p\} & \text{otherwise} \end{cases}$$

for each $q \in S_p$ includes the *MFP-Solution* with respect to the initial information $c_0 \in C$.

$$\forall q \in S_p, c_0 \in C.\ MFP_{c_0}(q) =_{df} l_{c_0}(q).$$

As in our application, this often leads to an algorithmic description. The well-known coincidence theorem of Kam and Ullman [KU77] bridges the gap between the *JOP*-Solution and the *MFP*-Solution.

Theorem 3.19. (Coincidence Theorem)
If all local abstract semantic functions $[\![t]\!]$ for $t \in \longrightarrow_p$ are additive, then the *MFP*-Solution is *correct* and *complete* with respect to the *JOP*-Solution, i.e.

$$\forall q \in S_p, c_0 \in C.\ JOP_{c_0}(q) = MFP_{c_0}(q).$$

Thus according to Lemma 3.15, we can compute the desired *JOP*-solution (cf. Proposition 3.17) by iteratively approximating the smallest solution of the equation system defined in Definition 3.18 in the following fashion:

Procedure 3.20. (Language Labelling)
Given $p \in \mathcal{P}$ and $I \in \mathcal{I}(p)$, the iterative *Language Labelling Procedure* works by successively enlarging approximative labellings according to the following two steps:

1. It initially labels p with $\mathcal{L}(I)$ and all the other states with the empty language.
2. If a state q of p is currently labelled by $\mathcal{L}$ and q' is one of its a-successors, then $\mathcal{E}^I_a(\mathcal{L})$ is joined to the current language labelling of q' until a fixed point is reached.

In terms of DFA, the Language Labelling Procedure computes the minimal fixed point solution with respect to the start information $\mathcal{L}(I)$.

The Language Labelling Procedure 3.20 has been implemented by means of a *workset algorithm*, as part of the METAFrame environment [SMC96]. Its time and space complexity can be estimated by the product of the number of transitions of p and the number of states of I.

As the algorithm does not exploit the structure of the representation of interface specifications; this proves Theorem 3.5.

4. Minimisation Method

In this section we develop the $\mathcal{RM}$-Method, which compositionally minimises finite state distributed systems, on top of a reduction operator Π. This method is *correct* in that it always produces processes that are smaller in the specification-implementation preorder (cf. Theorem 4.1), guaranteeing that even faulty interface specifications never allow us to establish wrong properties. Moreover, it is complete, in that it only produces semantically equivalent processes as long as the interface specifications are correct (cf. Theorem 4.2), guaranteeing that the reduction is consistent with the semantical equivalence in this case. The $\mathcal{RM}$-Method is illustrated by means of an example, where the apparent complexity is exponential in the number of components, whereas the algorithmic and the reduced complexity are linear.

The $\mathcal{RM}$-Method deals with processes of the form $\mathrm{P} = (p_1 \| \ldots \| p_n)\langle L\rangle$. This form, called standard concurrent form in CCS, is of particular interest, as it is responsible for the state explosion problem and, therefore, characterises the processes that are critical during analysis and verification. Our method expects the finite state system P to be annotated with interface specifications that describe the interface between the right hand process and the left hand process of the parallel operator they are attached to: $(p_1 \|_{I_1} p_2 \|_{I_2} \ldots \|_{I_{n-1}} p_n)\langle L\rangle$. It proceeds by successively constructing transition systems $\mathcal{P}_i$ as follows:

$$\underbrace{\underbrace{\underbrace{(\underbrace{p_1 \|_{I_1}}_{\mathcal{P}_1} p_2 \|_{I_2} \ldots \|_{I_{n-1}} p_n)\langle L\rangle}_{\mathcal{P}_2}}_{\vdots}}_{\mathcal{P}_n}$$

where $\mathcal{P}_i$ is defined by:

- $\mathcal{P}_1 =_{df} \mathcal{M}(\Pi_{I_1}(\mathcal{M}(p_1\langle \mathcal{A}_{I_1} \cup L\rangle)))$,
- $\mathcal{P}_i =_{df} \mathcal{M}(\Pi_{I_i}(\mathcal{M}((\mathcal{P}_{i-1} \| p_i)\langle \mathcal{A}_{I_i} \cup L\rangle)))$ for $2 \leq i \leq n-1$, and
- $\mathcal{P}_n =_{df} \mathcal{M}((\mathcal{P}_{n-1} \| p_n)\langle L\rangle)$.

In order to avoid unnecessarily large intermediate transition systems during the construction of the minimal transition system, it is important to minimise all the intermediate constructions as it is done above.

Note that our method covers the naive method (only using $\mathcal{M}$) and methods which only consider the correspondence of the parallel and the window operator (see Proposition 2.7). The new additional power of the $\mathcal{RM}$-Method is due to the reduction operator Π which minimises all intermediate transition systems $\mathcal{P}_i$ according to global constraints specified in terms of the interface specifications I_i (for $1 \leq i \leq n-1$).

In the remainder of this section let P and $\mathcal{P}_i$ be as defined above and $Q_i =_{df} (p_{i+1} \| \ldots p_n)$ for $1 \leq i \leq n$.[10] Then we obtain the following correctness result, which is independent of the correctness of the interface specifications.

Theorem 4.1. (Correctness of the $\mathcal{RM}$-Method)
$\forall 1 \leq i \leq n. (\mathcal{P}_i \| Q_i)\langle L \rangle \preceq \mathrm{P}$.

Using Definition 3.1(i) and Propositions 2.6, 2.7, 2.11, and 2.12, the proof can be done straightforwardly by induction on i, where the cases $2 \leq i \leq n-1$ and $i = n$ need to be distinguished (according to the definition of $\mathcal{P}_i$) during the induction step.

For $i = n$ Theorem 4.1 states that $\mathcal{P}_n \preceq \mathrm{P}$. This is already enough to guarantee the correctness of the method, i.e. that a proof of property for $\mathcal{P}_n$, which is consistent with the kernel of $\preceq$, is valid for P. Thus, wrong interface specifications never lead to wrong results. They may only prevent a successful verification of a valid statement. In order to guarantee the success of the method, the correctness of the interface specifications is sufficient.

Theorem 4.2. (Completeness of the $\mathcal{RM}$-Method)
$\forall 1 \leq i \leq n. (\forall j \leq i. I_j \in \mathcal{I}(p_1 \| \ldots \| p_j, Q_j))$ implies $(\mathcal{P}_i \| Q_i)\langle L \rangle \approx^{\mathrm{d}} \mathrm{P}$.

Proof. The proof, which is done by induction on i again, requires special attention: an instance of the induction hypothesis is necessary to establish an auxiliary statement concerning the correctness of the given interface specifications in the special proof context (see statement $(*)$ below).

For $i = 1$ we have:

		$(\mathcal{P}_1 \| Q_1)\langle L \rangle$
(def. $\mathcal{P}_1$)	$=$	$(\mathcal{M}(\Pi_{I_1}(\mathcal{M}(p_1\langle \mathcal{A}_{I_1} \cup L\rangle)))\| Q_1)\langle L \rangle$
(def. $\mathcal{M}$, Prop. 2.12)	$\approx^{\mathrm{d}}$	$(\Pi_{I_1}(\mathcal{M}(p_1\langle \mathcal{A}_{I_1} \cup L\rangle))\| Q_1)\langle L \rangle$
(Prop. 3.2, 2.12)	$\approx^{\mathrm{d}}$	$(\Pi_{I_1}(p_1\langle \mathcal{A}_{I_1} \cup L\rangle)\| Q_1)\langle L \rangle$
(Def. 3.1(ii), Prop. 2.17, 2.12)	$\approx^{\mathrm{d}}$	$(p_1\langle \mathcal{A}_{I_1} \cup L\rangle \| Q_1)\langle L \rangle$
(Theorem 2.7)	$=$	$(p_1 \| Q_1)\langle L \rangle$
(def. P)	$=$	P

The induction step, $i-1 \longrightarrow i$, needs the following auxiliary statement:

$(*)$ $\quad I_i \in \mathcal{I}(p_1 \| \ldots \| p_i, Q_i)$ implies $I_i \in \mathcal{I}(\mathcal{M}((\mathcal{P}_{i-1} \| p_i)\langle \mathcal{A}_{I_i} \cup L\rangle), Q_i)$.

The statement follows by Definition 2.14 considering

[10] Q_n denotes the empty process consisting of a single state, an empty alphabet, and no transition.

$$
\begin{array}{lcl}
 & & \mathcal{L}(I_i) \\
\text{(Def. 2.14)} & \supseteq & \mathcal{L}(((p_1 \| \ldots \| p_i) \| Q_i) \langle \mathcal{A}_{I_i} \rangle) \\
\text{(def. } Q_i \text{, Prop. 2.6, 2.12, La. 2.13(2.))} & = & \mathcal{L}((p_1 \| \ldots \| p_n) \langle \mathcal{A}_{I_i} \rangle) \\
\text{(ind. hyp. for } L = \mathcal{A}_{I_i} \text{, La. 2.13(2.))} & = & \mathcal{L}((\mathcal{P}_{i-1} \| Q_{i-1}) \langle \mathcal{A}_{I_i} \rangle) \\
\text{(def. } Q_{i-1} \text{, Prop. 2.6,2.12, La. 2.13(2.))} & = & \mathcal{L}(((\mathcal{P}_{i-1} \| p_i) \| Q_i) \langle \mathcal{A}_{I_i} \rangle) \\
\text{(Prop. 2.7, La. 2.13(2.))} & = & \mathcal{L}(((\mathcal{P}_{i-1} \| p_i) \langle \mathcal{A}_{I_i} \cup L \rangle) \| Q_i) \langle \mathcal{A}_{I_i} \rangle) \\
\text{(def. } \mathcal{M} \text{, Prop. 2.12, La. 2.13(2.))} & = & \mathcal{L}((\mathcal{M}((\mathcal{P}_{i-1} \| p_i) \langle \mathcal{A}_{I_i} \cup L \rangle) \| Q_i) \langle \mathcal{A}_{I_i} \rangle)
\end{array}
$$

Now, the case $2 \leq i \leq n-1$ proceeds as follows:

$$
\begin{array}{lcl}
 & & (\mathcal{P}_i \| Q_i) \langle L \rangle \\
\text{(def. } \mathcal{P}_i) & = & (\mathcal{M}(\Pi_{I_i}(\mathcal{M}((\mathcal{P}_{i-1} \| p_i) \langle \mathcal{A}_{I_i} \cup L \rangle))) \| Q_i) \langle L \rangle \\
\text{(def. } \mathcal{M} \text{, Prop. 2.12)} & \approx^{\mathrm{d}} & (\Pi_{I_i}(\mathcal{M}((\mathcal{P}_{i-1} \| p_i) \langle \mathcal{A}_{I_i} \cup L \rangle)) \| Q_i) \langle L \rangle \\
\text{((}*\text{), Def. 3.1(ii), Prop. 2.12)} & \approx^{\mathrm{d}} & (\mathcal{M}((\mathcal{P}_{i-1} \| p_i) \langle \mathcal{A}_{I_i} \cup L \rangle) \| Q_i) \langle L \rangle \\
\text{(def. } \mathcal{M} \text{, Prop. 2.12)} & \approx^{\mathrm{d}} & ((\mathcal{P}_{i-1} \| p_i) \langle \mathcal{A}_{I_i} \cup L \rangle \| Q_i) \langle L \rangle \\
\text{(Prop. 2.7)} & = & ((\mathcal{P}_{i-1} \| p_i) \| Q_i) \langle L \rangle \\
\text{(Prop. 2.6, 2.12, def. } Q_{i-1}\text{)} & \approx^{\mathrm{d}} & (\mathcal{P}_{i-1} \| Q_{i-1}) \langle L \rangle \\
\text{(ind. hyp.)} & \approx^{\mathrm{d}} & \mathrm{P}
\end{array}
$$

This completes the proof, as the case $i = n$ is an easy variant of the previous case where $(*)$ is not needed. □

In practice, P is usually totally defined. Then by applying Theorem 4.1 it is easy to see that the proof of $\mathcal{P}_n \approx^{\mathrm{d}} \mathrm{P}$ reduces to the verification of the total definedness of $\mathcal{P}_n$.

Corollary 4.3. (Total Definedness)
Whenever P is totally defined, we have: $\mathcal{P}_n \approx^{\mathrm{d}} \mathrm{P}$ iff $\mathcal{P}_n$ is totally defined.

Up to now, we have considered the reduction operator as a parameter. The following applications use the reduction operator $\overline{\Pi}$.

4.1. An Application of the $\mathcal{RM}$-Method

The application of our method to the system of Example 2.8 and the interface specifications defined in Example 2.15 leads to the successive computation of the following three processes:

$$
\begin{array}{lcl}
\mathcal{P}_1 & = & \mathcal{M}(\overline{\Pi}_{I_1}(\mathcal{M}(P_1 \langle \{\mathtt{tk1}, \mathtt{tk2}, \mathtt{rb1}, \mathtt{sb1}\} \rangle))), \\
\mathcal{P}_2 & = & \mathcal{M}(\overline{\Pi}_{I_2}(\mathcal{M}((\mathcal{P}_1 \| B) \langle \{\mathtt{tk1}, \mathtt{tk2}, \mathtt{rb2}, \mathtt{sb2}\} \rangle))), \text{ and} \\
\mathcal{P}_3 & = & \mathcal{M}((\mathcal{P}_2 \| P_2) \langle \{\mathtt{tk1}, \mathtt{tk2}\} \rangle).
\end{array}
$$

As suggested in Example 2.15, $\overline{\Pi}_{I_1}$ has no effect on P_1 due to the $\mathrm{IA}^{\mathrm{all}}_{P_1, I_1}(\cdot)$-sets (cf. Figure 5, top left corner):

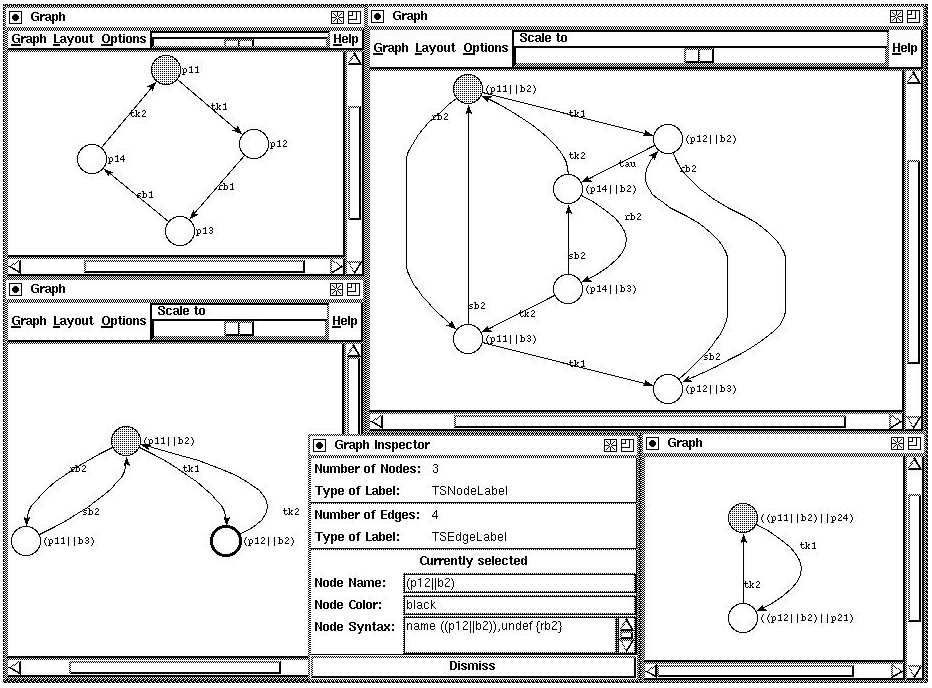


Fig. 5. Application of the method to our example

- $\mathrm{IA}^{\mathrm{all}}_{P_1,I_1}(p_{11}) = \{\mathtt{tk1}, \epsilon\}$,
- $\mathrm{IA}^{\mathrm{all}}_{P_1,I_1}(p_{12}) = \{\mathtt{rb1}, \mathtt{tk2}, \epsilon\}$,
- $\mathrm{IA}^{\mathrm{all}}_{P_1,I_1}(p_{13}) = \{\mathtt{sb1}, \epsilon\}$, and
- $\mathrm{IA}^{\mathrm{all}}_{P_1,I_1}(p_{14}) = \{\mathtt{rb1}, \mathtt{tk2}, \epsilon\}$.

Let us compute $\mathcal{P}_2$ stepwise. Computing $\mathcal{M}((\mathcal{P}_1 \| B)\langle\{\mathtt{tk1}, \mathtt{tk2}, \mathtt{rb2}, \mathtt{sb2}\}\rangle)$ leads to the process presented at the right top of Figure 5. The application of the algorithm of Section 3.3 provides the following $\mathrm{IA}^{\mathrm{all}}_{(P_1\|B),I_2}(\cdot)$-sets:

- $\mathrm{IA}^{\mathrm{all}}_{(P_1\|B),I_2}(p11\|b2) = \{\mathtt{tk1}, \mathtt{rb2}, \epsilon\}$,
- $\mathrm{IA}^{\mathrm{all}}_{(P_1\|B),I_2}(p12\|b2) = \{\mathtt{tk2}, \epsilon\} = \mathrm{IA}^{\mathrm{all}}_{(P_1\|B),I_2}(p14\|b2)$,
- $\mathrm{IA}^{\mathrm{all}}_{(P_1\|B),I_2}(p11\|b3) = \{\mathtt{sb2}, \epsilon\}$, and
- $\mathrm{IA}^{\mathrm{all}}_{(P_1\|B),I_2}(p12\|b3) = \emptyset = \mathrm{IA}^{\mathrm{all}}_{(P_1\|B),I_2}(p14\|b3)$.

$\mathcal{P}_2$ is presented at the bottom left corner of Figure 5 with a `rb2`-undefinedness at the state $(p12\|b2)$ and a `tk1`-undefinedness at the state $(p11\|b3)$.

The result of the reduction algorithm is $\mathcal{P}_3$ (cf. Figure 5, bottom right corner), and as $\mathcal{P}_3$ is totally defined, Corollary 4.3 yields that $\mathcal{P}_3 \approx^{\mathrm{d}} \mathit{System}$ holds. This reflects our intuition that an observer may only see the cyclical passing of the token on the channels `tk1` and `tk2`.

4.2. The Power of the $\mathcal{RM}$-Method

Let us consider a system guaranteeing the mutually exclusive access of n processes P_i to a common resource R as illustrated in Figure 6 for $n = 4$.[11] The idea behind the system is to pass a "token" via the communication channels tk_i, and to allow access to R only for the process that currently possesses the token. This process sends a request via ps_i to the resource R, which responds by transmitting the requested object. The corresponding transmission line is modelled by a buffer B_i. This choice is motivated by thinking of large objects whose transmission cannot be modelled by an atomic "handshake" communication.

In order to prove that the access of the resource is modelled as intended, we can hide everything except for the actions corresponding to the transmission of the token, and prove that the resulting process is equivalent to the process $Spec(n)$ that just repeatedly executes the sequence $tk_1, \ldots, tk_n$. Therefore, it is enough to show that

$$System(n) =_{df} (R \| P_1 \| B_1 \| \ldots \| P_n \| B_n) \langle \{tk_1, \ldots, tk_n\} \rangle \approx^{\mathrm{d}} Spec(n) \ .$$

It is easy to see that the apparent complexity of $System(n)$ is exponential in n, whereas its reduced complexity is linear. In fact, it is also possible to obtain an algorithmic complexity that is linear in n. This can be achieved by processing the system according to the structure indicated below, where the I_i denote the exact interface specifications presented in Figure 7:

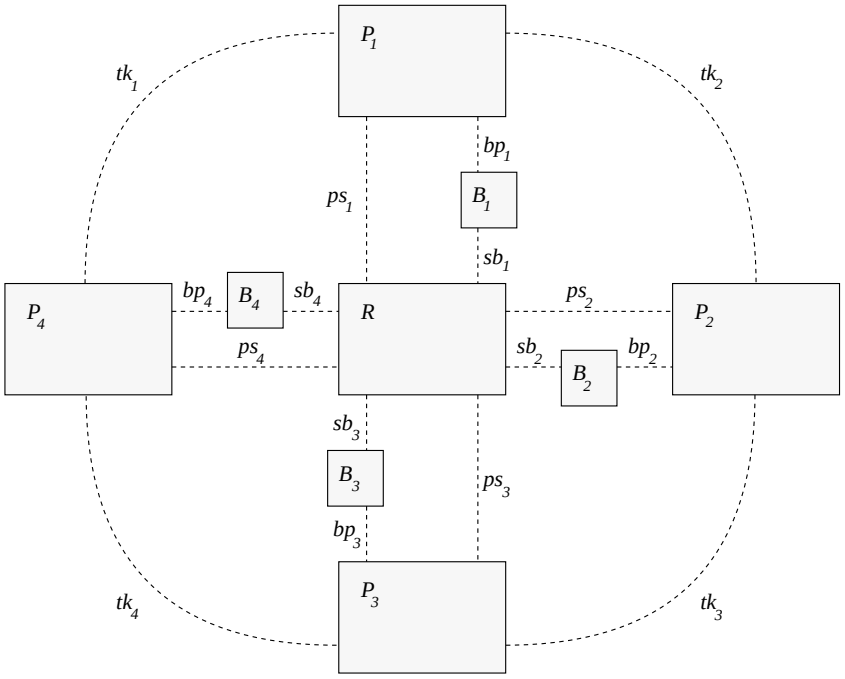
$$(\overbrace{R \| P_1 \| B_1}^{} \|_{I_1} \overbrace{P_2 \| B_2}^{} \|_{I_2} \ldots \|_{I_{n-1}} \overbrace{P_n \| B_n}^{}) \langle \{tk_1, \ldots, tk_n\} \rangle \ .$$

Table 1 summarises a quantitative evaluation of the effect of our method by means of the Aldebaran Verification Tool [Fer88]. It displays the size of the global state graph (its apparent complexity), the size of the maximal transition system constructed during stepwise minimisation when exploiting exact interface specifications (its algorithmic complexity), and the size of the minimised global state graph (its reduced complexity). It is worth mentioning that the method, which works just by stepwise composition and minimisation of components, encounters transition systems that are even larger than the global state graph (cf. Table 2). This stresses the importance of interface specifications for automatic proof techniques. Software designers should always provide these specifications as part of the implementation. We believe that besides enabling automatic verification, this requirement also leads to a transparent and well-structured programming. Note the similarity to the situation for *while*-programs, where automatic verification depends on *loop invariants* that also need to be provided by the programmer.

5. Conclusions

We have presented a method, called $\mathcal{RM}$-Method, for the compositional minimisation of finite state distributed systems which is intended to avoid the state explosion problem. This method can be used to support the verification of any property that is consistent with $\approx^{\mathrm{d}}$. However, the $\mathcal{RM}$-Method is not tailored

[11] In contrast to all the other P_i, which are displayed correctly, P_n is assumed to be initially in the bottom right state.



where

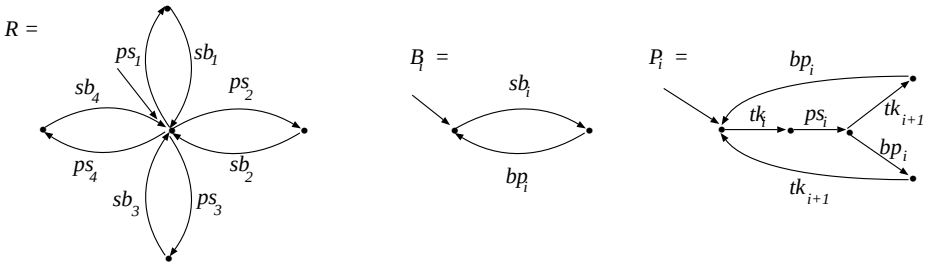


Fig. 6. Round-robin access

Table 1. Transition system sizes when using the $\mathcal{RM}$-Method

n	apparent complexity		algorithmic complexity		reduced complexity	
	states	trans.	states	trans.	states	trans.
4	144	368	20	29	4	4
5	361	1101	24	35	5	5
6	865	3073	28	41	6	6
7	2017	8177	32	47	7	7

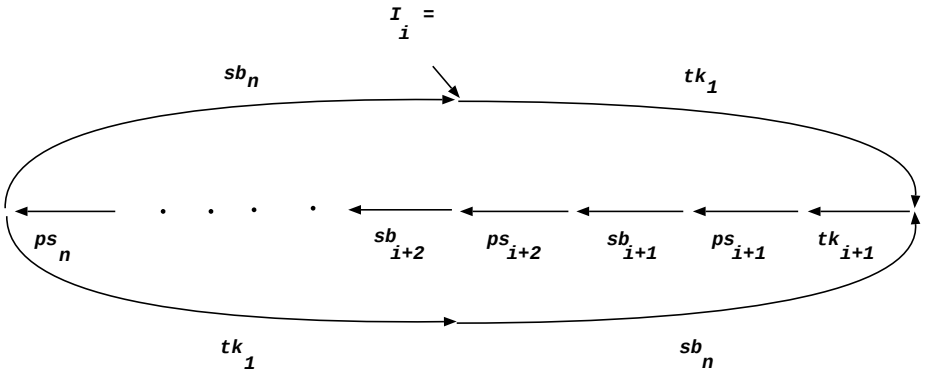


Fig. 7. Exact interface specifications

Table 2. Transition system sizes when using the naive method

n	states	trans.
4	96	243
5	324	927
6	972	3024
7	2916	9801

to this particular semantic equivalence. Other equivalences can be dealt with by adapting the preorder definition and the minimisation function accordingly. The $\mathcal{RM}$-Method is implemented as part of the METAFrame environment [SMC96] and the Aldebaran verification tool [Fer88] for the reduction operator $\overline{\Pi}$ and the semantical equivalence $\approx^{\mathrm{d}}$.

The *effect* of our method, which is intended to get the algorithmic complexity as close as possible to the reduced complexity, depends on interface specifications, which we assume to be given by the program designer. However, the *correctness* of the $\mathcal{RM}$-Method does not depend on the correctness of these interface specifications. Wrong interface specifications never lead to wrong results. They may only prevent a successful verification of a valid property. This is very important, because it allows the designer to simply "guess" interface specifications, while maintaining the reliability of a successful verification.

Indeed, a way to obtain interface specifications is by using the property to be verified as interface specification. This is what Clarke et al. [CLM89] had in mind. However, their approach only exploits the alphabet of the property under consideration. A refined treatment of property constraints using our notion of interface specification is under investigation.

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