Automata and Logic

Instructor: Radu Iosif
Hours: 32

This course is intended to introduce the student to basic decision methods for classical non-interpreted logics and integer arithmetic theories. The techniques studied are entirely based on automata theory. The relation of logic with automata theory was introduced by the seminal works of Büchi (1960) and Rabin (1969) and has had a deep impact in the fields of hardware and software verification.

The course is structured in three parts. In the first part, we recall the definition of classical first- and second-order logic, finite word and tree automata, closure properties and language emptiness. In the second part we will relate Weak Monadic Second Order Logic and finite automata, then study infinite word (Büchi, Müller) and tree (Rabin) automata, in relation with Monadic Second Order Logic. The third part of the course presents the logical theory of integer arithmetic. We establish decidability results for the additive (Presburger) and multiplicative (Skolem) sub-fragments of arithmetics, using word and tree automata, respectively.

1. **Preliminaries**
   (a) Words and Trees
   (b) First Order Logic
   (c) Second Order Logic
      - (Weak) Monadic Second Order Logic on words and trees
   (d) Finite Automata on Words
      - Determinism, closure properties, emptiness problem
      - Periodic and star-free languages
   (e) Finite Automata on Trees
      - Determinism, top-down vs. bottom-up
      - Closure properties, emptiness problem
2. **Classical Logics and Automata**
   (a) Automata on Infinite Words
      - Büchi Automata, Müller Automata

- Closure properties, emptiness problem
- Non-determinism, McNaughton's Theorem

(b) Logic and Automata on Words

- FOL and star-free languages
- Automata on Finite Words and WS1S
- Büchi Automata and S1S

(c) Logic and Automata on Trees

- FOL and star-free tree languages (open problem)
- Automata on Finite Trees and WSkS
- Rabin Automata and SkS

3. **Integer Arithmetic and Automata**

(a) Overview of Integer Arithmetic: decidability and undecidability results

(b) Presburger Arithmetic and Finite Automata

(c) Skolem Arithmetic and Tree Automata

4. **Concluding Remarks, Open Problems, Discussion**