Büchi Automata

Definition of Büchi Automata

Let $\Sigma = \{a, b, \ldots\}$ be a finite alphabet.

By Σ^ω we denote the set of all infinite words over Σ.

A *non-deterministic Büchi automaton* (NBA) over Σ is a tuple $A = \langle S, I, T, F \rangle$, where:

- S is a finite set of *states*,
- $I \subseteq S$ is a set of *initial states*,
- $T \subseteq S \times \Sigma \times S$ is a *transition relation*,
- $F \subseteq S$ is a set of *final states*.

Acceptance Condition

A *run* of a Büchi automaton is defined over an infinite word $w : \alpha_1\alpha_2\ldots$ as an infinite sequence of states $\pi \ : \ s_0 s_1 s_2 \ldots$ such that:

- $s_0 \in I$ and
- $(s_i, \alpha_{i+1}, s_{i+1}) \in T$, for all $i \in \mathbb{N}$.

Let $\inf(\pi) = \{s \mid s$ appears infinitely often on $\pi\}$.

Run π of A is said to be *accepting* iff $\inf(\pi) \cap F \neq \emptyset$.

Examples

Let $\Sigma = \{0, 1\}$. Define Büchi automata for the following languages:

1. $L = \{\alpha \mid 0 \text{ occurs in } \alpha \text{ exactly once}\}$
2. $L = \{\alpha \mid \text{after each } 0 \text{ in } \alpha \text{ there is } 1\}$
3. $L = \{\alpha \mid \alpha \text{ contains finitely many 1's}\}$
4. $L = (01)^n \Sigma^\omega$
5. $L = \{\alpha \mid 0 \text{ occurs on all even positions in } \alpha\}$

Closure Properties

Closure under union and projections are like in the finite automata case.

Intersection is a bit special.

Complementation of non-deterministic Büchi automata is a complex result.

Deterministic Büchi automata are not closed under complement.

Closure under Intersection

Let $A_1 = \langle S_1, I_1, T_1, F_1 \rangle$ and $A_2 = \langle S_2, I_2, T_2, F_2 \rangle$

Build $A_\cap = \langle S, I, T, F \rangle$:

- $S = S_1 \times S_2 \times \{1, 2, 3\}$,
- $I = I_1 \times I_2 \times \{1\}$,
- the definition of T is the following:
  - $((s_1, s_1', 1), a, (s_2, s_2', 1)) \in T$ iff $(s_i, a, s_i') \in T_i$, $i = 1, 2$ and $s_1' \notin F_1$
  - $((s_1, s_1', 2), a, (s_2, s_2', 2)) \in T$ iff $(s_i, a, s_i') \in T_i$, $i = 1, 2$ and $s_2' \notin F_2$
  - $((s_1, s_1', 1), a, (s_2, s_2', 2)) \in T$ iff $(s_i, a, s_i') \in T_i$, $i = 1, 2$ and $s_1' \in F_1$
  - $((s_1, s_1', 2), a, (s_2, s_2', 3)) \in T$ iff $(s_i, a, s_i') \in T_i$, $i = 1, 2$ and $s_2' \in F_2$
  - $((s_1, s_1', 3), a, (s_2, s_2', 1)) \in T$ iff $(s_i, a, s_i') \in T_i$, $i = 1, 2$
- $F = F_1 \times F_2 \times \{3\}$

The Büchi Characterization Theorems

Lemma 1 *If $L \subseteq \Sigma^*$ is a rational language, there exists a DFA $A = \langle S, \{s_0\}, T, F \rangle$ such that s_0 has no incoming transition, and $L = \mathcal{L}(A)$.*

If $U, V \subseteq \Sigma^*$ define $U \cdot V = \{uv \mid u \in U, \ v \in V\}$.

If $W \subseteq \Sigma^\omega$ we denote $W^\omega = \{w_1 w_2 \ldots \mid w_i \in W, i \in \mathbb{N}\}$.

Theorem 1 *Let $W, V \subseteq \Sigma^*$ be rational languages. Then the language $V \cdot W^\omega$ is Büchi recognizable.*

Is it possible that, for some non-rational language $W \subseteq \Sigma^*$, W^ω is Büchi recognizable?

First Büchi Characterization Theorem

Let $A = \langle S, I, T, F \rangle$ be a Büchi automaton and $s, s' \in S$ be two states.

Let $W_{s,s'} = \{w \in \Sigma^* \mid s \xrightarrow{w} s'\}$.

The language $W_{s,s'} \subseteq \Sigma^*$ is rational, for any $s, s' \in S$.

Theorem 2 *An ω-langage $L \subseteq \Sigma^\omega$ is Büchi recognizable iff L is a finite union of ω-languages $V \cdot W^\omega$, where $V, W \subseteq \Sigma^*$ are rational languages.*

"$\Rightarrow$" $L = \bigcup_{s \in I, s' \in F} W_{s,s'} \cdot W^\omega_{s',s'}$

Second Büchi Characterization Theorem

Theorem 3 *An ω-langage $L \subseteq \Sigma^\omega$ is Büchi recognizable iff L is a finite union of ω-languages $V \cdot W^\omega$, where $V, W \subseteq \Sigma^*$ are rational languages such that $W \cdot W \subseteq W$.*

"$\Rightarrow$" $\mathcal{L}(A) = \bigcup_{s \in I, s' \in F} W_{s,s'} W^\omega_{s',s'}$ and $W_{s',s'} \cdot W_{s',s'} \subseteq W_{s',s'}$ for all $s' \in F$

Corollary 1 *Any non-empty Büchi-recognizable language contains an ultimately periodic word of the form $uvvv\ldots$.*

Complementation of Büchi Automata

Congruences

Definition 1 *An equivalence relation* $R \subseteq \Sigma^* \times \Sigma^*$ *is said to be a* left-congruence *iff for all* $u, v, w \in \Sigma^*$ *we have* $u \cong v \Rightarrow wu \cong wv$*.*

Definition 2 *An equivalence relation* $R \subseteq \Sigma^* \times \Sigma^*$ *is said to be a* right-congruence *iff for all* $u, v, w \in \Sigma^*$ *we have* $u \; R \; v \Rightarrow uw \; R \; vw$*.*

Definition 3 *An equivalence relation* $R \subseteq \Sigma^* \times \Sigma^*$ *is said to be a* congruence *iff it is both a left- and a right-congruence.*

Ex: the Myhill-Nerode equivalence $\sim_L$ is a right-congruence.

Congruences

Let $A = \langle S, I, T, F \rangle$ be a Büchi automaton.

For $s, s' \in S$ and $w \in \Sigma^*$, denote $s \rightarrow_w^F s'$ iff $s \xrightarrow{w} s'$ visiting a state from F

Let $W_{s,s'}^F$ be the set of all words such that $s \rightarrow_w^F s'$.

For any two words $u, v \in \Sigma^*$ we have $u \cong v$ iff for all $s, s' \in S$ we have:

- $s \xrightarrow{u} s' \iff s \xrightarrow{v} s'$, and
- $s \rightarrow_u^F s' \iff s \rightarrow_v^F s'$.

The relation $\cong$ is a congruence of finite index on Σ^*

Congruences

Let $[w]_\cong$ denote the equivalence class of $w \in \Sigma^*$ w.r.t. $\cong$.

Lemma 2 *For any $w \in \Sigma^*$, $[w]_\cong$ is the intersection of all sets of the form $W_{s,s'}, W^F_{s,s'}, \overline{W_{s,s'}}, \overline{W^F_{s,s'}}$, containing w.*

$$T_w = \bigcap_{w \in W_{s,'s}} W_{s,s'} \cap \bigcap_{w \in W^F_{s,'s}} W^F_{s,s'} \cap \bigcap_{w \in \overline{W_{s,s'}}} \overline{W_{s,s'}} \cap \bigcap_{w \in \overline{W^F_{s,s'}}} \overline{W^F_{s,s'}}$$

We show that $[w]_\cong = T_w$.

"$\subseteq$" If $u \cong w$ then clearly $u \in T_w$.

Congruences

"$\supseteq$" Let $u \in T_w$

- if $s \xrightarrow{w} s'$, then $w \in W_{s,s'}$, hence $u \in W_{s,s'}$, then $s \xrightarrow{u} s'$ as well.
- if $s \stackrel{w}{\not\rightarrow} s'$, the $w \in \overline{W_{s,s'}}$, hence $u \in \overline{W_{s,s'}}$, then $s \stackrel{u}{\not\rightarrow} s'$.

Also,

- if $s \rightarrow_w^F s'$, then $w \in W_{s,s'}^F$, hence $u \in W_{s,s'}^F$, then $s \rightarrow_u^F s'$ as well.
- if $s \not\rightarrow_w^F s'$, then $w \in \overline{W_{s,s'}^F}$, hence $u \in \overline{W_{s,s'}^F}$, then $s \not\rightarrow_u^F s'$.

Then $u \cong w$.

Outline of the proof

We prove that:

$$\mathcal{L}(A) = \bigcup_{VW^\omega \cap \mathcal{L}(A) \neq \emptyset} VW^\omega$$

where V, W are $\cong$-equivalence classes

Then we have

$$\Sigma^\omega \setminus \mathcal{L}(A) = \bigcup_{VW^\omega \cap \mathcal{L}(A) = \emptyset} VW^\omega$$

Finally we obtain an algorithm for complementation of Büchi automata

Saturation

Definition 4 *A congruence relation* $R \subseteq \Sigma^* \times \Sigma^*$ saturates *an* ω*-language* L *iff for all* R*-equivalence classes* V *and* W*, if* $VW^\omega \cap L \neq \emptyset$ *then* $VW^\omega \subseteq L$*.*

Lemma 3 *The congruence relation* $\cong$ *saturates* $\mathcal{L}(A)$*.*

Every word belongs to some VW^ω

Let $\alpha \in \Sigma^\omega$ be an infinite word for the rest of this section.

By $\alpha(n, m)$, we denote $\alpha(n)\alpha(n+1)\ldots\alpha(m-1)$, $n \leq m$.

We will build two $\cong$-equivalence classes V and W such that $\alpha \in V \cdot W^\omega$

Together with the saturation lemma, this proves

$$\mathcal{L}(A) = \bigcup_{VW^\omega \cap \mathcal{L}(A) \neq \emptyset} VW^\omega$$

Merging of positions

Definition 5 *Two positions* $k, k' \in \mathbb{N}$ *are said to* merge at m, $m > k$ *and* $m > k'$ *iff* $\alpha(k, m) \cong \alpha(k', m)$. *We say that* k *and* k' *are* $\cong_\alpha$*-equivalent, denoted* $k \cong_\alpha k'$ *iff they merge at* m*, for some* $m > k, k'$.

If k and k' merge at m then they also merge at m', for all $m' \geq m$.

$k \cong_\alpha k'$ (m) is an equivalence relation on $\mathbb{N}$ of finite index.

Merging of positions

There exists infinitely many positions $0 < k_0 < k_1 < \ldots$, all $\cong_\alpha$-equivalent.

Consider the sequence $\alpha(k_0, k_1), \alpha(k_0, k_2), \alpha(k_0, k_3) \ldots$

There exist $\alpha(k_0, k_{i_0}), \alpha(k_0, k_{i_1}), \alpha(k_0, k_{i_2}) \ldots$ all $\cong$-equivalent

There exist $k_{j_0}, k_{j_1}, k_{j_2}, \ldots$ such that for all $i \leq j$ $k_i \cong_\alpha k_j (k_{j+1})$

There exists infinitely many positions $0 < k_0 < k_1 < k_2 < \ldots$ such that

1. $\alpha(k_0, k_i) \cong \alpha(k_0, k_j)$ for all $i, j \in \mathbb{N}$
2. $k_i \cong_\alpha k_j (k_{j+1})$ for all $i \leq j$.

Defining V and W

Let $V = [\alpha(0, k_0)]_{\cong}$ and $W = [\alpha(k_0, k_1)]_{\cong}$

By (1) $\alpha(k_0, k_1) \cong \alpha(k_0, k_i)$ for all $i > 0$

By (2) $\alpha(k_0, k_{i+1}) \cong \alpha(k_i, k_{i+1})$, for all $i > 0$

Hence $\alpha(k_0, k_1) \cong \alpha(k_i, k_{i+1})$, for all $i > 0$.

Therefore $\alpha \in V \cdot W^{\omega}$

Complementation of Büchi Automata

Theorem 4 *For any Büchi automaton A there exists a Büchi automaton $\overline{A}$ such that $\mathcal{L}(\overline{A}) = \Sigma^\omega \setminus \mathcal{L}(A)$.*

$$\mathcal{L}(A) = \bigcup_{VW^\omega \cap \mathcal{L}(A) \neq \emptyset} VW^\omega$$

where V, W are $\cong$-equivalence classes

$$\Sigma^\omega \setminus \mathcal{L}(A) = \bigcup_{VW^\omega \cap \mathcal{L}(A) = \emptyset} VW^\omega$$

By the Büchi Characterization Theorem(s) there exists a Büchi automaton recognizing $\Sigma^\omega \setminus \mathcal{L}(A)$ $\Box$

Deterministic Büchi Automata

ω-languages recognized by NBA $\supset$ ω-languages recognized by DBA

Let $W \subseteq \Sigma^*$. Define $\overrightarrow{W} = \{\alpha \in \Sigma^\omega \mid \alpha(0, n) \in W \text{ for infinitely many } n\}$

Theorem 5 *A language $L \subseteq \Sigma^\omega$ is recognizable by a deterministic Büchi automaton iff there exists a rational language $W \subseteq \Sigma^*$ such that $L = \overrightarrow{W}$.*

If $L = \mathcal{L}(A)$ then $W = \mathcal{L}(A')$ where A' is the DFA with the same definition as A, and with the finite acceptance condition.

Deterministic Büchi Automata

Theorem 6 *There exists a Büchi recognizable language that can be recognized by no deterministic Büchi automaton.*

$\Sigma = \{a, b\}$ and $L = \{\alpha \in \Sigma^\omega \mid \#_a(\alpha) < \infty\} = \Sigma^* b^\omega$.

Suppose $L = \overrightarrow{W}$ for some $W \subseteq \Sigma^*$.

$b^\omega \in L \Rightarrow b^{n_1} \in W$

$b^{n_1} a b^\omega \in L \Rightarrow b^{n_1} a b^{n_2} \in W$

$$\dots$$

$b^{n_1} a b^{n_2} a \dots \in \overrightarrow{W} = L$, contradiction.

Deterministic Büchi Automata are not closed under complement

Theorem 7 *There exists a DBA A such that no DBA recognizes the language $\Sigma^\omega \setminus \mathcal{L}(A)$.*

$\Sigma = \{a, b\}$ and $L = \{\alpha \in \Sigma^\omega \mid \#_a(\alpha) < \infty\} = \Sigma^* b^\omega$.

Let $V = \Sigma^* a$. There exists a DFA A such that $\mathcal{L}(A) = V$.

There exists a deterministic Büchi automaton B such that $\mathcal{L}(A) = \overrightarrow{V}$

But $\Sigma^\omega \setminus \overrightarrow{V} = L$ which cannot be recognized by any DBA.

Decidability

Theorem 8 *The emptiness problem for Büchi automata is decidable.*

Let $A = \langle S, I, T, F \rangle$ be a Büchi automaton.

$\mathcal{L}(A) \neq \emptyset \iff$ exists $s \in S$ accessible from some $s_0 \in I$ and from itself

Theorem 9 *The equivalence and universality problems for Büchi automata are decidable.*

How about the infinity problem?

Büchi Automata and S1S

Let $\Sigma = \{a, b, \ldots\}$ be a finite alphabet.

Any finite word $w \in \Sigma^*$ induces the *infinite* sets $p_a = \{p \mid w(p) = a\}$.

- $x \leq y$: x is less than y,
- $S(x) = y$: y is the successor of x,
- $p_a(x)$: a occurs at position x in w

Remember that $\leq$ and S can be defined one from another.

Problem Statement

Let $\mathcal{L}(\varphi) = \{w \mid \mathfrak{m}_w \models \varphi\}$

A language $L \subseteq \Sigma^*$ is said to be S1S-*definable* iff there exists a S1S formula φ such that $L = \mathcal{L}(\varphi)$.

1. Given a Büchi automaton A build an S1S formula φ_A such that $\mathcal{L}(A) = \mathcal{L}(\varphi)$
2. Given an S1S formula φ build a Büchi automaton A_φ such that $\mathcal{L}(A) = \mathcal{L}(\varphi)$

The Büchi recognizable and S1S-definable languages coincide

From Automata to Formulae

Let $A = \langle S, I, T, F \rangle$ with $S = \{s_1, \ldots, s_p\}$, and $\Sigma = \{0,1\}^m$.

Build $\Phi_A(X_1, \ldots, X_m)$ such that $\forall w \in \Sigma^* \,.\, w \in \mathcal{L}(A) \iff w \models \Phi_A$

$\Phi_A(X_1, \ldots, X_m) = \exists Y_1 \ldots \exists Y_p \,.\, \Phi_S(\mathbf{Y}) \wedge \Phi_I(\mathbf{Y}) \wedge \Phi_T(\mathbf{Y}, \mathbf{X}) \wedge \Phi_F(\mathbf{Y})$

$$\Phi_F(\mathbf{Y}) = \forall x \exists y \,.\, x \leq y \wedge x \neq y \wedge \bigvee_{s_i \in F} Y_i(y)$$

Consequences

Theorem 10 *A language $L \subseteq \Sigma^\omega$ is definable in S1S iff it is Büchi recognizable.*

Corollary 2 *The SAT problem for S1S is decidable.*

Lemma 4 *Any S1S formula $\phi(X_1, \ldots, X_m)$ is equivalent to an S1S formula of the form $\exists Y_1 \ldots \exists Y_p \,.\, \varphi$, where φ does not contain other set variables than $X_1, \ldots, X_m, Y_1, \ldots, Y_p$.*

Müller and Rabin Word Automata

Müller Automata

Let $\Sigma = \{a, b, \ldots\}$ be a finite alphabet.

Definition 6 *A* Müller automaton *over* Σ *is* $A = \langle S, s_0, T, \mathcal{F} \rangle$, *where:*

- S *is the finite* set of states
- $s_0 \in S$ *is the* initial state
- $T : S \times \Sigma \mapsto S$ *is the* transition table
- $\mathcal{F} \subseteq 2^S$ *is the set of* accepting sets

Acceptance Condition

A *run* of a Müller automaton is defined over an infinite word $w : \alpha_1 \alpha_2 \ldots$ as an infinite sequence of states $\pi \ : \ s_0 s_1 s_2 \ldots$ such that:

- $T(s_i, \alpha_{i+1}) = s_{i+1}$, for all $i \in \mathbb{N}$.

Let $\inf(\pi) = \{s \mid s$ appears infinitely often on $\pi\}$.

Run π of A is said to be *accepting* iff $\inf(\pi) \in \mathcal{F}$.

$L \subseteq \Sigma^\omega$ is *Müller-recognizable* iff there exists a MA A such that $L = \mathcal{L}(A)$.

Deterministic Büchi $\subseteq$ Müller

Theorem 11 *For each deterministic Büchi automaton A there exists a Müller automaton B such that $\mathcal{L}(A) = \mathcal{L}(B)$*

Let $A = \langle S, \{s_0\}, T, F\rangle$ be a deterministic Büchi automaton.

Define $B = \langle S, s_0, T, \{G \in 2^S \mid G \cap F \neq \emptyset\}\rangle$

Closure Properties

Theorem 12 *The class of Müller-recognizable languages is closed under union, intersection and complement.*

Let $A = \langle S, s_0, T, \mathcal{F} \rangle$ be a Müller automaton.

Define $B = \langle S, s_0, T, 2^S \setminus \mathcal{F} \rangle$.

We have $\mathcal{L}(B) = \Sigma^\omega \setminus \mathcal{L}(A)$.

Closure Properties

Let $A_i = \langle S_i, s_{0,i}, T_i, \mathcal{F}_i \rangle$, $i = 1, 2$ be Müller automata.

Define $B = \langle S, s_0, T, \mathcal{F} \rangle$ where:

- $S = S_1 \times S_2$,
- $s_0 = \langle s_{0,1}, s_{0,2} \rangle$,
- $T(\langle s_1, s_2 \rangle, a) = \langle T(s_1, a), T(s_2, a) \rangle$
- $\mathcal{F} = \{\{\langle s_1, s_1' \rangle, \ldots, \langle s_k, s_k' \rangle\} \mid \{s_1, \ldots, s_k\} \in \mathcal{F}_1 \text{ or } \{s_1', \ldots, s_k'\} \in \mathcal{F}_2\}$

We have $\mathcal{L}(B) = \mathcal{L}(A_1) \cup \mathcal{L}(A_2)$.

Characterization of Müller-recognizable languages

A language $L \subseteq \Sigma^\omega$ is Müller-recognizable iff L is a Boolean combination of sets $\overrightarrow{W}$, $W \subseteq \Sigma^*$, i.e. $L = \bigcup_i \left(\bigcap_j \overrightarrow{W_{ij}} \cap \bigcap_k (\Sigma^\omega \setminus \overrightarrow{W_{ik}}) \right)$.

"$\Leftarrow$" Any set $\overrightarrow{W_{ij}}$ is recognized by a deterministic Büchi automaton, hence also by a Müller automaton.

"$\Rightarrow$" Let $A = \langle S, s_0, T, \mathcal{F} \rangle$ be a Müller automaton recognizing L.

Let $A_q = \langle S, s_0, T, \{q\} \rangle$, $q \in S$, and $W_q = \mathcal{L}(A_q)$.

$$L = \bigcup_{Q \in \mathcal{F}} \left(\bigcap_{q \in Q} \overrightarrow{W_q} \cap \bigcap_{q \in S \setminus Q} (\Sigma^\omega \setminus \overrightarrow{W_q}) \right)$$

Rabin Word Automata

Let $\Sigma = \{a, b, \ldots\}$ be a finite alphabet.

Definition 7 *A* Rabin automaton *over* Σ *is* $A = \langle S, s_0, T, \Omega \rangle$, *where:*

- S *is the finite* set of states
- $s_0 \in S$ *is the* initial state
- $T : S \times \Sigma \mapsto S$ *is the* transition table
- $\Omega = \{\langle N_1, P_1 \rangle, \ldots, \langle N_k, P_k \rangle\}$ *is the set of* accepting pairs, $N_i, P_i \subseteq S$.

Run π of A is said to be *accepting* iff

$$\inf(\pi) \cap N_i = \emptyset \text{ and } \inf(\pi) \cap P_i \neq \emptyset$$

for some $1 \leq i \leq k$.

From Rabin to Müller

Given a Rabin automaton $A = \langle S, s_0, T, \Omega \rangle$, there exists a Müller automaton B such that $\mathcal{L}(A) = \mathcal{L}(B)$

Let $\Omega = \{\langle N_1, P_1 \rangle, \ldots, \langle N_k, P_k \rangle\}$.

Let $A_i = \langle S, s_0, T, P_i \rangle$, and $B_i = \langle S, s_0, T, N_i \rangle$.

$$\mathcal{L}(A) = \bigcup_{i=1}^{k} \left(\overrightarrow{\mathcal{L}(A_i)} \cap (\Sigma^\omega \setminus \overrightarrow{\mathcal{L}(B_i)}) \right)$$

From Müller to Rabin

Given a Müller automaton $A = \langle S, s_0, T, \mathcal{F} \rangle$, there exists a Rabin automaton B such that $\mathcal{L}(A) = \mathcal{L}(B)$

Let $\mathcal{F} = \{Q_1, \ldots, Q_k\}$

Let $B = \langle S', s'_0, T', \Omega' \rangle$ where:

- $S' = 2^{Q_1} \times \ldots \times 2^{Q_k} \times S$
- $s'_0 = \langle \emptyset, \ldots, \emptyset, s_0 \rangle$

From Müller to Rabin

- $T'(\langle S_1, \ldots, S_k, s\rangle, a) = \langle S'_1, \ldots, S'_k, s'\rangle$ where:
  - $s' = T(s, a)$
  - $S'_i = \emptyset$ if $S_i = Q_i$, $1 \leq i \leq k$
  - $S'_i = (S_i \cup \{s'\}) \cap Q_i$, $1 \leq i \leq k$

- $P_i = \{\langle S_1, \ldots, S_i, \ldots, S_k, s\rangle \mid S_i = Q_i\}$, $1 \leq i \leq k$

- $N_i = \{\langle S_1, \ldots, S_i, \ldots, S_k, s\rangle \mid s \notin Q_i\}$, $1 \leq i \leq k$

The McNaughton Theorem

For every Büchi automaton there exists a Müller (Rabin) automaton with the same language.

Equivalence between Büchi, Müller and Rabin automata.

$$L = \bigcup_i \left(\bigcap_j \overrightarrow{W_{ij}} \cap \bigcap_k (\Sigma^\omega \setminus \overrightarrow{W_{ik}}) \right)$$

Büchi recognizable languages can be accepted by deterministic automata, closed under complement.

Müller-recognizable languages are closed under projection.