Crypto Engineering - verifying security protocols

Cristian Ene
thanks Jannik Dreier for some slides!

Grenoble Alpes University, Verimag

Master 2: 1st Semester 2024-2025

Administrative information and contents

Administrative information:

- E-mail (me): Cristian.Ene@univ-grenoble-alpes.fr
- Web page: `www-verimag.imag.fr/~enec/m2p/`
- Tamarin related small exam on Thusday, January 9, 2025

Schedule

1. Security Protocols - introduction and a simple protocol
2. Security Protocols - a few attacks and the need for proofs
3. A formal language for security protocols
4. An introduction to Tamarin
5. Formalizing security properties in Tamarin
6. Indistinguishability and Security Notions
7. Link Between Computational and Symbolic

From crypto primitives to secure distributed applications

- RSA, ElGamal, , AES, ChaCha20, SHA-1 . . . provide provably correct cryptographic primitives.
- How can we construct secure distributed applications with these primitives?
  - E-commerce
  - E-banking
  - E-voting
  - Mobile communication
  - Digital contract signing
- Even if cryptography is hard to break, this is not a trivial task

Example: Securing an e-banking application

$A \longrightarrow B$: "Send 10000 dollars to account C"
$B \longrightarrow A$: "I just did it!"

- How does B know the message originated from A?
- How does B know A just said it?
- Confidentiality, integrity, accountability, non-repudiation, ...?
- We need security protocols like IPsec, Kerberos, SSH, TLS, EMV, 5G AKA ...

Communication Protocols

- **What are communication protocols?**
  - A set of rules that governs the interaction and transmission of data exchanged between agents or principals. In short, a distributed algorithm whose goal is communication.
  - **Examples**
    - TCP and UDP govern computer interactions over an IP network
    - HTTP governs the exchange of data in the hypertext format.
    - SMTP governs the exchange of e-mail.

Security Protocols

- **What are security protocols?**
  - Communication protocols that operate in an untrusted network or among (some) untrusted agents (principals), and that are used to achieve security goals against a threat model
  - Ingredients:
    - Principal: a protocol participant, typically human or computer
    - Security Goal: the confidentiality or integrity of data, the authentication of a principal
    - Threat model: the capabilities of the attacker
  - They use cryptographic primitives as basic primitives
  - Examples: TLS, IPsec, SSH, WPA, Kerberos, Needham-Schroeder,...

Example - Online Banking

- Cryptographic protocol: TLS (HTTPS)
  - Principals: Web Browser, Bank Website
  - Security Goal: the confidentiality and integrity of data, server authentication
  - Threat model: network attacker, phishing website
- Password-based authentication protocol
  - Principals: Bank Client, Bank Website
  - Security Goal: client authentication
  - Threat model: dishonest client

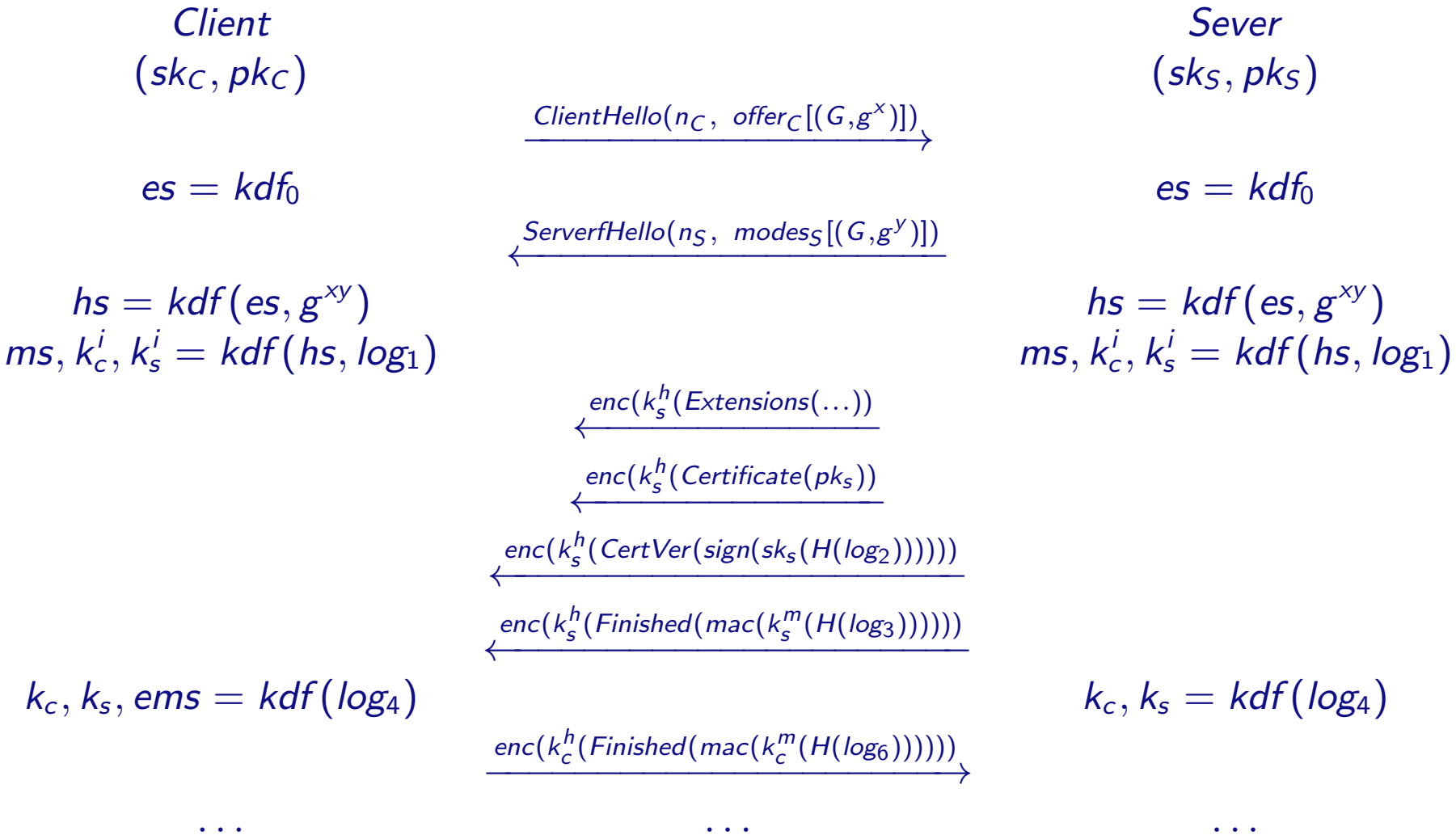
Credit card payment

- Online Transaction Authorization
  - Principals: Credit Card, Terminal, Bank Website
  - Security Goal: transaction data integrity, card authentication
  - Threat model: fake card, tampered terminal
- Cardholder verification (PIN Entry)
  - Principals: Client, Credit Card, Terminal
  - Security Goal: client authentication
  - Threat model: stolen credit card

Modelling security protocols

- Security protocols are small (but **critical**) components embedded within large distributed applications: for example, TLS within a web browser
- the security of the system depends on their correctness
- small recipes, but non trivial to design and understand: a long history of attacks on academic or real word protocols (many attacks, several years later after deployment!)
- why is it so difficult to correctly design security protocols?
  - *cryptografic guarantees* provided by primitives are often misunderstood
  - *rich threat models* are difficult to reason about and to test

TLS 1.3 (a small part of)



Building a secret establishment protocol

- an attempt to design a simple good protocol
- principals:
  - a set of users, denoted by a, b, c, ... (not all are necessarily honest!)
  - an **honest** server denoted by s
- security goal: to establish a new secret value k
  - **secrecy**: At the end of the protocol, k is **known** only to a and b, and possibly s, but to no other parties
  - **freshness**: a and b know that k is freshly generated
- k can be used to generate new session keys (further communication protocols between the same principals may be based on these keys)

A secret establishment protocol - notations

- we consider a protocol with 3 roles A, B (intended to be played by users) and S (intended to be played by the server)
- a role A describes how a participant (e.g. $a, b, \ldots$) playing this role should behave during a session of the protocol
- we suppose that there is an intruder i (but nobody knows that i is not honest!)
- A protocol is **informally** specified as a sequence of messages m exchanged between roles (principals) A and B

$$A \longrightarrow B : \quad m \qquad \text{(also called Alice-Bob notation)}$$

A secret establishment protocol - first attempt

- first attempt: a protocol that consists of 3 messages

$1. A \longrightarrow S : \ A, B$	1. A contacts S by sending the identities of the 2 parties who are going to share the secret
$2. S \longrightarrow A : \ K_{AB}$	2. S sends the secret K_{AB} to A
$3. A \longrightarrow B : \ K_{AB}, A$	3. A passes K_{AB} together with its identity to B

- K_{AB} does not contain information about A and B, it is simply a symbolic name for the bit-string representing the secret intended to be shared by the principals playing the roles A and B
- notice the significantly incomplete protocol specification
- we use both M, N and $\langle M, N \rangle$ to denote concatenation

A secret establishment protocol - a naive attempt

- Is the protocol correct? Recall that only the users that play the roles A and B may learn the secret
- In an ideal world, maybe... but a realistic assumption on typical communication networks is:

  Security Assumption 1: the intruder is able to steal all messages sent in the protocol.

$\Rightarrow$ Use cryptography (symmetric/asymmetric encryption, signatures, hashes ...).

A secret establishment protocol - using crypto

- assume that the server playing the role S initially shares a secret key $sk(U, S)$ with each user in the system that can play the role U

$$
\begin{aligned}
&1.A \longrightarrow S: \ A, B \\
&2.S \longrightarrow A: \ \{\, K \,\}_{sk(A,S)}, \{\, K \,\}_{sk(B,S)} \\
&3.A \longrightarrow B: \ \{\, K \,\}_{sk(B,S)}, A
\end{aligned}
$$

  - where $\{\, m \,\}_{sk}$ denotes symmetric encryption of message m using the key sk

- Problems? No in case of a **passive adversary**, since:

  Perfect Cryptography Assumption : an encrypted message can be decrypted only with the appropriate key.

- Is really B, sharing K (only) with A?

  Security Assumption 2: the intruder is able to intercept and send messages to anybody, under any sender name (that is, he can impersonate any other participant in the protocol).

A secret establishment protocol - a simple attack

- so basically, we assume two things (such an adversary is called a Dolev-Yao adversary):
  - the adversary has complete control over the network
    - **interception** $A \longrightarrow O(B) : \quad m$
    - **injection** $O(A) \longrightarrow B \quad : \quad m$
  - the adversary cannot break cryptography
- we denote by $m.n$ the nth step of the mth session

$$
\begin{array}{ll}
1.1 & a \longrightarrow s : \ a, b \\
1.2 & s \longrightarrow a : \ \{\, k \,\}_{sk(a,s)}, \{\, k \,\}_{sk(b,s)} \\
1.3 & a \longrightarrow i(b) : \ \{\, k \,\}_{sk(b,s)}, a \\
2.3 & i(x) \longrightarrow b : \ \{\, k \,\}_{sk(b,s)}, x
\end{array}
$$

- The intruder i intercepts the message from a to b and replaces x for a's identity (x is any agent name).

$\rightarrow$ Problem: b believes that he is sharing the key with x, whereas he is sharing it with a.

A secret establishment protocol - a simple attack (II)

$$
\begin{array}{ll}
1.1 & a \longrightarrow s : \ a, b \\
1.2 & s \longrightarrow a : \ \{\, k \,\}_{sk(a,s)}, \{\, k \,\}_{sk(b,s)} \\
1.3 & a \longrightarrow i(b) : \ \{\, k \,\}_{sk(b,s)}, a \\
2.3 & i(x) \longrightarrow b : \ \{\, k \,\}_{sk(b,s)}, x
\end{array}
$$

- Consequences: it depends on the context in which the protocol is used, for example, b can give away to a information which should have been shared only with x
- Hence, even if i does not get k, the protocol is broken since it does not satisfy the requirement that the users should know who else knows the session key.

A secret establishment protocol - a clever attack

$$
\begin{array}{lll}
1.1 & a \longrightarrow i(s): & a, b \\
2.1 & i \longrightarrow s: & i, a \\
2.2 & s \longrightarrow i: & \{\, k \,\}_{sk(i,s)}, \{\, k \,\}_{sk(a,s)} \\
1.2 & i(s) \longrightarrow a: & \{\, k \,\}_{sk(a,s)}, \{\, k \,\}_{sk(i,s)} \\
1.3 & a \longrightarrow i(b): & \{\, k \,\}_{sk(i,s)}, a
\end{array}
$$

- i intercepts the message intended to be for s, and replaces b's identity with its own identity, so that s generates a key (a bit-string) k, and encrypts it with the keys of the two supposed users i and a
- Since a cannot distinguish between encrypted messages (if she does not have the good key!), she will not detect the alteration

A secret establishment protocol - a clever attack (II)

$$
\begin{array}{lll}
1.1 & a \longrightarrow i(s): & a, b \\
2.1 & i \longrightarrow s: & i, a \\
2.2 & s \longrightarrow i: & \{\, k \,\}_{sk(i,s)}, \{\, k \,\}_{sk(a,s)} \\
1.2 & i(s) \longrightarrow a: & \{\, k \,\}_{sk(a,s)}, \{\, k \,\}_{sk(i,s)} \\
1.3 & a \longrightarrow i(b): & \{\, k \,\}_{sk(i,s)}, a
\end{array}
$$

- Hence, a will belive that the session has been successfully completed with b, and that she is sharing the key k with b, whereas i knows this key; so i can further masquerade as b and learn all information that a intends to send to b.
- We suppose that i is a legitimate user registered to s.

Security Assumption 3: the intruder can be a legitimate protocol participant, or is able to corrupt legitimate participants (an insider), an external party (an outsider) or a combination of both.

A secret establishment protocol - a clever attempt

- to prevent this kind of attacks, a good principle is to bound cryptographically the names of the participants A and B to the key k intended to be shared by them.

$$
\begin{array}{ll}
1.A \longrightarrow S: & A, B \\
2.S \longrightarrow A: & \{\, K, B \,\}_{sk(A,S)}, \{\, K, A \,\}_{sk(B,S)} \\
3.A \longrightarrow B: & \{\, K, A \,\}_{sk(B,S)}
\end{array}
$$

- Neither of the previous two attacks does work. Can you justify that the intruder is unable to attack it by eavesdropping, altering and injecting his own messages?
- Still a problem: What about old keys...?

The intruder has memory

- The problem is that the quality of long-term encrypting keys is much better than that of the session keys generated during each exection of the protocol.
- Consequences: communications in different sessions should be separated; in particular, it should be impossible to replay messages from old sessions.

Security Assumption 4: the intruder is able to obtain the value of any "sufficiently old" session key Ks generated in a previous run of the protocol.

The intruder has memory(II)

1.1	$a \longrightarrow s$:	a, b
1.2	$s \longrightarrow a$:	$\{ k_{ab}, b \}_{sk(a,s)}, \{ k_{ab}, a \}_{sk(b,s)}$
1.3	$a \longrightarrow b$:	$\{ k_{ab}, a \}_{sk(b,s)}$

. . . extinction of dinosaurs . . .

3000.1	$a \longrightarrow i(s)$:	a, b
3000.2	$i(s) \longrightarrow a$:	$\{ k_{ab}, b \}_{sk(a,s)}, \{ k_{ab}, a \}_{sk(b,s)}$
3000.3	$a \longrightarrow b$:	$\{ k_{ab}, a \}_{sk(b,s)}$

- i masquerades as s, and replays a "very old" key k_{ab}
- Consequences:
  - By **Assumption 1**, i can be expected to know the older encrypted messages $\{ k_{ab}, b \}_{sk(a,s)}$ and $\{ k_{ab}, a \}_{sk(b,s)}$.
  - By **Assumption 4**, i can be expected to know the value of k_{ab} (by cryptanalysis ... or blackmail).
  - Thus i is able to decrypt subsequent messages encrypted with k_{ab} or even to inject messages whose integrity must be protected by k_{ab}.

The intruder has memory(III)

- Even if i does not obtain k_{ab}, the previous attack can be regarded as successful
  - i has succeded in making a and b to accept an old session key!
  - i can now replay messages protected by k_{ab} in the previous session.
- Imagine a further step in the context of the protocol:

$$4.A \longrightarrow B : \quad \{ request \}_{K_{AB}}$$

The intruder has memory(IV)

$$
\begin{array}{lll}
1.1 & a \longrightarrow s: & a, b \\
1.2 & s \longrightarrow a: & \{\, k_{ab}, b \,\}_{sk(a,s)}, \{\, a, k_{ab} \,\}_{sk(b,s)} \\
1.3 & a \longrightarrow b: & \{\, k_{ab}, a \,\}_{sk(b,s)} \\
1.4 & a \longrightarrow b: & \{\, \text{transfer 10000 euro to i} \,\}_{k_{ab}} \\
\ldots & & \\
3000.3 & i(a) \longrightarrow b: & \{\, k_{ab}, a \,\}_{sk(b,s)} \\
3000.4 & i(a) \longrightarrow b: & \{\, \text{transfer 10000 euro to i} \,\}_{k_{ab}}
\end{array}
$$

- You could say: is b memoryless (he should remember k_{ab})?

Security Assumption 5: "The Principals don't think", but they only follow the protocol.

A secret establishment protocol - using timestamps

- To prevent replays, we could add a timestamp T to each message:

$$
\begin{aligned}
1.&A \longrightarrow S: && A, B \\
2.&S \longrightarrow A: && \{ K_{AB}, B, T \}_{sk(A,S)}, \{ K_{AB}, A, T \}_{sk(B,S)} \\
3.&A \longrightarrow B: && \{ K_{AB}, A, T \}_{sk(B,S)}
\end{aligned}
$$

- B should reject messages that are older than some threshold Δ
- This requires the clocks at A and B to be synchronized
- It still leaves a window of opportunity for replay attacks (within Δ)
- Another solution: use nonces (fresh randomly-generated values intended to be used only once).

A secret establishment protocol - using nonces

$$
\begin{aligned}
1.&A \longrightarrow S: && A, B, N_A \\
2.&S \longrightarrow A: && \{\, K_{AB}, B, N_A, \{\, K_{AB}, A \,\}_{sk(B,S)} \,\}_{sk(A,S)} \\
3.&A \longrightarrow B: && \{\, K_{AB}, A \,\}_{sk(B,S)} \\
4.&B \longrightarrow A: && \{\, N_B \,\}_{K_{AB}} \\
5.&A \longrightarrow B: && \{\, N_B - 1 \,\}_{K_{AB}}
\end{aligned}
$$

- A sends her nonce N_A to S in order to get a new key
- B, once he get the new key, challenges A with a fresh nonce N_B
- N_A and N_B do not identify who created them, are just variable names!
- This is a famous protocol proposed by Needham and Schroeder. Is it secure?

A secret establishment protocol - still attacks

1. $a \longrightarrow s$: a, b, n_a
2. $s \longrightarrow a$: $\{ k_{ab}, b, n_a, \{ k_{ab}, a \}_{sk(b,s)} \}_{sk(a,s)}$
3. $a \longrightarrow b$: $\{ k_{ab}, a \}_{sk(b,s)}$
4. $b \longrightarrow a$: $\{ n_b \}_{k_{ab}}$
5. $a \longrightarrow b$: $\{ n_b - 1 \}_{k_{ab}}$

...

3000.3 $i(a) \longrightarrow b$: $\{ k_{ab}, a \}_{sk(b,s)}$

3000.4. $b \longrightarrow i(a)$: $\{ n'_b \}_{k_{ab}}$

3000.5. $i(a) \longrightarrow b$: $\{ n'_b - 1 \}_{k_{ab}}$

- the same attack as before: i masquerades a and can convince b to use an old key

A secret establishment protocol - a good protocol?

$$
\begin{aligned}
1.&B \longrightarrow A: && B, N_B \\
2.&A \longrightarrow S: && A, B, N_A, N_B \\
3.&S \longrightarrow A: && \{ K_{AB}, B, N_A \}_{sk(A,S)}, \{ K_{AB}, A, N_B \}_{sk(B,S)} \\
4.&A \longrightarrow B: && \{ K_{AB}, A, N_B \}_{sk(B,S)}
\end{aligned}
$$

- Is "the good" protocol?
- The protocol avoids all previous attack, as long as the encryption provides confidentiality and integrity...
- It is not a proof... it is just intuition.
- But neither A nor B can deduce at the end of a session that the other actually received the key K_{AB}

A secret establishment protocol - a perfect protocol?

$$
\begin{aligned}
1.&B \longrightarrow A: && B, N_B \\
2.&A \longrightarrow S: && A, B, N_A, N_B \\
3.&S \longrightarrow A: && \{\, K_{AB}, B, N_A \,\}_{sk(A,S)}, \{\, K_{AB}, A, N_B \,\}_{sk(B,S)} \\
4.&A \longrightarrow B: && \{\, K_{AB}, A, N_B \,\}_{sk(B,S)}, \{\, A, N_B \,\}_{K_{AB}} \\
5.&B \longrightarrow A: && \{\, N_B - 1 \,\}_{K_{AB}}
\end{aligned}
$$

- We added a final communication exchange that allows to both A and B to check that the other received the key K_{AB}
- Is this protocol "perfect"?

Other attacks

- All previous attacks were logical or symbolic attacks: they supposed that cryptography is perfect!
- Suppose now that K_{AB} is a password of 8 characters, and that the encryption algorithm is deterministic.
  The protocol (recall).

$$
\begin{array}{ll}
\ldots & \\
1. & b \longrightarrow a : \quad b, n_b \\
\ldots & \\
4. & a \longrightarrow b : \quad \ldots, \{\, a, n_b \,\}_{k_{ab}} \\
\ldots &
\end{array}
$$

- i generates all strings of 8 characters pwd, and since he knows both a, n_b and the encryption algorithm, he can check for which value of pwd, the message $\{\, a, n_b \,\}_{k_{ab}}$ sent during the execution of the protocol and the "computed" message $\{\, a, n_b \,\}_{pwd}$ are equal. This is called a "dictionary attack".

Needham-Schroeder protocol

- assume that each user U has a pair of public/private keys $(pk(U), sk(U))$ such that everybody knows $pk(U)$, but U is the only one to know $sk(U)$. Moreover, we denote by $[\![m]\!]_{pk}$ the asymmetric encryption of m using the public key pk.
- NS protocol (1978)

$$
\begin{array}{lll}
1. & A \longrightarrow B: & [\![A, N_A]\!]_{pk(B)} \\
2. & B \longrightarrow A: & [\![N_A, N_B]\!]_{pk(A)} \\
3. & A \longrightarrow B: & [\![N_B]\!]_{pk(B)}
\end{array}
$$

- Is really B, sharing N_A, N_B (only) with A?
- Gawin Lowe found an attack in 1995 (17 years later!). Can you see it?

Needham-Schroeder protocol - an attack

- man-in-the-middle attack (1995)

$$
\begin{array}{lll}
1.1. & a \longrightarrow i: & \{\!|a, n_a|\!\}_{pk(i)} \\
2.1. & i(a) \longrightarrow b: & \{\!|a, n_a|\!\}_{pk(b)} \\
2.2. & b \longrightarrow i(a): & \{\!|n_a, n_b|\!\}_{pk(a)} \\
1.2. & i \longrightarrow a: & \{\!|n_a, n_b|\!\}_{pk(a)} \\
1.3. & a \longrightarrow i: & \{\!|n_b|\!\}_{pk(i)} \\
2.3. & i(a) \longrightarrow b: & \{\!|n_b|\!\}_{pk(b)}
\end{array}
$$

- b "thinks" he is talking to a, while he is talking to i, and moreover, n_b (which maybe is intended to be the shared session secret) was disclosed to i

Needham-Schroeder-Lowe protocol

- Fixing the Lowe attack

$$\begin{array}{lll} 1. & A \longrightarrow B: & [\![A, N_A]\!]_{pk(B)} \\ 2. & B \longrightarrow A: & [\![N_A, N_B, B]\!]_{pk(A)} \\ 3. & A \longrightarrow B: & [\![N_B]\!]_{pk(B)} \end{array}$$

- ... and a type-flaw attack

$$\begin{array}{lll} 1.1. & i(a) \longrightarrow b: & [\![a, i]\!]_{pk(b)} \\ 1.2. & b \longrightarrow i(a): & [\![i, n_b, b]\!]_{pk(a)} \\ 2.1. & i \longrightarrow a: & [\![i, \langle n_b, b\rangle]\!]_{pk(a)} \\ 2.2. & a \longrightarrow i: & [\![\langle n_b, b\rangle, n_a, a]\!]_{pk(i)} \\ 1.3. & i(a) \longrightarrow b: & [\![n_b]\!]_{pk(b)} \end{array}$$

- To fix: change $[\![N_A, N_B, B]\!]_{pk(A)}$ to $[\![B, N_A, N_B]\!]_{pk(A)}$ or encode messages correctly (tag each field with its type)

The perfect cryptography and the real world

- And if cryptography is not perfect?
- Let us take a simple El-Gamal encryption scheme:
  - a cyclic group G of order q and generator g
  - $pk(U) = g^{sk(u)}$, and messages are elements of G: $m = g^{m'}$ for some m'
  - $[\![m]\!]_{pk} = (pk^r \times g^{m'}, g^r)$ for some randomly sampled r (where $g^{m'}$ is the encoding of m).
  - let us assume that $\langle m_1, m_2 \rangle$ is obtained by concatenating the bit-strings, i.e. is mapped to $g^{m'_1 + 2^{|m'_1|} \times m'_2}$
  - then encryption is malleable: if one knows $|m|$ and $n = g^{n'}$, then one can transform $[\![m, n]\!]_{pk}$ into $[\![m, p]\!]_{pk}$ for any known $p = g^{p'}$: if $[\![m, n]\!]_{pk} = (bs_1, bs_2)$, then $[\![m, p]\!]_{pk} = (bs_1 \times (g^{n'})^{-2^{|m'|}} \times (g^{p'})^{2^{|m'|}}, bs_2)$.

A computational attack

- ... and we get a computational attack

$$
\begin{array}{lll}
1.1. & a \longrightarrow i : & \{\!|a, n_a|\!\}_{pk(i)} \\
2.1. & i(a) \longrightarrow b : & \{\!|a, n_a|\!\}_{pk(b)} \\
2.2. & b \longrightarrow a : & \{\!|n_a, n_b, b|\!\}_{pk(a)} \\
\textit{computation step} : & & i \textit{ computes } \{\!|n_a, n_b, i|\!\}_{pk(a)} \\
1.2. & i \longrightarrow a : & \{\!|n_a, n_b, i|\!\}_{pk(a)} \\
1.3. & a \longrightarrow i : & \{\!|n_b|\!\}_{pk(i)} \\
2.3. & i(a) \longrightarrow b : & \{\!|n_b|\!\}_{pk(b)}
\end{array}
$$

- b is convinced that he is sharing a secret value n_b and that is talking with a
- Symbolic model vs. Computational model

Attacks on “simple” protocols

- although generally there at most 4 or 5 messages exchanged in a legitimate run of the prootcol, there are an infinite number of variations in which the adversary can participate
- there are several sources of infinity: number of sessions, number of messages that adversary can create, number of nonces

Types of protocol attacks

- Man-in-the-middle attack: $a \leftrightarrow i \leftrightarrow b$
- Replay (or freshness) attack: reuse parts of previous messages
- Masquerading attack: pretend to be another principal
- Reflection attack: send transmitted information back to originator
- Oracle attack: take advantage of normal protocol responses as encryption and decryption "services"
- Binding attack: using messages for a different purpose than originally intended
- Type flaw attack: substitute a different type of message field

These attack types are not formally defined and there may be overlaps between them.

Security protocols - general goals

- **Confidentiality:** Can the intruder i learn a secret that is meant to be shared only by a and b?
- **Integrity:** Can the intruder i modify a message from a and get it accepted by b ?
- **Authentication:** Can the intruder i convince b that he is talking to a?
- **Anonymity:** If a wishes to remain anonymous, can the intruder i disclose its identity?
- **Non-repudiation:** If a sends a message, can she later denies that she sent the message? Or, if b receives a message, can he later denies that he got the message?
- **Fairness:** Can one of a or b obtain an unfair advantage before the transaction is completed?

E-vote protocols - specific goals

‘

‘It’s not who votes that counts, it’s who counts the votes “
(Joseph Stalin)
“Indeed, you won the elections, but I won the count”
(Anastasio Somoza)

- **Eligibility:** Only legitimate voter can vote, and only once.
- **Vote privacy:** the vote of any honest voter is not revealed
- **Receipt-freenes:** a voter cannot prove she vote in a certain way
- **Coercion-resistance:** a voter cannot prove she vote in a certain way, even if the intruder ineracts with the voter during the voting process
- **Individual verifiability:** a voter can verify that her vote was counted
- **Universal verifiability:** the result of the vote was correctly computed
- **Fairness:** no partial results can be obtained before the end of the process vote

An e-vote protocol : Belenios - Participants

The registrar:

- generates and sends privately a signing key to each voter
- sends the corresponding verification keys to the voting server

The voters:

- select their vote, use their voting device to encrypt and sign their vote
- the resulting ballot is sent on an authenticated channel to the voting server (thanks to a login and password mechanism)
- may check at any time that their ballot is present on the bulletin board (BB)
- may revote, in which case only the last ballot is retained

The voting server:

- maintains the bulletin board (BB), that is, the list of accepted ballots
- upon receiving a ballot from a voter, it checks that the ballot is valid (e.g. the signature is valid) and adds it to the bulleting board

Decryption trustees:

a set of m decryption trustees are selected, out of which $t + 1$ are needed to decrypt the result of the election

Belenios protocol - Phase 1 : Setup

Setup

- for each voter id, the registrar generates a signing key $sk_{id} \xleftarrow{R} \mathbb{Z}_q$ and sends it privately (in practice, by email) to the voter
- the registrar transmits the corresponding list of verification keys $vk_{id1} \ldots vk_{idn}$ to the voting server, in some random order, where $vk_{id} = g^{sk_{id}}$
- the voting server publishes the list of verification keys
- the voting server generates a password pwd_{id} for each voter id and sends it privately (in practice, again by email)
- the m trustees run a "Pedersen Distributed Key Generation protocol" in order to generate the public encryption key pk of the election and pairs of private/public keys $(dk_i, pk_i^{dk_i})$ for each trustee i
- pk is published and each trustee i sends pk_i to the server.

Belenios protocol - Phase 2 : Voting

Voting

- the list of accepted ballots BB is public and is initially empty
- to vote, a voter *id* simply encrypts her vote v yielding a ciphertext $c = enc(v, pk, r)$, produces a proof $\pi = proofv(v, r, enc(v, pk, r), pk, vk_{id})$ that the vote belongs to the set of valid votes, and signs c, yielding a signature $s = sign(c, sk_{id})$. The ballot $b = ((c, \pi, s), vk_{id})$ is sent to the voting server over an authenticated channel thanks to the password pwd_{id}
- upon receiving a ballot from voter *id*, the server checks its consistency with respect to 1) the one-to-one association of the verification key vk wrt to *id* (using a *log* mechanism), 2) the signature, 3) the proof inside the ballot. Then:
  - possibly adds (id, vk) to *log*
  - if there is a ballot of the form $(b0, vk) \in$BB, then this ballot is removed
  - finally, the new (b, vk) ballot is added to BB
- at any time, voters may check that their last submitted ballot appears in the public board BB

Belenios protocol - Phase 3 : Tally

Tally

- once the voting phase is over, the list BB of accepted ballots is of the form $((c_1, \pi_1, s_1), vk_1), \ldots, ((c_p, \pi_p, s_p), vk_p)$, where the vk_j are all distinct, the proofs π_j and the signatures s_j are valid
- since the encryption used (ElGamal) to produce the ciphertexts c_i is homomorphic, anyone can compute the encrypted result

$$res_e = \prod_{i=1}^{p} c_i = enc\ (\sum_{i=1}^{p} v_i,\ pk,\ \sum_{i=1}^{p} r_i)$$

- then each trustee i (needed at least $t+1$ honest trustees) contributes to the decryption by providing $res_e^{dk_i}$ together with a proof pok of correct decryption
- from these contributions, it is possible to compute the decryption of res_e, that is $\sum_{i=1}^{p} v_i$

Belenios protocol - Properties 1

Ballot privacy

ballot privacy requires that any coalition of at most t trustees together with any coalition of corrupted users cannot infer information from ballots cast by honest voters, even after the election result is announced

Belenios **guarantees vote privacy** provided that both the registrar and the voting server are honest, that the voting device of the voter is honest, and that at most t decryption trustees are corrupted

Belenios **does not satisfy receipt-freeness**, but variants of it (BeleniosRF, BeleniosVS) do

Belenios protocol - Properties 2

Verifiability

A voting scheme is verifiable if the result corresponds to the votes of

- all honest voters that have checked that their vote was cast correctly
- at most n valid votes where n is the number of corrupted voters (i.e. the attacker may only use the corrupted voters to cast valid votes);
- a subset of the votes cast by honest voters that did not check their vote was cast correctly

Subnotions:

- Individual Verifiability (cast as intended / recorded as cast)
- Universal Verifiability (tallied as recorded)
- Eligibility Verifiability (avoid ballot stuffing)

Belenios **guarantees verifiability** provided that the registrar or the voting server are honest and the voting device of the voter is honest. The decryption trustees may all be corrupted.

Cryptographic primitives : symmetric encryption

- **Notation** $\{\!|\, m \,|\!\}_{sk}$
- **Decrypting:** one uses the same key for encryption and decryption: $dec(\{\!|\, m \,|\!\}_{sk}, sk) = m$
- **Security property:** Informally, a ciphertext cannot be decrypted without knowing the key.
- It is fast, useful to encrypt large messages
- ... but any pair of users should share a secret key
- **Examples:** DES, 3DES, AES, RC4 ,...

Cryptographic primitives : asymmetric encryption

- **Notation** $[\![m]\!]_{pk}$
- **Decrypting:** $dec([\![m]\!]_{pk}, sk) = m$, where $pk = sk^{-1}$
- **Security property:** Informally, a ciphertext encrypted with the public key cannot be decrypted without knowing the inverse private key.
- It is slower, but allows to communicate with an unknown participant (by means of a public key infrastructure PKI)
- Key exchange protocols use asymmetric encryption in order to establish a secret key that can be used by a symmetric encryption scheme
- **Examples:** RSA, ElGamal, Cramer-Shoup, OAEP+,...

Cryptographic primitives : signature

- **Notation** $[m]_{sk}$
- **Verifying:** $ver([m]_{sk}, pk) = true$, where $sk = pk^{-1}$
- **Security property:** Informally,
  - a signature that can be verified using a public key pk, cannot be created without knowing the inverse private key sk
  - a signature is associated to a unique document: it can not be used to authenticate another document
  - a signed document can not be modified
- **Examples:** RSA, DSA, ECDSA,...

Other cryptographic primitives

- A hashing function h produces for a large message m, a small message $h(m)$ called hash. **Informal "idealized" security properties:**
  - If $m_1 \neq m_2$, then $h(m_1) \neq h(m_2)$
  - Given only $h(m)$, it is impossible to find m
- Nonces, denoted $n_a, n_b, ..., n_1, n_2, ...$ are randomly generated large values. **Informal "idealized" security properties:**
  - $n_1 \neq n_2$ for any independent generated nonces
  - Given only partial information about n, it is impossible to find entire n

From attacks to proofs

- Can we be confident that a protocol given in the Alice-Bob notation is correct?
- How can we rigorously prove that the protocol achieves his security goals?
  - What does $A \longrightarrow B$ mean? How A and B parse the received messages and what exactly they do?
  - How we specify formally the security goals?
  - How do we capture the threat model?
- Using our informal notation, we can find and describe attacks.
- To prove mathematically that a given protocol satisfy a security goal, we have to move to a more formal setting

Proofs of protocols : the symbolic model

- The **Dolev-Yao model** was proposed in 1983:
  - messages are terms in a free algebra: $\{\, m \,\}_k$, $[\![m]\!]_k$, $[m]_k$, $h(m)$...
  - no secret value (nonce or key) can be guessed
  - the intruder can only apply functions in the given signature
  - ... but has complete control of the network
- One can add equations between primitives (functions), but anyway, we assume that the only equalities are those given by these equations
- Proofs in this model can be automated

Proofs of protocols : the computational model

- The **computational model** was proposed in the '80s:
  - the messages are *bitstrings*
  - the cryptographic primitives are *operations on bitstrings*
  - the intruder is any *probabilistic polynomial-time Turing machine*
  - has complete control of the network
- The model is much closer to reality, but proofs in this model are mostly manual

Proofs of protocols : (no) side channels

- The **computational model** is still an abstraction, which does not exactly match the reality. It ignores:
  - timing, power consumption, noise
  - tampering attacks
  - errors in implementation: what to answer if a message does not have the expected form?
- In this course we will ignore such kind of attacks

Proofs of protocols in the symbolic model

- One must compute **the set of terms** that the attacker can obtain.
- This set can be infinite for several reasons:
  - unbounded number of sessions executed in parallel
  - unbounded "size" of messages
  - unbounded number of fresh values (nonces)

Complexity

- If everything is unbounded, then proving security is undecidable.
- If everything is bounded, then we get a finite state transition system, and we can apply model checking!
- Bounding only the number of sessions: insecurity is typically **NP-complete**

Solutions to undecidability in the general case

- Uses approximations (generally preserving soundness):
  - abstract interpretation
  - typing
- Allow non-termination

Relevance of the symbolic model

- Advantages:
  - Numerous attacks have been found
  - An attack in the symbolic model can be easily translated in a practical atttack in the computational model
  - Proofs are simpler and can be automated
- Drawback: in general, a proof in the symbolic model, does not imply a proof in the computational model

Link between the two models

- Computational soundness theorems:

Proof in the symbolic model $\Longrightarrow$ Proof in the computational model

modulo additional assumptions (initiated by Abadi and Rogaway[2000]).

Proofs of protocols in the symbolic model

- A decidability result for intruder deduction
  - Deduction is PTIME complete
- A decidability result for secrecy for bounded number of sessions
  - Bounded protocol security is NP complete
- A undecidability result for secrecy for unbounded number of sessions
  - Proof techniques that require user input or allow non-termination

Real-world protocol standards: ISO/IEC 9798

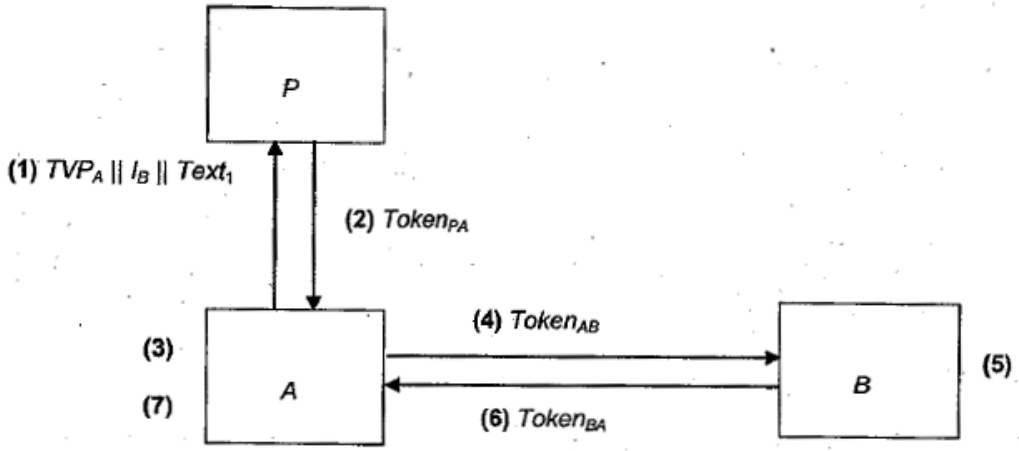


Figure 5 — Mechanism 5 — Four-pass authentication

The form of the token ($Token_{PA}$), sent by P to A, is:

$$Token_{PA} = Text_4 \| e_{K_{AP}}(TVP_A \| K_{AB} \| I_B \| Text_3) \| e_{K_{BP}}(TN_P \| K_{AB} \| I_A \| Text_2)$$

The form of the token ($Token_{AB}$), sent by A to B, is:

$$Token_{AB} = Text_6 \| e_{K_{BP}}(TN_P \| K_{AB} \| I_A \| Text_2) \| e_{K_{AB}}(TN_A \| I_B \| Text_5)$$

The form of the token ($Token_{BA}$), sent by B to A, is:

$$Token_{BA} = Text_8 \| e_{K_{AB}}(TN_B \| I_A \| Text_7)$$

The choice of using either time stamps or sequence numbers in this mechanism depends on the capabilities of the entities involved as well as on the environment.

Real-world protocol specifications: IKE RFC

When using encryption for authentication, Main Mode is defined as follows.

```
     Initiator                          Responder
     -----------                        -----------
     HDR, SA                    -->
                                <--     HDR, SA
     HDR, KE, [ HASH(1), ]
       <IDii_b>PubKey_r,
         <Ni_b>PubKey_r         -->
                                                    HDR, KE, <IDir_b>PubKey_i,
                                <--                    <Nr_b>PubKey_i
     HDR*, HASH_I               -->
                                <--     HDR*, HASH_R
```

Aggressive Mode authenticated with encryption is described as follows:

```
     Initiator                          Responder
     -----------                        -----------
     HDR, SA, [ HASH(1),] KE,
       <IDii_b>Pubkey_r,
        <Ni_b>Pubkey_r          -->
                                                    HDR, SA, KE, <IDir_b>PubKey_i,
                                <--                 <Nr_b>PubKey_i, HASH_R
     HDR, HASH_I                -->
```

Real-world protocol specifications: IKE RFC


```
  For signatures:            SKEYID = prf(Ni_b | Nr_b, g^xy)
  For public key encryption: SKEYID = prf(hash(Ni_b | Nr_b), CKY-I |
CKY-R)
  For pre-shared keys:       SKEYID = prf(pre-shared-key, Ni_b |
Nr_b)
```

The result of either Main Mode or Aggressive Mode is three groups of authenticated keying material:

```
   SKEYID_d = prf(SKEYID, g^xy | CKY-I | CKY-R | 0)
   SKEYID_a = prf(SKEYID, SKEYID_d | g^xy | CKY-I | CKY-R | 1)
   SKEYID_e = prf(SKEYID, SKEYID_a | g^xy | CKY-I | CKY-R | 2)
```

and agreed upon policy to protect further communications. The values of 0, 1, and 2 above are represented by a single octet. The key used for encryption is derived from SKEYID_e in an algorithm-specific manner (see appendix B).

To authenticate either exchange the initiator of the protocol generates HASH_I and the responder generates HASH_R where:

```
 HASH_I = prf(SKEYID, g^xi | g^xr | CKY-I | CKY-R | SAi_b | IDii_b )
 HASH_R = prf(SKEYID, g^xr | g^xi | CKY-R | CKY-I | SAi_b | IDir_b )
```

For authentication with digital signatures, HASH_I and HASH_R are signed and verified; for authentication with either public key encryption or pre-shared keys, HASH_I and HASH_R directly authenticate the exchange. The entire ID payload (including ID type, port, and protocol but excluding the generic header) is hashed into both HASH_I and HASH_R.

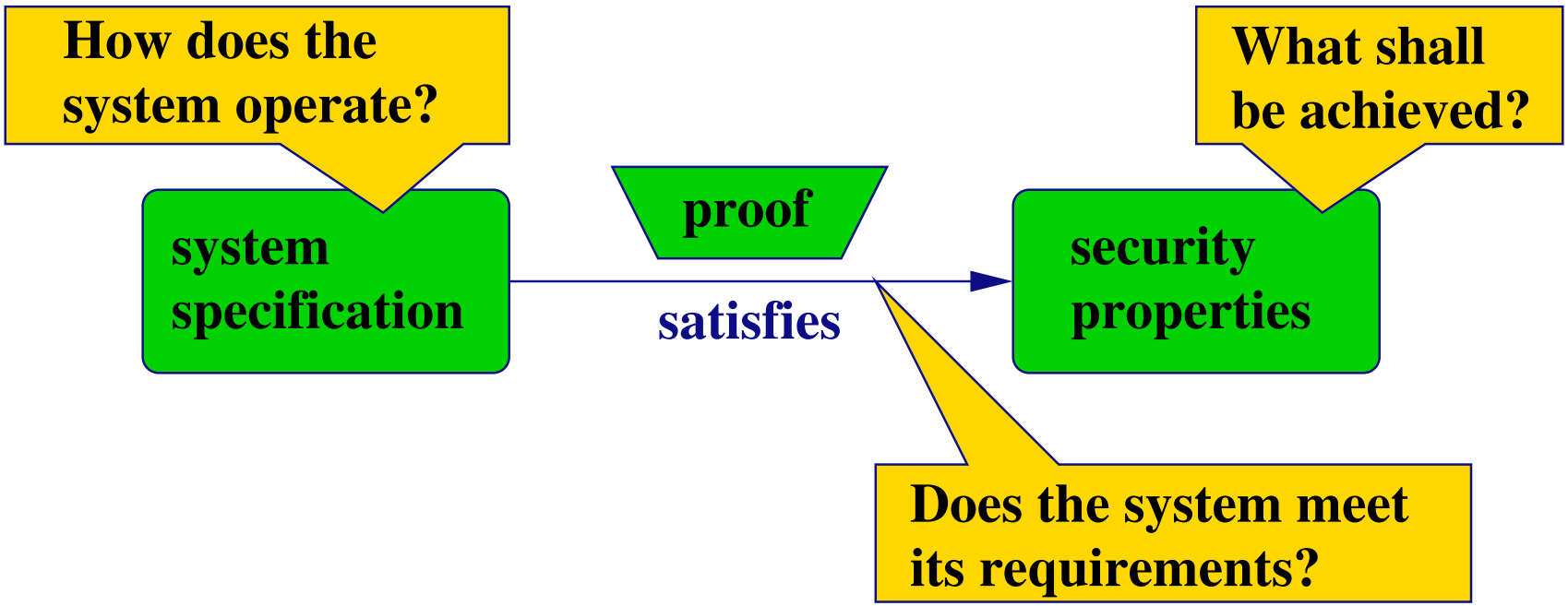
What are formal models?

- A **language** is **formal** when it has a well-defined syntax and semantics. Additionally there is often a deductive system for determining the truth of statements.
- **Examples:** propositional logic, first-order logic.
- A **model** (or **construction**) is **formal** when it is specified in a formal language.
- Standard protocol notation is not formal.
- We will see how to formalize such notations.

Formal modeling and analysis of protocols

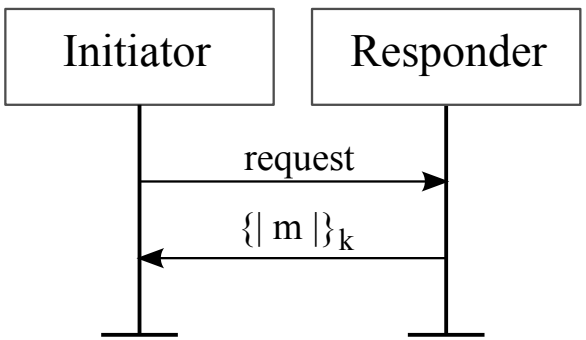
Goal:	formally model protocols and their properties and provide a mathematically sound means for reasoning about these models.
Basis:	suitable abstraction of protocols.
Analysis:	with formal methods based on mathematics and logic.

Formal Methods



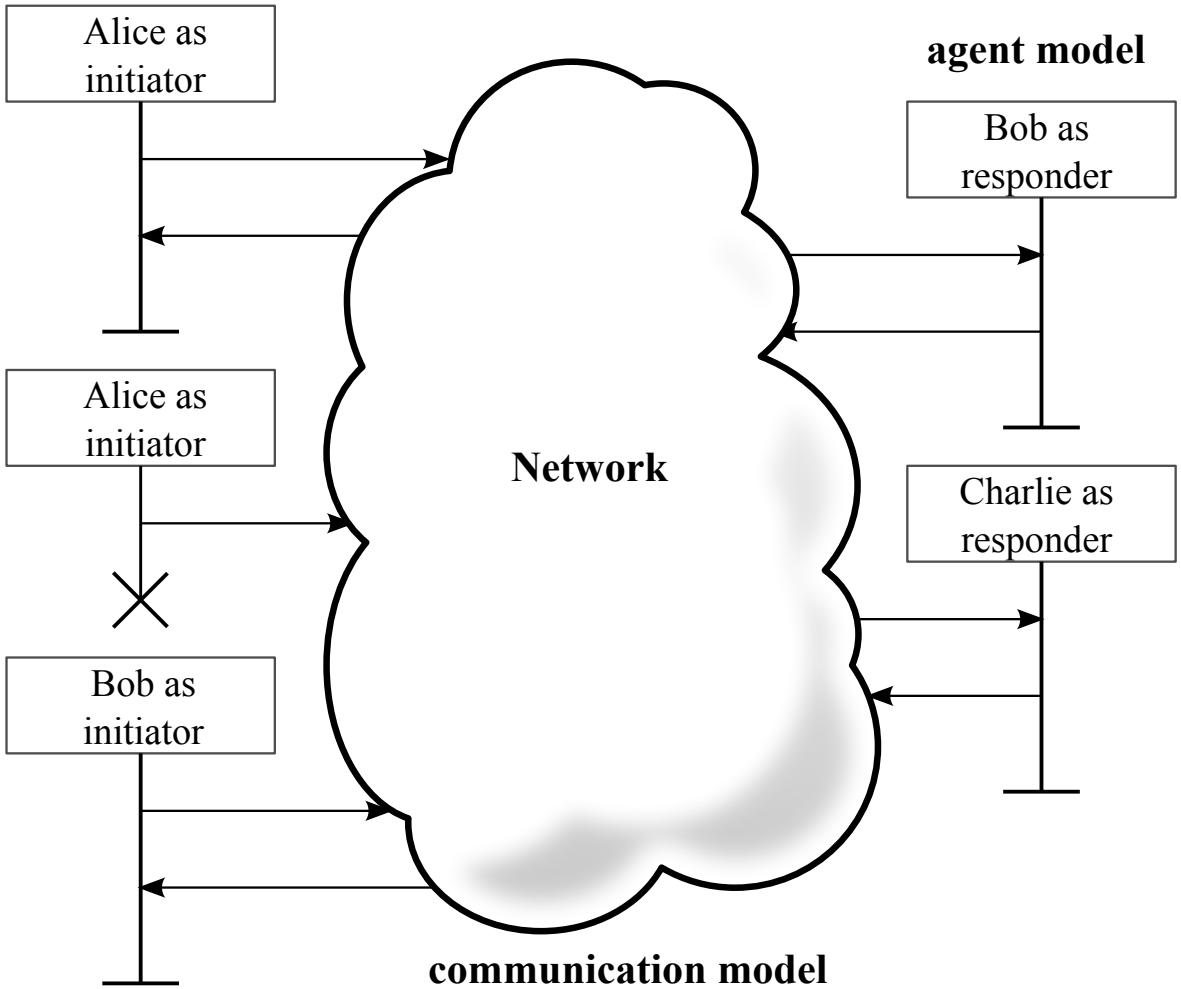
From protocol sequence charts to protocol execution

Protocol specification



cryptographic primitives

Protocol execution



Tamarin - a Term Rewriting-based tool

- Protocols and adversaries are specified as multiset rewriting rules
- Security properties are defined as trace properties, checked against the traces of the transition system

Rewriting theory

A labelled rewriting theory is a tuple $R = (\Sigma, E, \mathcal{R}, \mathcal{L})$, with:

- (Σ, E) an equational theory with Σ a signature, E a set of equations
- $\mathcal{R}$ a set of labeled rewriting rules of the general form

$$t - [lset] \rightarrow t'$$

where *lset* is a set of labels in $\mathcal{L}$ and t, t' are Σ-terms

Signature

Definition (Signature)

An unsorted signature Σ is a set of function symbols, each having an arity $n \geq 0$. We call function symbols of arity 0 constants.

Example (Peano notation for natural numbers)

$\Sigma = \{0, s, +\}$, where 0 is a constant, s has arity 1 and represents the successor function, and $+$ has arity 2 and represents addition. Note that for binary operators we sometimes will use infix notation.

Term Algebra

Definition (Term Algebra)

Let Σ be a signature, $\mathcal{X}$ a set of variables, and $\Sigma \cap \mathcal{X} = \emptyset$. We call the set $\mathcal{T}_\Sigma(\mathcal{X})$ the term algebra over Σ. It is the least set such that:

- $\mathcal{X} \subseteq \mathcal{T}_\Sigma(\mathcal{X})$.
- If $t_1, \ldots, t_n \in \mathcal{T}_\Sigma(\mathcal{X})$ and $f \in \Sigma$ with arity n, then $f(t_1, \ldots, t_n) \in \mathcal{T}_\Sigma(\mathcal{X})$.

The set of ground terms $\mathcal{T}_\Sigma$ consists of terms built without variables, i.e., $\mathcal{T}_\Sigma := \mathcal{T}_\Sigma(\emptyset)$.

Remark: constants are included in $\mathcal{T}_\Sigma$ and $\mathcal{T}_\Sigma(\mathcal{X})$.

Example (Peano notation for natural numbers (ctd.))

$s(0) \in \mathcal{T}_\Sigma$ $\quad$ $s(s(0)) + s(X) \in \mathcal{T}_\Sigma(\mathcal{X})$ $\quad$ $+s(0)+ \notin \mathcal{T}_\Sigma(\mathcal{X})$

Cryptographic Messages

We generally denote variables with upper case names $X, Y, \ldots$, and function symbols (including constants) with lower case names $a, b, f, g \ldots$

Definition (Messages)

A message is a term in $\mathcal{T}_\Sigma(\mathcal{X})$, where $\Sigma = \mathcal{A} \cup \mathcal{F} \cup \mathit{Func} \cup \{\mathit{pair}, \mathit{pk}, \mathit{aenc}, \mathit{senc}\}$. We call

$\mathcal{X}$	the set of variables A, B, X, Y, Z, ...,
$\mathcal{A}$	the set of agents a, b, c, ...,
$\mathcal{F}$	the set of fresh values na, nb, k (nonces, keys, ...),
Func	the set of user-defined functions (hash, exp, ...),
$\mathit{pair}(t_1, t_2)$	pairing, also denoted by $\langle t_1, t_2 \rangle$,
$\mathsf{pk}(t)$	public key,
$\mathit{aenc}(t_1, t_2)$	asymmetric encryption, also denoted by $\{t_2\}_{t_1}$,
$\mathit{senc}(t_1, t_2)$	symmetric encryption, also denoted by $\{\!|t_1|\!\}_{t_2}$.

Free Algebra

Definition (Free Algebra)

In the free algebra every term is interpreted by itself (syntactically).

- $t_1 =_{\text{free}} t_2$ iff $t_1 =_{\text{syntactic}} t_2$
- $a \neq_{\text{free}} b$ for any different constants a and b

Example (But symmetric cryptography enjoys the following equation E)

$\Sigma = \mathcal{A} \cup \mathcal{F} \cup \{senc, sdec\}$, with $senc$ and $sdec$ of arity 2.
(E: $sdec(senc(M, K), K) = M$)

- For above example: $sdec(senc(X, Y), Y) \neq_{\text{free}} X$.

This is too coarse, as we obviously want to identify those two terms, which means we will need to reason modulo equations.

Equational Theory

Definition (Equation)

An equation is a pair of terms, written: $t = t'$, and a set of equations E is called an equational theory (Σ, E). An equation can be oriented as $t \to t' \in \vec{E}$ or as $t \leftarrow t' \in \overleftarrow{E}$.

Equations are usually oriented from left to right for use in simplification.

Example (Peano natural numbers (ctd.))

The equations E defining the Peano natural numbers are:
$X + 0 = X$
$X + s(Y) = s(X + Y)$
Using $\vec{E}$ on $s(s(0)) + s(0)$ yields the equational derivation:
$s(s(0)) + s(0) = s(s(s(0)) + 0) = s(s(s(0)))$.

Algebraic Properties

Example (Equations E_0)

$$\{\{M\}_K\}_{(K)^{-1}} = M \qquad ((K)^{-1})^{-1} = K$$
$$\{\{\!|\{\!|M|\!\}_K|\!\}_K = M \qquad \exp(\exp(B,X),Y) = \exp(\exp(B,Y),X)$$

Definition (Congruence, Equivalence, Quotient)

A set of equations E induces a congruence relation $=_E$ on terms and thus the equivalence class $[t]_E$ of a term modulo E. The quotient algebra $\mathcal{T}_\Sigma(\mathcal{X})/_{=_E}$ interprets each term by its equivalence class.

- Two terms are semantically equal iff $t_1 =_E t_2$.
- For the above example equations (E_0):
  - $a \neq_{E_0} b$ for any distinct constants a and b
  - If $m_1 \neq_{E_0} m_2$ then $h(m_1) \neq_{E_0} h(m_2)$ (since h does not have any equation)
  - $\{\{M\}_{(K)^{-1}}\}_K =_{E_0} M$ (since $K =_{E_0} ((K)^{-1})^{-1}$)
  - $\{\{\!|\{\!|M|\!\}_{\exp(\exp(g,Y),X)}|\!\}_{\exp(\exp(g,X),Y)} =_{E_0} M$

Substitution

Definition (Substitution)

A substitution σ is a function $\sigma : \mathcal{X} \to \mathcal{T}_\Sigma(\mathcal{X})$ where $\sigma(x) \neq x$ for finitely many $x \in \mathcal{X}$.
We write substitutions in postfix notation and homomorphically extend them to a mapping $\sigma : \mathcal{T}_\Sigma(\mathcal{X}) \to \mathcal{T}_\Sigma(\mathcal{X})$ on terms:

$$f(t_1, \ldots, t_n)\sigma = f(t_1\sigma, \ldots, t_n\sigma)$$

Example (Applying a substitution)

Given substitution $\sigma = \{X \mapsto \mathit{senc}(M, K)\}$ and the term $t = \mathit{sdec}(X, K)$ we can apply the substitution and get $t\sigma = \mathit{sdec}(\mathit{senc}(M, K), K)$.

Substitution (ctd.)

Definition (Substitution composition)

We denote with $\sigma\tau$ the composition of substitutions σ and τ, i.e., $\tau \circ \sigma$.

Example (Substitution composition)

For substitutions $\sigma = [x \mapsto f(y), y \mapsto z]$ and $\tau = [y \mapsto a, z \mapsto g(b)]$ we have $\sigma\tau = [x \mapsto f(a), y \mapsto g(b), z \mapsto g(b)]$.

Position

Definition (Position)

A position p is a sequence of positive integers. The subterm $t|_p$ of a term t at position p is obtained as follows.

- If $p = [\,]$ is the empty sequence, then $t|_p = t$.
- If $p = [i\,] \cdot p'$ for a positive integer i and a sequence p', and $t = f(t_1, \ldots, t_n)$ for $f \in \Sigma$ and $1 \leq i \leq n$ then $t|_p = t_i|_{p'}$, else $t|_p$ does not exist.

Example (Position in a term)

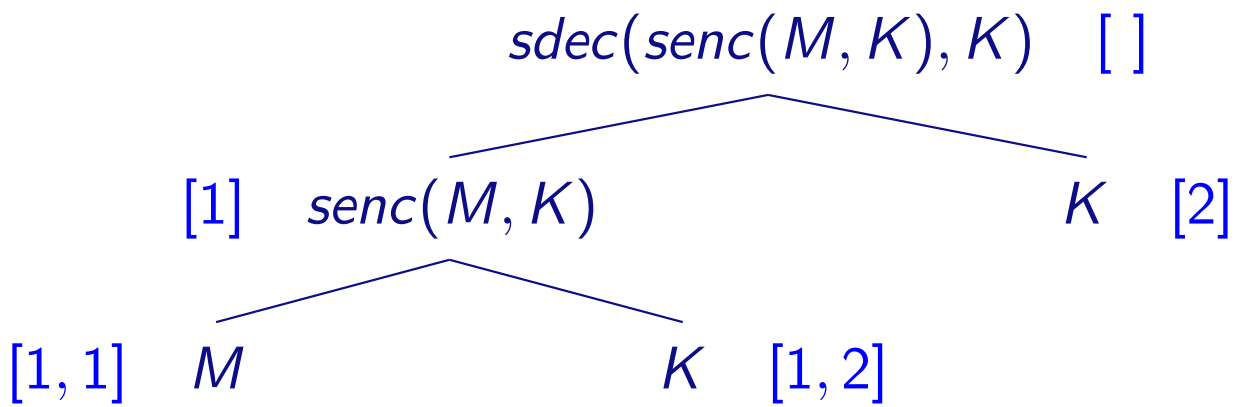
For the term $t = sdec(senc(M, K), K)$ we have five subterms:

$t|_{[\,]} = t$ $\quad$ $t|_{[1]} = senc(M, K)$ $\quad$ $t|_{[1,1]} = M$

$t|_{[1,2]} = K$ $\quad$ $t|_{[2]} = K$

Graphical representation of positions in a term

Tree of subterms of $sdec(senc(M, K))$ and their positions.



Matching and Application

Definition (Matching)

A term t matches another term l if there is a subterm of t, i.e., $t|_p$, such that there is a substitution σ so that $t|_p = l\sigma$. We call σ the matching substitution.

Example (Matching)

The term $t = A(B(a, b), c)$ matches with $l = B(x, y)$ since there is position $p = [1]$ and substitution $\sigma = [x \mapsto a, y \mapsto b]$ such that $t|_p = l\sigma$.

Matching and Application

Definition (Application of a rule)

A rule (oriented equation) $l \to r$ is applicable on a term t, if t matches l.

Given a matching substitution σ, the result of such a rule application to t is the term obtained from t by replacing the subterm $l\sigma$ at position p, by the right-hand side of the rule instantiated with the matching substitution $r\sigma$, term denoted by $t[r\sigma]_p$.

Example (Applcation of a rule)

The rule $B(x, y) \to C(y, x)$ can be applied to term $t = A(B(a, b), c)$ using the matching substitution $\sigma = [x \mapsto a, y \mapsto b]$ and it produces the term $A(C(b, a), c)$, obtained by replacing $B(a, b) = (B(x, y))\sigma$ with $C(b, a) = (C(y, x))\sigma$ in the term t.

Unification

Definition (Unification)

We say that $t \stackrel{?}{=} t'$ is unifiable in (Σ, E) for $t, t' \in \mathcal{T}_\Sigma(\mathcal{X})$, if there is a substitution σ such that $t\sigma =_E t'\sigma$ and we call σ a unifier.

For syntactic unification ($E = \emptyset$) there is a most general unifier for two unifiable terms, and it is decidable whether they are unifiable.

Unification modulo theories ($E \neq \emptyset$) is much more complicated: undecidable in general, or potentially (infinitely) many unifiers.

This is no good for automated analysis: we need to restrict ourselves.

Unification modulo theories

- When considering other algebras, unifiability is in general undecidable, e.g., associativity and distributivity.
- Even when decidable, there is in general no unique most general unifier, e.g., $\{\exp(X, Y), \exp(X', c)\}$ ...
- Some unification problems are decidable but infinitary: in general, there is an infinite set of most general unifiers, e.g., associativity.

Equational Proofs

Definition (Equality Relation)

Given (Σ, E), an E-equality step for $u, v \in \mathcal{T}_\Sigma(\mathcal{X})$ is defined as $u \rightarrow_{(\vec{E} \cup \overleftarrow{E})} v$ and denoted as $u \leftrightarrow_E v$.
The transitive-reflexive closure of $\leftrightarrow_E$ is the E-equality relation $=_E$.

Definition (Equality Proof)

A sequence of steps $t_0 \leftrightarrow_E t_1 \leftrightarrow_E \ldots \leftrightarrow_E t_n$, witnessing n-step equality of $t_0 \leftrightarrow_E^+ t_n$ is an equality proof.

Equality for Peano natural numbers

Example (Peano natural numbers - remind)

The equations E defining the Peano natural numbers are:
$X + 0 = X$
$X + s(Y) = s(X + Y)$

Example (Equality reasoning for Peano natural numbers)

Consider how to prove $s(s(0)) + s(0) = s(0) + s(s(0))$:

$\underline{s(s(0)) + s(0)} = s(\underline{s(s(0)) + 0}) = s(s(\underline{s(0)}))$

$= s(\underline{s(s(0) + 0)}) = \underline{s(s(0) + s(0))} = s(0) + s(s(0))$

Complicated! Using termination and confluence, we could have instead computed the normal form of both sides, and simply compared them! (See next slides.)

Termination of $\vec{E}$

Definition (Termination)

$(\Sigma, \vec{E})$ has infinite computations, if there is a function $a : \mathbb{N} \to \mathcal{T}_\Sigma(\mathcal{X})$ such that

$$a(0) \to_{\vec{E}} a(1) \to_{\vec{E}} a(2) \to_{\vec{E}} \ldots \to_{\vec{E}} a(n) \to_{\vec{E}} a(n+1) \ldots$$

We say it is terminating, when it does not have infinite computations.

Example (Termination)

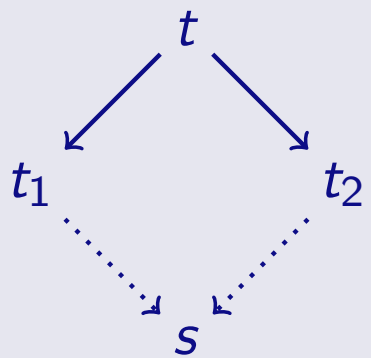
For $E = \{a = b\}$, $\vec{E}$ is terminating.
For $E = \{a = b, b = a\}$, $\vec{E}$ is not terminating.

Confluence of $\vec{E}$

Definition (Confluence)

Confluence is the property that guarantees the order of applying equalities is immaterial, formally:

$\forall t, t_1, t_2 . t \rightarrow^* t_1 \wedge t \rightarrow^* t_2 \Rightarrow \exists s . t_1 \rightarrow^* s \wedge t_2 \rightarrow^* s$



Example (Confluence)

For $E = \{a = b, a = c\}$, we have that $\vec{E}$ is not confluent, as b and c are reachable from a, but not joinable.

For $E = \{a = b, a = c, b = c\}$, then $\vec{E}$ is confluent.

Tamarin - syntax overview

Example (Tamarin Syntax)

```
theory Equations
begin
functions: h/1, senc/2, sdec/2
equations: sdec(senc(m,k),k) = m
builtins: diffie-hellman, bilinear-pairing, multiset

/* Other builtins: hashing, symmetric-encryption,
    asymmetric-encryption, signing, revealing-signing    */

/* rules, restrictions,  lemmas ... */
end
```

Tamarin supports

- any user defined equational theory that is convergent (confluent and terminating) and has the finite variant property
- built-in theories: DH exponentiation, bilinear pairing, multisets, XOR

Multiset rewriting in Tamarin

A protocol defines a *setup* and the behavior of a set of roles. Every role has a name R and consists of a set of rules, specifying the sending and receiving of messages, branching and looping conditions, and the generation of fresh constants.

In Tamarin, protocols are modeled using rewrite rules operating on multisets of facts:

$$l \xrightarrow{a} r$$

where l, a, and r are multisets of facts, l is called the left hand side, r the right hand side, and a the actions of the rule.

The left and right hand side of the rule specify which facts are consumed or produced when executing the rule, the actions are recorded as event labels on the trace and are used to specify properties.

Multiset rewriting in Tamarin: example

Example

- rule 1: $\xrightarrow{\text{Init}()} \text{A}('5'), \text{C}('3')$ ('x' is a constant)
- rule 2: $\text{A}(x) \xrightarrow{\text{Step}(x)} \text{B}(x)$

or in Tamarin syntax:

```
rule 1: [ ] --[ Init() ]-> [ A('5'), C('3') ]
rule 2: [ A(x) ] --[ Step(x) ]-> [ B(x) ]

// A rule without action:
rule 3: [ C(x) ] --> [ D(x) ]
```

Terms

Fresh terms

Agents generate fresh terms using fresh facts, denoted by Fr. These fresh terms represent randomness being used, are assumed unguessable and unique, i.e., can represent nonces.
There is a countable supply of fresh terms, each as argument of a fresh fact, usable in rules.
In Tamarin, fresh variables are prefixed with a ~, e.g., ~r.

Public terms

We define public terms to be terms known to all participants of a protocol. These include all agent names and all constants.
In Tamarin, public variables are prefixed with a $, e.g., $X.

Facts

Facts

A fact is represented by $F(t_1, \ldots, t_k)$, where F is a fact symbol and $t_1, \ldots, t_k$ are terms.

- facts can be linear or persistent: linear facts can only be consumed once, persistent facts infinitely often
- by default, facts are linear, persistent facts are marked with a !
- there are special facts:
  - *Fr* - for fresh data
  - *In* and *Out* - for protocol inputs and outputs
  - K - for attacker knowledge

Example (Key revealing and encryption)

```
rule key-reveal:
 [ !Ltk(~k) ] --[ Reveal(~k) ]-> [ Out(~k) ]

rule sym-enc: [ C(x), In(y) ] --> [ Out( senc(y,x) ) ]
```

Fresh rule

Definition (Fresh rule)

We define a special rule for the creation of fresh values. This is the only rule allowed to produce fresh values and has no precondition:

$$[] \rightarrow [\mathrm{Fr}(\sim N)]$$

Note that each created nonce N is fresh, and thus unique.

Well-formedness

Protocol rules have to be well-formed.

Definition (Well-formedness)

For a protocol rule $l \xrightarrow{a} r$ to be well-formed, the following conditions must be satisfied:

1. Only In, Fr, and other protocol facts occur in l.
2. Only Out and protocol facts occur in r.
3. Every variable in r or a that is not public must occur in l.
4. All occurrences of the same fact have the same arity, and the same persistence.

Outlook

We will define a trace semantics for protocols in terms of labeled transition systems.

Labeled Multiset Rewriting

Definition (Multiset)

A multiset is a set of elements, each imbued with a multiplicity. Instead of stating an explicit multiplicity, we may also simply write elements multiple times.
We use $\setminus^\sharp$ for the multiset difference, and $\cup^\sharp$ for the union.

Definition (Labeled multiset rewriting)

A labeled multiset rewriting rule is a triple, l, a, r, each of which is a multisets of facts, and written as:

$$l \xrightarrow{a} r$$

State

Definition (State)

A state is a multiset of facts.

Example (State)

$$\mathrm{St\_R\_1}(A, id, k_1, k_2), \mathrm{Out}(k_1), \mathrm{Out}(k_2), \mathrm{Out}(k_2)$$

Ground substitution

Definition (Ground substitutition)

A substitution is called ground when each variable is mapped to a ground term.

Definition (Ground instances)

We call the ground instances of a term t all those terms $t\sigma$ that are ground for some (ground) substitution σ .
A fact F is ground if all its terms are ground. The set of all multisets of ground facts is $\mathcal{G}^\sharp$.
For a rule, its ground instances are those where all facts are ground, and we use

$$ginsts(R)$$

for the set of all ground instances of the set of rules R.

Labeled operational semantics - single step

Definition (Steps)

For a multiset rewrite system R we define the labeled transition relation step, $\mathit{steps}(R) \subseteq \mathcal{G}^\sharp \times \mathit{ginsts}(R) \times \mathcal{G}^\sharp$, as follows:

$$\frac{I \xrightarrow{a} r \in \mathit{ginsts}(R), \qquad I \subseteq^\sharp S, \qquad S' = (S \setminus^\sharp I) \cup^\sharp r}{(S, I \xrightarrow{a} r, S') \in \mathit{steps}(R)}$$

Executions

Definition (Execution)

An execution of R is an alternating sequence

$$S_0, (l_1 \xrightarrow{a_1} r_1), S_1, \ldots, S_{k-1}(l_k \xrightarrow{a_k} r_k), S_k$$

of states and multiset rewrite rule instances with

1. $S_0 = \emptyset$
2. $\forall i : S_{i-1}, (l_i \xrightarrow{a_i} r_i), S_i \in \mathit{steps}(R)$
3. Fresh names are unique, i.e., for n fresh, and $(l_i \xrightarrow{a_i} r_i) = (l_j \xrightarrow{a_j} r_j) = ([] \rightarrow [\mathrm{Fr}(n)])$ it holds that $i = j$.
   (two different applications of the "fresh rule" produce two different unique values!)

Trace

Definition (Trace)

The trace of an execution

$$S_0, (l_1 \xrightarrow{a_1} r_1), S_1, \ldots, S_{k-1}(l_k \xrightarrow{a_k} r_k), S_k$$

is defined by the sequence of the multisets of its action labels, i.e.:

$$a_1; a_2; \ldots; a_k$$

Semantics of a rule

Two parts:

- State transition
- Trace event

Example (Transition example)

$$[\text{St\_I\_2}(A, 17, k), \text{In}(m)] \xrightarrow{\text{Recv}(A,m)} [\text{St\_I\_3}(A, 17, k, m)]$$

Effects:

- Agent state changes: $\text{St\_I\_2}(A, 17, k)$ is removed and $\text{St\_I\_3}(A, 17, k, m)$ is added to the set of facts representing the global state of the system
- $\text{In}(m)$ fact is consumed
- $\text{Recv}(A, m)$ action is added to the trace

Multiset rewriting in Tamarin: example

Example (Rules in Tamarin syntax:)

```
rule 1: [ ] --[ Init() ]-> [ A('3'), A('5'), C('3') ]
rule 2: [ A(x) ] --[ StepA(x) ]-> [ !B(x) ]
rule 3: [ B(x), B(x) ] --> [ D(x) ]                          /* silent rule */
rule 4: [ A(x), C(x) ] --[ StepA2(x), StepC2(x) ]-> [ D(x) ]
```

Example (A possible execution:)

```
[ ] --[ Init() ]->
[ A('3'), A('5'), C('3') ] --[ StepA('3') ]->
[ A('5'), C('3'), !B('3') ] -->                              /* silent step */
[ A('5'), C('3'), !B('3'), D('3') ] --[ Init() ]->
[ A('3'), A('5'), C('3'), A('5'), C('3'), !B('3'), D('3') ]
                        --[ StepA2('3'), StepC2('3') ]->
[ A('5'), A('5'), C('3'), !B('3'), D('3'), D('3') ]
```

Example (and its associated trace:)

```
[ Init() ] ; [ StepA('3') ] ; [ Init() ] ; [ StepA2('3'), StepC2('3') ]
```

Dolev-Yao adversary

- Intruder controls the network and can:
  - intercept messages
  - modify messages
  - block messages
  - generate new messages
  - insert new messages
- Perfect cryptography:
  - Abstraction with terms algebra
  - Decryption only if inverse key is known
- Protocol has
  - Arbitrary number of principals
  - Arbitrary number of parallel sessions
  - Messages with arbitrary size

Dolev-Yao Deduction

Definition (Adversary Knowledge)

We represent the adversary knowing a term t by a fact $\mathsf{K}(t)$. The adversary's knowledge $\mathcal{K}$ contains all facts of the form $\mathsf{K}(t)$, all of which are persistent.

Definition (Adversary Knowledge Derivation)

The adversary can use the following inference rules on the state:

$$\frac{\mathsf{Fr}(x)}{\mathsf{K}(x)} \quad \frac{\mathsf{Out}(x)}{\mathsf{K}(x)} \quad \frac{\mathsf{K}(x)}{\mathsf{In}(x)} \quad \frac{\mathsf{K}(t_1) \ldots \mathsf{K}(t_k)}{\mathsf{K}(f(t_1, ..., t_k))} \; \forall f \in \Sigma(\text{k-ary})$$

Terms are used modulo the equational theory

Definition (Adversary Knowledge Derivation as rewrite rules)

$$[\mathsf{Fr}(x)] \to [\mathsf{K}(x)]$$
$$[\mathsf{Out}(x)] \to [\mathsf{K}(x)]$$
$$[\mathsf{K}(x)] \xrightarrow{\mathsf{K}(x)} [\mathsf{In}(x)]$$
$$[\mathsf{K}(t_1), \ldots, \mathsf{K}(t_k)] \to [\mathsf{K}(f(t_1, \ldots, t_k))] \quad \forall f \in \Sigma(\text{k-ary})$$

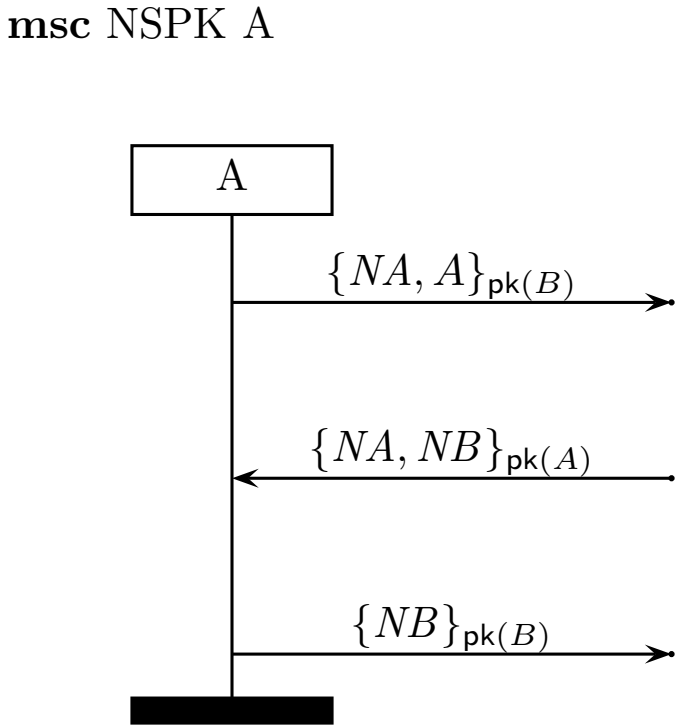
Example

Given $\mathsf{K}(x), \mathsf{K}(\{\!|b,n|\!\}_k), \mathsf{K}(k), \mathsf{K}(m) \in \mathcal{K}$. Use the equational theory E (containing decryption and pairing) to derive $\mathsf{K}(\{\!|m|\!\}_{prf(n,x)})$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\mathsf{K}(\{\!|b,n|\!\}_k) \quad \mathsf{K}(k)}{\mathsf{K}(\{\!|\{\!|b,n|\!\}_k|\!\}_k)}\,E}{\mathsf{K}(b,n)}}{\mathsf{K}(snd(b,n))}\,E}{\mathsf{K}(n)} \quad \dfrac{\mathsf{K}(x)}{\mathsf{K}(prf(n,x))}}{\mathsf{K}(m) \qquad \mathsf{K}(\{\!|m|\!\}_{prf(n,x)})}$$

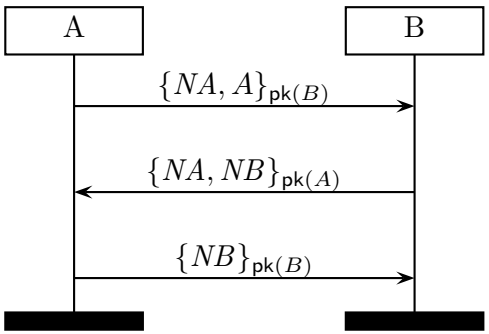
Example protocol: NSPK

Graphical:



PKIs and longterm data

msc NSPK

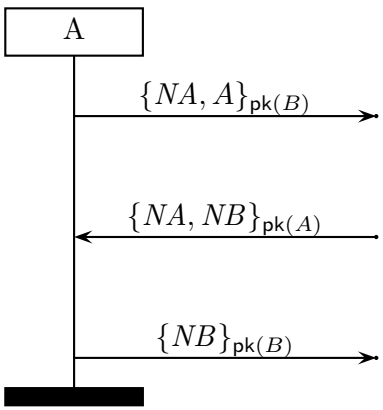


Generate longterm keys and public keys.

$$[\mathsf{Fr}(\sim skR)] \rightarrow [!\mathsf{Ltk}(R, \sim skR), \mathsf{Out}(pk(\sim skR))]$$

Initialization of protocol roles

msc NSPK A

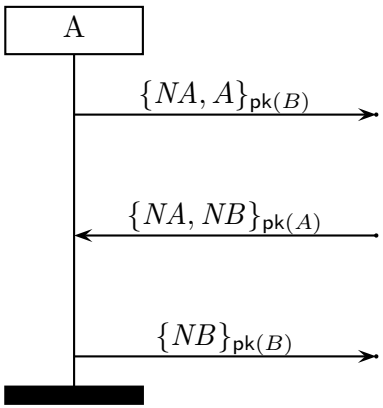


For each role R there must be an initialization rule which is instantiated with a name A and a thread identifier id:

$$[\mathsf{Fr}(\sim id), !\mathsf{Ltk}(A, skA), !\mathsf{Ltk}(B, skB)] \xrightarrow{\mathsf{Create\_I(A,\sim id)}} [\mathsf{St\_A\_1}(A, \sim id, skA, pk(skB))]$$

Protocol rules

msc NSPK A



$$[\mathsf{St\_A\_1}(A, tid, skA, pk(skB)), \quad \mathsf{Fr}(\sim NA)] \rightarrow$$
$$[\mathsf{St\_A\_2}(A, tid, skA, pk(skB), \sim NA), \quad \mathsf{Out}(\{\sim NA, A\}_{pk(skB)})]$$

$$[\mathsf{St\_A\_2}(A, tid, skA, pk(skB), NA), \quad \mathsf{In}(\{NA, NB\}_{pk(skA)})] \rightarrow$$
$$[\mathsf{St\_A\_3}(A, tid, skA, pk(skB), NA, NB)]$$

$$[\mathsf{St\_A\_3}(A, tid, skA, pk(skB), NA, NB)] \rightarrow$$
$$[\mathsf{St\_A\_4}(A, tid, skA, pk(skB), NA, NB), \quad \mathsf{Out}(\{NB\}_{pk(skB)})]$$

Be careful: pattern matching!

Protocol Properties and Correctness

What does it mean?

Properties

- Semantics of a security protocol P is a set of traces $\|P\| = \mathit{traces}(P)$. (Traces may be finite or infinite, state- or event-based.)
- Security goal / property ϕ also denotes a set of traces $\|\phi\|$.

Correctness

has an exact meaning

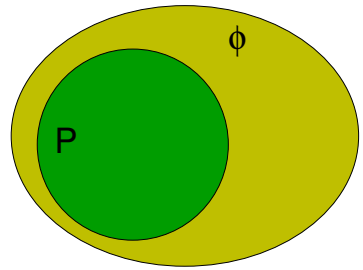
- Protocol P satisfies property ϕ, written $P \models \phi$, iff

$$\|P\| \subseteq \|\phi\|$$

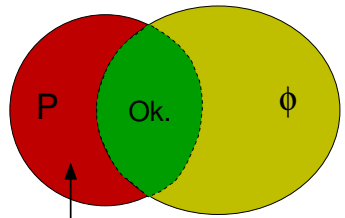
- Attack traces are those in

$$\|P\| - \|\phi\|$$

- Every correctness statement is either true or false.
- Later: how do we find attacks or prove correctness?

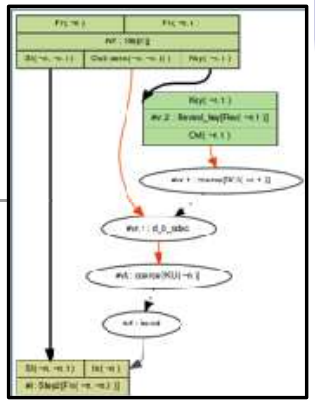


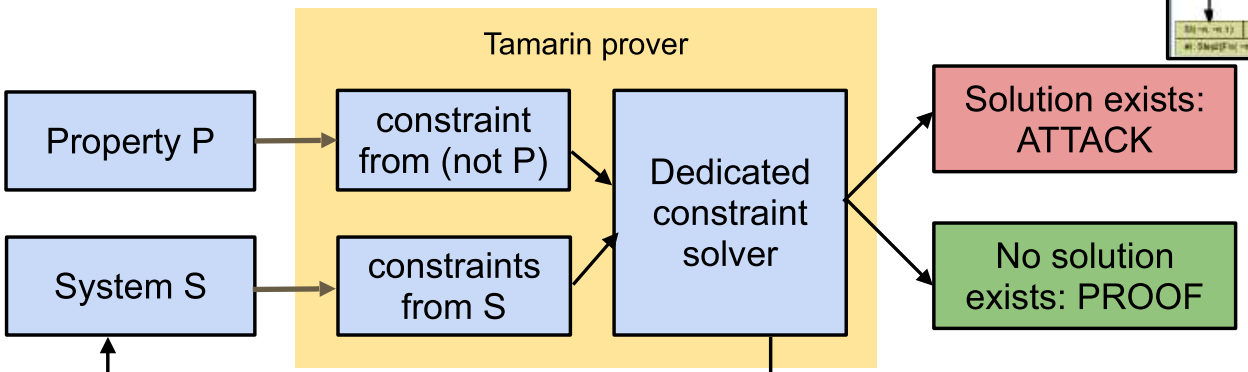
Ok, no attacks.

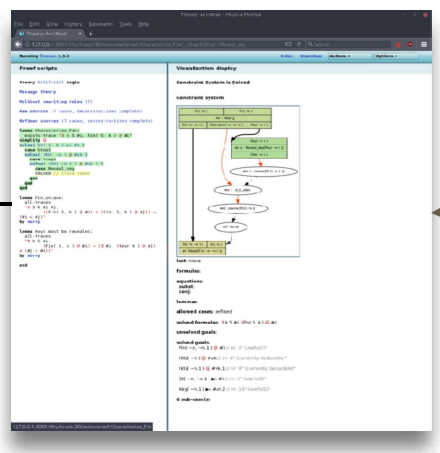


Attacks.

Tamarin Prover







Provide **hints** for the prover (e.g. invariants)

Interactive mode
Inspect partial proof

9

Taken from "Analyzing Payment Protocols with Tamarin" by David Basin

Protocol Goals

Goals

What the protocol should achieve, e.g.,

- Authenticate messages, binding them to their originator
- Ensure timeliness of messages (recent, fresh, ...)
- Guarantee secrecy of certain items (e.g., generated keys)

Most common goals

- secrecy
- authentication (many different forms)

Other goals

- anonymity, non-repudiation (of receipt, submission, delivery), fairness, availability, sender invariance, ...

Specifying properties in Tamarin

Tamarin's *property specification language* is a guarded fragment of a many-sorted first-order logic with a sort for timepoints (prefixed with #). This logic supports quantification over both messages and timepoints:

- `All` for universal quantification (temporal variables are prefixed with #)
- `Ex` for existential quantification (temporal variables are prefixed with #)
- `==>` for implication, `not` for negation
- `|` for disjunction ("or"), `&` for conjunction ("and")
- `f @ i` for action constraints (the sort prefix for the temporal variable 'i' is optional)
- `i < j` for temporal ordering (the sort prefix for the temporal variables 'i' and 'j' is optional)
- `#i = #j` for an equality between temporal variables 'i' and 'j'
- `x = y` for an equality between message variables 'x' and 'y'

Example

Example

The property that the argument `n` is distinct in all applications of a fictitious rule generating an action fact Act1(n):

```
lemma distinct_nonces:
    "All n #i #j. Act1(n)@i & Act1(n)@j ==> #i=#j"
```

or equivalently

```
lemma distinct_nonces:
  all-traces
    "All n #i #j. Act1(n)@i & Act1(n)@j ==> #i=#j"
```

These lemmas require that the property holds for all traces, we can also express that there exists a trace for which the property does not hold:

```
lemma not_distinct_nonces:
  exists-trace
    "not All n #i #j. Act1(n)@i & Act1(n)@j ==> #i=#j"
```

Guardedness

- All action fact symbols may be used in formulas.
- All variables must be guarded.

Guardedness

For universally quantified variables:

- all variables have to occur in an action constraint right after the quantifier and
- the outermost logical operator inside the quantifier has to be an implication

For existentially quantified variables:

- all variables have to occur in an action constraint right after the quantifier and
- the outermost logical operator inside the quantifier has to be a conjunction

Formalizing Security Properties

Two approaches

Direct formulation

- Formulate property ϕ directly in terms of actions occurring in protocol traces, i.e., as a set of (or predicate on) traces.
- Drawback: standard properties like secrecy and authentication become highly protocol-dependent, since they need to refer to the concrete protocol messages.

Protocol instrumentation

- Idea: insert special claim events into the protocol roles:

  $$\text{Claim\_}\texttt{claimtype}(R, t)$$

  where R is the executing role, `claimtype` indicates the type of claim, and t is a message term.
- Serve as interface to express properties independently of protocol.
- Example: $\text{Claim\_}\texttt{secret}(A, N_A)$ claims that N_A is a secret for role A, i.e., not known to the intruder.

Claim Events

Claim events are part of the protocol rules as actions.

Properties of claim events

- Their only effect is to record facts or claims in the protocol trace.
- They cannot be observed, modified, or generated by the intruder.

Expressing properties using claim events

- Properties of traces *tr* are expressed in terms of claim events and other actions (e.g., adversary knowledge K) occuring in *tr*.
- Properties are formulated from the point of view of a given role, thus yielding security guarantees for that role.
- We concentrate on secrecy and various forms of authentication, though the approach is not limited to these properties.

Role Instrumentation for Secrecy

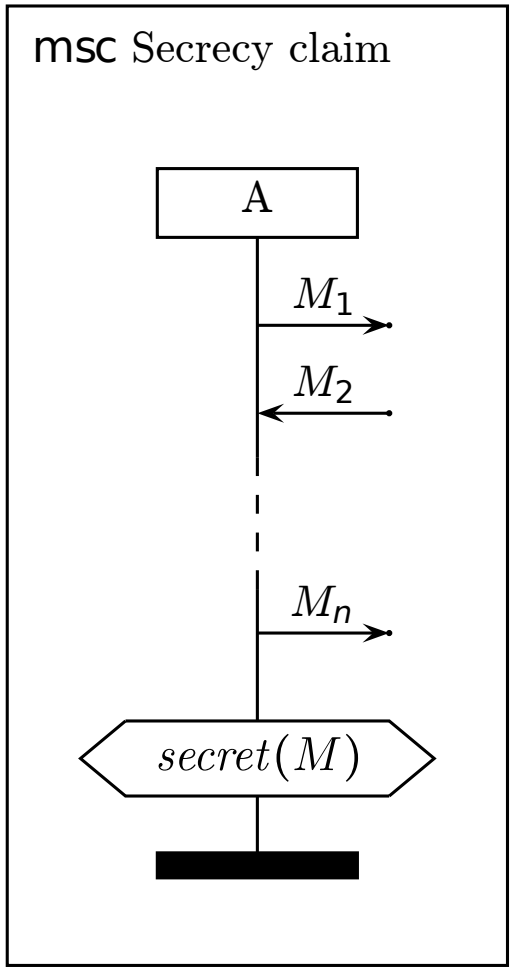
Definition (Secrecy, informally)

The intruder cannot discover the data (e.g., key, nonce, etc.) that is intended to be secret.

Role instrumentation

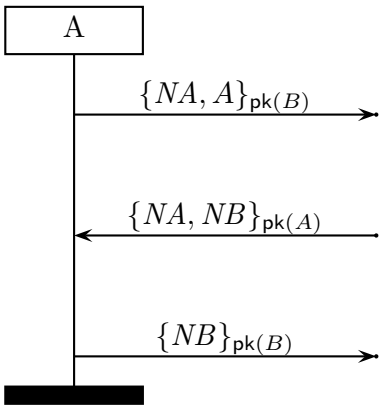
- Insert the claim event Claim_*secret*(A, M) into role A to claim that the message M used in the run remains secret.
- Position: At the end of the role.
- For instance, in NSPK, the nonces *na* and *nb* should remain secret.

Note: In the graphs, where the executing role is clear from the context, we abbreviate Claim_*claimtype*(A, t) to *claimtype*(t) inside a hexagon.



Instrumentation for Secrecy of Role A in NSPK

msc NSPK A



$$[\mathsf{St\_A\_1}(A, tid, skA, pk(skB)), \quad \mathsf{Fr}(NA)] \rightarrow$$
$$[\mathsf{St\_A\_2}(A, tid, skA, pk(skB), NA), \quad \mathsf{Out}(\{NA, A\}_{pk(skB)})]$$

$$[\mathsf{St\_A\_2}(A, tid, skA, pk(skB), NA), \quad \mathsf{In}(\{NA, NB\}_{pk(skA)})] \rightarrow$$
$$[\mathsf{St\_A\_3}(A, tid, skA, pk(skB), NA, NB)]$$

$$[\mathsf{St\_A\_3}(A, tid, skA, pk(skB), NA, NB)] \xrightarrow{\mathsf{Claim\_secret}(A,NB)}$$
$$[\mathsf{St\_A\_4}(A, tid, skA, pk(skB), NA, NB), \quad \mathsf{Out}(\{NB\}_{pk(skB)})]$$

Adding Claim_secret to role A with respect to NB

Formalization of Secrecy

Definition (Secrecy, first attempt)

The secrecy property consists of all traces tr satisfying

$$\forall A, M, i.\ \text{Claim\_}secret(A, M)@i \Rightarrow \neg(\exists j.\mathrm{K}(M)@j)$$

- Let $tr = tr_1; tr_2; \ldots; tr_k; \ldots; tr_n$. We write $x@k$ as a shorthand for $x \in tr_k$.
- Can only require M to remain secret if A runs the protocol with another honest agent, i.e.,
- Trivially broken whenever A or B is instantiated with a compromised agent, since then the adversary rightfully knows M.
- This definition is fine for a passive adversary, who only observes network traffic, but does not act as a protocol participant.

Compromised Agent

Definition (Compromised Agent)

A compromised agent is under adversary control, i.e., sharing all its information with the adversary and participating in protocols upon its direction. We model this by having the agent give its initial secret information to the adversary, which can then mimic the agent's actions.

We note the fact that an agent A is compromised by a Rev event in the trace, attached to the rule that passes its initial secrets (here the private key $\mathrm{Ltk}(A, skA)$) to the adversary:

$$[!\mathrm{Ltk}(A, skA)] \xrightarrow{\mathrm{Rev(A)}} [\mathrm{Out}(skA)]$$

Exercise: convince yourself that, given the agent's secret, the adversary is capable of performing all of the agent's send and receive steps.

Formalization of Secrecy

Definition (Honesty)

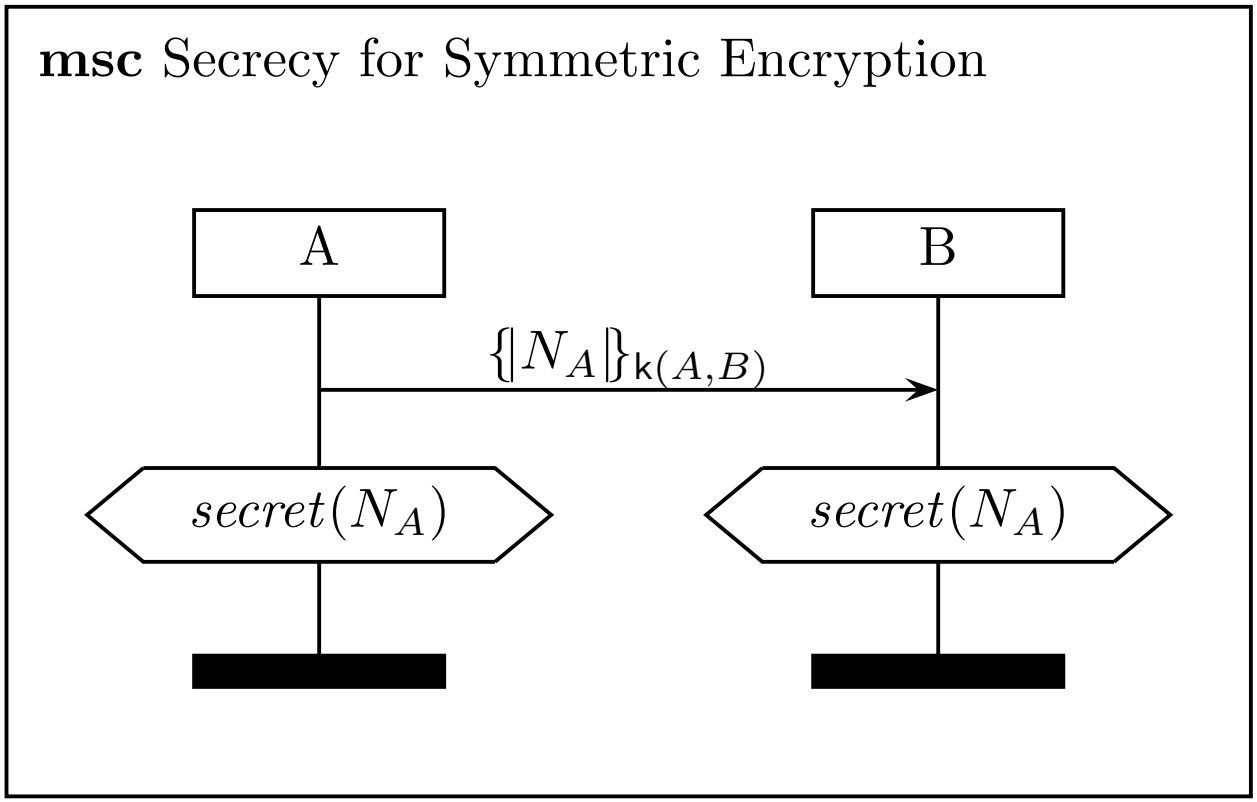
An agent A is honest in a trace tr when $\mathrm{Rev}(A) \notin tr$.
When making a claim in a rule action, all parties B that are expected to be honest need to be listed with a $\mathrm{Honest}(B)$ action in that rule.

Definition (Secrecy)

The secrecy property consists of all traces tr satisfying

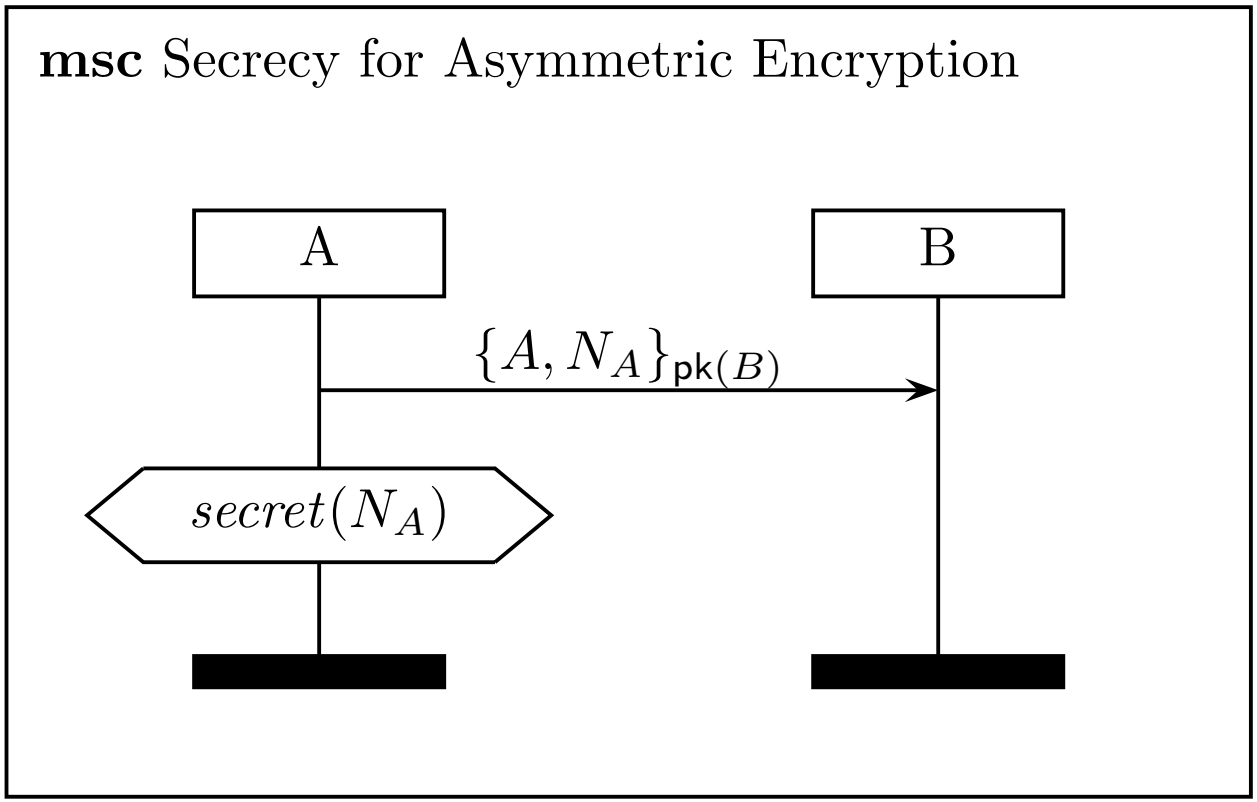
$$\forall A\ M\ i.\ (\mathrm{Claim\_}secret(A, M)@i)$$
$$\Rightarrow (\neg(\exists j.\mathrm{K}(M)@j) \vee (\exists B\ j.\mathrm{Rev}(B)@j \wedge \mathrm{Honest}(B)@i))$$

Secrecy Example #1



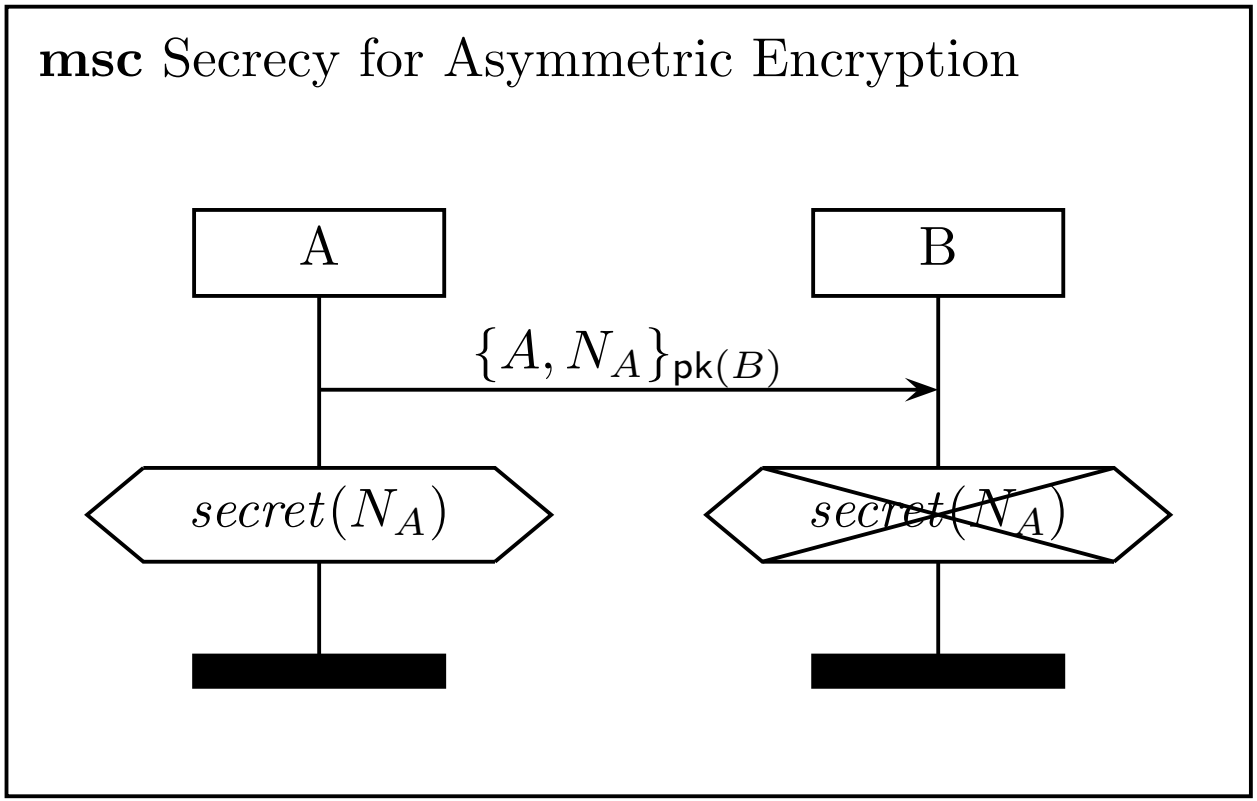
- This is fine: secrecy holds for both A and B.
- We omit the obvious annotations Honest(A), Honest(B) in message sequence charts for 2-party protocols.

Secrecy Example #2



- Secrecy holds for A: she knows that only B can decrypt message.

Secrecy Example #2



- Secrecy fails for B: he does not know who encrypted message!
- ... unless one can authenticate the origin of the message

Authentication

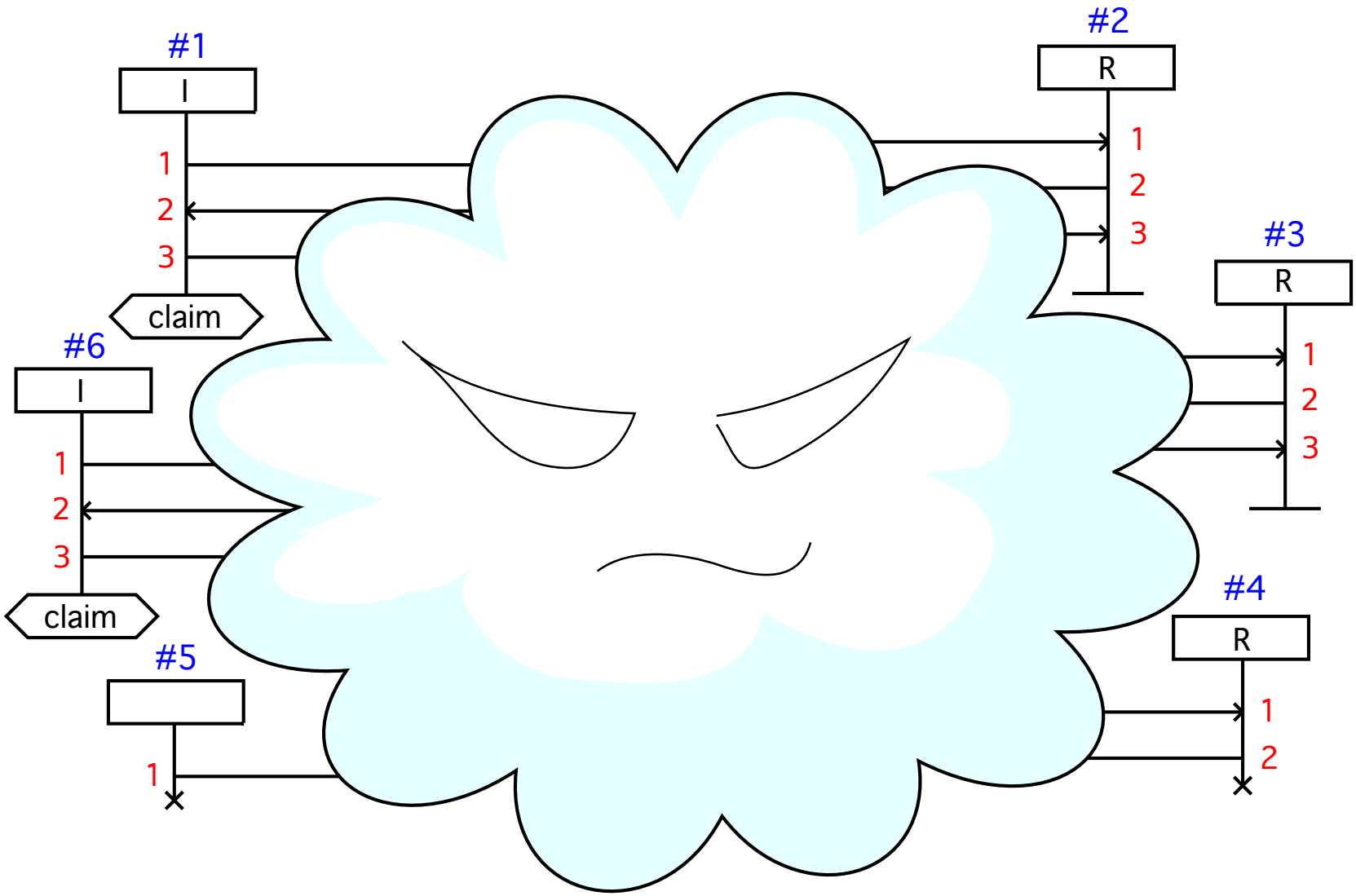
Which authentication are you talking about?

- No unique definition of authentication, but a variety of different forms.
- Considerable effort has been devoted to specifying and classifying, semi-formally or formally, different forms of authentication (e.g., by Cervesato/Syverson, Clark/Jacob, Gollmann, Lowe, Cremers et al.).

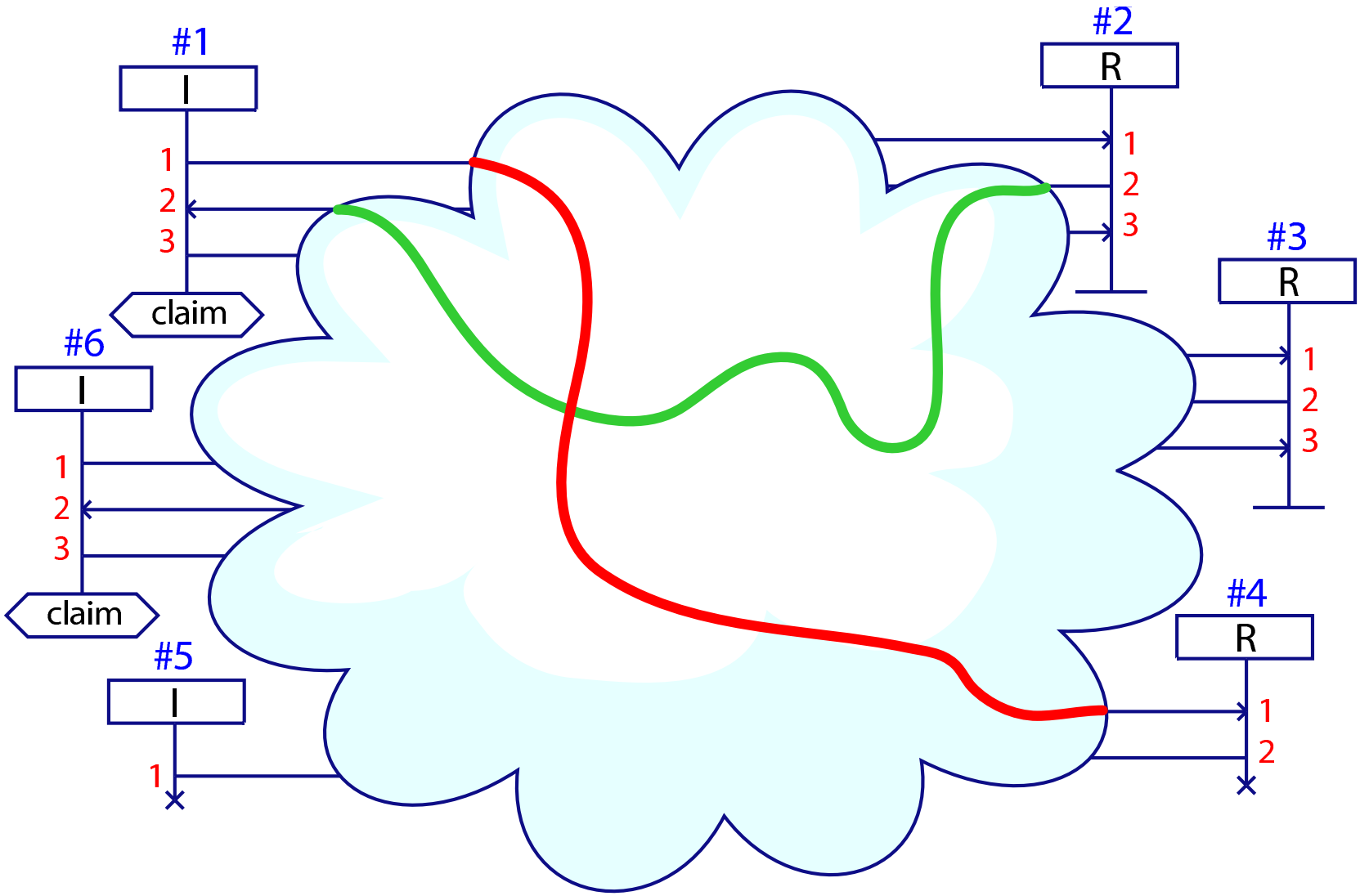
Examples

- ping authentication, aliveness, weak agreement, non-injective agreement, injective agreement, weak and strong authentication, synchronization, and matching histories.

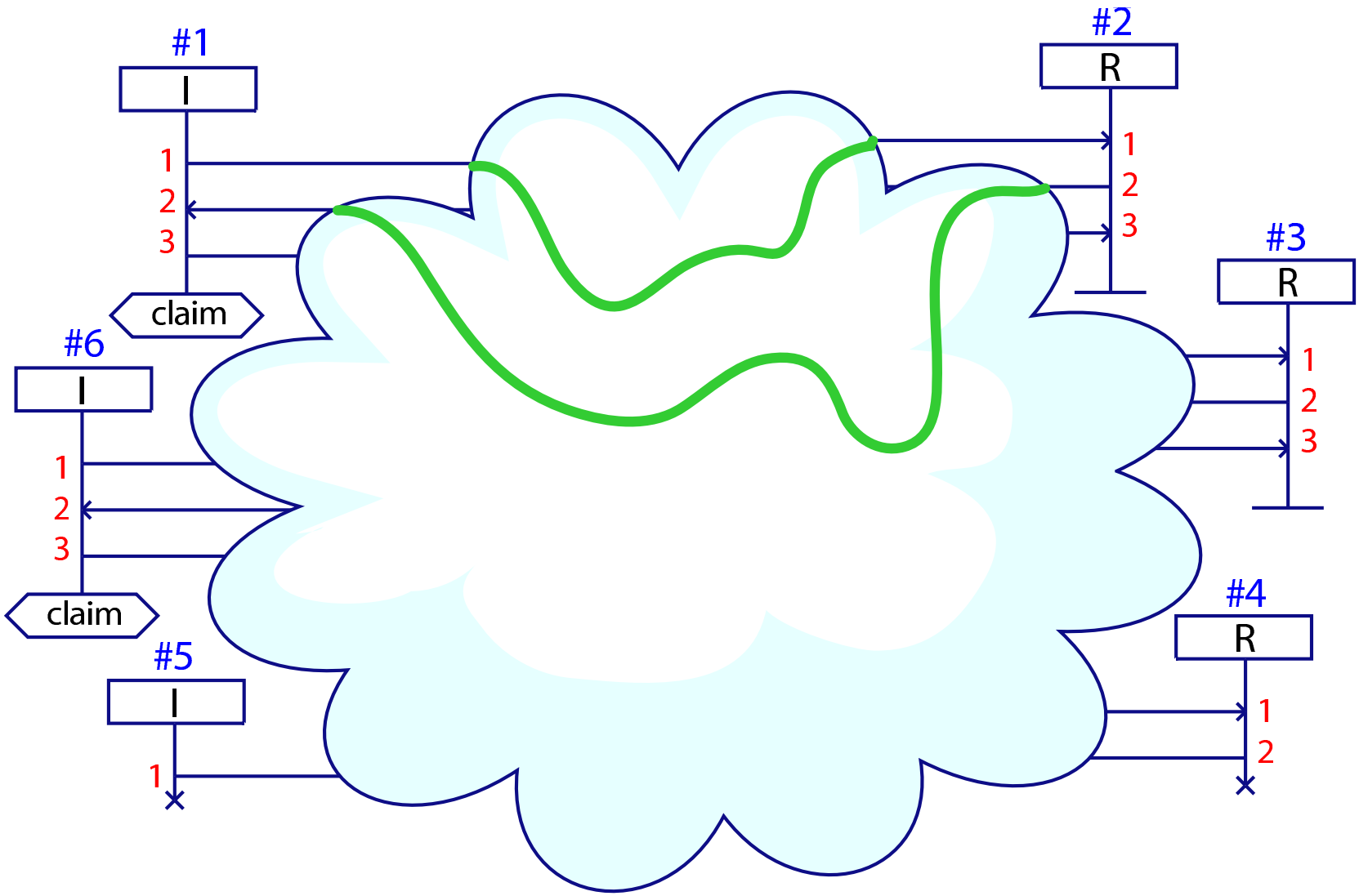
A Picture of the World



Failed Authentication



Successful Authentication



A Hierarchy of Authentication Specifications (1)

[Gavin Lowe, 1997]

Gavin Lowe has defined the following hierarchy of increasingly stronger authentication properties[1]:

Aliveness A protocol guarantees to an agent a in role A aliveness of another agent b if, whenever a completes a run of the protocol, apparently with b in role B, then b has previously been running the protocol.

Weak agreement A protocol guarantees to an agent a in role A weak agreement with another agent b if, whenever agent a completes a run of the protocol, apparently with b in role B, then b has previously been running the protocol, apparently with a.

[1] Terminology and notation slightly adapted to our setting.

A Hierarchy of Authentication Specifications (2)

[Gavin Lowe, 1997]

Non-injective agreement A protocol guarantees to an agent a in role A non-injective agreement with an agent b in role B on a message M if, whenever a completes a run of the protocol, apparently with b in role B, then b has previously been running the protocol, apparently with a, and b was acting in role B in his run, and the two principals agreed on the message M.

Injective agreement is non-injective agreement where additionally each run of agent a in role A corresponds to a unique run of agent b.

Also versions including recentness: insist that B's run was recent (e.g., within t time units).

These are quite complex properties. How can we formalize them?

Role Instrumentation for Authentication

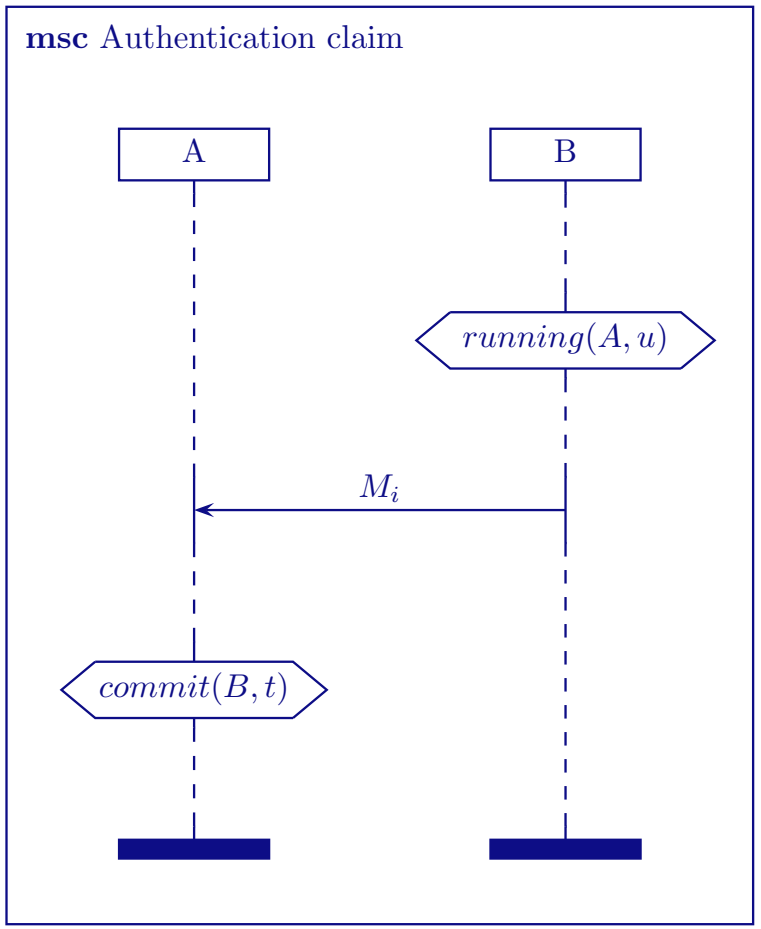
We use two claims to express that role A authenticates role B on t:

In role A:

- Insert a commit claim event Claim_commit(A, B, t).
- Position: after A can construct t. Typically, at end of A's role.

In role B:

- Insert a running claim event Claim_running(B, A, u).
- Term u is B's view of t.
- Position: after B can construct u and causally preceding Claim_commit(A, B, t).



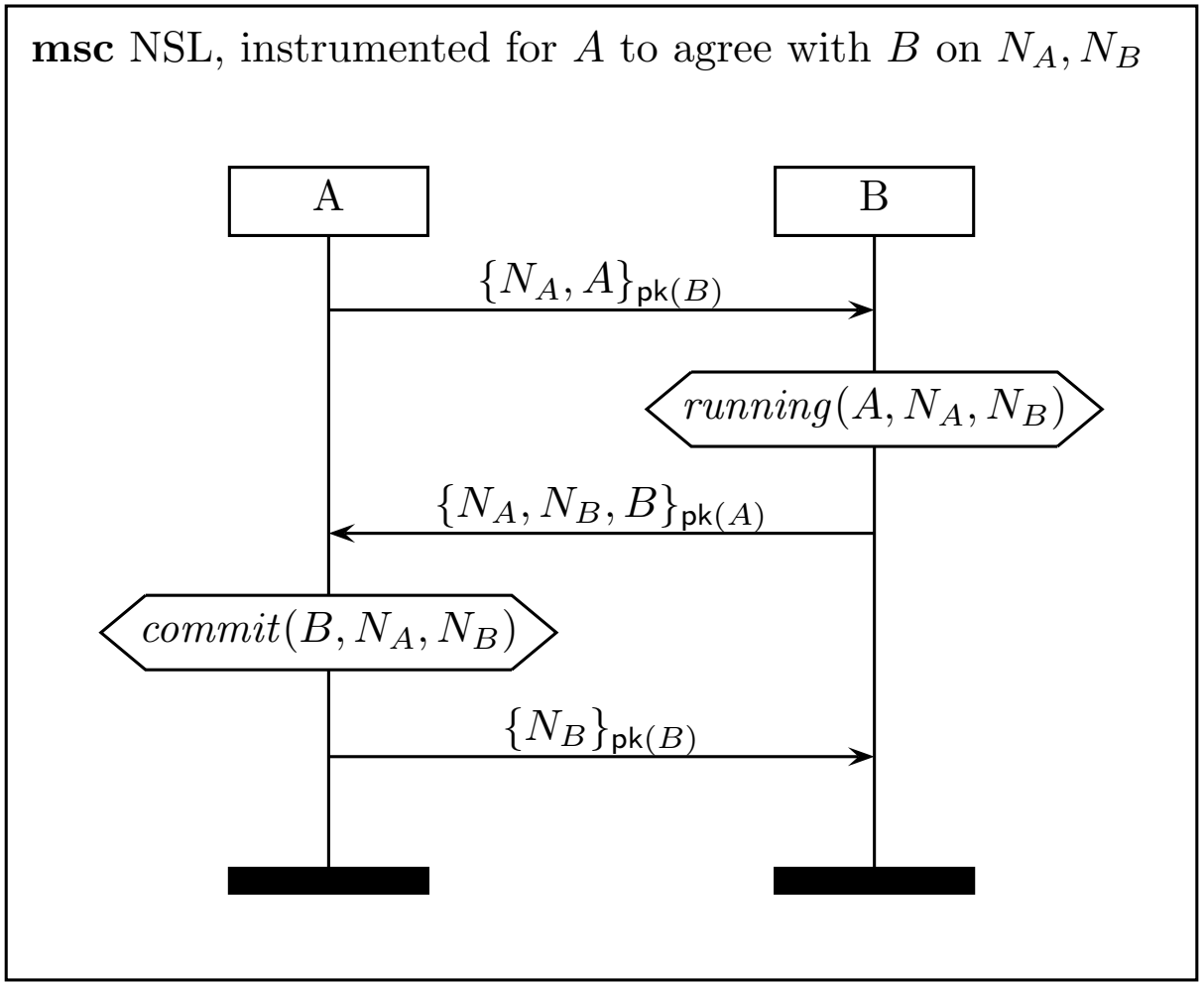
Definition (Non-injective agreement)

The property $Agreement_{NI}(A, B, t)$ consists of all traces satisfying

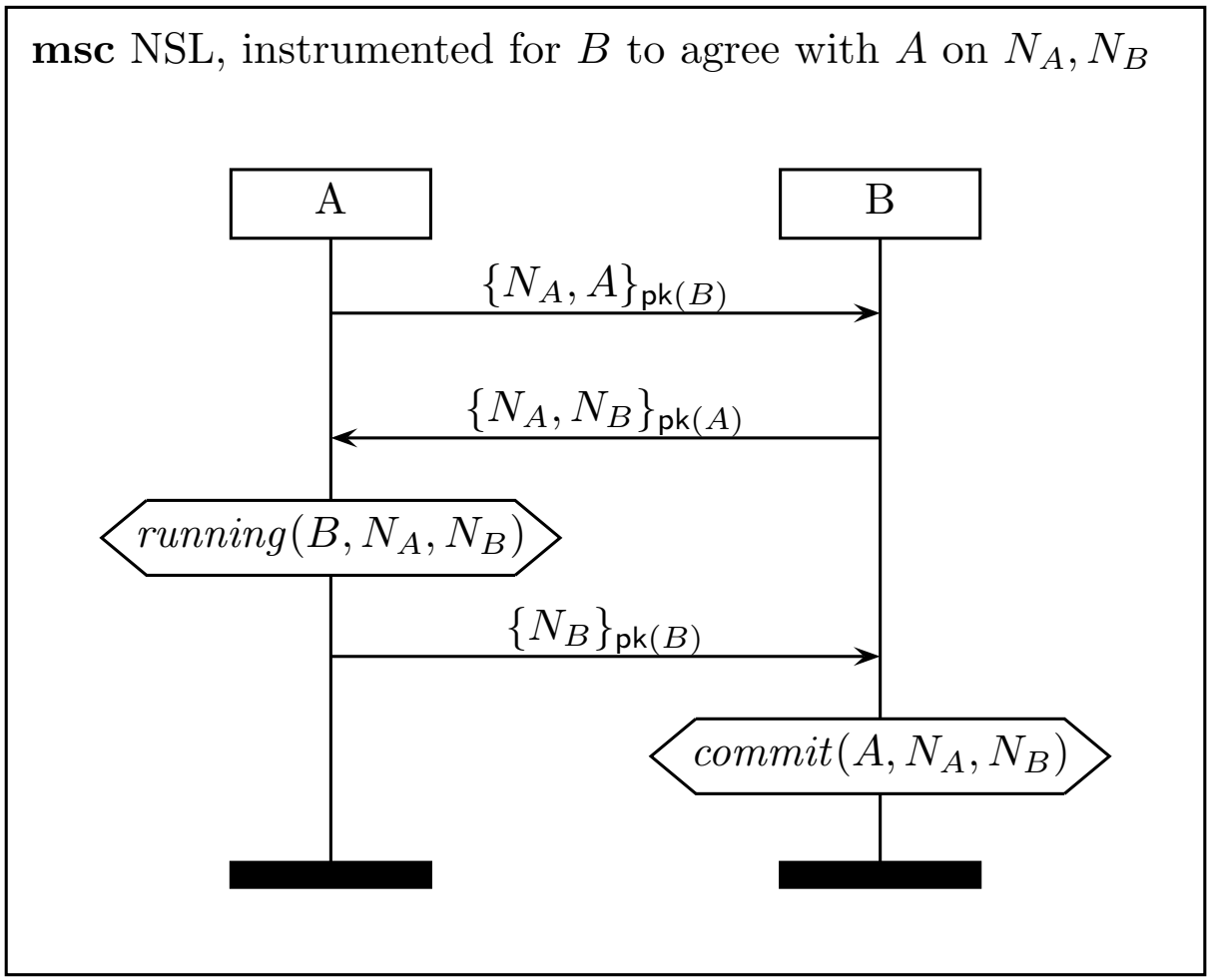
$$\begin{aligned}\forall a\ b\ t\ i.\quad & \text{Claim\_}\texttt{commit}(a, b, \langle A, B, t\rangle)@i \\ & \Rightarrow (\exists j.\text{Claim\_}\texttt{running}(b, a, \langle A, B, t\rangle)@j \wedge j < i) \\ & \quad \vee (\exists X\ r.Rev(X)@r \wedge Honest(X)@i)\end{aligned}$$

- Whenever a commit claim is made with honest agents a and b, then the peer b must be running with the same parameter t, or the adversary has compromised at least one of the two agents.

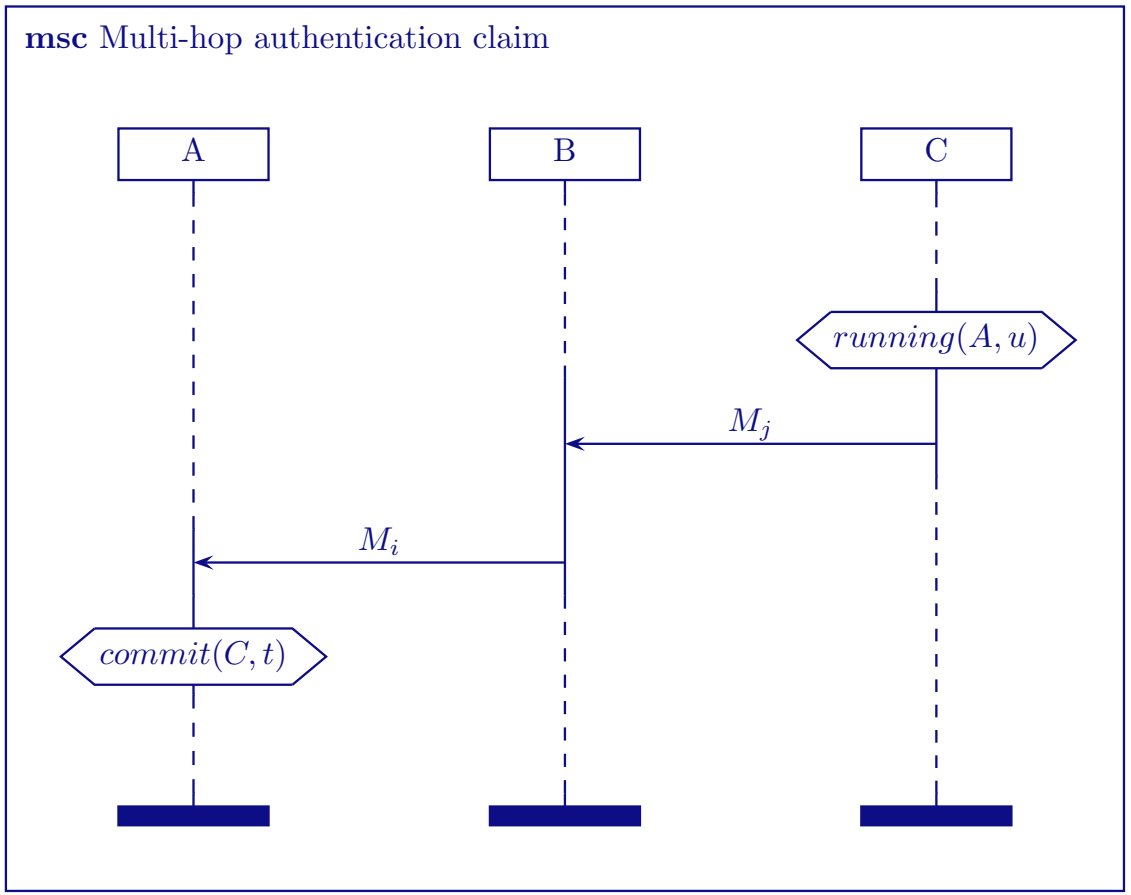
Example: NSL Protocol (1/2)



Example: NSL Protocol (2/2)

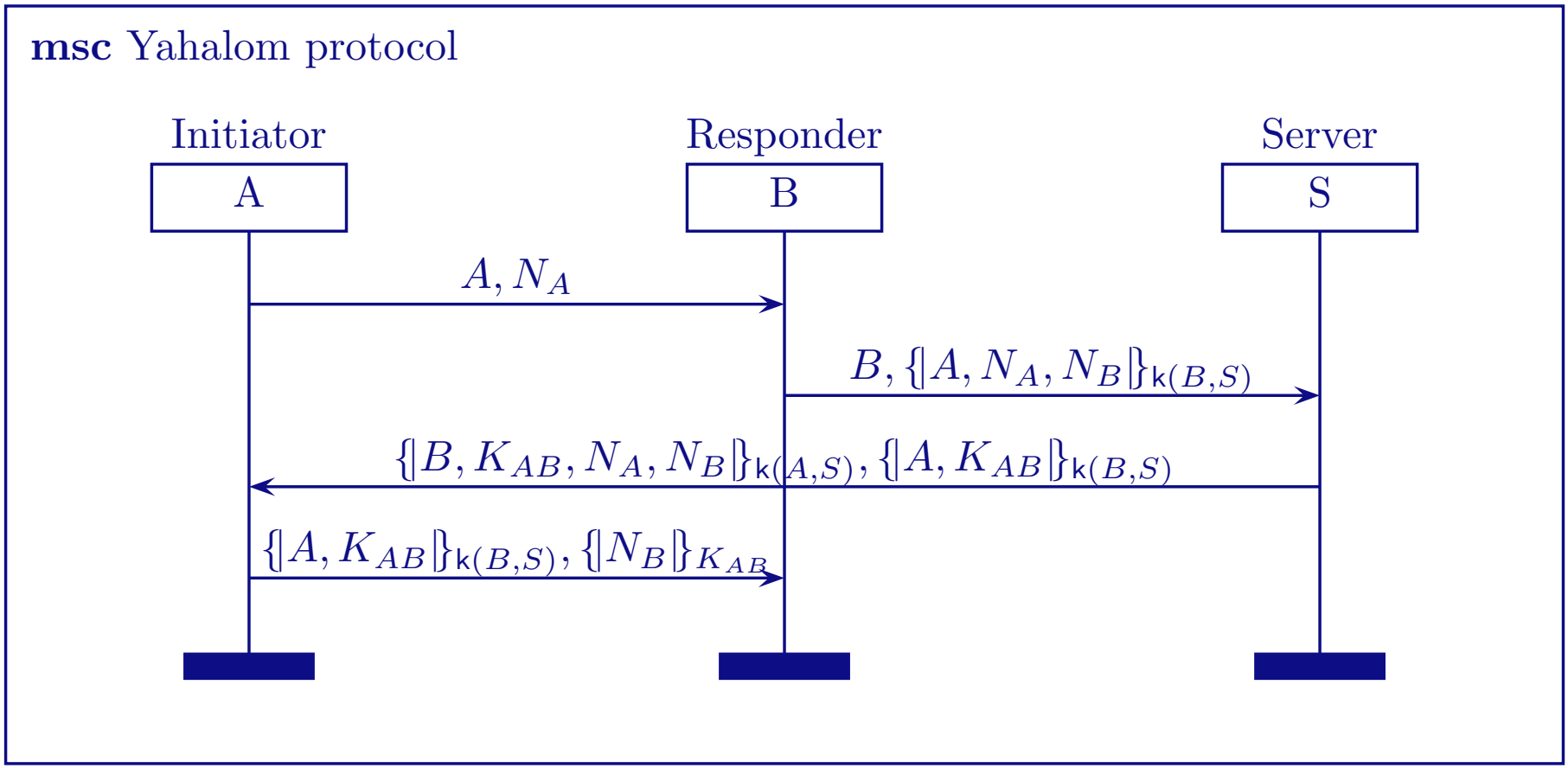


Role Instrumentation for Authentication (cont.)

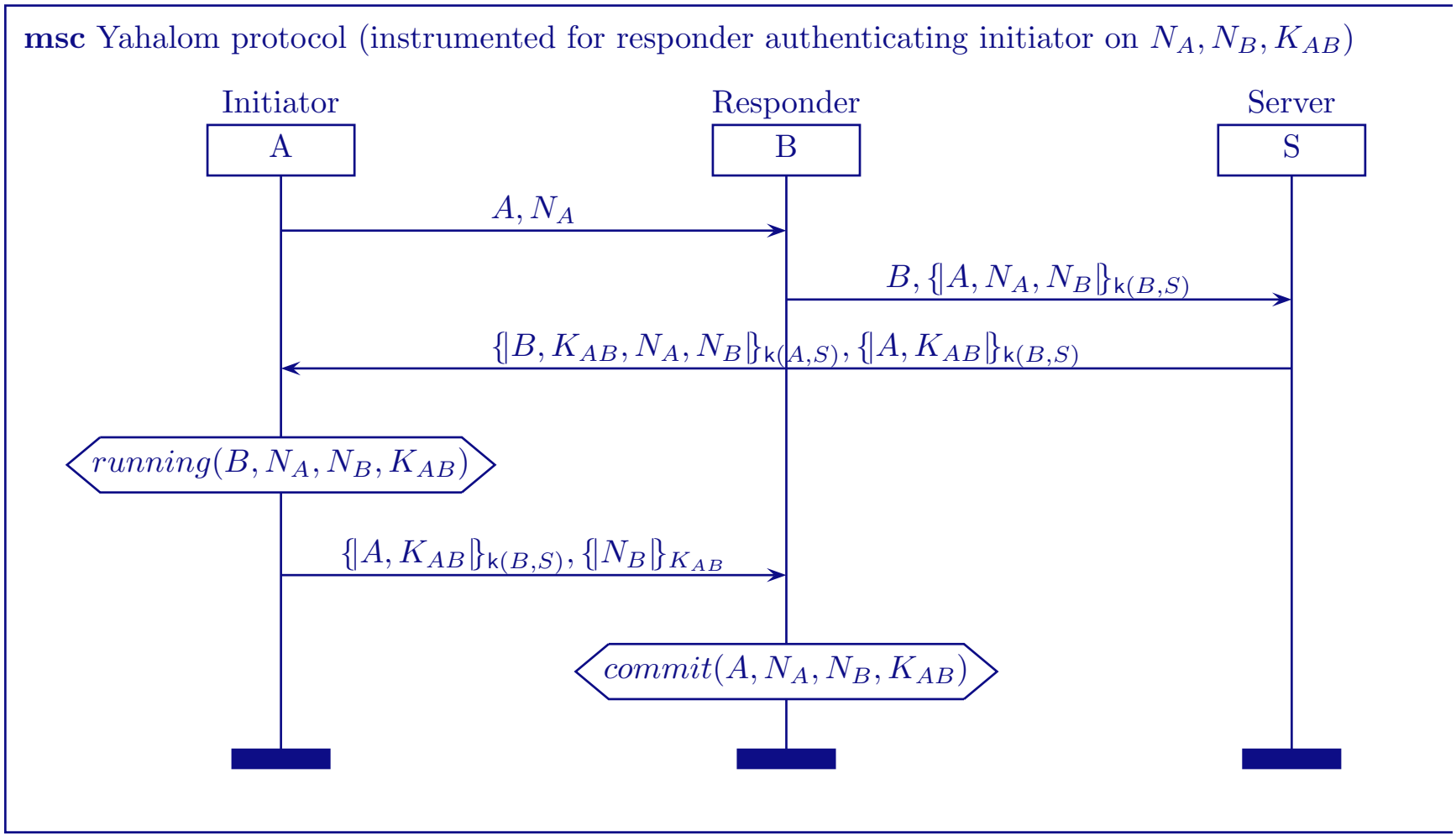


Event causality in multi-hop authentication claims: The *running* event must causally precede the *commit* event and the messages t and u must be known at the position of the claim event in the respective role.

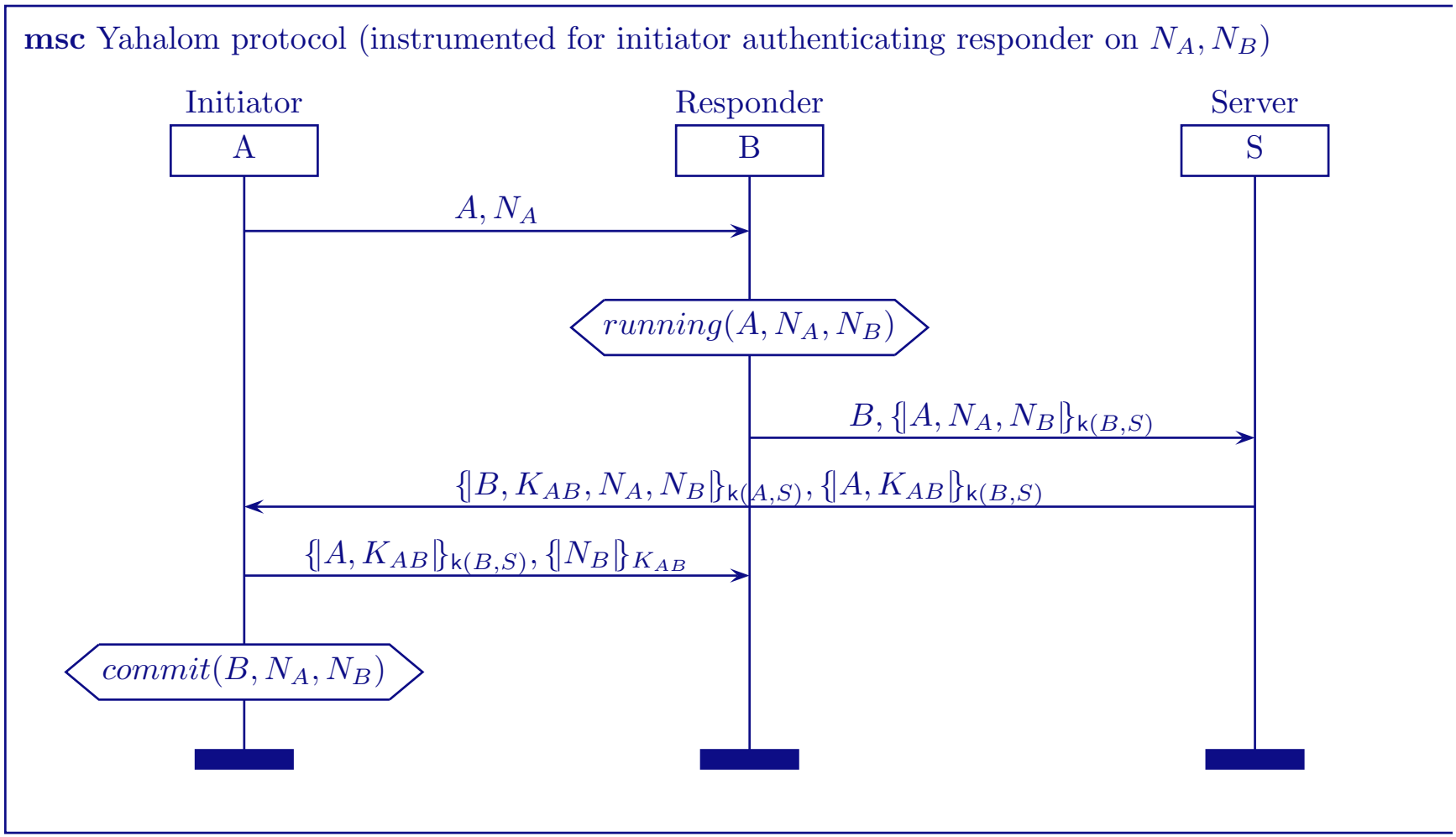
Example: Yahalom Protocol (1/3)



Example: Yahalom Protocol (2/3)



Example: Yahalom Protocol (3/3)



Note: agreement for A on K_{AB} is not possible, since B gets K_{AB} after A.

Definition (Injective agreement)

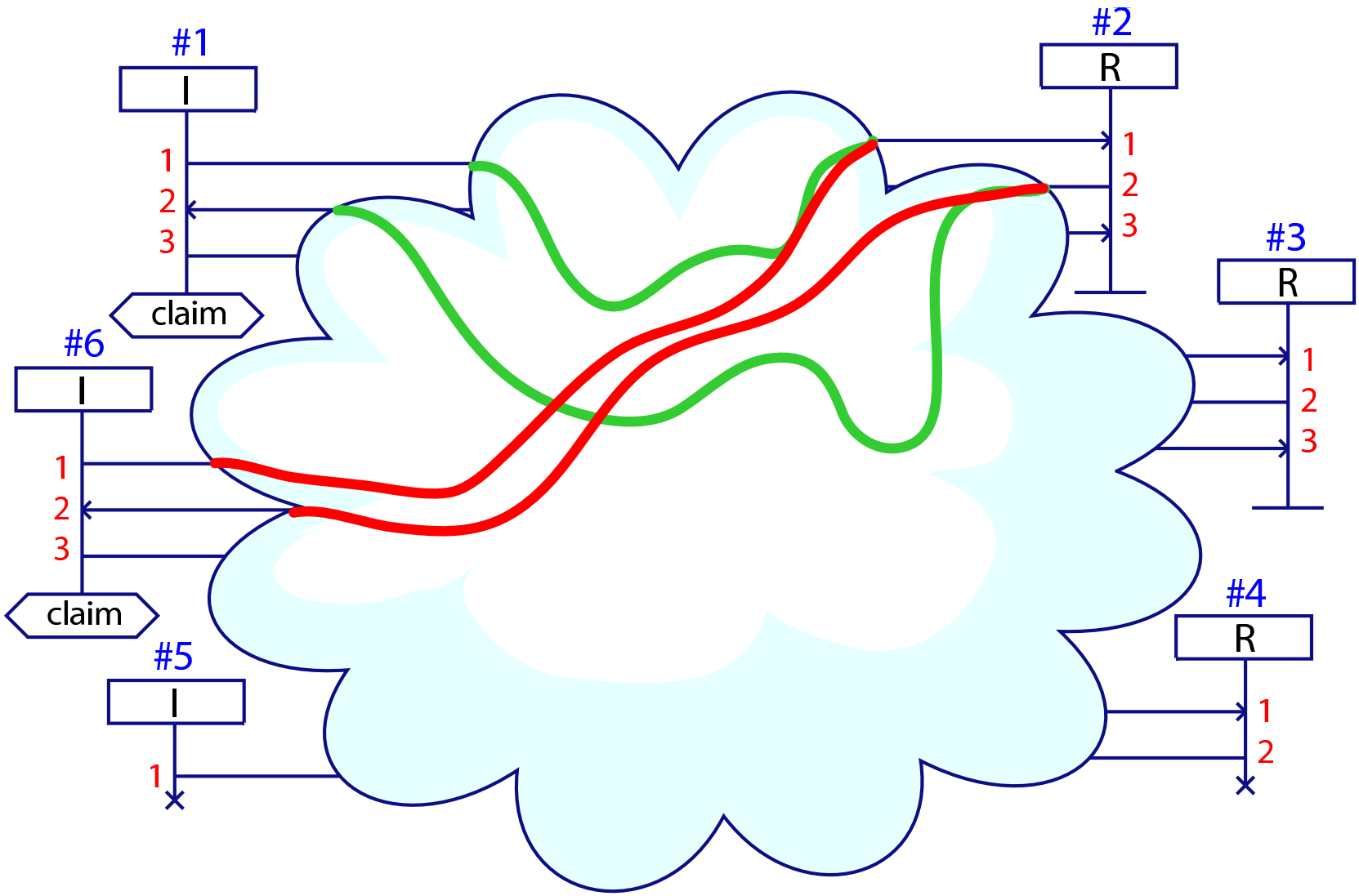
The property $\mathit{Agreement}(A, B, t)$ consists of all traces satisfying:

$$
\begin{aligned}
\forall a\ b\ t\ i.\quad & \mathrm{Claim\_commit}(a, b, \langle A, B, t\rangle)@i \\
& \Rightarrow (\exists j.\mathrm{Claim\_running}(b, a, \langle A, B, t\rangle)@j \wedge j < i \\
& \quad \wedge\neg(\exists a_2\, b_2\, i_2.\mathrm{Claim\_commit}(a_2, b_2, \langle A, B, t\rangle)@i_2 \\
& \qquad \wedge\neg(i_2 = i) \\
& \quad) \\
&) \\
& \vee(\exists X\ r.\mathit{Rev}(X)@r \wedge \mathit{Honest}(X)@i)
\end{aligned}
$$

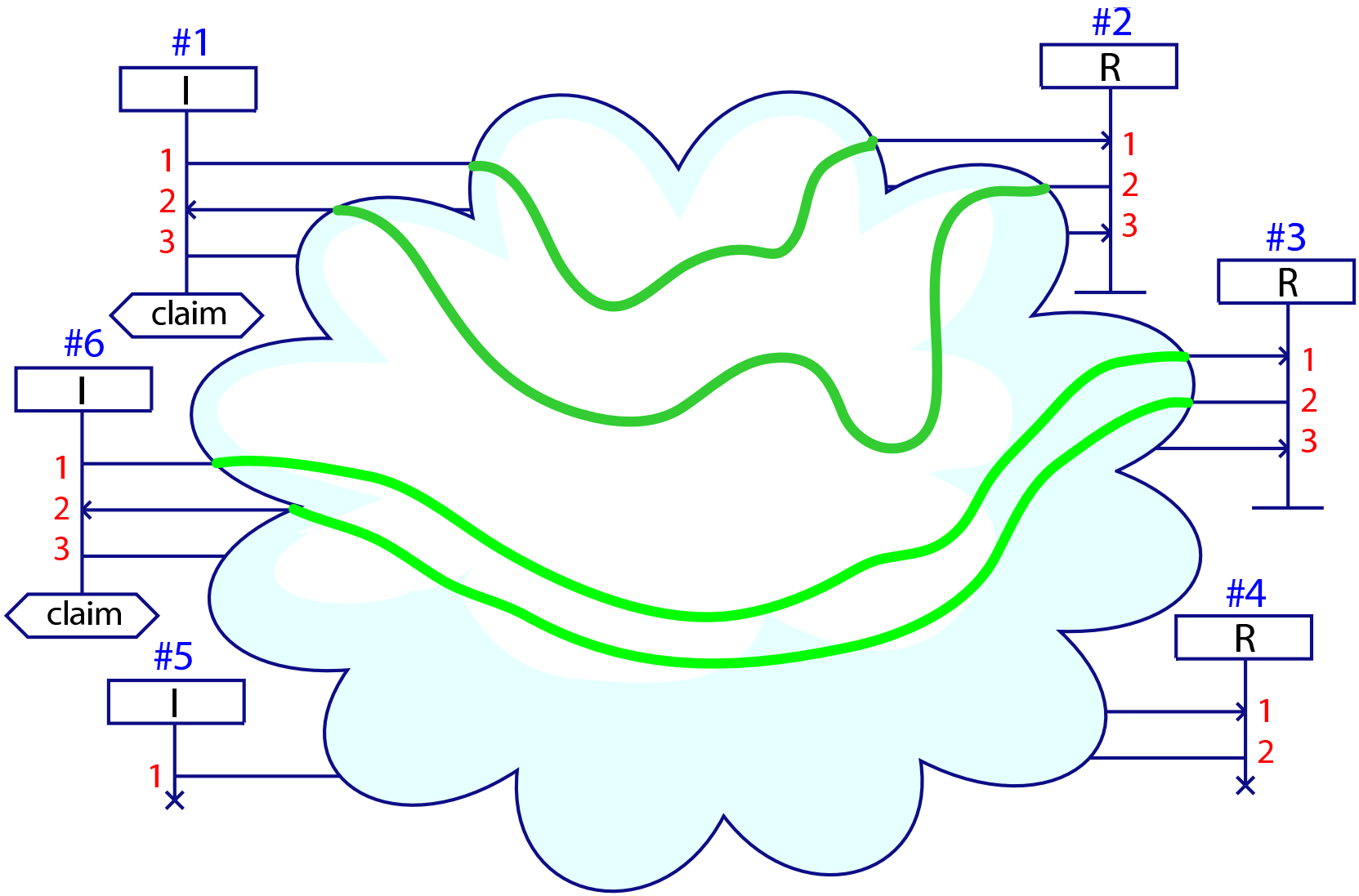
Remarks

- For each commit by a in role A on the trace there is a unique matching b executing role B.

Failed Injective Authentication

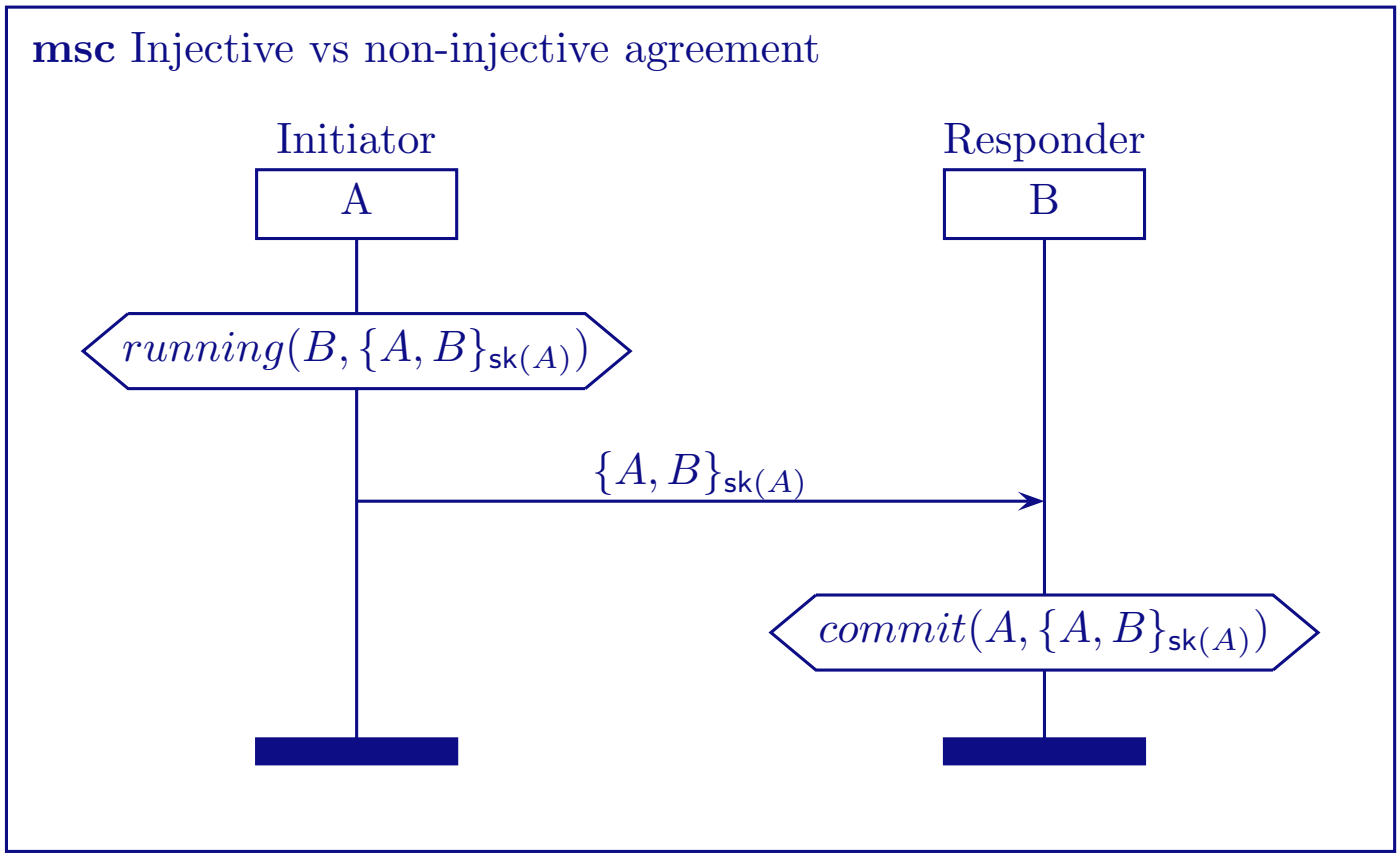


Successful Injective Authentication



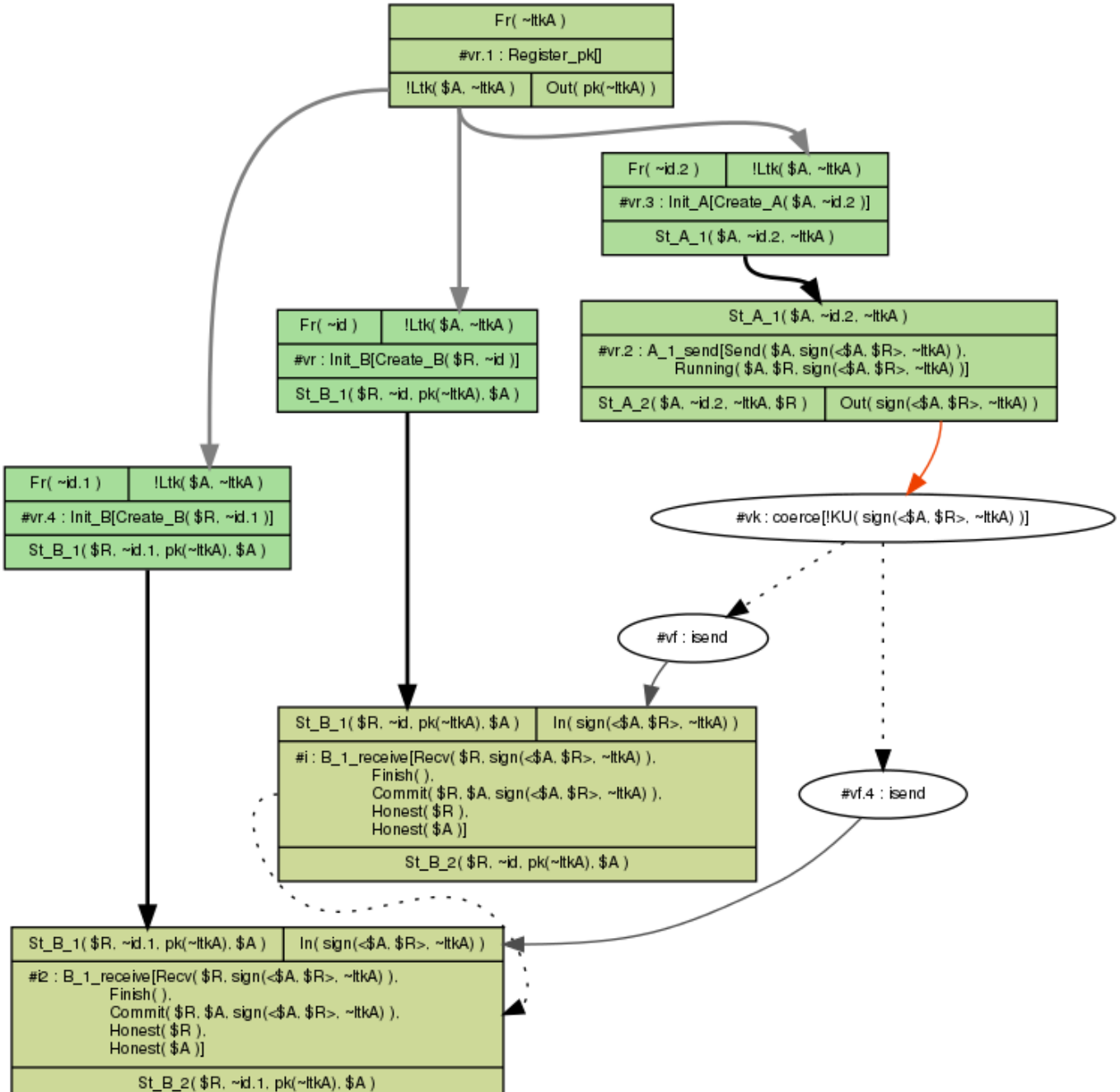
Injective vs Non-injective Agreement

Separating Example



- Non-injective agreement holds.
- Injective agreement fails, since the adversary can replay message to several threads in responder role B.

Injective Agreement counter-example



Definition (Weak agreement)

A trace tr satisfies the property $\mathit{WeakAgreement}(A, B)$ iff

$$\begin{aligned}\forall a\ b\ i.\quad & \mathsf{Claim\_commit}(a, b, \langle\rangle)@i \\ & \Rightarrow (\exists j.\mathsf{Claim\_running}(b, a, \langle\rangle)@j) \\ & \quad \vee (\exists X\ r.\mathit{Rev}(X)@r \wedge \mathit{Honest}(X)@i)\end{aligned}$$

It is sufficient that the agents agree they are communicating, it is not required that they play the right roles. Note also the empty list of data $\langle\rangle$ that is agreed upon, i.e., none.

Definition (Aliveness)

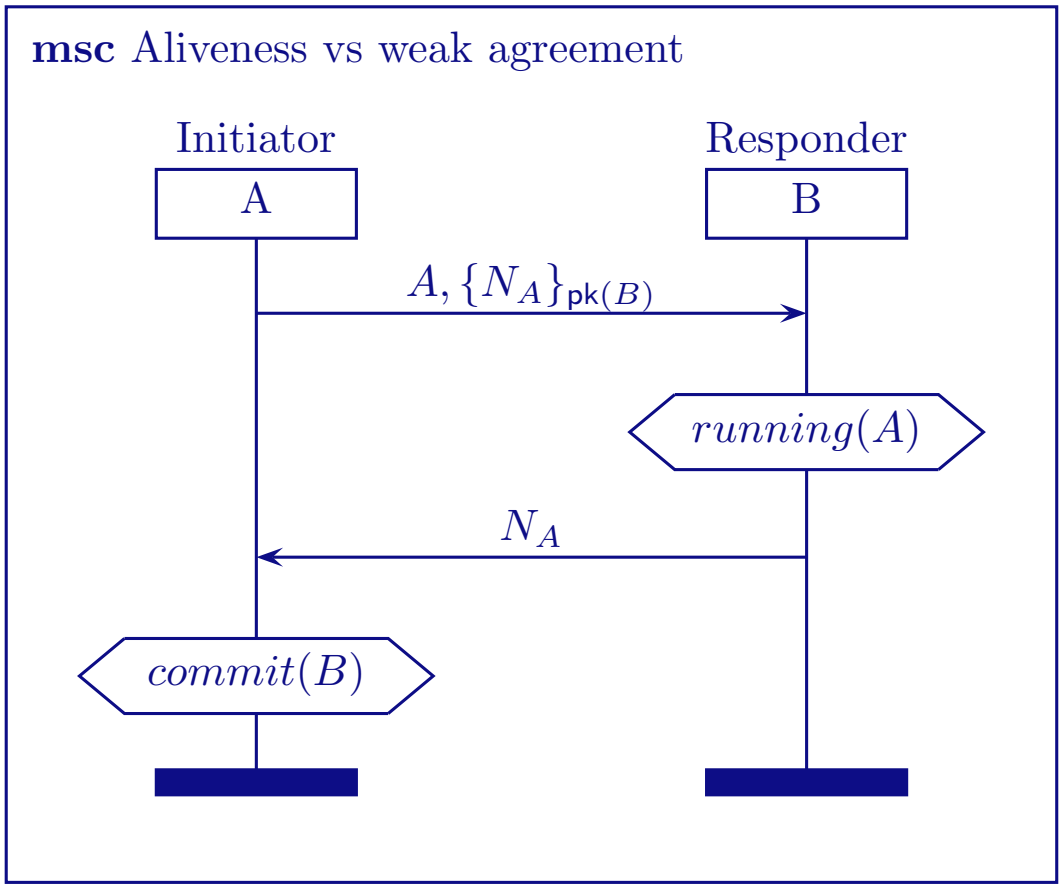
A trace tr satisfies the property $Alive(A, B)$ iff

$$\begin{aligned}\forall a\ b\ i.\quad & \mathsf{Claim\_commit}(a, b, \langle\rangle)@i \\ & \Rightarrow (\exists j\ id.\mathsf{Create\_B}(b, id)@j \vee \mathsf{Create\_A}(b, id)@j) \\ & \quad \vee (\exists X\ r.Rev(X)@r \wedge Honest(X)@i)\end{aligned}$$

It is neither required that the agent b, believed to instantiate role B by agent a, really plays role B, nor that he believes to be talking to a.

Aliveness vs Weak Agreement

Separating Example



- Aliveness holds: only B can have decrypted the fresh nonce N_A.
- Weak agreement fails, since adversary may modify unprotected identity A to C in first message so that B thinks he is talking to C.

Weak Agreement counter-example

```
theory ALIVEvsWEAKAGREE
begin
builtins: asymmetric-encryption

/* Public key infrastructure */
rule Register_pk:
   [ Fr(~ltkA) ] -->
   [ !Ltk($A, ~ltkA),  Out(pk(~ltkA)) ]

rule Reveal_ltk:
   [ !Ltk(A, ltkA) ] --[ Reveal(A) ]-> [ Out(ltkA) ]

/* We formalize the following protocol protocol
    1. A -> B: A,{na}_pk(B)
    2. A <- B: na
*/
```

Weak Agreement counter-example

```
/* new session thread creation */
rule Init_A:
  [ Fr(~id), !Ltk(R, ltkR) ]
  --[ Create_A($I, ~id) ]->
  [ St_A_1($I, ~id, pk(ltkR), R) ]

rule Init_B:
  [ Fr(~id), !Ltk(R, ltkR) ]
  --[ Create_B(R, ~id) ]->
  [ St_B_1(R, ~id, ltkR) ]

/*    1. A -> B: A,{na}_pk(B) */
rule A_1_send:
  [ St_A_1(I, ~id, pkltkR, R), Fr(~ni) ]
--[ Send(I, <I, aenc{~ni}pkltkR>), OUT_I_1(aenc{~ni}pkltkR)]->
  [ St_A_2(I, ~id, pkltkR, R, ~ni), Out(<I, aenc{~ni}pkltkR>) ]

rule B_1_receive:
  [ St_B_1(R, ~id, ltkR), In(<I, aenc{ni}pk(ltkR)>) ]
--[ Recv(R, <I, aenc{ni}pk(ltkR)>), IN_R_1_ni(ni, aenc{ni}pk(ltkR)) ]->
  [ St_B_2(R, ~id, ltkR, I, ni) ]
```

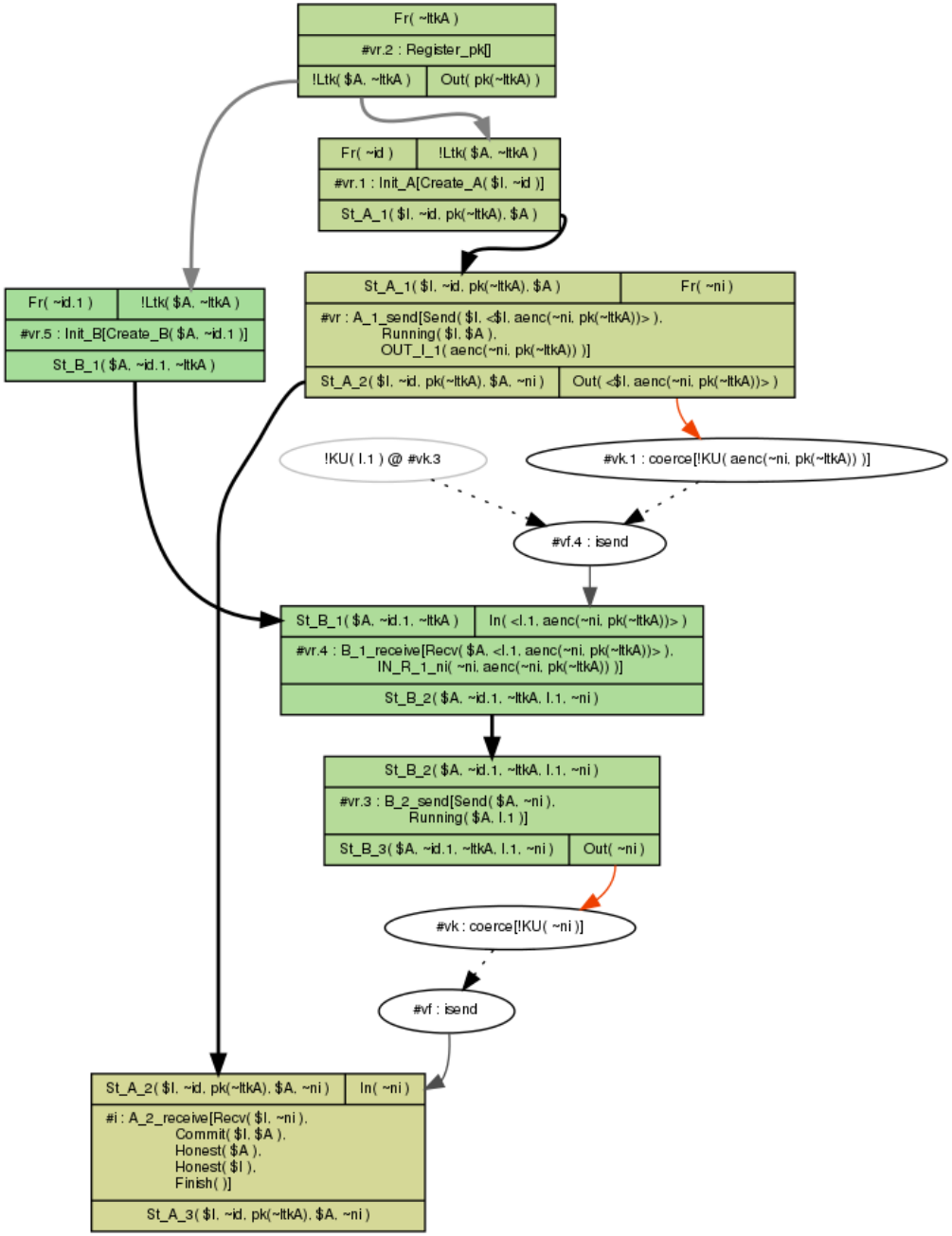
Weak Agreement counter-example

```
/*    2. A <- B: na */
rule B_2_send:
  [ St_B_2(R, ~id, ltkR, I, ni) ]
--[ Send(R, ni), Running(R, I) ]->
  [ St_B_3(R, ~id, ltkR, I, ni), Out(ni) ]

rule A_2_receive:
  [ St_A_2(I, ~id, pkltkR, R, ~ni), In(~ni) ]
--[ Recv(I, ~ni), Commit(I, R), Honest(R), Honest(I), Finish()]->
  [ St_A_3(I, ~id, pkltkR, R, ~ni) ]

/* it helps to deal with infinite loops in applying rules */
lemma types [sources]:
"(All ni m1 #i. IN_R_1_ni( ni, m1) @ i ==>
  ((Ex #j. KU(ni) @ j & j < i) | (Ex #j. OUT_I_1( m1 ) @ j))
)"
```

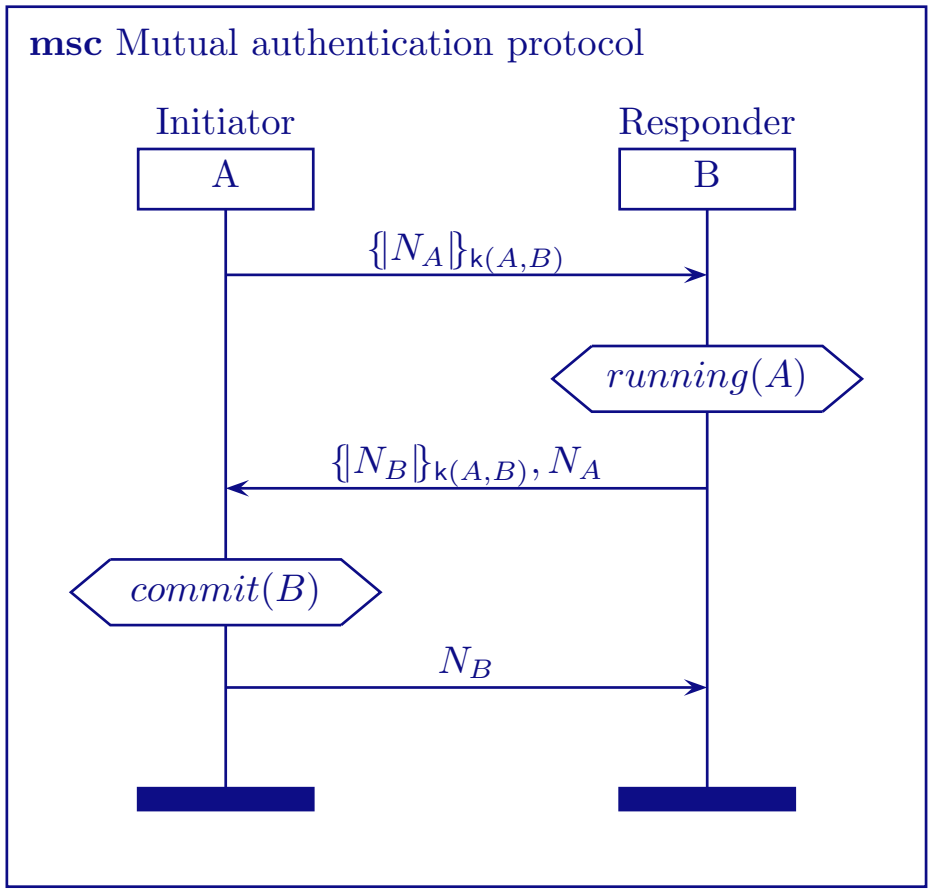
Weak Agreement counter-example

```
/* protocol is executable */
lemma executable:
exists-trace
  "Ex #i. Finish() @i & not (Ex X #j. Reveal(X) @j) "

/* succeeds */
lemma aliveness:
all-traces
  "All A B #i. Commit(A,B) @ i  ==>
    (Ex #j id. Create_B(B,id) @j) | (Ex #j id. Create_A(B,id)@j)
    | (Ex X #r. Reveal(X) @ r & Honest(X) @i)"

/* fails as expected */
lemma weakagree:
 all-traces
  "All A B #i. Commit(A,B) @i ==>
  (Ex #j. Running(B,A) @j)| (Ex X #r. Reveal(X) @ r & Honest(X) @i)"
end
```

Weak Agreement counter-example

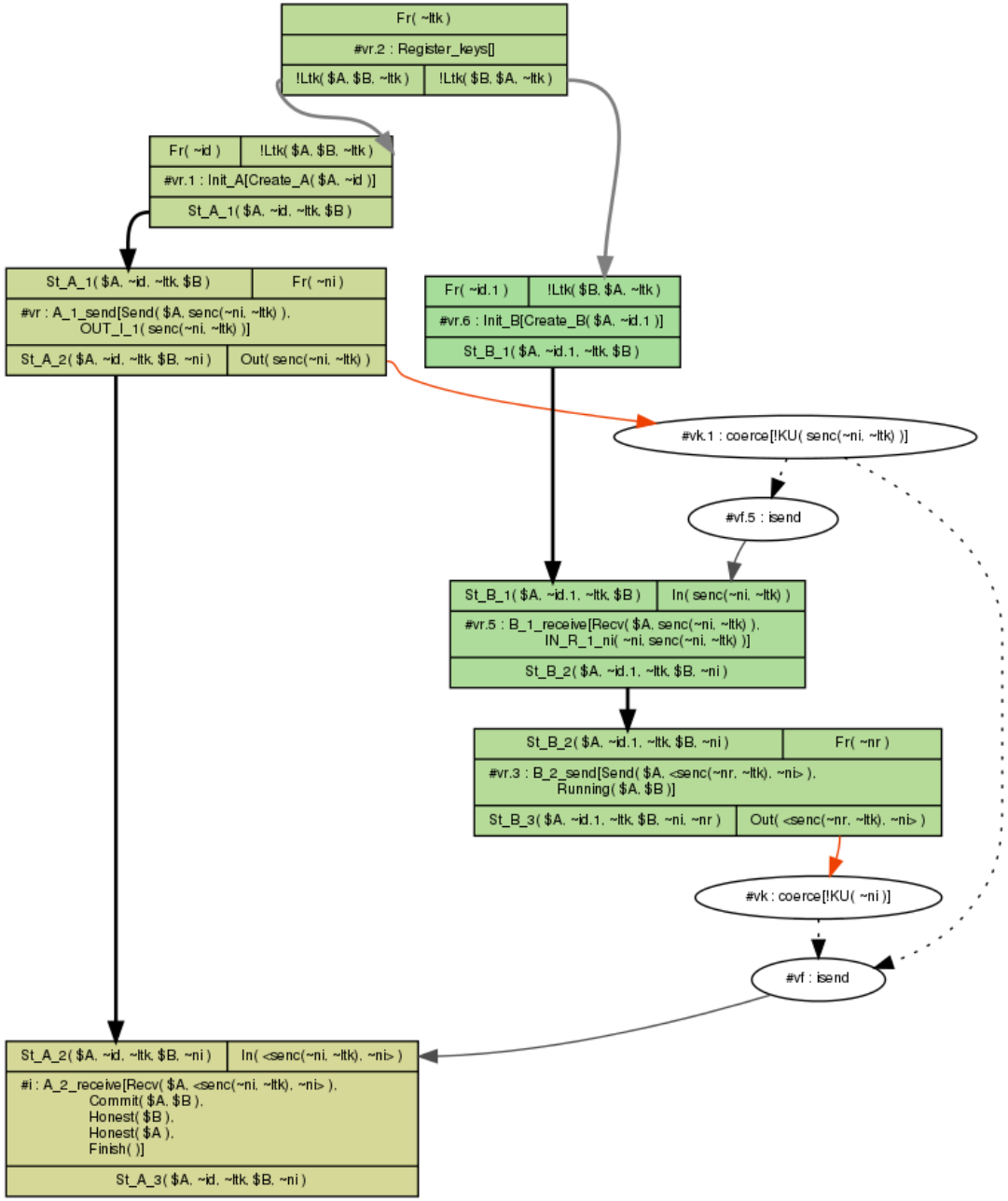


When Even Aliveness Fails ...



- Reflection attack: A may complete run without B's participation.
- Hence, aliveness fails.

Attack found by Tamarin



Key-related Properties

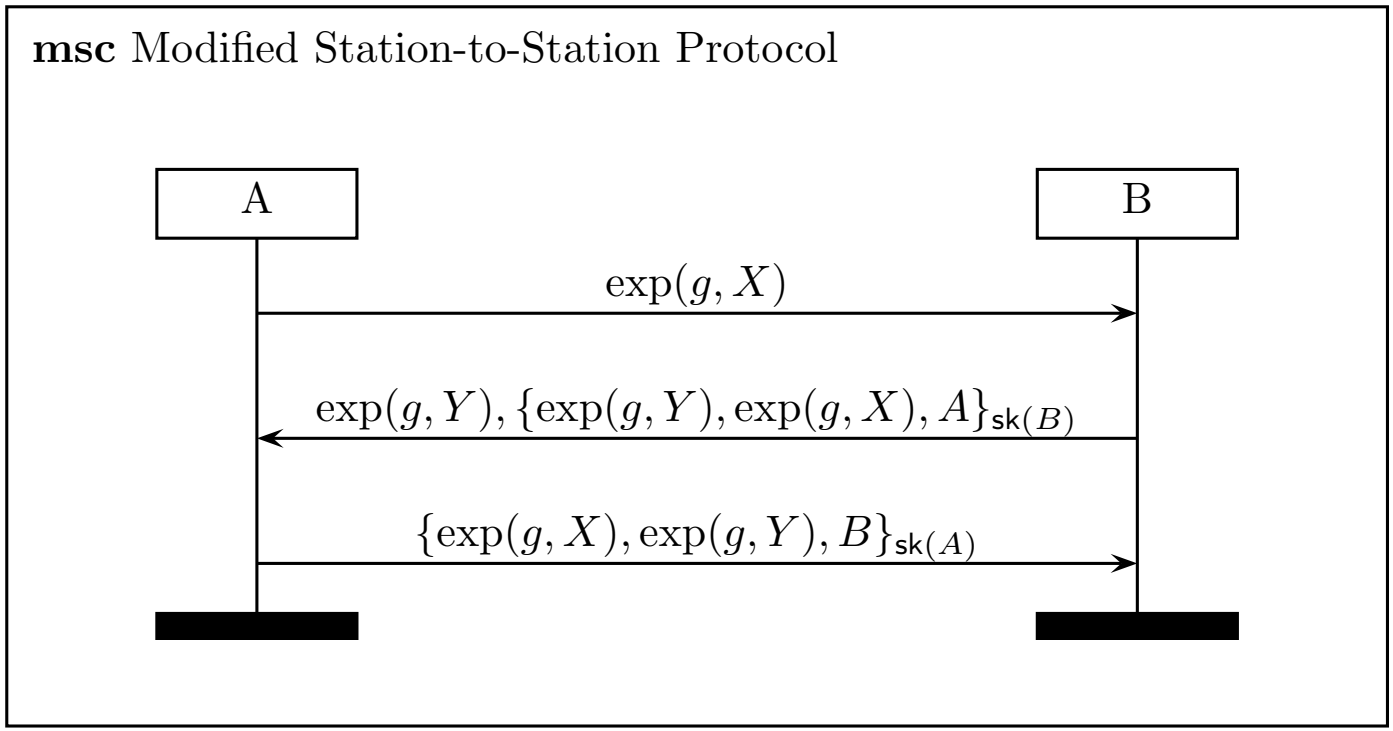
Basic key-oriented goals

- key freshness
- (implicit) key authentication: a key is only known to the communicating agents A and B and mutually trusted parties
- key confirmation of A to B is provided if B has assurance that agent A has possession of key K
- explicit key authentication = key authentication + key confirmation $\Rightarrow$ expressible in terms of secrecy and agreement

Goals concerning compromised keys

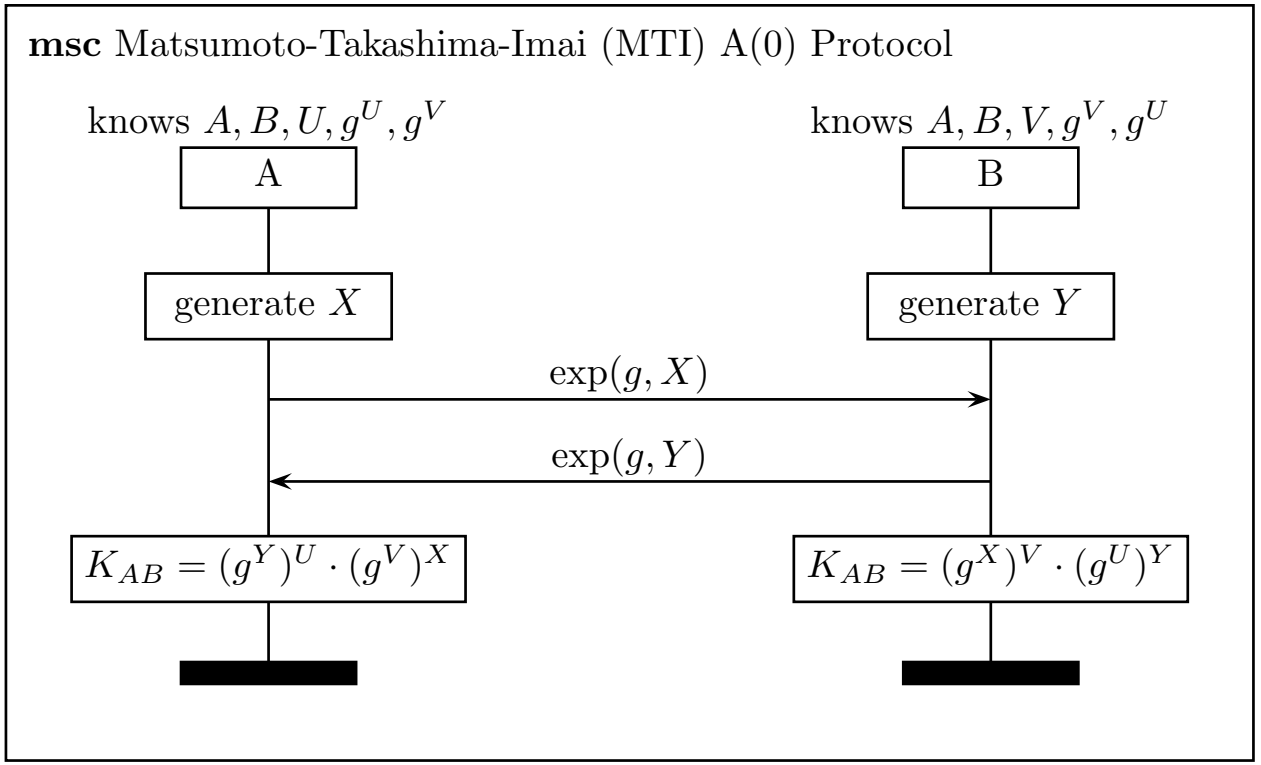
- (perfect) forward secrecy: compromise of long-term keys of a set of principals does not compromise the session keys established in previous protocol runs involving those principals
- resistance to key-compromise impersonation: compromise of long-term key of an agent A does not allow the adversary to masquerade to A as a different principal.

Forward Secrecy: Example 1



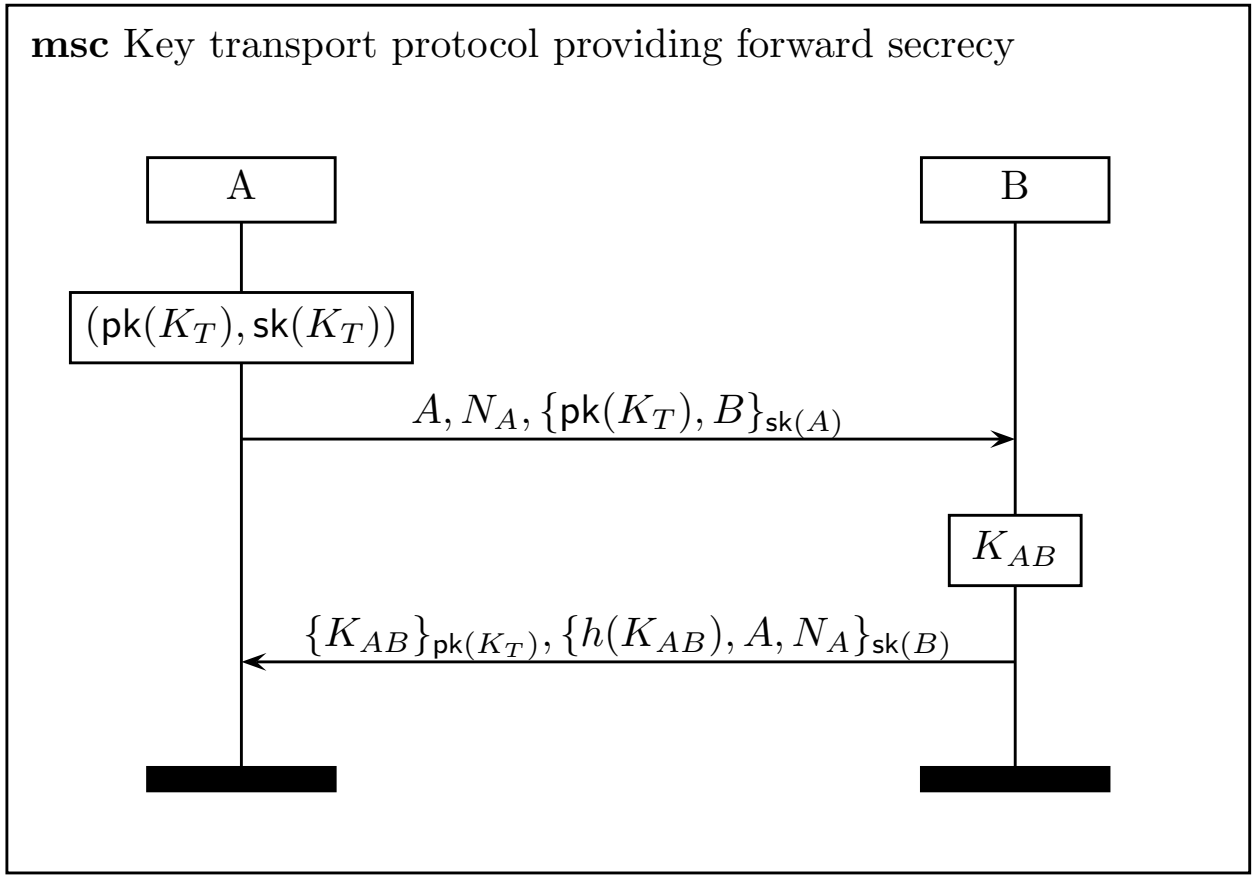
- Signatures are used to authenticate the Diffie-Hellman public keys $\exp(g, X)$ and $\exp(g, Y)$.
- Protocol provides forward secrecy: The adversary cannot derive session key $K_{AB} = \exp(\exp(g, X), Y)$ by compromise of signing keys.

Forward Secrecy: Example 2



- Message exchange as in basic DH; protocol combines long-term and ephemeral DH keys to authenticate exchanged DH public keys.
- Protocol does not provide forward secrecy: The adversary can construct the session key $K_{AB} = g^{VX+UY}$ as $(g^X)^V \cdot (g^Y)^U$ from observed messages and long-term private keys U and V.

Forward Secrecy: Example 3



- A generates an ephemeral asymmetric key pair $(\mathsf{pk}(K_T), \mathsf{sk}(K_T))$.
- Protocol provides forward secrecy without using Diffie-Hellman keys: Adversary cannot learn session key by compromise of signing keys.

Built-in message theories 1

hashing

Functions: h/1

Equations: No equations

asymmetric-encryption

Functions: aenc/2, adec/2, pk/1

Equations: adec(aenc(m, pk(sk)), sk) = m

signing

Functions: sign/2, verify/3, pk/1, true/0

Equations: verify(sign(m,sk),m,pk(sk)) = true

symmetric-encryption

Functions: senc/2, sdec/2

Equations: sdec(senc(m,k),k) = m

Built-in message theories 2

diffie-hellman

Functions: inv/1, 1/0, ^/2, */2

Equations:
$(x\hat{}y)\hat{}z = x\hat{}(y*z)$ $\quad x\hat{}1 = x$ $\quad x*y = y*x$
$(x*y)*z = x*(y*z)$ $\quad x*1 = x$ $\quad x*\mathrm{inv}(x) = 1$

xor

Functions: XOR/2, zero/0

Equations:

$x \mathrm{\ XOR\ } y = y \mathrm{\ XOR\ } x$ $\quad (x \mathrm{\ XOR\ } y) \mathrm{\ XOR\ } z = x \mathrm{\ XOR\ } (y \mathrm{\ XOR\ } z)$
$x \mathrm{\ XOR\ zero} = x$ $\quad x \mathrm{\ XOR\ } x = \mathrm{zero}$

multiset

Functions: +/2

Equations: $x + y = y + x$ $\quad (x + y) + z = x + (y + z)$

Then you can define a "Smaller" predicate by
predicates: Smaller(x,y) $\Leftrightarrow$ Ex z. $x + z = y$

Restrictions

Restrictions can be used exclude undesired traces: Tamarin will ignore all traces that violate a restriction

- Take care not to exclude attacks!
- Many standard applications:
  - Equality
  - Inequality
  - LessThan
  - GreaterThan
  - OnlyOnce
- Essentially same syntax as lemmas

Restriction Example: Only Once

```
restriction once:
  "All #i #j. OnlyOnce()@#i & OnlyOnce()@#j
                                    ==> #i=#j"
```

Given the following rules:

```
rule 1: [ ]      --[ OnlyOnce() ]-> [ A('5') ]
rule 2: [ A(x) ] --[ Step(x) ]->    [ B(x) ]
```

The restriction forbids e.g. the following execution (valid without restriction):

```
[ ] --[ OnlyOnce() ]--> [ A('5')]
--[ OnlyOnce() ]-> [ A('5'), A('5') ]
--[ Step('5')]-> [ A('5'), B('5') ]
```

But the following is still allowed:

```
[ ] --[ OnlyOnce() ]-> [ A('5') ]
--[ Step('5')]-> [ B('5') ]
```

Restriction Example: Inequality

```
restriction InEq:
  "All x #i. Neq(x,x)@#i ==> F"
```

Given the following rules:

```
rule 1: [ ] --[ A1() ]-> [ A('1') ]
rule 2: [ ] --[ A2() ]-> [ A('2') ]
rule 3: [ A(x), A(y) ] --[ Neq(x,y) ]-> [ B(x,y) ]
```

The restriction forbids e.g. the following execution (valid without restriction):

```
[ ] --[ A1() ]-> [ A('1')]
--[ A1() ]-> [ A('1'), A('1') ]
--[ Neq('1','1') ]-> [ B('1','1') ]
```

But the following is still allowed:

```
[ ] --[ A1() ]-> [ A('1')]
--[ A2()]-> [ A('1'), A('2') ]
--[ Neq('1','2') ]-> [ B('1','2') ]
```

Command line parameters

`tamarin-prover --help` or manual to see all parameters.
Most important options:

- `--prove` Attempt to prove the lemmas
- `--stop-on-trace[=DFS|BFS|NONE]` How to search for traces (default DFS)
- `-b --bound[=INT]` Bound the depth of the proofs
- `--heuristic[=(s|S|o|O|p|P|l|c|C|i|I)+]` Sequence of goal rankings to use (default 's')
- `--diff` Turn on observational equivalence mode using diff terms.
- `--quit-on-warning` Strict mode that quits on any warning that is emitted. (Good for debugging!)
- `--parse-only` Just parse the input file and pretty print it as-is (to find syntax errors)
- `-V --version` Print version information (for bug reports)

Exporting and importing proofs and attacks

Running Tamarin 1.2.3 — Index | Download | Actions » | Options »

Proof scripts

```
Multiset rewriting rules and restrictions (8)

Raw sources (10 cases, deconstructions complete)

Refined sources (10 cases, deconstructions complete)

lemma Client_session_key_secrecy:
  all-traces
  "¬(∃ S k #i #j.
          ((SessKeyC( S, k ) @ #i) ∧ (K( k ) @ #j)) ∧
          (¬(∃ #r. LtkReveal( S ) @ #r)))"
simplify
solve( Client_1( S, k ) ▶₀ #i )
  case Client_1
  solve( !KU( ~k ) @ #vk.1 )
    case Client_1
    solve( !KU( ~ltk ) @ #vk.2 )
      case Reveal_ltk
      by contradiction /* from formulas */
    qed
  qed
qed

lemma Client_auth:
  all-traces
  "∀ S k #i.
         (SessKeyC( S, k ) @ #i) ⇒
         ((∃ #a. AnswerRequest( S, k ) @ #a) ∨
          (∃ #r. (LtkReveal( S ) @ #r) ∧ (#r < #i)))"
by sorry

lemma Client_auth_injective:
  all-traces
  "∀ S k #i.
         (SessKeyC( S, k ) @ #i) ⇒
         ((∃ #a.
            (AnswerRequest( S, k ) @ #a) ∧
            (∀ #j. (SessKeyC( S, k ) @ #j) ⇒ (#i = #j))) ∨
          (∃ #r. (LtkReveal( S ) @ #r) ∧ (#r < #i)))"
by sorry

lemma Client_session_key_honest_setup:
  exists-trace
  "∃ S k #i.
         (SessKeyC( S, k ) @ #i) ∧ (¬(∃ #r. LtkReveal( S ) @ #r))"
by sorry

end
```

Case: Reveal_ltk

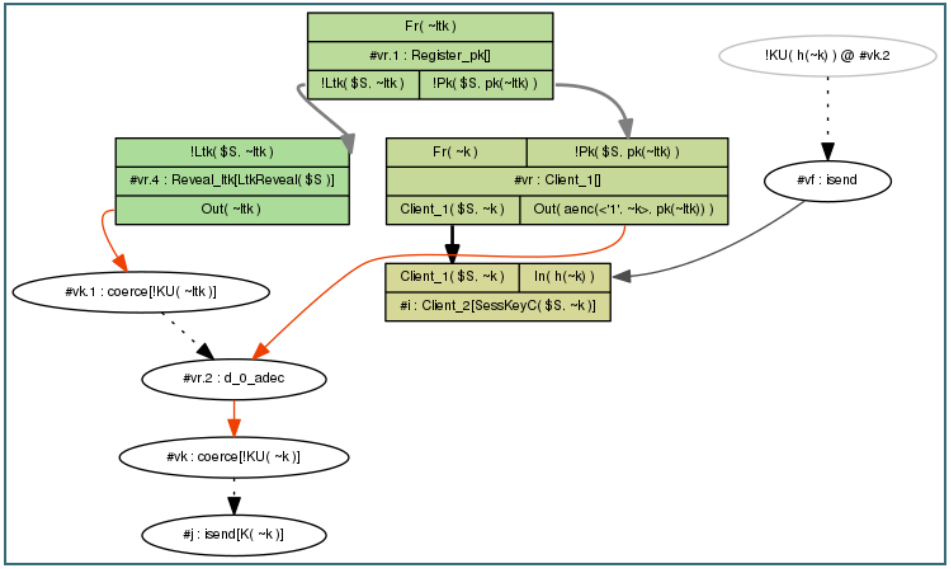
Applicable Proof Methods: Goals sorted according to the 'smart' heuristic (loop breakers delayed)

1. **contradiction** /* from formulas */

2. **solve(** !KU(h(~k)) @ #vk.2 **)** // nr. 4

a. **autoprove** (A. **for all solutions**)
b. **autoprove** (B. **for all solutions**) with proof-depth bound 5

Constraint system



last: none

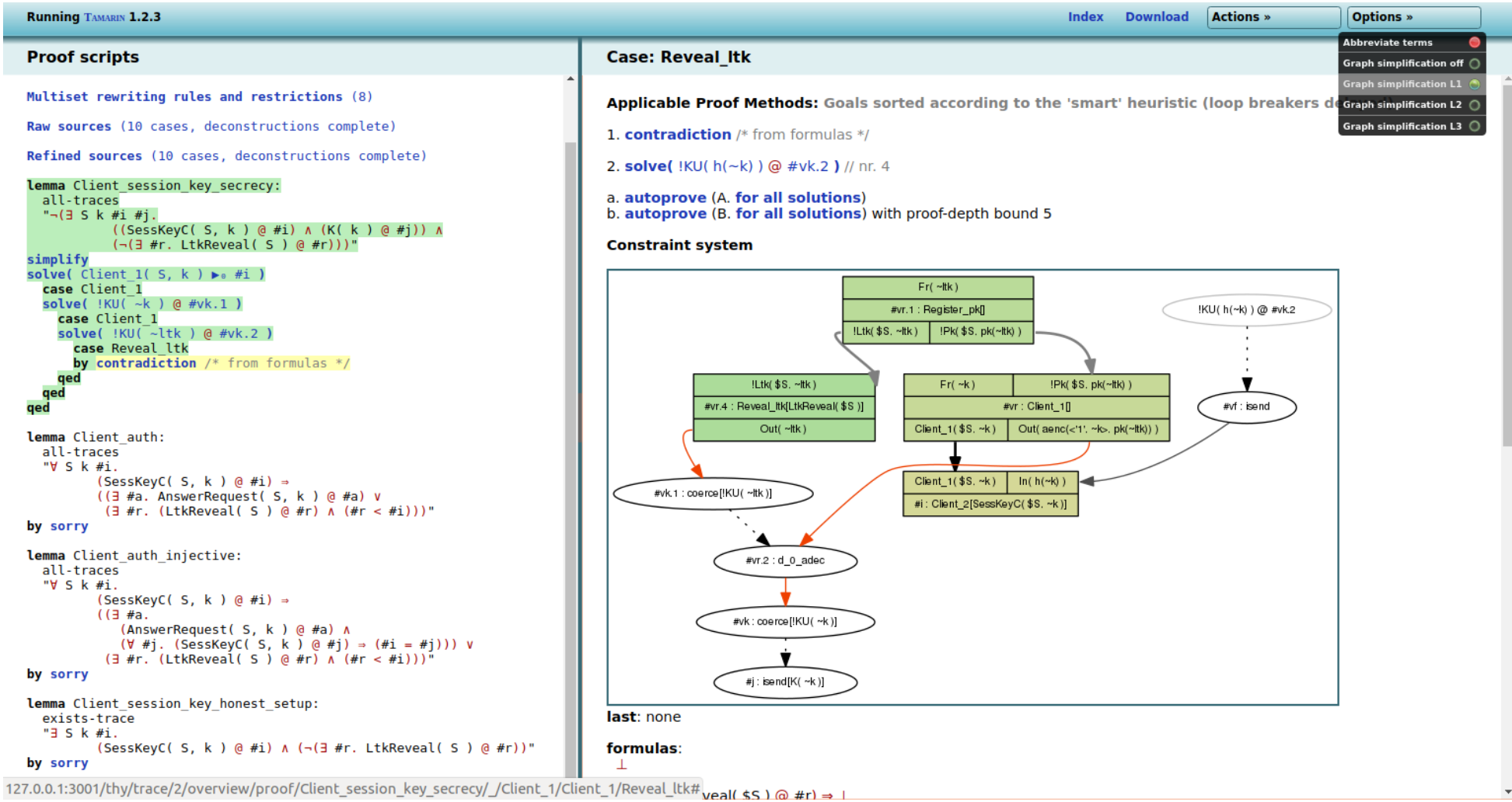
formulas:

⊥

∀ #r. (LtkReveal($S) @ #r) ⇒ ⊥

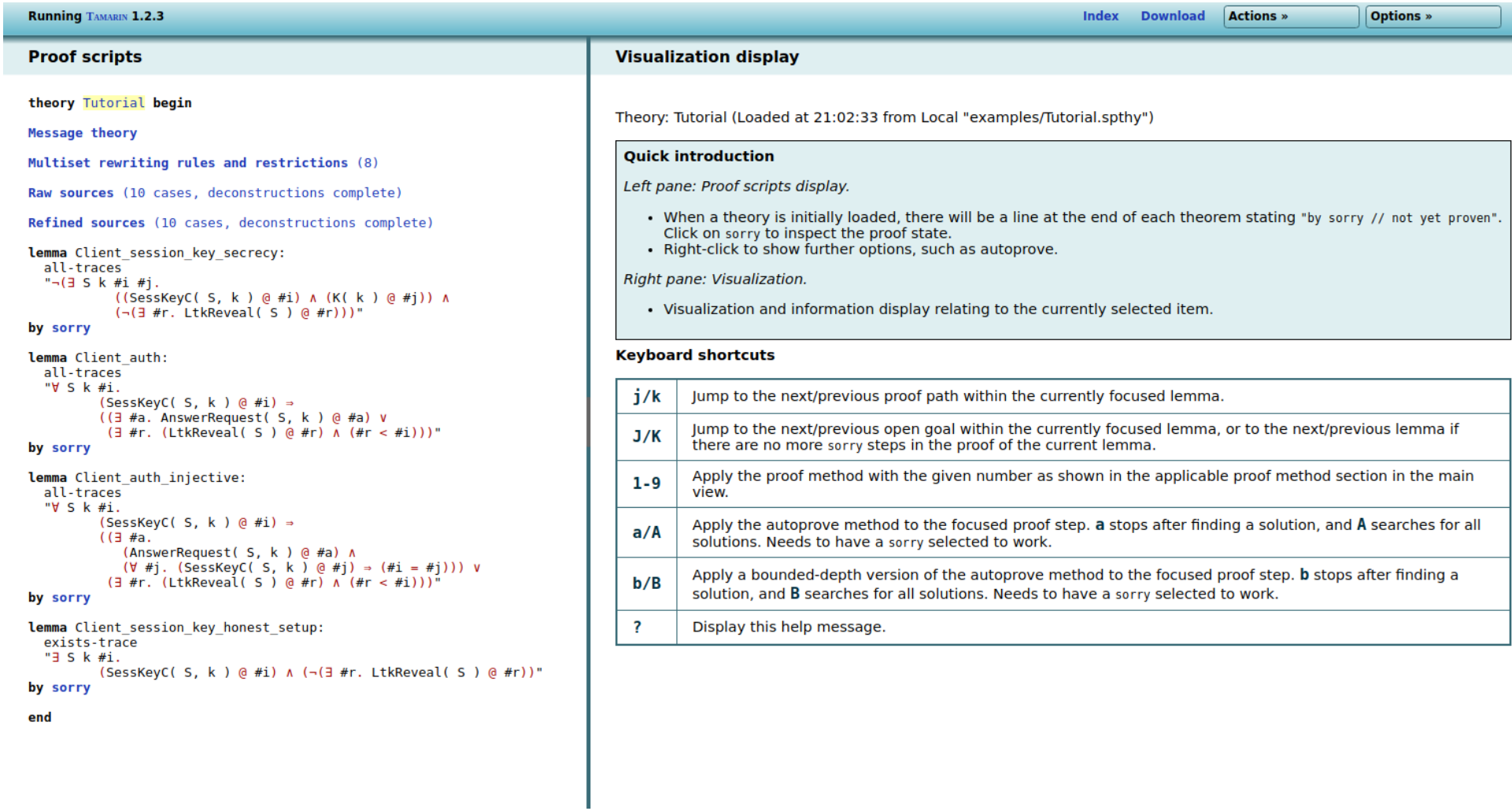
"Download" allows to export proofs. Gives a valid `.spthy` containing spec and proof, which is again verified on loading.

GUI configuration



Allows to choose the level of details in graph visualization.

Keyboard shortcuts



Makes proof inspection quicker.

Syntactic sugar: let statements

```
rule Serv_1:
    [ !Ltk($S, ~ltkS)
    , In( request )
    ]
  --[ Eq(fst(adec(request, ~ltkS)), '1')
    ]->
    [ Out( h(snd(adec(request, ~ltkS))) ) ]
```

can be written aus

```
rule Serv_1:
  let d = adec(request, ~ltkS) in
    [ !Ltk($S, ~ltkS)
    , In( request )
    ]
  --[ Eq(fst(d), '1')
    ]->
    [ Out( h(snd(d)) ) ]
```

Syntactic sugar: formal comments

Normal comments are lost when exporting a proof.

```
section{* This comment is kept even
          when exporting proofs. *}
```

Debugging your model

Problem: How to be convinced that your model is correct?

- Check for warnings and errors
- Inspect message theory and case distinctions
- Use sanity lemmas, e.g., to verify that the normal protocol execution is possible
- Change details and check whether the expected happens

Observational Equivalence - Motivation

Two types of properties:

- **Trace properties**:
  - (Weak) secrecy as reachability
  - Authentication as correspondence

- **Observational equivalence** $\approx$

Why observational equivalence?

Consider classic **Dolev-Yao** intruder for deterministic public-key encryption:

$$\frac{enc(x, pk(k)) \quad k}{x}$$

- Intruder can only decrypt if he knows the secret key

Now consider a simple **voting** system:

- Voter chooses $v =$ "Yes" or $v =$ "No"
- Encrypt v using server's public key $pk(k)$: $c = enc(v, pk(k))$
- Send c to server

Is the vote **secret**?

- Dolev-Yao: **Yes**, intruder does not know server's secret key
- Reality: **No**, encryption is deterministic and there are only two choices
  - **Attack:** encrypt "Yes", and compare to c

Observational Equivalence vs Reachability

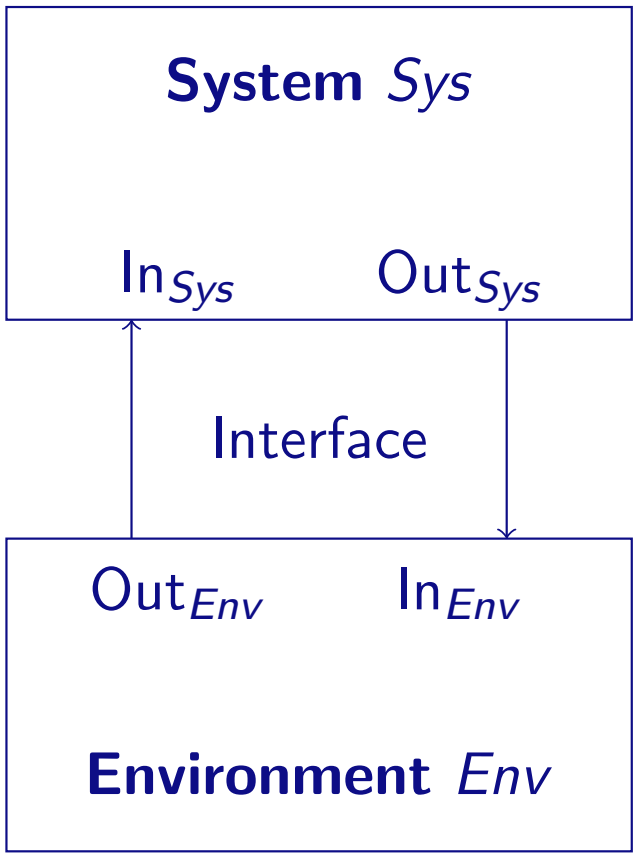
- **Reachability**-based (weak) secrecy is insufficient
- **Stronger** notion: intruder cannot distinguish
  - a system where the voter votes "Yes" from
  - a system where the voter votes "No"
- **Observational equivalence** between two systems
- Can be used to express:
  - Strong secrecy
  - Privacy notions
  - Game-based notions, e.g., ciphertext indistinguishability

Running Example

- **Auction** system
- Property: **strong secrecy** of bids
- Property *violated*: **Shout-out auction**
  - Broadcast bid (e.g., A or B)
  - Send "A" in first system
  - Send "B" in second system
  - Observer knows if he is observing first or second system
- Property holds: using **shared symmetric key**
  - Shared symmetric key k between bidder and auctioneer
  - Send "$\{A\}_k$" in first system
  - Send "$\{B\}_k$" in second system
  - Observer has no access to k, does not know which system he is observing

System and environment

- We separate **environment** and **system**
  - System: agents running according to protocol
  - Environment: adversary acting according to its capabilities
- Environment can observe:
  - Output of the system
  - If system reacts at all



Defining observational equivalence

- **Two** system **specifications** given as set of rules
  - One rule per role action (send/receive)
  - Running example shout-out auction:

System 1: $\dfrac{}{\mathrm{Out}_{Sys}(A)}$ System 2: $\dfrac{}{\mathrm{Out}_{Sys}(B)}$

- Interface and environment/adversary rule(s):

$$\frac{\mathrm{Out}_{Sys}(X)}{\mathrm{In}_{Env}(X)} \qquad \frac{\mathrm{Out}_{Env}(X)}{\mathrm{In}_{Sys}(X)} \qquad \frac{\mathrm{In}_{Env}(X) \quad K(X)}{\mathrm{Out}_{Env}(true)}$$

  - $K(X)$ represents that environment knows term X
  - last rule models comparisons by the adversary
- Each specification yields a labeled transition system
- Observational equivalence is a kind of **bisimulation** accounting for the adversaries' **viewpoint** and **capabilities**
  - Our definition can be instantiated for various adversaries

Diff terms

- General definition **difficult** to verify: requires inventing simulation relation
- Idea: **specialize** for cryptographic protocols
  - Consider strong bid secrecy:
    - both systems differ in **secret bid only**, i.e.
    - both specifications contain **same rule(s)** which differ only in **some terms**
  - Exploit this similarity in description and proof
- Approach: two systems described by one specification – using **diff**-terms
  - Running example

$$\overline{\mathrm{Out}_{Sys}(A)} \qquad \overline{\mathrm{Out}_{Sys}(B)}$$

  - Is equivalent to one rule with a **diff**-term

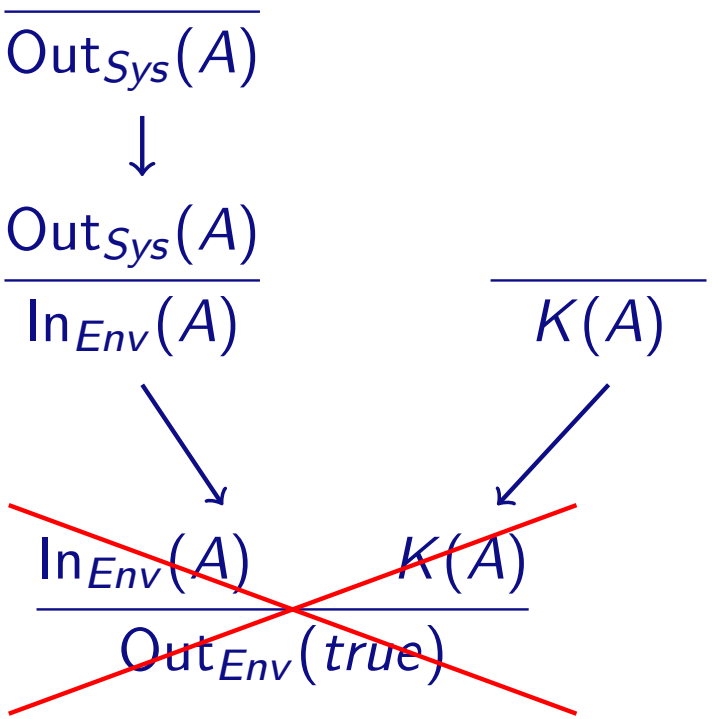
$$\overline{\mathrm{Out}_{Sys}(\mathbf{diff}(A, B))}$$

Approximating observational equivalence using mirroring

- Both systems contain the same rules modulo diff-terms
- Idea: assume that each rule simulates itself
- **Mirrors** each execution into the other system
- If the mirrors are **valid executions**, we have **observational equivalence** (sound approximation)
- We represent executions using *dependency graphs*
  - Computed via backwards constraint solving

Dependency graphs and mirrors

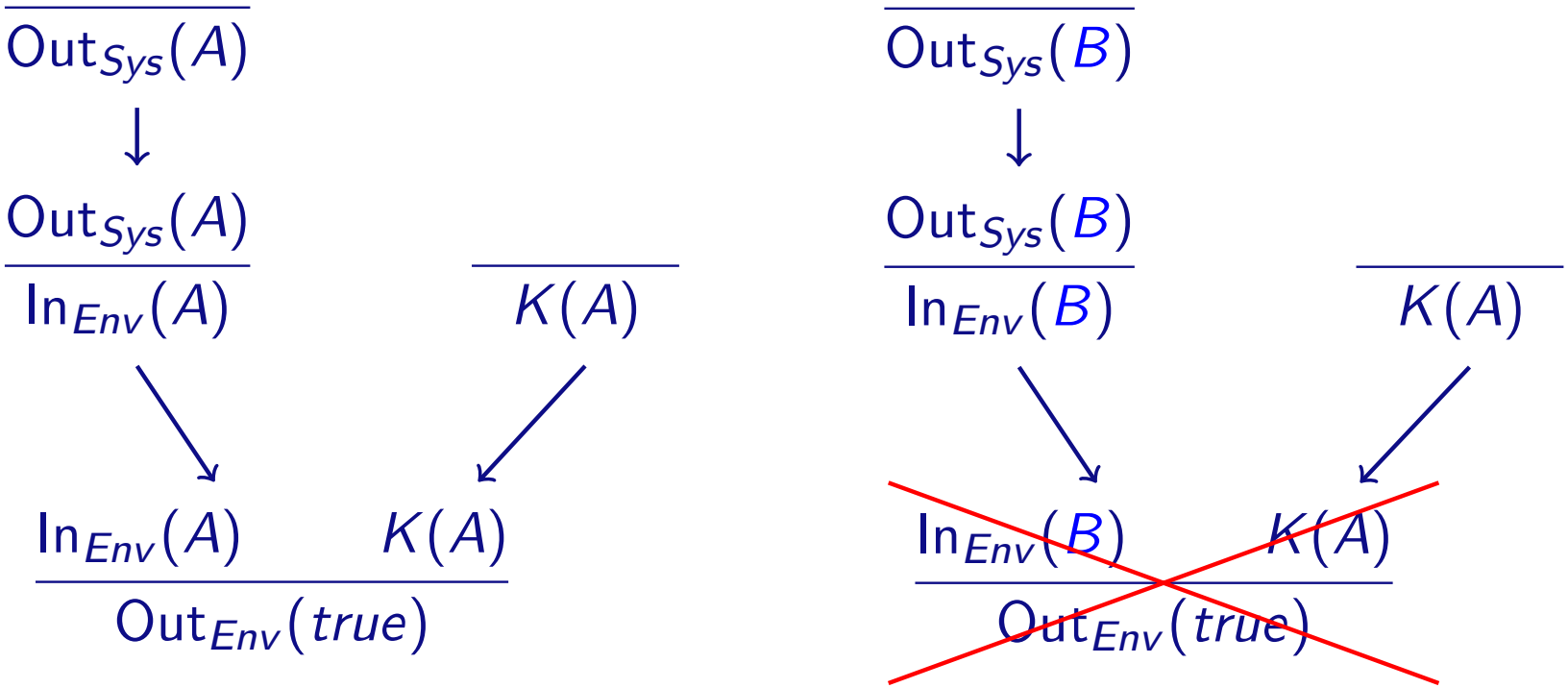
Bidder picks A, observer compares to public value A



- Dependency graph mirror for bidder choice B is **invalid**
  - Adversary choices stay fixed, comparison is with A

Invalid mirrors and attacks

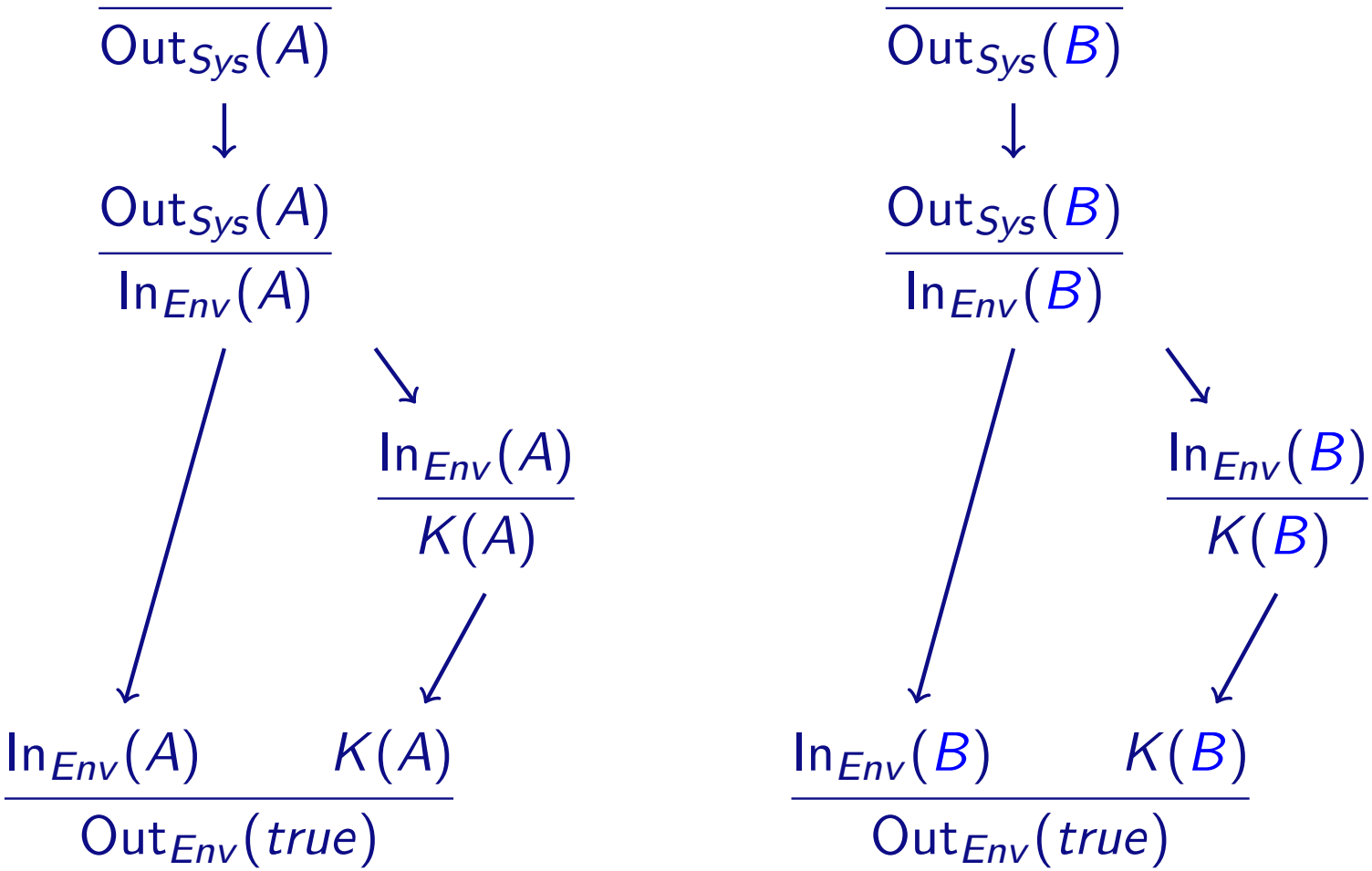
Bidder picks A/B, observer compares to public value A



- **Counter example** to observational equivalence of the given systems

Valid mirror

Observer compares system output to itself



- **All** mirrors need to be valid for observational equivalence

Dependency graph equivalence

A **diff**-system is *dependency graph equivalent* if mirrors of all dependency graphs rooted in any rule on both sides are valid.

- Sound but incomplete approximation
- Efficient and sufficient in practice

Input:

- Protocol specification
- Property: equivalence given two choices for some term(s)
  - Example: random value vs expected value

Output:

- **Yes**, observational equivalent
- **No**, dependency graph with invalid mirror
- Non-termination possible

Probabilistic encryption

Given equational theory for decryption

$$pdec(penc(m, pk(k), r), k) \simeq m\,.$$

1. Agent knows k, published $pk(k)$ previously
2. Adversary provides value x, agent selects random r_1, r_2
   Agent sends either one of
   - r_1
   - $penc(x, pk(k), r_2)$

Can adversary distinguish random value from encryption?

- No!
- Tamarin verifies this automatically in 0.2 seconds
- Iterates over all rules
- Simple, illustrative toy example to show approach

Weak Secrecy

```
theory probenc
begin
functions: penc/3, pdec/2, pk/1
equations: pdec(penc(m, pk(k), r), k)=m

rule R_pk:
[ Fr(~k) ]  --> [ !Ltk($A,~k), !Pk($A, pk(~k)) ]

rule Out_pk:
[ !Pk($A, pubk) ] --> [ Out(pubk) ]

rule Send_:
[ !Ltk($A, k), Fr(~x),  Fr(~r2) ]
--[ WSecret(~x) ]->
[ Out(penc(~x , pk(k), ~r2) ) ]

//Weak secrecy
lemma Secret:
"
  All B #i. WSecret(B)@i ==> not( Ex #j. K(B)@j )
"
end
```

Strong Secrecy

```
theory probenc
begin
functions: penc/3, pdec/2, pk/1
equations: pdec(penc(m, pk(k), r), k)=m

rule R_pk:
[ Fr(~k) ]  --> [ !Ltk($A,~k), !Pk($A, pk(~k)) ]

rule Out_pk:
[ !Pk($A, pubk) ] --> [ Out(pubk) ]

rule Send_:
[ !Ltk($A, k), In(x), Fr(~r1), Fr(~r2) ]
--[ SSecret(x) ]->
[ Out(diff( ~r1,  penc(x , pk(k), ~r2) )) ]

//Strong secrecy implicitely encoded by the equivalence of two systems
```

Probabilistic encryption - reasoning

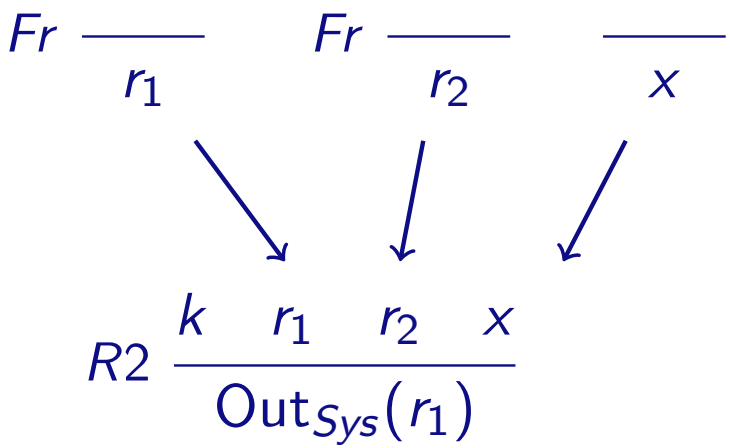
Setup rule (1) that picks long-term key and sends public key:

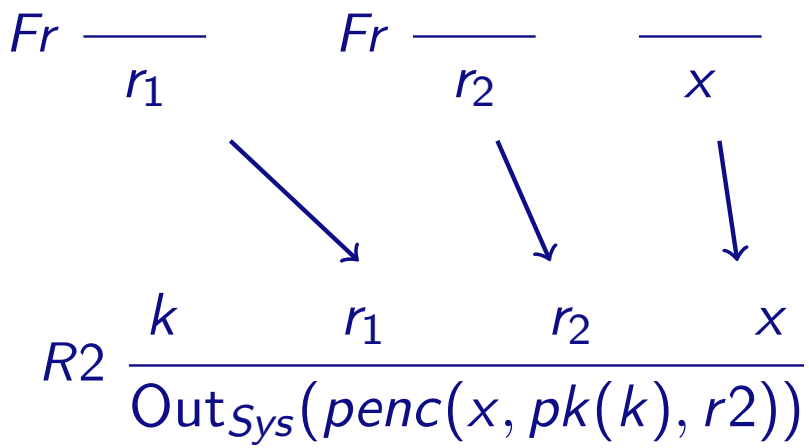
- No choice in the rule
- Easy to complete dependency graph
- Trivially mirrors itself

$$\frac{Fr\ \dfrac{}{k}\ \Big\downarrow\ k}{\mathrm{Out}_{Sys}(pk(k))}$$

Probabilistic encryption - reasoning

Rule (2) sending out random or real encryption, taking value x as input and having a stored key k:





- Same premise in both cases
- Outputs do not matter in root
  - Considered in equality check
- Identical shape of dependency graphs in both cases:
  - Input and key are independent
- Mirror each other

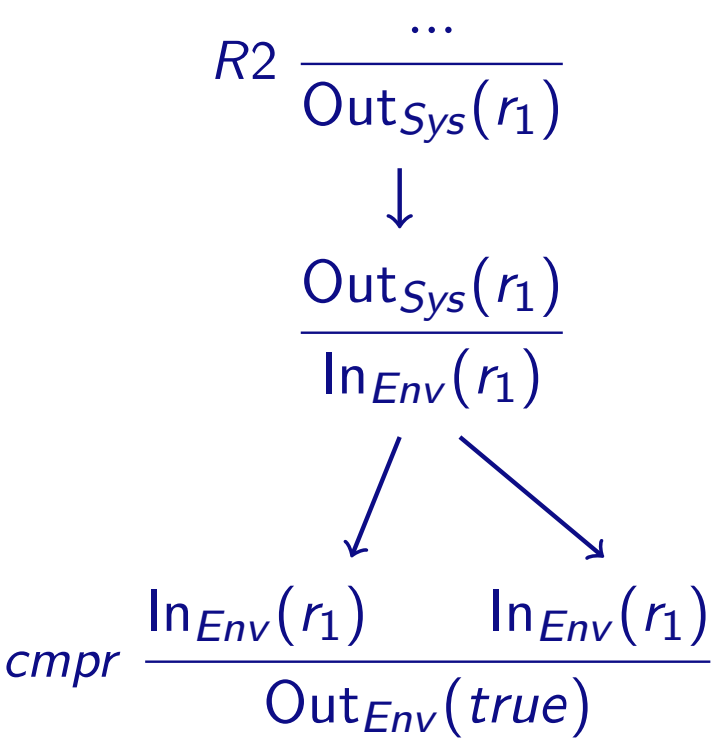
Probabilistic encryption - reasoning

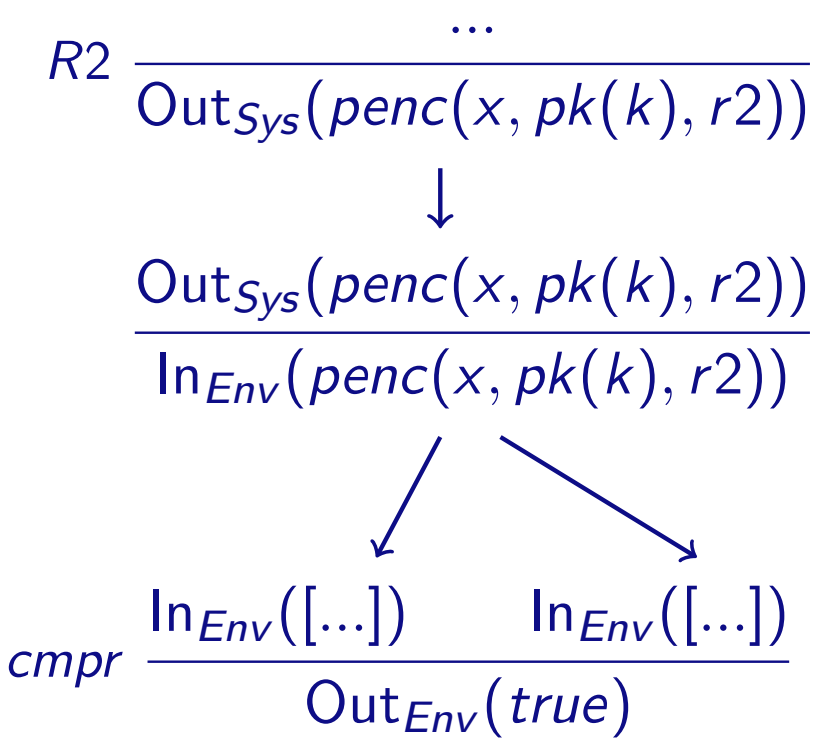
Equality rule: considers two derivations to get some value x

- Both x adversary-generated: trivially mirrors
- x as result of (1)
  - key k is fresh, never known to adversary
  - only derive same x from same source
  - mirrors itself
- x as result of (2)
  - k, r_1, r_2 fresh, not known to adversary
  - only derive same x from same source
  - mirrored - next slide

Probabilistic encryption - reasoning

Truncated graphs shown:





Equivalence Intruder Deduction

Added new message deduction rule:

$$\mathit{IEquality} : \mathrm{K}^{\downarrow}(x), \mathrm{K}^{\uparrow}(x) \texttt{--}\,[]\,\texttt{-->}[]\,.$$

Allows to test whether an equality holds.

- If a side can construct the same value twice this rule is applicable
- Ensures that the same equalities hold on both sides

Algorithm

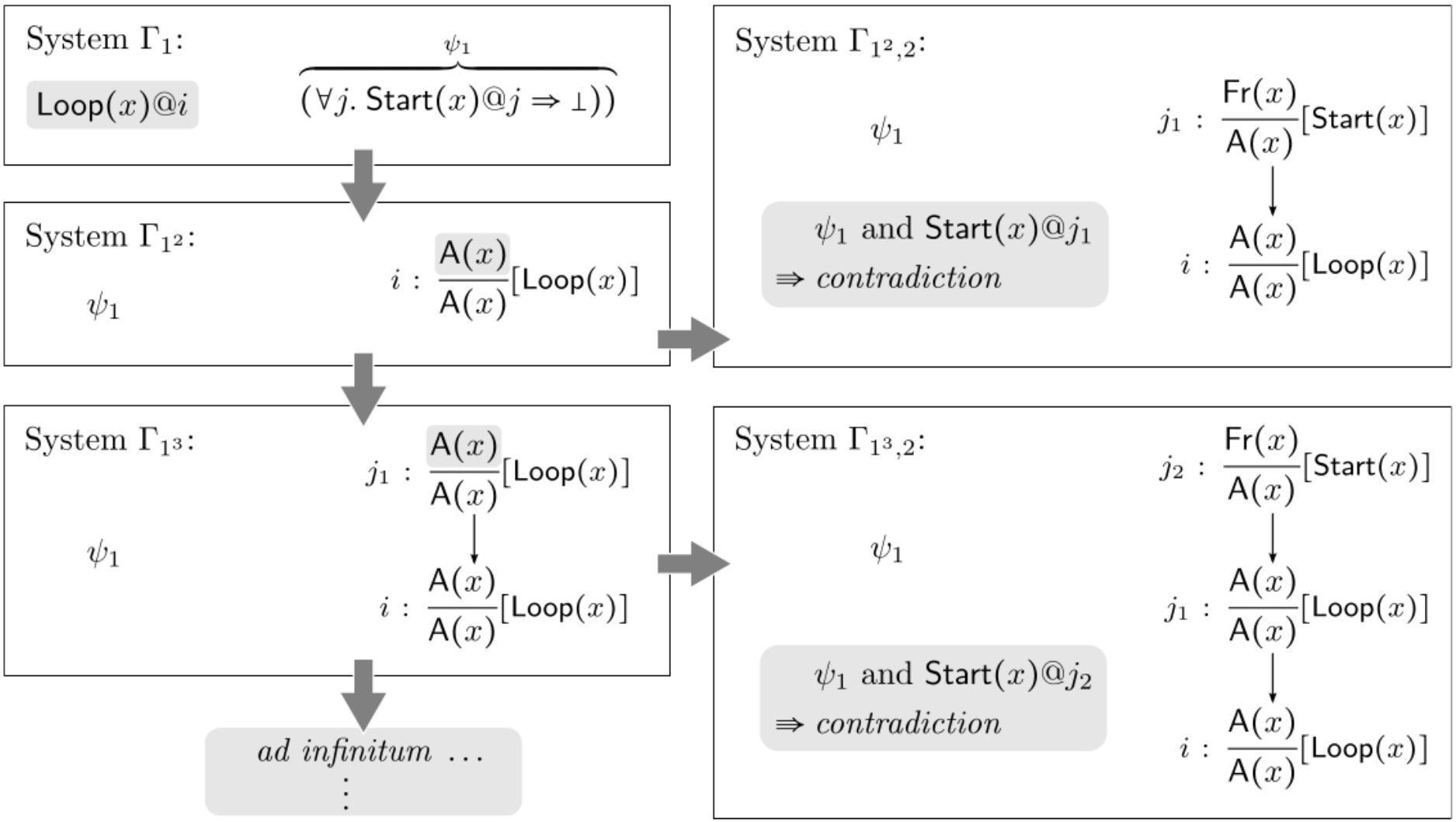
1: **function** VERIFY(S)
2: $RU \leftarrow L(S) \cup R(S) \cup IF \cup Env$
3: **while** $RU \neq \emptyset$ **do**
4: choose $r \in RU$, $RU \leftarrow (RU \setminus \{r\})$
5: compute $DG \leftarrow dgraphs(r)$ by constraint solving
6: **if** $\exists dg \in DG$ s.t. $mirrors(dg)$ lacks ground instances
7: **then return** "potential attack found: ", dg
8: **return** "verification successful"

Induction Example

$$R_{loop} := \left\{ \frac{\mathsf{Fr}(x)}{\mathsf{A}(x)}[\mathsf{Start}(x)],\ \frac{\mathsf{A}(x)}{\mathsf{A}(x)}[\mathsf{Loop}(x)] \right\}$$

- Proof goal: $\forall x\ \#i.\ \mathsf{Loop}(x)@i \Rightarrow \exists \#j.\ \mathsf{Start}(x)@j$
- Such properties are needed:
  - looping constructs (counters etc.)
  - "Sources" lemmas
- Normal constraint solving does not work

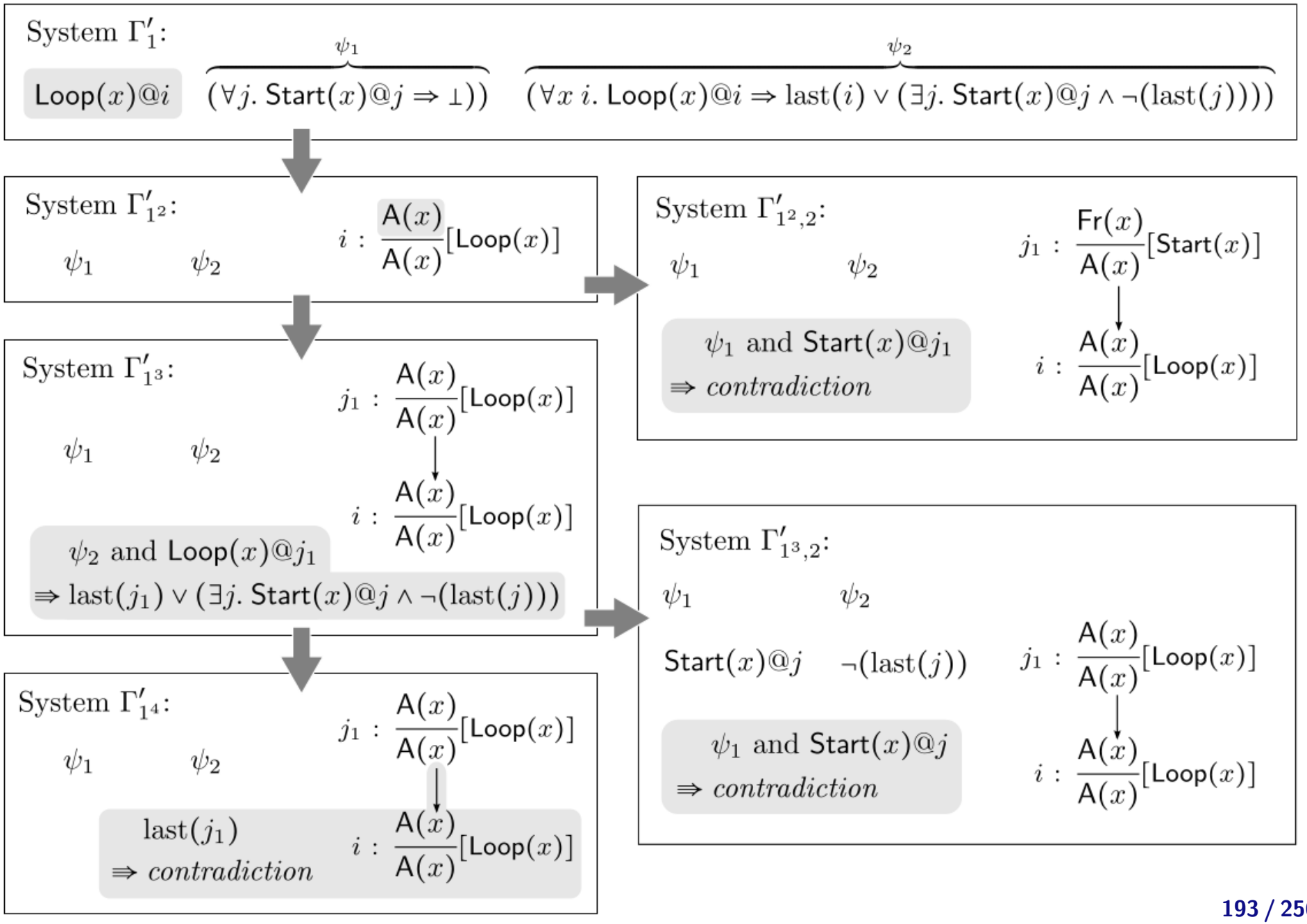
Why standard constraint solving fails



Induction on time points

- Intuitively, induction could help
- Formally, for IH ϕ:
  - Check if ϕ holds for empty trace
  - Consider special `last` rule index on trace
    Assume ϕ holds at all non-last indices, and prove for `last`
- Added constraint reduction rules for last atoms
- Allows proof of previous example

Inductive proof



Induction on time points

- Required for all "sources" lemmas
- Helps for all looping constructs, used in e.g.:
  - YubiKey
  - TPM
  - PKCS11
  - Group protocols
  - Counters
- Can be activated using lemma annotations or in the interactive mode: `lemma counter [use_induction]: ...`

Applicable Proof Methods: Goals sorted according to the 'smart' heuristic (loop breakers delayed)

1. **simplify**

2. **induction**

a. **autoprove** (A. **for all solutions**)
b. **autoprove** (B. **for all solutions**) with proof-depth bound 5

Message Deduction

$$MD_\Sigma = \left\{ \begin{array}{cccc} \dfrac{\mathsf{Out}(x)}{\mathsf{K}(x)} & \dfrac{\mathsf{K}(x)}{\mathsf{In}(x)}[\mathsf{K}(x)] & \dfrac{\mathsf{Fr}(x:fr)}{\mathsf{K}(x:fr)} & \dfrac{}{\mathsf{K}(x:pub)} \\ & \dfrac{\mathsf{K}(x_1) \quad \ldots \quad \mathsf{K}(x_k)}{\mathsf{K}(f(x_1,\ldots,x_k))} \text{ for all } f \in \Sigma & & \end{array} \right\}$$

Nice in theory, but not in practice...

Problems with Message Deduction

$$1:\ \frac{\mathsf{K}(\langle a, b\rangle)}{\mathsf{K}(a)}$$

$$\downarrow$$

$$2:\ \frac{\mathsf{K}(a)\quad \mathsf{K}(c)}{\mathsf{K}(\langle a, c\rangle)}$$

$$\downarrow$$

$$3:\ \frac{\mathsf{K}(\langle a, c\rangle)}{\mathsf{K}(a)}$$

$$\downarrow$$

$$4:\ \frac{\mathsf{K}(a)\quad \mathsf{K}(d)}{\mathsf{K}(\langle a, d\rangle)}$$

$$1:\ \frac{\mathsf{K}^{\uparrow}(a)\quad \mathsf{K}^{\uparrow}(c)}{\mathsf{K}^{\uparrow}(\langle a, c\rangle)}$$

$$\not\downarrow$$

$$2:\ \frac{\mathsf{K}^{\downarrow}(\langle a, c\rangle)}{\mathsf{K}^{\downarrow}(a)}$$

$$1:\ \frac{\mathsf{K}^{\downarrow}(\langle a, b\rangle)}{\mathsf{K}^{\downarrow}(a)}$$

$$\downarrow$$

$$2:\ \frac{\mathsf{K}^{\downarrow}(a)}{\mathsf{K}^{\uparrow}(a)}$$

$$\downarrow$$

$$3:\ \frac{\mathsf{K}^{\uparrow}(a)\quad \mathsf{K}^{\uparrow}(d)}{\mathsf{K}^{\uparrow}(\langle a, d\rangle)}$$

Idea: Avoid loops using $\uparrow$ and $\downarrow$ by converting only $\downarrow$ to $\uparrow$, but not the other way round

Normal Message Deduction

$$ND_\Sigma = \left\{ \begin{array}{c} \dfrac{\mathrm{Out}(x)}{\mathrm{K}^{\downarrow}(x)} \quad \dfrac{\mathrm{K}^{\uparrow}(x)}{\mathrm{In}(x)}[\mathrm{K}(x)] \quad \text{Coerce} : \dfrac{\mathrm{K}^{\downarrow}(x)}{\mathrm{K}^{\uparrow}(x)} \\ \\ \dfrac{\mathrm{Fr}(x : fr)}{\mathrm{K}^{\uparrow}(x : fr)} \quad \dfrac{}{\mathrm{K}^{\uparrow}(x : pub)} \\ \\ \dfrac{\mathrm{K}^{\downarrow}(\langle x, y \rangle)}{\mathrm{K}^{\downarrow}(x)} \quad \dfrac{\mathrm{K}^{\downarrow}(\langle x, y \rangle)}{\mathrm{K}^{\downarrow}(y)} \\ \\ \text{plus other deconstruction rules} \\ \text{depending on the equations} \\ \\ \dfrac{\mathrm{K}^{\uparrow}(x_1) \quad \ldots \quad \mathrm{K}^{\uparrow}(x_k)}{\mathrm{K}^{\uparrow}(f(x_1, \ldots, x_k))} \text{ for all } f \in \Sigma \end{array} \right\}$$

We would like to have a program V with ...

- Input:
  - some description of a program P
  - some description of a functional specification S
- Output: *Yes* if P satisfies S, and *No* otherwise.
- Optional extra: in the *No* case, give a counter-example, i.e. an input on which P violates the specification.

Forget it:

Theorem (Rice)

Let S be any non-empty, proper subset of the computable functions. Then the verification problem for S (the set of programs P that compute a function in S) is undecidable.

The Sources of Infinity

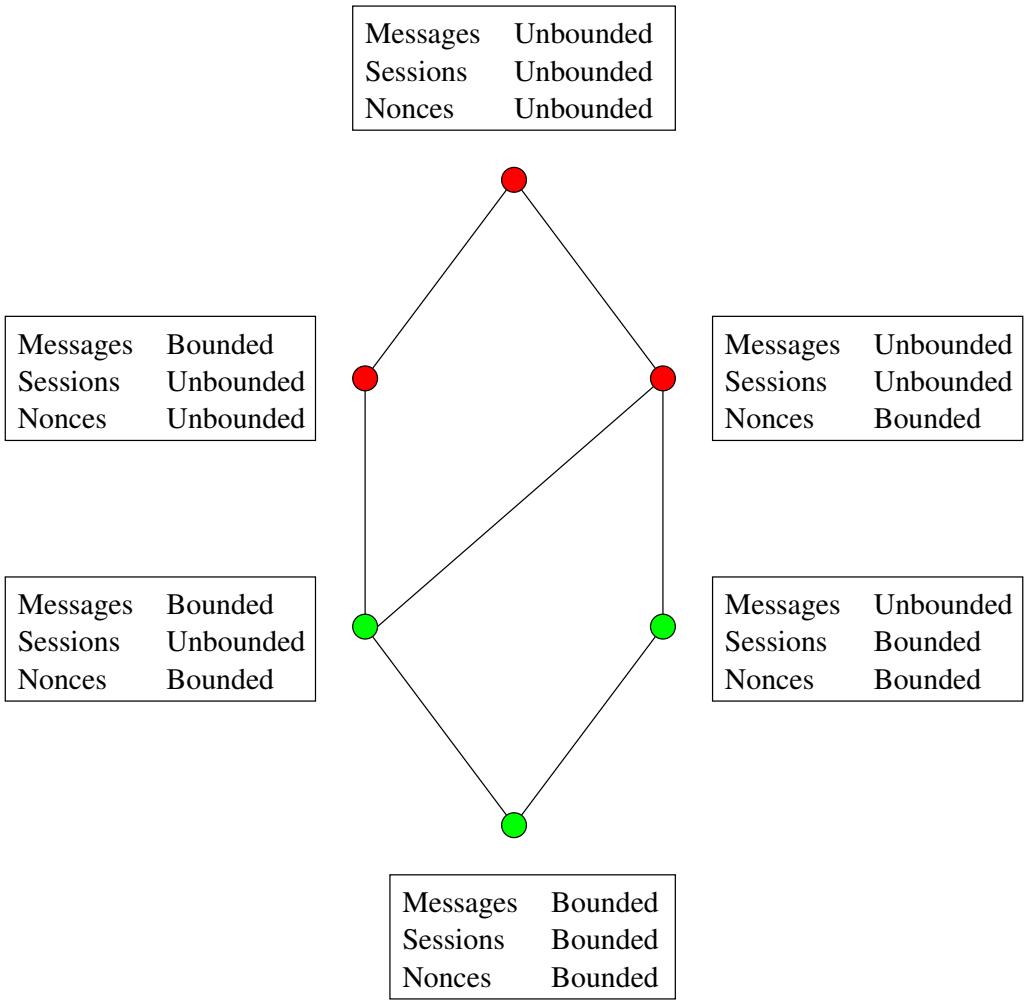
For security protocols, the state space can be infinite for (at least) the following reasons:

Messages The intruder can compose arbitrarily complex messages from his knowledge, e.g., $i, h(i), h(h(i)), \ldots$.

Sessions Any number of sessions (or threads) may be executed.

Nonces Unbounded number of fresh nonces generated.

(Un)decidability: Complete picture



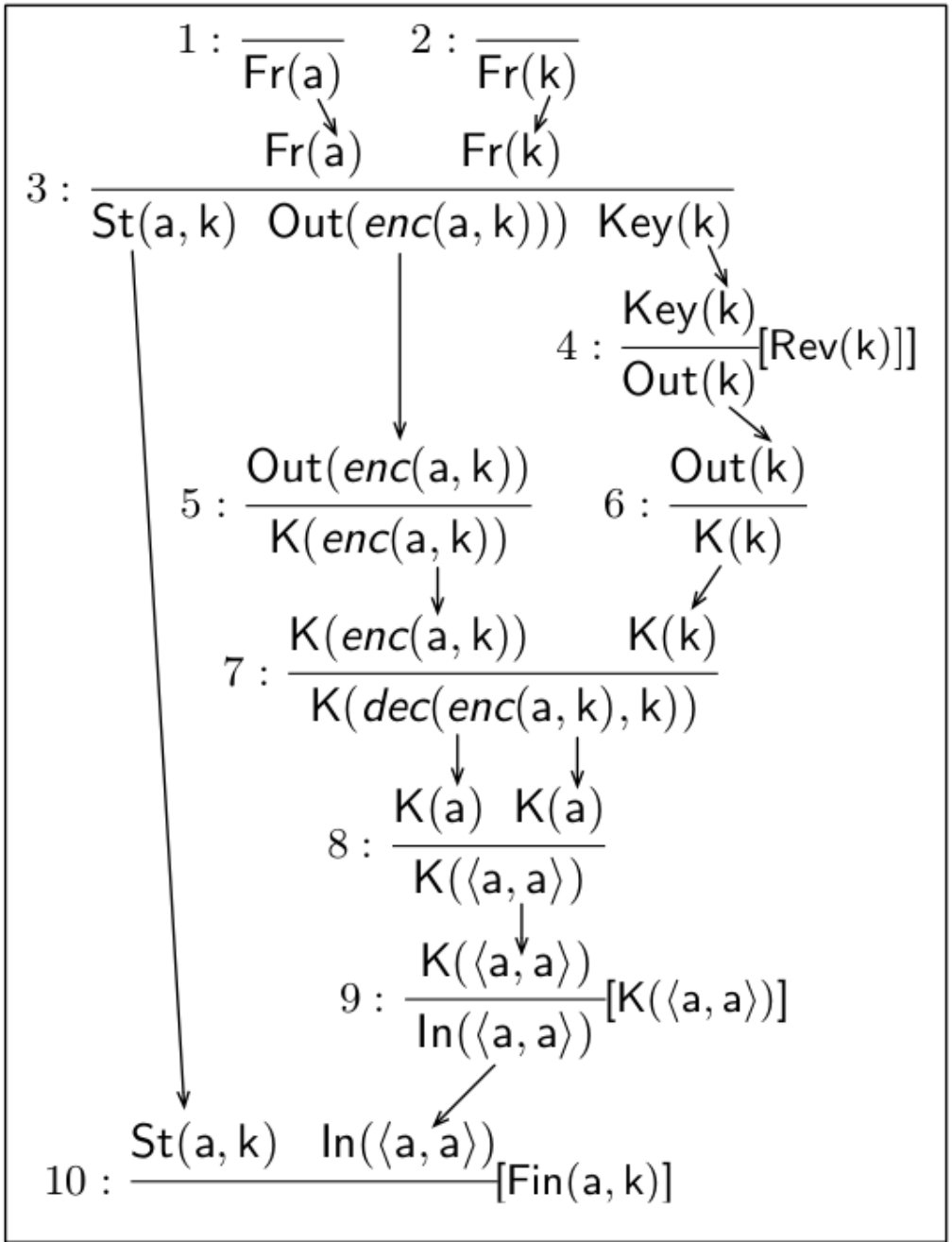
Bottom line: need at least two bounded parameters for decidability.

Tamarin overview

- Use multiset rewriting to represent protocol
- Adversary message deduction rules given as multiset rewriting rules
- Properties specified in first-order logic
  - Allows quantification over messages and timepoints
- Verification algorithm is proven sound and complete

- Backwards reachability analysis – searching for insecure states
  - Negate security property, search for *solutions*
- Constraint solving
  - Uses dependency graphs
  - Normal dependency graphs for state-space reduction – efficiency despite undecidability

Dependency graph example



Bibliography

- David Basin, Sebastian Mödersheim, and Luca Viganò. *OFMC: A symbolic model checker for security protocols.* International Journal of Information Security, 4(3), 2005.
- Hubert Comon, Véronique Cortier, John Mitchell. *Tree automata with one memory, set constraints, and ping-pong protocols*. ICALP 2001.
- Shimon Even and Oded Goldreich. *On the security of multi-party ping-pong protocols*, Symposium on Foundations of Computer Science, IEEE Computer Society, 1983.
- N. Durgin, P. Lincoln, J. Mitchell, and A. Scedrov. *Undecidability of bounded security protocols.* In Workshop on Formal Methods and Security Protocols (FMSP '99), 1999.
- J. Millen and V. Shmatikov. *Constraint Solving for Bounded-Process Cryptographic Protocol Analysis.* CCS 2001.
- Michaël Rusinowitch and Mathieu Turuani. *Protocol Insecurity with Finite Number of Sessions is NP-complete.* CSFW, 2001.

Precomputation

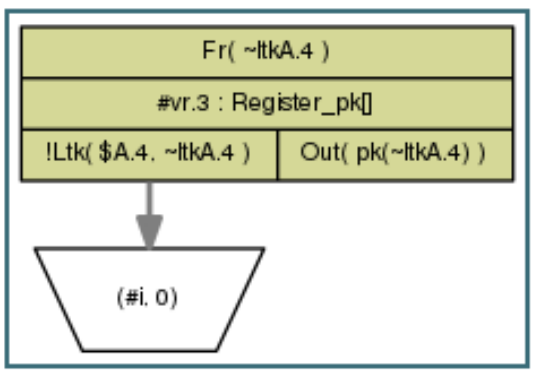
- **Idea:** for all facts in rule premises precompute their *possible sources* (improves efficiency)
- Sources are (combinations of) rules yielding that fact as (part of the) result
- Initial precomputations are called *raw sources*
- Sometimes these precomputations are incomplete, and give *partial deconstructions*
- GUI shows both raw and *refined sources*:

```
theory sources begin

Message theory

Multiset rewriting rules (5)

Raw sources (8 cases, 6 partial deconstructions left)

Refined sources (8 cases, deconstructions complete)
```

Examples of sources

Sources of "!Ltk(t.1, t.2) ▶₀ #i" (1 cases)

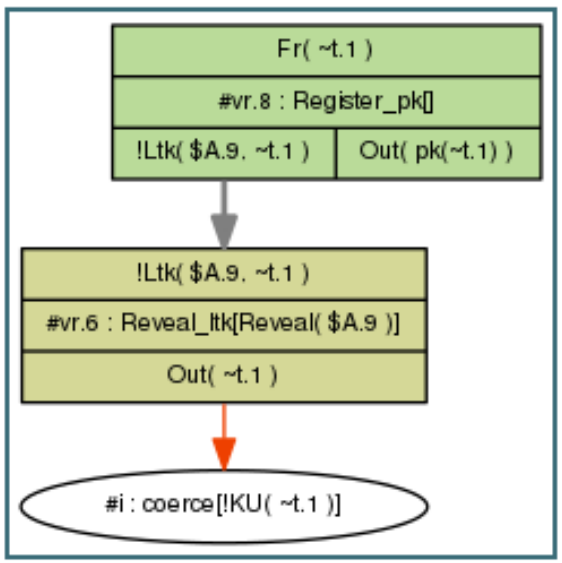
Source 1 of 1 / named "Register_pk"



"!Ltk(t.1, t.2) ▶₀ #i"

Sources of "!KU(~t.1) @ #i" (6 cases)

Source 1 of 6 / named "Reveal_ltk"



"!KU(~t.1) @ #i"

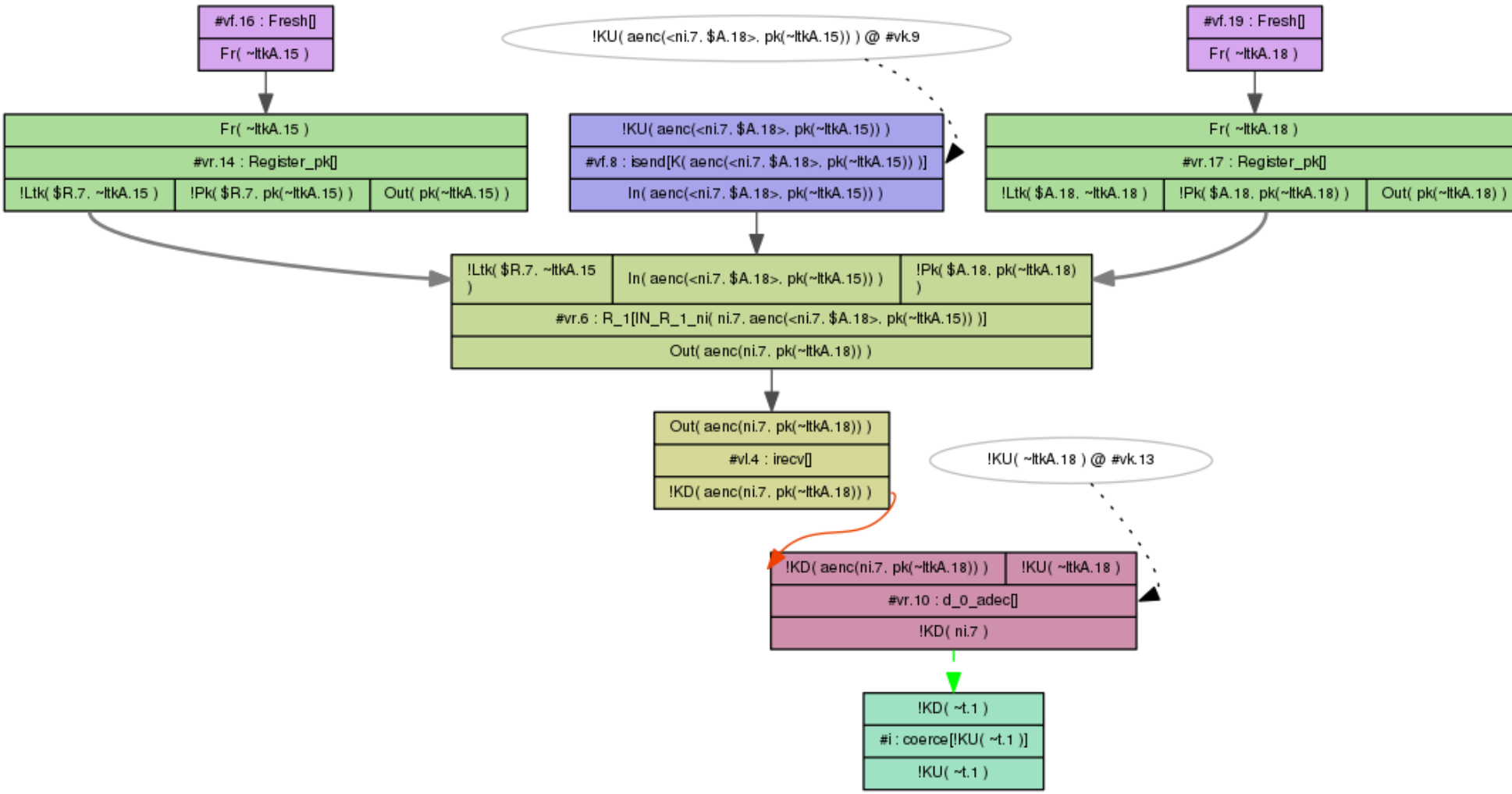
Partial deconstructions: example protocol

1: $I \to R$: $\{ni, I\}_{pk(R)}$
2: $R \to I$: $\{ni\}_{pk(I)}$

```
rule I_1:
 let m1 = aenc{~ni, $I}pkR
 in
 [ Fr(~ni) , !Pk($R, pkR) ]
 --[ OUT_I_1(m1) ]->
 [ Out( m1 ) ]

rule R_1:
 let m1 = aenc{ni, I}pk(ltkR)
     m2 = aenc{ni}pkI
 in
 [ !Ltk($R, ltkR) , In( m1 ), !Pk(I, pkI) ]
 --[ IN_R_1_ni( ni, m1 ) ]->
 [ Out( m2 ) ]
```

(This looks like a decryption oracle for values ni.)

Partial deconstruction: example



Partial deconstructions - issues

- Source for *"everything"*
- Complicates proofs and can easily cause non-termination
- Need to resolve such partial deconstructions
- *Realization:* result not "everything", but only values actually sent by legitimate party or adversary-generated terms
- Claim (and then prove) this using *sources lemma*

Example:

```
lemma types [sources]:
 "(All ni m1 #i.
  IN_R_1_ni( ni, m1) @ i
 ==>
  ( (Ex #j. K(ni) @ j & j < i)
  | (Ex #j. OUT_I_1( m1 ) @ j) ) ) "
```

Sources lemmas

Explain where terms can come from or what their form must be.

How Tamarin proceeds:

1. Determine possible sources (raw)
2. Apply sources lemma to raw sources to get refined sources
3. Prove sources lemma with respect to raw sources
4. Prove other lemmas with respect to refined sources

Termination issues

Problem: What to do if Tamarin does not terminate?

- Use interactive mode to inspect the proof (use bounded autoprover!)
- If you find a loop, try to avoid it:
  - Find an intermediate helping lemma:
    `lemma help [reuse]: ...`
  - Tweak your model: add or remove details, ...
- To find a known attack: use BFS

Conclusions

- Getting security protocols is not easy
- Symbolic analysis can help
- Tamarin has an interesting set of features and applications, many of whom I did not talk about (Human interaction security protocols, user-defined heuristics, parallelization, preprocessors, SAPIC, . . .)
- . . . but is still under development!

(-: Happy Proving :-)

Proofs of protocols : the symbolic model

- The **Dolev-Yao model** was proposed in 1983:
  - messages are terms in a free algebra: (m_1, m_2), $\{m\}_k$, $[m]_k$, $h(m)$...
  - no secret value (nonce or key) can be guessed
  - the intruder can only apply functions in the given signature
  - ... but has complete control of the network
  - choice in semantics : non - deterministic
- One can add equations between primitives (functions), but anyway, we assume that the only equalities are those given by these equations.
- Proofs in this model are simpler and can be automated.

Proofs of protocols : the computational model

- The **computational model** was proposed in the '80s:
  - the messages are *bitstrings*
  - the cryptographic primitives (encryption, decryption, key-generation, signing..) are *probabilistic polynomial-time algorithms on bitstrings*
  - secrets can be guessed ... with *very small* probability
  - the intruder is any *interactive probabilistic polynomial-time algorithme*
  - has complete control of the network
- The model is much closer to reality, but proofs in this model are mostly manual.

Proofs

- Computational world:
  Done by hand, long and often complex and hard to understand. (Sometime proofs are error-prone). Often based on reduction to cryptographic assumptions.
- Symbolic world:
  Based on constraint solving, First-order Logic, Tree automata etc ... these proofs are automatic.

Exemple: asymmetric encryption

In the symbolic world:

(AD) $\dfrac{T_0 \vdash \{u\}_{pk(v)} \qquad T_0 \vdash sk(v)}{T_0 \vdash u}$ **(AC)** $\dfrac{T_0 \vdash u \qquad T_0 \vdash pk(v)}{T_0 \vdash \{u\}_{pk(v)}}$

In the real world: ElGamal Encryption:

- a cyclic group G of order q and generator g (g and q deppend on some integer η - the security parameter)
- $\mathcal{K}_\eta() = x \leftarrow^R \mathbb{Z}_q; sk \leftarrow x; pk \leftarrow g^x; return(sk, pk)$
- $\{m\}_{pk} = r \leftarrow^R \mathbb{Z}_q; return(pk^r \times g^m, g^r)$.

But what can we assume about this primitive?

IND-CPA encryption

After having chosen two plaintexts m_0 and m_1, upon receiving the encryption of m_b (for a random bit b), it shoud be hard to guess which message has been encrypted:

$$Adv_{\eta}^{IND-CPA}(\mathcal{A}) = Pr[b' = 1 | (sk, pk) \leftarrow^R \mathcal{K}_{\eta}(); (m_0, m_1) \leftarrow^R \mathcal{A}(pk); c = \{m_1\}_{pk}; b' \leftarrow^R \mathcal{A}(c)] - Pr[b' = 1 | (sk, pk) \leftarrow^R \mathcal{K}_{\eta}(); (m_0, m_1) \leftarrow^R \mathcal{A}(pk); c = \{m_0\}_{pk}; b' \leftarrow^R \mathcal{A}(c)]$$

The encryption is IND-CPA, if the advantage $Adv_{\eta}^{IND-CPA}(\mathcal{A})$ is negligible for any ppt-adversary $\mathcal{A}$.
Let us take it slowly...

Security parameter

In practice, we use keys, nonces, messages... they all have some length. Also, the algorithms we use, take some time to execute. So, the **security parameter**, denoted η, is an integer such that:

- the key/plaintext length in asymmetric encryption and signing can be η
- the key/block length in block ciphers/symmetric encryption can be η
- the key/tag length in MACs or output length in hash functions can be η
- the execution time of honest participants and of the intruder is **polynomial** in η

Computational indistinguishability - definition

- Let $\mathcal{D}^0 = \{\mathcal{D}^0_\eta\}_{\eta\in\mathbb{N}}$ and $\mathcal{D}^1 = \{\mathcal{D}^1_\eta\}_{\eta\in\mathbb{N}}$ be two (families of) distributions over bit-strings. The advantage $Adv^{\mathcal{D}_0,\mathcal{D}_1}(\mathcal{A})$ of an adversary $\mathcal{A}$ is defined by

$$Adv^{\mathcal{D}^0,\mathcal{D}^1}(\mathcal{A}) = Pr[b' = 1 | d \leftarrow^R \mathcal{D}^0_\eta; b' \leftarrow^R \mathcal{A}(\eta, d)]$$
$$- Pr[b' = 1 | d \leftarrow^R \mathcal{D}^1_\eta; b' \leftarrow^R \mathcal{A}(\eta, d)]$$

  The distributions $\mathcal{D}^0$ and $\mathcal{D}^1$ are **computationally indistinguishable** (denoted $\mathcal{D}^0 \approx \mathcal{D}^1$), if the advantage $Adv^{\mathcal{D}_0,\mathcal{D}_1}(\mathcal{A})$ is negligible for any ppt-algorithm $\mathcal{A}$.
- $f : \mathbb{N} \to \mathbb{R}$ is **negligible** if $\forall n.\exists N_n.\forall \eta \geq N_n.f(\eta) \leq \frac{1}{\eta^n}$ (for all polynomials p, $lim_{\eta\to\infty} f(\eta) \times p(\eta) = 0$)

Computational indistinguishability - exemples

- statistical indistinguishability implies computational indistinguishability

$$[d \leftarrow^R \{0,1\}^\eta : d] \approx [d \leftarrow^R \{0,1\}^\eta;\ \mathit{if}\ d = 0^\eta\ \mathit{then}\ d = 1^\eta : d]$$

- Decisional Diffie Hellman Problem is (assumed) difficult.
  Let G a cyclic group of order q and generator g (g and q deppend on η). Then

$$[x, y \leftarrow^R \mathbb{Z}_q : (g^x, g^y, g^{x \times y})] \approx [x, y, z \leftarrow^R \mathbb{Z}_q : (g^x, g^y, g^z)]$$

  We can prove that ElGamal encryption is IND-CPA secure, assuming that the Decisional Diffie Hellman Problem is hard.

Computational indistinguishability : transitivity

Theorem

If $\mathcal{D}^0 \approx \mathcal{D}^1$ and $\mathcal{D}^1 \approx \mathcal{D}^2$ then $\mathcal{D}^0 \approx \mathcal{D}^2$.

Proof.

- Suppose that $\mathcal{D}^0 \not\approx \mathcal{D}^2$, and let $\mathcal{A}$ be a ppt-adversary that can distinguish $\mathcal{D}^0$ and $\mathcal{D}^2$ with non-negligible advantage.
- For $i \in \{0, 1, 2\}$, let $p^i_\eta = Pr[b' = 1 | d \leftarrow^R \mathcal{D}^i_\eta; b' \leftarrow^R \mathcal{A}(\eta, d)]$.
- There is a polynomial q, such that for infinitely many η, $p^0_\eta - p^2_\eta \geq 1/q(\eta)$.
- Then, either $p^0_\eta - p^1_\eta \geq 1/(q(\eta)/2)$ or $p^1_\eta - p^2_\eta \geq 1/(q(\eta)/2)$ for infinitely many η.
- Hence $\mathcal{A}$ distinguishes either $\mathcal{D}^0$ and $\mathcal{D}^1$, or $\mathcal{D}^1$ and $\mathcal{D}^2$ with non-negligible advantage.

□

Computational indistinguishability : proof by reduction

Definition

A family of distributions $\mathcal{E}$ is **polynomial-time constructible**, if there is a ppt-algorithm $\Psi_{\mathcal{E}}$, such that the output of $\Psi_{\mathcal{E}}(\eta)$ is distributed identically to $\mathcal{E}_\eta$.

Definition

Given two families of distributions $\mathcal{D}$ and $\mathcal{E}$, we define $\mathcal{D}\|\mathcal{E}$ by

$$(\mathcal{D}\|\mathcal{E})_\eta = [x \leftarrow^R \mathcal{D}_\eta; y \leftarrow^R \mathcal{E}_\eta : \ (x, y)]$$

Theorem

If $\mathcal{D}^0 \approx \mathcal{D}^1$ and $\mathcal{E}$ is polynomial-time constructible, then $(\mathcal{D}^0\|\mathcal{E}) \approx (\mathcal{D}^1\|\mathcal{E})$.

Computational indistinguishability : proof by reduction

Theorem. If $\mathcal{D}^0 \approx \mathcal{D}^1$ and $\mathcal{E}$ is polynomial-time constructible, then $(\mathcal{D}^0 \| \mathcal{E}) \approx (\mathcal{D}^1 \| \mathcal{E})$.

Proof.

- Suppose that $(\mathcal{D}^0 \| \mathcal{E}) \not\approx (\mathcal{D}^1 \| \mathcal{E})$, and let $\mathcal{A}$ be a ppt-adversary that can distinguish $(\mathcal{D}^0 \| \mathcal{E})$ and $(\mathcal{D}^1 \| \mathcal{E})$ with non-negligible advantage.
- Define an adversary B by

$$\mathrm{B}(\eta, x) = [y \leftarrow^R \Psi_{\mathcal{E}}(\eta); b' \leftarrow^R \mathcal{A}(\eta, (x, y)) : \ b']$$

- We can see that if x is distributed according to $\mathcal{D}^i_\eta$, then the argument of $\mathcal{A}$ is distributed according to $(\mathcal{D}^i \| \mathcal{E})_\eta$.
- Hence the advantage of B is equal to the advantage of $\mathcal{A}$.

□

Computational indistinguishability : hybrid argument

Definition

Let $\mathcal{D}$ be a family of distributions, and p be a positive polynomial. We define the family of distributions $\overrightarrow{\mathcal{D}}$ by

$$\overrightarrow{\mathcal{D}}_\eta = [x_1 \leftarrow^R \mathcal{D}_\eta; \ldots; x_{p(\eta)} \leftarrow^R \mathcal{D}_\eta : \ (x_1, \ldots, x_{p(\eta)})]$$

To sample $\overrightarrow{\mathcal{D}}_\eta$, sample $\mathcal{D}_\eta$ $p(\eta)$ times, and construct the tuple of sampled values.

Theorem

If $\overrightarrow{\mathcal{D}^0} \approx \overrightarrow{\mathcal{D}^1}$ then $\mathcal{D}^0 \approx \mathcal{D}^1$.

Theorem

If $\mathcal{D}^0 \approx \mathcal{D}^1$ and $\mathcal{D}^i$ are polynomial-time constructible, then $\overrightarrow{\mathcal{D}^0} \approx \overrightarrow{\mathcal{D}^1}$.

Computational indistinguishability : hybrid argument 1

If $\overrightarrow{\mathcal{D}^0} \approx \overrightarrow{\mathcal{D}^1}$ then $\mathcal{D}^0 \approx \mathcal{D}^1$.

Theorem.

Proof.

- Suppose that $\mathcal{D}^0 \not\approx \mathcal{D}^1$, and let $\mathcal{A}$ be a ppt-adversary that can distinguish $\mathcal{D}^0$ and $\mathcal{D}^1$ with non-negligible advantage.
- Define an adversary B by

$$\mathrm{B}(\eta, (x_1, \ldots, x_{p(\eta)})) = [b' \leftarrow^R \mathcal{A}(\eta, x_1) : \ b']$$

- We can see that if $(x_1, \ldots, x_{p(\eta)})$ is distributed according to $\overrightarrow{\mathcal{D}^i}_\eta$, then the argument x_1 of $\mathcal{A}$ is distributed according to $\mathcal{D}^i_\eta$.
- Hence the advantage of B is equal to the advantage of $\mathcal{A}$.

□

Computational indistinguishability : hybrid argument 2

Theorem. If $\mathcal{D}^0 \approx \mathcal{D}^1$ and $\mathcal{D}^i$ are polynomial-time constructible, then $\overrightarrow{\mathcal{D}^0} \approx \overrightarrow{\mathcal{D}^1}$.

Proof.

- Suppose that $\overrightarrow{\mathcal{D}^0} \not\approx \overrightarrow{\mathcal{D}^1}$, and let $\mathcal{A}$ be a ppt-adversary that can distinguish $\overrightarrow{\mathcal{D}^0}$ and $\overrightarrow{\mathcal{D}^1}$ with non-negligible advantage.
- Hence for some polynomial q, and for infinitely many η, $Pr[\mathcal{A}(\eta, x) = 1 | x \leftarrow^R \overrightarrow{\mathcal{D}^0}_\eta] - Pr[\mathcal{A}(\eta, x) = 1 | x \leftarrow^R \overrightarrow{\mathcal{D}^1}_\eta] \geq 1/q(\eta)$.
- Assume for now that p is a constant.
- For $k \in \{0, \ldots, p\}$, define $\overrightarrow{\mathcal{E}^k}_\eta$ by $\overrightarrow{\mathcal{E}^k}_\eta = [x_1 \leftarrow^R \mathcal{D}^0_\eta; \ldots; x_k \leftarrow^R \mathcal{D}^0_\eta; x_{k+1} \leftarrow^R \mathcal{D}^1_\eta; \ldots; x_p \leftarrow^R \mathcal{D}^1_\eta : (x_1, \ldots, x_p)]$
- Thus $\overrightarrow{\mathcal{E}^0}_\eta = \overrightarrow{\mathcal{D}^1}$ and $\overrightarrow{\mathcal{E}^p}_\eta = \overrightarrow{\mathcal{D}^0}$. Define $P^k_\eta = Pr[\mathcal{A}(\eta, x) = 1 | x \leftarrow^R \overrightarrow{\mathcal{E}^k}_\eta]$.
- Then for infinitely many η, $1/q(\eta) \leq P^p_\eta - P^0_\eta = \sum_{i=1}^p (P^i_\eta - P^{i-1}_\eta)$.
- Then for some j and infinitely many η, $(P^j_\eta - P^{j-1}_\eta)] \geq 1/(p \times q(\eta))$.
- Then for some j, $\overrightarrow{\mathcal{E}^{j-1}}_\eta \not\approx \overrightarrow{\mathcal{E}^j}_\eta$, and $\mathcal{A}$ can distinguish them.

Computational indistinguishability : hybrid argument 3

Proof.

(Continuation 1)

- There is some j, such that $\overrightarrow{\mathcal{E}^{j-1}}_\eta \not\approx \overrightarrow{\mathcal{E}^{j}}_\eta$, and $\mathcal{A}$ can distinguish them.
- Now if $\mathcal{A}$ can distinguish $\overrightarrow{\mathcal{E}^{j-1}}_\eta$ and $\overrightarrow{\mathcal{E}^{j}}_\eta$, how can we construct B that can distinguish $\overrightarrow{\mathcal{D}^0} = \overrightarrow{\mathcal{E}^p}_\eta$ and $\overrightarrow{\mathcal{D}^1} = \overrightarrow{\mathcal{E}^0}_\eta$.
- Define B as follows. On input (η, x):
  1. Let $x_1 \leftarrow^R \mathcal{D}^0_\eta; \ldots; x_{j-1} \leftarrow^R \mathcal{D}^0_\eta$.
  2. Let $x_j := x$.
  3. Let $x_j \leftarrow^R \mathcal{D}^1_\eta; \ldots; x_p \leftarrow^R \mathcal{D}^1_\eta$.
  4. Call $b' \leftarrow^R \mathcal{A}(\eta, (x_1, \ldots, x_p))$ and return b'.
- The advantage of this distinguisher is at least $1/(p \times q(\eta))$.
- Remark: the above prove was not constructive.

□

Computational indistinguishability : being constructive

Proof.

(Continuation 2)

- Suppose that $\overrightarrow{\mathcal{D}^0} \not\approx \overrightarrow{\mathcal{D}^1}$, and let $\mathcal{A}$ be a ppt-adversary that can distinguish $\overrightarrow{\mathcal{D}^0}$ and $\overrightarrow{\mathcal{D}^1}$ with non-negligible advantage.
- Hence for some polynomial q, and for infinitely many η,
  $Pr[\mathcal{A}(\eta, x) = 1 | x \leftarrow^R \overrightarrow{\mathcal{D}^0}_\eta] - Pr[\mathcal{A}(\eta, x) = 1 | x \leftarrow^R \overrightarrow{\mathcal{D}^1}_\eta] \geq 1/q(\eta)$.
- Define B as follows. On input (η, x):
  0. Let $j \leftarrow^R \{1, \ldots, p\}$.
  1. Let $x_1 \leftarrow^R \mathcal{D}^0_\eta; \ldots; x_{j-1} \leftarrow^R \mathcal{D}^0_\eta$.
  2. Let $x_j := x$.
  3. Let $x_j \leftarrow^R \mathcal{D}^1_\eta; \ldots; x_p \leftarrow^R \mathcal{D}^1_\eta$.
  4. Call $b' \leftarrow^R \mathcal{A}(\eta, (x_1, \ldots, x_p))$ and return b'.
- The advantage of this distinguisher is at least $1/(p \times q(\eta))$.

□

Computational indistinguishability : p depends on η

Proof.

(Continuation 3)

- Suppose that $\overrightarrow{\mathcal{D}^0} \not\approx \overrightarrow{\mathcal{D}^1}$, and let $\mathcal{A}$ be a ppt-adversary that can distinguish $\overrightarrow{\mathcal{D}^0}$ and $\overrightarrow{\mathcal{D}^1}$ with non-negligible advantage.
- Hence for some polynomial q, and for infinitely many η,
  $Pr[\mathcal{A}(\eta, x) = 1 | x \leftarrow^R \overrightarrow{\mathcal{D}^0}_\eta] - Pr[\mathcal{A}(\eta, x) = 1 | x \leftarrow^R \overrightarrow{\mathcal{D}^1}_\eta] \geq 1/q(\eta)$.
- Define B as follows. On input (η, x):
  0. Let $j \leftarrow^R \{1, \ldots, p(\eta)\}$.
  1. Let $x_1 \leftarrow^R \mathcal{D}^0_\eta; \ldots; x_{j-1} \leftarrow^R \mathcal{D}^0_\eta$.
  2. Let $x_j := x$.
  3. Let $x_j \leftarrow^R \mathcal{D}^1_\eta; \ldots; x_{p(\eta)} \leftarrow^R \mathcal{D}^1_\eta$.
  4. Call $b' \leftarrow^R \mathcal{A}(\eta, (x_1, \ldots, x_{p(\eta)}))$ and return b'.
- The advantage of this distinguisher is at least $1/(p(\eta) \times q(\eta))$.

□

Back to the symbolic world

Messages are defined by the following grammar

$$
\begin{aligned}
\text{Keys} &= \{k, k_1, k_2, \ldots, \} \\
\text{Coins} &= \{r, s, r_1, r_2, \ldots, \} \\
\text{Bits} &= \{0, 1\} \\
e ::= & \; k \in \text{Keys} \\
& \mid b \in \text{Bits} \\
& \mid (e_1, e_2) \\
& \mid \{e\}_k^r
\end{aligned}
$$

- If $\{e\}_k^r$ and $\{e'\}_{k'}^r$ both occur in the same context, then $e = e'$ and $k = k'$.

Recall: Dolev-Yao Deduction System

We denote $T_0 \vdash e$ for "message e is deducible from the set of messages T_0".

Deduction System for Dolev Yao theory: $T_0 \vdash^? s$

(C0) $\dfrac{}{T_0 \vdash 0}$

(C1) $\dfrac{}{T_0 \vdash 1}$

(A) $\dfrac{u \in T_0}{T_0 \vdash u}$

(UL) $\dfrac{T_0 \vdash (u, v)}{T_0 \vdash u}$

(P) $\dfrac{T_0 \vdash u \qquad T_0 \vdash v}{T_0 \vdash (u, v)}$

(UR) $\dfrac{T_0 \vdash (u, v)}{T_0 \vdash v}$

(C) $\dfrac{T_0 \vdash u \qquad T_0 \vdash v}{T_0 \vdash \{u\}_v^r}$

(D) $\dfrac{T_0 \vdash \{u\}_v^r \qquad T_0 \vdash v}{T_0 \vdash u}$

Symbolic world: meaning of a message

- Now we want do define an equivalence $\sim$ between messages, such that $e_1 \sim e_2$ when e_1 and e_2 *"look the same"*. Intuitevely, by doing any permitted operation, we cannot distinguish e_1 and e_2.
- Examples:
  - $\{0\}_v^r \sim \{1\}_v^{r'}$.
  - $(\{0\}_v^r, v) \not\sim (\{1\}_v^{r'}, v)$.
  - $(\{0\}_v^r, (\{1\}_v^s) \sim (\{1\}_v^{r'}, \{1\}_u^{s'})$.
- How can we define formally $\sim$?

Key Recovery **rec** function

- We define $\mathbf{rec}(E)$ as being the set of keys that can be recovered from E using information available in E only: $\mathbf{rec}(E) = \{k \in \mathsf{Keys} | E \vdash k\}$.
- Example:

$$\mathbf{rec}((((\{\{k_2\}^r_{k_1}\}^{r'}_{k_1}, \{(k_3, 0)\}^{r''}_{k_2}), k_1)) = \{k_1, k_2, k_3\}$$

- But we want a constructive definition of $\mathbf{rec}(E)$!

Key Recovery Function in one pass $\mathbf{F}_{kr}(E, K)$

- Following the approach of [Micciancio,Warinschi], for an expression E and a set K of keys, we define Key Recovery Function $\mathbf{F}_{kr}(E, K)$ as follows:

$$\begin{aligned}
\mathbf{F}_{kr}(b, K) &= K \\
\mathbf{F}_{kr}(k, K) &= \{k\} \cup K \\
\mathbf{F}_{kr}((E_1, E_2), K) &= \mathbf{F}_{kr}(E_1, K) \cup \mathbf{F}_{kr}(E_2, K) \\
\mathbf{F}_{kr}(\{E\}_k, K) &= \begin{cases} K & \text{if } k \notin K \\ \mathbf{F}_{kr}(E, K) & \text{otherwise.} \end{cases}
\end{aligned}$$

Example

$E = ((\{\{\{k_2\}^{r}_{k_1}\}^{r'}_{k_1}, \{(k_3, 0)\}^{r''}_{k_2}), k_1)$

1. $\mathbf{F}_{kr}(E, \emptyset) = \{k_1\}$
2. $\mathbf{F}_{kr}(E, \{k_1\}) = \{k_1, k_2\}$
3. $\mathbf{F}_{kr}(E, \{k_1, k_2\}) = \{k_1, k_2, k_3\}$
4. $\mathbf{F}_{kr}(E, \{k_1, k_2, k_3\}) = \{k_1, k_2, k_3\}$

Inductive Definition of Key Recovery **rec**

- Let E be an expression, then we define **rec** by $\mathbf{rec}(E) = \bigcup_i G_i(E) = G_{|E|}(E)$ where

$$G_0(E) = \emptyset$$
$$G_i(E) = \mathbf{F}_{kr}(E, G_{i-1}(E)).$$

Example

$E = ((\{\{k_2\}^r_{k_1}\}^{r'}_{k_1}, \{(k_3, 0)\}^{r''}_{k_2}), k_1)$

1. $G_1(E) = \mathbf{F}_{kr}(E, \emptyset) = \{k_1\}$
2. $G_2(E) = \mathbf{F}_{kr}(E, \{k_1\}) = \{k_1, k_2\}$
3. $G_3(E) = \mathbf{F}_{kr}(E, \{k_1, k_2\}) = \{k_1, k_2, k_3\}$
4. $G_4(E) = \mathbf{F}_{kr}(E, \{k_1, k_2, k_3\}) = \{k_1, k_2, k_3\} = G_3(E) = \mathbf{rec}(E)$

Patterns

- *Patterns* are defined by the following grammar

$$
\begin{aligned}
\text{Keys} &= \{k, k_1, k_2, \ldots, \} \\
\text{Coins} &= \{r, s, r_1, r_2, \ldots, \} \\
\text{Bits} &= \{0, 1\} \\
P ::= & \; k \in \text{Keys} \\
& \mid b \in \text{Bits} \\
& \mid (P_1, P_2) \\
& \mid \{P\}_k^r \\
& \mid \square^r
\end{aligned}
$$

- Remark: a message is a pattern with no occurrence of $\square$
- e.g., $\{(\{\square^r\}_{k_2}^s, 1)\}_{k_1}^{s'}$ is a pattern

The pattern of an expression

- We denote by **keys**(E) the set of all keys occurring in E, and let **hidden**$(E) =$ **keys**$(E) -$ **rec**(E). Define $\textsc{Pat}(E, K)$ as follows:

$$\begin{aligned}
\textsc{Pat}(b, K) &= b \\
\textsc{Pat}(k, K) &= k \\
\textsc{Pat}((E_1, E_2), K) &= (\textsc{Pat}(E_1, K), \textsc{Pat}(E_2, K)) \\
\textsc{Pat}(\{E\}^r_k, K) &= \begin{cases} \{\textsc{Pat}(E, K)\}^r_k & \text{if } k \in K \\ \Box^r & \text{otherwise} \end{cases}
\end{aligned}$$

  Finally, we define **pat** by **pat**$(E) = \textsc{Pat}(E,$ **rec**$(E))$. Intuitively, **pat**(E) is the **pattern** corresponding to E, given the knowledge of any keys that may be recovered from E.

Examples

$$\mathbf{pat}(E) = \textsc{Pat}(E, \mathbf{rec}(E))$$

$$E = ((\{\{k_2\}^{r}_{k_1}\}^{r'}_{k_1}, \{k_3\}^{r''}_{k_2}), k_1)$$

$$\mathbf{rec}(E) = \{k_1, k_2, k_3\}$$

$$\mathbf{pat}(E) = E$$

$$E = ((\{\{k_2\}^{r}_{k_1}\}^{r'}_{k_1}, \{k_3\}^{r''}_{k_2}), k_2)$$

$$\mathbf{rec}(E) = \{k_2, k_3\}$$

$$\mathbf{pat}(E) = (\Box^{r'}, \{k_3\}^{r''}_{k_2}, k_2)$$

Pattern - exercise

Example (Exercise)

$$
\begin{aligned}
pat((\mathbf{0},\mathbf{1})) &= ? \\
pat((\{\mathbf{0}\}^{r}_{K_1},(\{\mathbf{1}\}^{r'}_{K_2},K_1))) &= ? \\
pat((\{\{K_1\}^{r'}_{K_2}\}^{r''}_{K_3},K_3)) &= ?
\end{aligned}
$$

Pattern Equivalence

- We define E to be **symbolically equivalent** to F (and we write $E \sim F$) if there is a renaming (a bijection) σ_K of keys of F and a renaming σ_R of random coins of F such that $\mathbf{pat}(E) = \mathbf{pat}(F\sigma_K\sigma_R)$

Examples:

- $k' \sim k$
- $(k_1, k_2) \not\sim (k_3, k_3)$ (no bijection !)
- but $(k_1, k_2) \sim (k_4, k_1)$
- $(\{0\}_k^r, \{0\}_k^{r'}) \sim (\{0\}_k^{r'}, \{1\}_k^s)$
- but $(\{0\}_k^r, \{0\}_k^r) \not\sim (\{0\}_k^{r'}, \{1\}_k^s)$
- $\{((1,1),(1,1))\}_k^r \sim \{1\}_k^r$
- $(\{0\}_k^{r'}, \{0\}_k^s) \sim (\{0\}_{k'}^{r'}, \{0\}_{k''}^s)$

Terms and the computational world

What is the meaning of a term in the computational world?

- $0, 1$? ... are constants, hence deterministic
- $((0,1),1)$ ... using some deterministic concatenation
- k ... keys are randomly sampled
- r ... coins are randomly sampled
- $\{0\}_k^r$ ... in general, encryption is probabilistic

Hence to a term, we will associate a **distribution**.

Implementation - primitives

- Let $\langle u, v \rangle : \{0,1\}^* \times \{0,1\}^* \mapsto \{0,1\}^*$ be an easily computable and invertible injective function.
- Let $\Pi = (\mathcal{K}, \mathcal{E}, \mathcal{D})$ be a symmetric encryption scheme:
  - $\mathcal{K}(1^\eta)$: generates random key.
  - $\mathcal{E}^r(1^\eta, k, x)$: encrypts x with k using random coins r.
  - $\mathcal{D}(1^\eta, k, y)$: decrypts y with k.
  - Correctness:

$$\forall \eta, \forall x, \forall r$$

$$k := \mathcal{K}(1^\eta); y := \mathcal{E}^r(1^\eta, k, x); x' := \mathcal{D}(1^\eta, k, y) :$$

$$(x = x')?$$

Implementation of formal terms

Let $\Pi = (\mathcal{K}, \mathcal{E}, \mathcal{D})$ be a symmetric encryption scheme and $\eta \in \mathbb{N}$ be the security parameter. We associate to the formal term M a distribution $[\![M]\!]_\eta$ and by consequence a family of distributions $[\![M]\!]$.

- For each $k \in \mathit{Keys}(M)$ do $\tau_K(k) \xleftarrow{R} \mathcal{K}_\eta()$.
- For each $r \in \mathit{Coins}(M)$ do $\tau_R(r) \xleftarrow{R} \{0,1\}^\omega$.
- $[\![k]\!]_\eta = \tau_K(k)$.
- For $b \in \{0,1\}$, $[\![b]\!]_\eta = b$.
- $[\![(e_1, e_2)]\!]_\eta = \langle [\![e_1]\!]_\eta, [\![e_2]\!]_\eta \rangle$.
- $[\![\{e\}_k^r]\!]_\eta = \mathcal{E}^{\tau_R(r)}(1^\eta, \tau_K(k), [\![e]\!]_\eta)$.

Computational equivalence of terms

We say that two terms E and F are **computationally equivalent**, and we write $E \approx F$, when the associated distributions are computationally indistinguishable

$$[\![E]\!] \approx [\![F]\!]$$

Corectness

Do we have that $E \sim F$ implies that $E \approx F$?
YES, if we avoid key cycle in terms, and if encryption is *type-0*-secure.

Encryption Cycles - examples:

- $\{k\}^r_k$
- $\{k_2\}^r_{k_1}, \{k_1\}^{r'}_{k_2}$
- $\{k_2\}^r_{k_1}, \{k_3\}^{r'}_{k_2}, \{k_1\}^{r''}_{k_3}$
- etc ...

Definition

Definition (Key Cycles)

Let $k, k' \in$ Keys. We say that k encrypts k' in the term M, denoted by $k \succ_M k'$, if $\{N\}^r_k \in st(M)$ and $k' \in st(N)$.
A term M is acyclic iff the relation $\succ_M$ is acyclic.

Example

.

- Let $M = (\{\{k_1\}^{r1}_{k_2}\}^{r2}_{k_3}, \mathbf{0})$. $\succ_M$ is defined by $k_3 \succ_M k_2 \succ_M k_1$. M is acyclic.
- Let $M = \{k\}^r_k$. We get $k \succ_M k$. M has a cycle of size 1.
- Let $M = (\{k_1\}^{r1}_{k_2}, \{k_2\}^{r2}_{k_1})$. We have that $k_1 \succ_M k_2$ and $k_2 \succ_M k_1$. M has a cycle of size 2.

Type-0 Scheme - intuition

- A *type-0* encryption scheme has the following properties:
  1. message-hiding: given two messages m and m' of equal length and a cipher-text c of one of them, one cannot tell which of m or m' corresponds to c (IND-CPA)
  2. Which-key-hiding: given cipher-texts c, c', one cannot tell if they were encrypted under the same key
  3. Message-length-hiding: given cipher-text c, one cannot determine the length of the corresponding plain-text

Type-0 Scheme - formal definition

Definition

Let $\Pi = (\mathcal{K}, \mathcal{E}, \mathcal{D})$ be an encryption scheme and $\mathcal{A}$ be an adversary:

$$Adv^0_{\Pi,\eta}(\mathcal{A}) = Pr[\mathcal{A}^{\mathcal{O}^0_1(.),\mathcal{O}^0_2(.)}(\eta) = 1 | k \leftarrow^R \mathcal{K}_\eta()]$$

$$- Pr[\mathcal{A}^{\mathcal{O}^1_1(.),\mathcal{O}^1_2(.)}(\eta) = 1 | k_1, k_2 \leftarrow^R \mathcal{K}_\eta()]$$

Π is **Type-0 secure** if $Adv^0_{\Pi,\eta}(\mathcal{A})$ is negligible, for any ppt $\mathcal{A}$.

- $\mathcal{A}^{\mathcal{O}}$ is an adversary having acces to a (or several) oracle(s) $\mathcal{O}$.
- $\mathcal{O}^1_1(.)$ is an oracle that takes a message m and answers $c \leftarrow^R \mathcal{E}_{k_1}(m)$.
- $\mathcal{O}^1_2(.)$ is an oracle that takes a message m and answers $c \leftarrow^R \mathcal{E}_{k_2}(m)$.
- $\mathcal{O}^0_1(.)$ is an oracle that takes a message m and answers $c \leftarrow^R \mathcal{E}_k(0)$.
- $\mathcal{O}^0_2(.)$ is an oracle that takes a message m and answers $c \leftarrow^R \mathcal{E}_k(0)$.

Soundness

Theorem

Soundness If e_1,e_2 do not contain encryption cycles, and if encryption is implemented using a type-0 secure scheme, then

$$e_1 \sim e_2 \quad \textit{implies} \quad e_1 \approx e_2$$

Proof is by a hybrid reduction-style argument.

From now on we suppose that expressions do not contain encryption cycles, and that encryption is implemented using a type-0 secure scheme.

Implementation of formal patterns

- For each $k \in \mathit{Keys}(M)$ do $\tau_K(k) \xleftarrow{R} \mathcal{K}_\eta()$.
- $k_\square \xleftarrow{R} \mathcal{K}_\eta()$.
- For each $r \in \mathit{Coins}(M)$ do $\tau_R(r) \xleftarrow{R} \{0,1\}^\omega$.
- $[\![k]\!]_\eta = \tau_K(k)$.
- For $b \in \{0,1\}$, $[\![b]\!]_\eta = b$.
- $[\![(e_1, e_2)]\!]_\eta = \langle [\![e_1]\!]_\eta, [\![e_2]\!]_\eta \rangle$.
- $[\![\{e\}^r_k]\!]_\eta = \mathcal{E}^{\tau_R(r)}(1^\eta, \tau_K(k), [\![e]\!]_\eta)$.
- $[\![\square^r]\!]_\eta = \mathcal{E}^{\tau_R(r)}(1^\eta, k_\square, 0)$.

Replacing one key

For a key $k_0 \in \text{Keys}$, define

- $\mathit{replace}(k, k_0) = k$.
- $\mathit{replace}(b, k_0) = b$.
- $\mathit{replace}((e_1, e_2), k_0) = (\mathit{replace}(e_1, k_0), \mathit{replace}(e_2, k_0))$.
- $\mathit{replace}(\{e\}^r_k, k_0) = \{\mathit{replace}(e, k_0)\}^r_k$ if $k \neq k_0$.
- $\mathit{replace}(\{e\}^r_{k_0}, k_0) = \square^r$.
- $\mathit{replace}(\square^r, k_0) = \square^r$.

Replacing one key is sound

Theorem

Soundness Let e an expression, and k_0 a key occuring in e only as encryption key. Then $[\![e]\!] \approx [\![replace(e, k_0)]\!]$.

Proof.

Assume that B distinguishes $[\![e]\!]$ from $[\![replace(e, k_0)]\!]$, and let construct an adversary $\mathcal{A}^{\mathcal{O}}$ that wins against the security of the type-0 encryption scheme. $\mathcal{A}^{\mathcal{O}}(1^\eta)$ works as follows:

- For each $k \in Keys(e)$ do $\tau_K(k) \xleftarrow{R} \mathcal{K}_\eta()$.
- For each $r \in Coins(e)$ do $\tau_R(r) \xleftarrow{R} \{0,1\}^\omega$.
- Let $L = \{\}$ (the empty mapping).
- Compute the "semantics" v of e by invoking $\mathbf{S}em^{\mathcal{O}}(e)$.
- Return $\mathrm{B}(1^\eta, v)$.

□

Replacing one key is sound

Theorem Let e an expression, and k_0 a key occuring in e only as encryption key. Then $[\![e]\!] \approx [\![\mathit{replace}(e, k_0)]\!]$.

Proof.

(Continuation)
$\mathbf{Sem}^{\mathcal{O}}(e)$ is: case e of

- k: return $\tau_K(k)$ (note that $k \neq k_0$).
- b: return b.
- (e_1, e_2): Let $v_i = \mathbf{Sem}^{\mathcal{O}}(e_i)$; return $\langle v_1, v_2 \rangle$.
- $\square^r$: return $\mathcal{O}_2(0)$.
- $\{e\}^r_k$: Let $w = \mathbf{Sem}^{\mathcal{O}}(e)$;
  - If $k \neq k_0$, then return $\mathcal{E}^{\tau_R(r)}(1^\eta, \tau_K(k), w)$.
  - If $k = k_0$ and $L(r)$ is not defined, then $L(r) = \mathcal{O}_1(w)$; return $L(r)$.

One can see that $\mathbf{Sem}^{\mathcal{O}^1}(e) = [\![e]\!]$ and $\mathbf{Sem}^{\mathcal{O}^0}(e) = [\![\mathit{replace}(e, k_0)]\!]$, hence $\mathcal{A}$ distinguishes $\mathcal{O}^0$ from $\mathcal{O}^1$, with the same advantage that B distinguishes $[\![e]\!]$ from $[\![\mathit{replace}(e, k_0)]\!]$.

□

Soundness

Theorem If e_1, e_2 do not contain encryption cycles, and if encryption is implemented using a type-0 secure scheme, then

$$e_1 \sim e_2 \quad \textit{implies} \quad e_1 \approx e_2$$

Proof.

- If $\{k_1, \ldots, k_n\} = \mathit{Keys}(e)$, and for all i, j, $i < j \Longrightarrow k_j \not\succ_e k_i$, then
$$\mathit{replace}(\cdots \mathit{replace}(\mathit{replace}(e, k_1), k_2) \cdots, k_n) = \mathbf{pat}(e)$$

- Apply the previous lemma to each key in $\mathbf{hidden}(e)$, we get
$$[\![e]\!] \approx [\![\mathbf{pat}(e)]\!]$$

- Permuting the formal keys and coins does not change the generated probability distribution over bit-strings. If $e_1 \sim e_2$, then $\mathbf{pat}(e_1) = \mathbf{pat}(e_2\sigma_K\sigma_R)$ for some permutations σ_k on keys and σ_R on coins, and by transitivity
$$[\![e_1]\!] \approx [\![\mathbf{pat}(e_1)]\!] = [\![\mathbf{pat}(e_2\sigma_K\sigma_R)]\!] = [\![\mathbf{pat}(e_2)]\!] \approx [\![e_2]\!],$$

hence $e_1 \approx e_2$. □

Example

$$[\![((\{k_4, 0\}^{r_1}_{k_3}, \{k_3\}^{r_2}_{k_2}), (\{\{11\}^{r_4}_{k_4}\}^{r_3}_{k_1}, k_1))]\!]$$

$$\approx$$

$$[\![((\{k_4, 0\}^{r_1}_{k_3}, \Box^{r_2}), (\{\{11\}^{r_4}_{k_4}\}^{r_3}_{k_1}, k_1))]\!]$$

$$\approx$$

$$[\![((\Box^{r_1}, \Box^{r_2}), (\{\{11\}^{r_4}_{k_4}\}^{r_3}_{k_1}, k_1))]\!]$$

$$\approx$$

$$[\![((\Box^{r_1}, \Box^{r_2}), (\{\Box^{r_4}\}^{r_3}_{k_1}, k_1))]\!]$$

$$\approx$$

$$[\![((\{1\}^{r_1}_{k_2}, \Box^{r_2}), (\{\{0\}^{r_4}_{k_2}\}^{r_3}_{k_1}, k_1))]\!]$$

$$\approx$$

$$[\![((\{1\}^{r_1}_{k_2}, \{k_2\}^{r_2}_{k_3}), (\{\{0\}^{r_4}_{k_2}\}^{r_3}_{k_1}, k_1))]\!]$$

Abadi Rogaway 2000 Paper

- Introduced in [AR 00] M. Abadi et P. Rogaway. Reconciling two views of cryptography (the computational soundness of formal encryption). Journal of Cryptology, 15(2):103–127, 2002.
- Models security of encryption against a *passive* computational adversary
- Adversary observes two *expressions* which have the same *pattern* assuming secure encryption – goal is to distinguish them with non-negligible probability

RESULT:
Symbolic indistinguishability implies computational security

(Some) Extensions

- [Micciancio, Warinschi] give completeness for modified (authenticated) encryption, and also soundness for active adversaries
- [Micciancio, Panjwani] give a soundness theorem for an adversary which can adaptively attack the encryption scheme
- [Backes, Pfitzmann, Waidner] give an cryptographic implementation of Dolev-Yao terms in a general (UC) setting
- [Canneti, Herzog] give a formal (automated) approach to universal composability