

Series 1.bis (more about S.O.S.)

## Exercise 1

Let  $S$  be the statement 'while **true** do skip;'. Show that its execution loops in every state according to the natural semantics first, and then according to the structural operational semantics. Can you exhibit a statement of **While** such that there exists a state in which it loops in natural semantics and not in structural operational semantics, or vice versa? Why?

## Exercise 2

1. Write down the mathematical phrase corresponding to stating that S.O.S is deterministic.
2. Show that it is true.
3. Deduce that there is exactly one derivation sequence starting in configuration  $\langle S, s \rangle$ ; and then that a statement of **While** cannot both terminate and loop on a state  $s$ .

## Exercise 3

(Useful lemma to prove the equivalence of semantics functions of S.O.S. and natural semantics.)

1. Suppose that  $\langle S_1; S_2, s \rangle \Rightarrow^* \langle S_2, s' \rangle$ . Show that it is *not* necessary that  $\langle S_1, s \rangle \Rightarrow^* s'$
2. Prove that if  $\langle S_1, s \rangle \Rightarrow^k s'$  then  $\langle S_1; S_2, s \rangle \Rightarrow^k \langle S_2, s' \rangle$ .

## Exercise 4

1. Prove that if  $\langle S_1; S_2, s \rangle \Rightarrow^k s''$ , then there exists a state  $s'$  and natural numbers  $m$  and  $n$  such that  $k = m + n$ ,  $\langle S_1, s \rangle \Rightarrow^m s'$  and  $\langle S_2, s' \rangle \Rightarrow^n s''$ .
2. (uses exercise 2)  
Let  $S_1, S_2, S_3$  be statements in **While**.  
Show that  $(S_1; S_2); S_3$  and  $S_1; (S_2; S_3)$  are semantically equivalent with respect to the S.O.S. semantics. (First, what does it mean to be semantically equivalent?...)