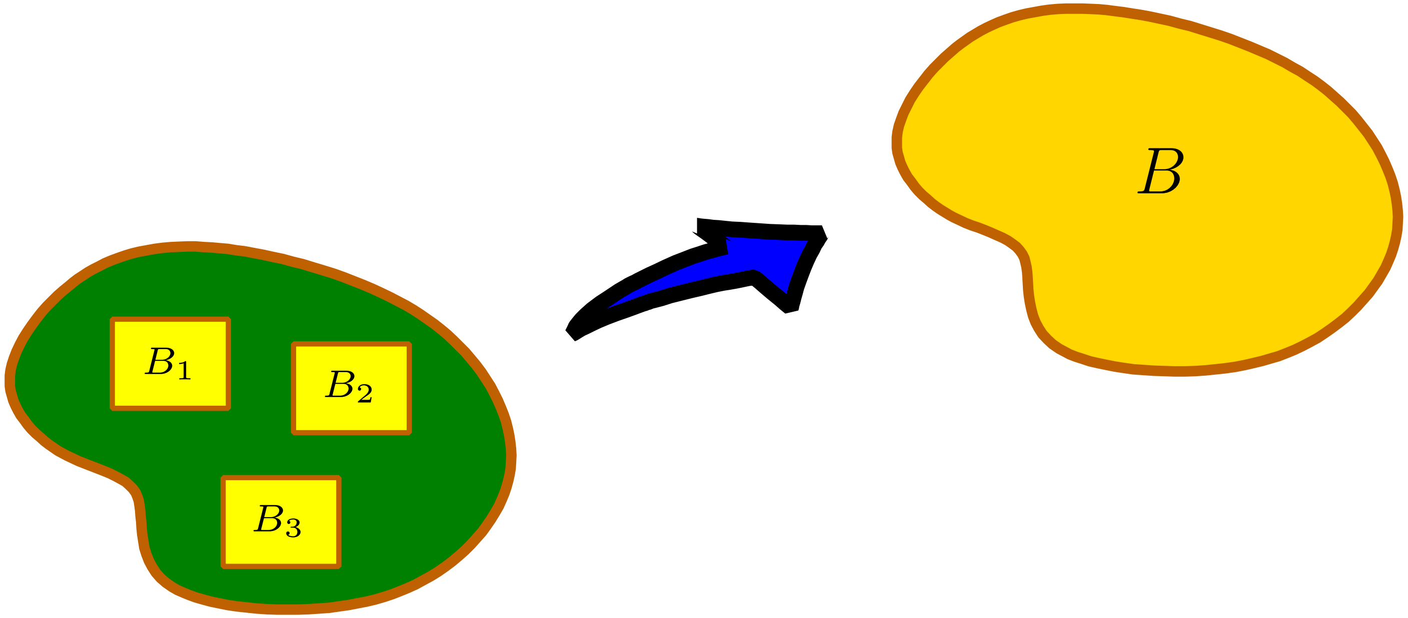
A Notion of Glue Expressiveness for Component-Based Systems

Simon Bliudze, Joseph Sifakis
{bliudze, sifakis}@imag.fr
VERIMAG, Grenoble

Presentation Outline

- Glue Expressiveness
- Labelled Transition Systems
- Structural Operational Semantics Glue
- Comparison of Classical Glues

Motivation: Component-Based Systems



- Components are assembled from smaller (atomic) ones by application of glue.
  - A semantic behaviour domain $\mathcal{B}$.
  - A set $\mathcal{G}$ of glue operators $2^{\mathcal{B}} \to \mathcal{B}$.
- How do we compare two glues $G_1, G_2 \subseteq \mathcal{G}$?
  - Comparison is made by flattening, i.e. directly on $\mathcal{B}$: $G_1(\mathcal{B}) \stackrel{?}{=} G_2(\mathcal{B})$.
  - Not satisfactory: most formalisms are Turing complete.
- **Goal:** develop a framework to **compare glue**, i.e. on $(\mathcal{B}, \mathcal{G})$.

Comparison of Glue

Assumptions:

- A semantic behaviour domain $\mathcal{B}$ with a relation $\simeq \subseteq \mathcal{B} \times \mathcal{B}$
- A set $\mathcal{G}$ of glue operators $2^{\mathcal{B}} \to \mathcal{B}$

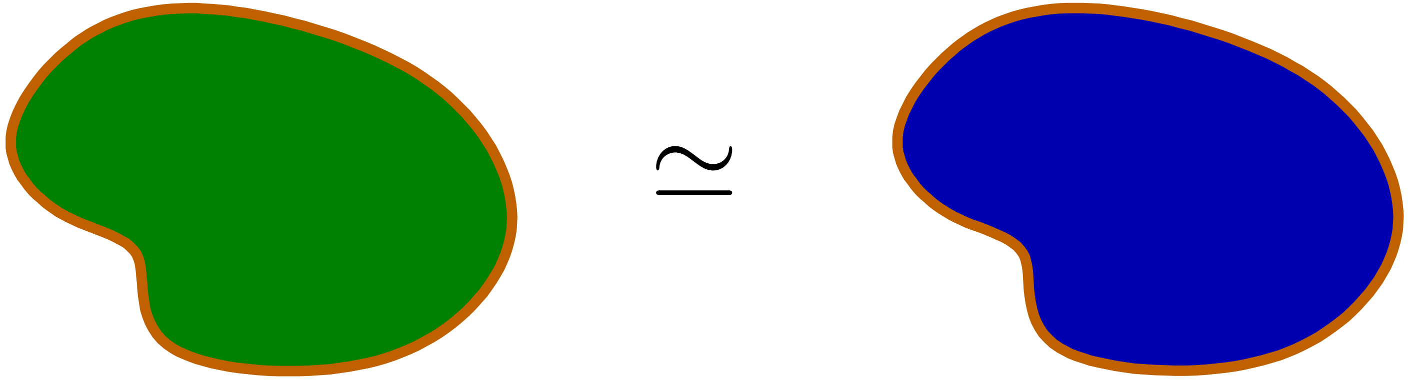
Comparison:

- (Very strong) $\simeq$ induces a relation on $\mathcal{G}$:

$$G_1 \preccurlyeq G_2 \quad \stackrel{def}{\Longleftrightarrow} \quad \forall g_1 \in G_1,\ \exists g_2 \in G_2 : g_1 \simeq g_2\,.$$

- (Strong) First choose the behaviours, then the operator g_2.
- (Weak) Allow some additional coordination behaviour.

Relation Induced on Glue Operators

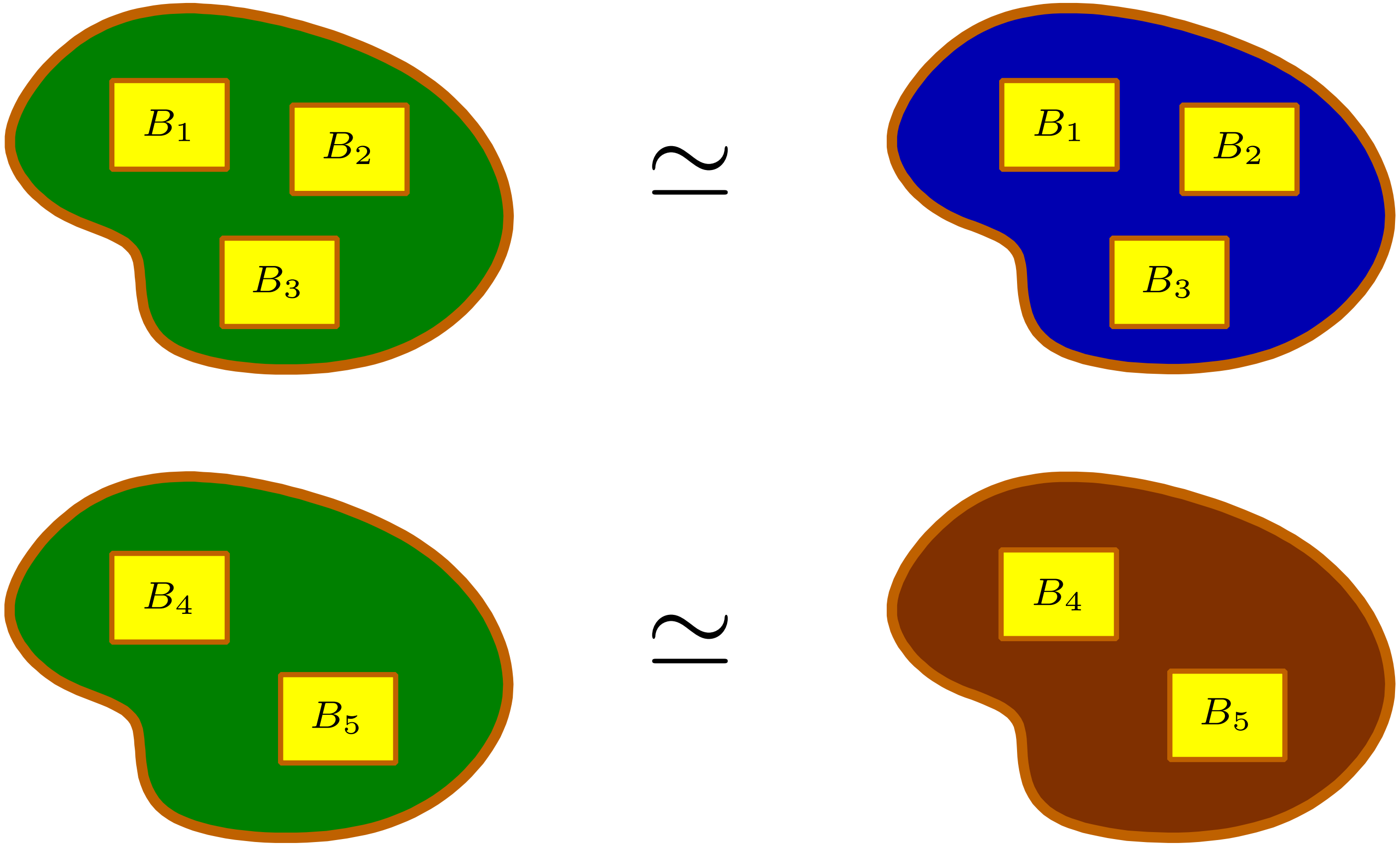


$$g_1 \simeq g_2 \quad \overset{def}{\Longleftrightarrow} \quad \forall \mathbf{B} \subset \mathcal{B},\ g_1(\mathbf{B}) \simeq g_2(\mathbf{B})$$

$$G_1 \preccurlyeq G_2 \quad \overset{def}{\Longleftrightarrow} \quad \forall g_1 \in G_1,\ \exists g_2 \in G_2 : g_1 \simeq g_2$$

$$\Longleftrightarrow \quad \forall g_1 \in G_1,\ \exists g_2 \in G_2 : \forall \mathbf{B} \subset \mathcal{B},\ g_1(\mathbf{B}) \simeq g_2(\mathbf{B})$$

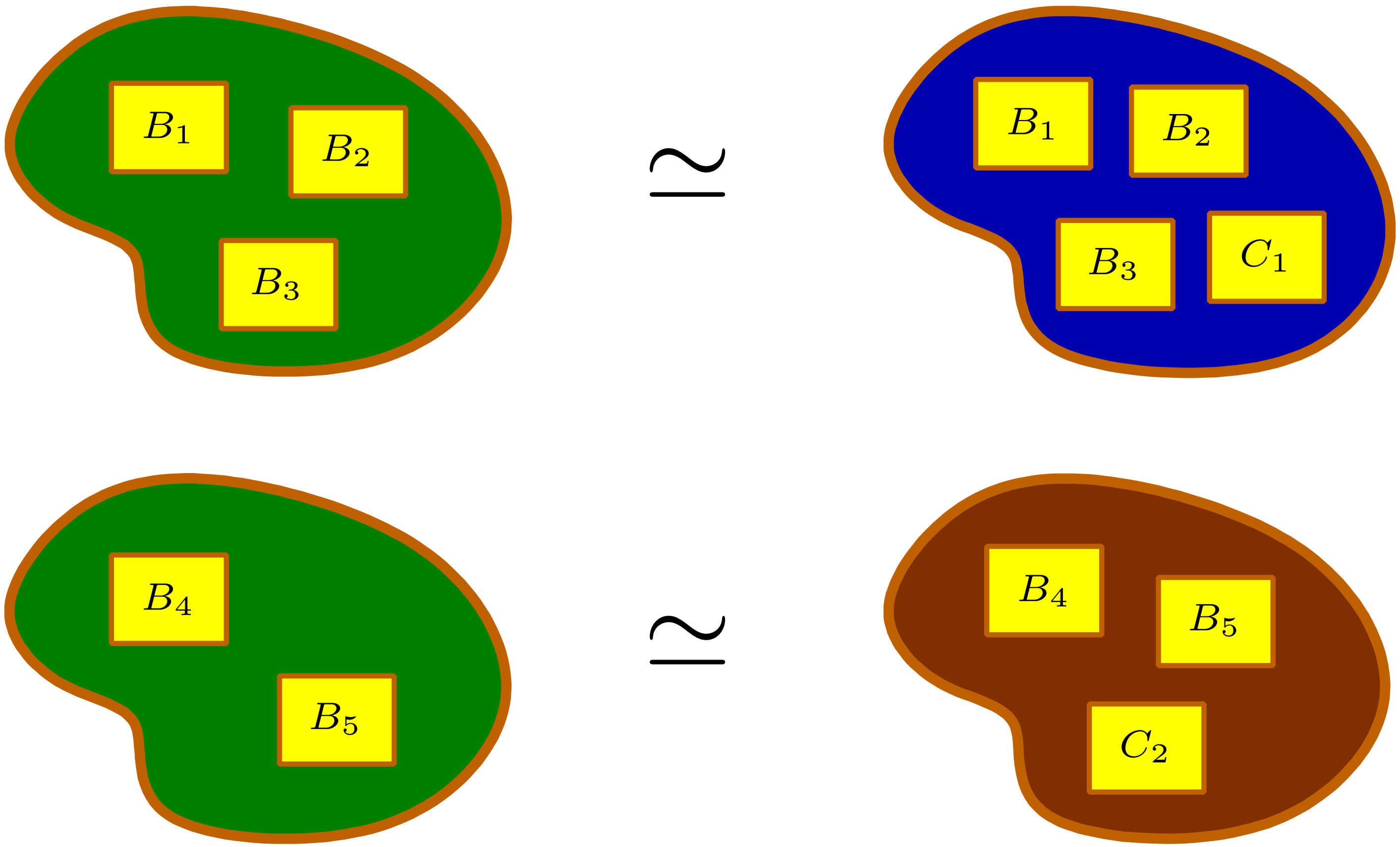
Strong Expressiveness Preorder



$$G_1 \preccurlyeq_S G_2 \quad \overset{def}{\Longleftrightarrow} \quad \forall g_1 \in G_1, \forall \mathbf{B} \subset \mathcal{B},\ \exists g_2 \in G_2 : g_1(\mathbf{B}) \simeq g_2(\mathbf{B})$$

$$\Big/ \text{recall } G_1 \preccurlyeq G_2 \quad \Longleftrightarrow \quad \forall g_1 \in G_1, \underline{\exists g_2 \in G_2 : \forall \mathbf{B} \subset \mathcal{B},}\ g_1(\mathbf{B}) \simeq g_2(\mathbf{B}) \Big/$$

Weak Expressiveness Preorder



$G_1 \preccurlyeq_W G_2 \quad \stackrel{def}{\Longleftrightarrow}$

- there exists a **finite** subset $\mathcal{C} \subset \mathcal{B}$ of coordination behaviours, such that
- $\forall g_1 \in G_1, \forall \mathbf{B} \subset \mathcal{B},\ \exists \mathbf{C} \subset \mathcal{C}, g_2 \in G_2 :\ g_1(\mathbf{B}) \simeq g_2(\mathbf{B}, \mathbf{C})$

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Labelled Transition Systems

$B = (Q, P, \rightarrow)$, where $\rightarrow \subseteq Q \times 2^P \times Q$

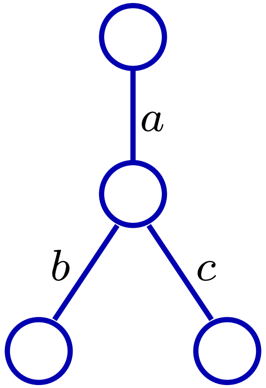
Relations:

- $\sqsubseteq_S$ simulation preorder,
- $\simeq_S$ simulation equivalence,
- $\sqsubseteq_{RS}$ ready simulation preorder,
- $\simeq_{RS}$ ready simulation equivalence,
- $\underline{\leftrightarrow}$ bisimulation.

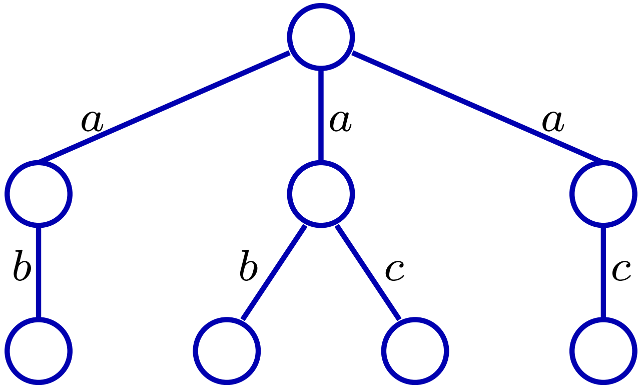
$$\begin{array}{ccccc} \underline{\leftrightarrow} & \subseteq & \simeq_{RS} & \subseteq & \simeq_S \\ & & \cap\!\mid & & \cap\!\mid \\ & & \sqsubseteq_{RS} & \subseteq & \sqsubseteq_S \end{array}$$

Some counterexamples

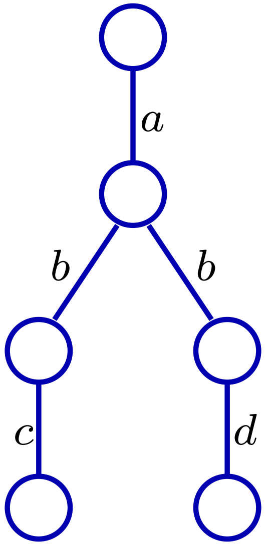
Simulation but not Ready Simulation equivalent:



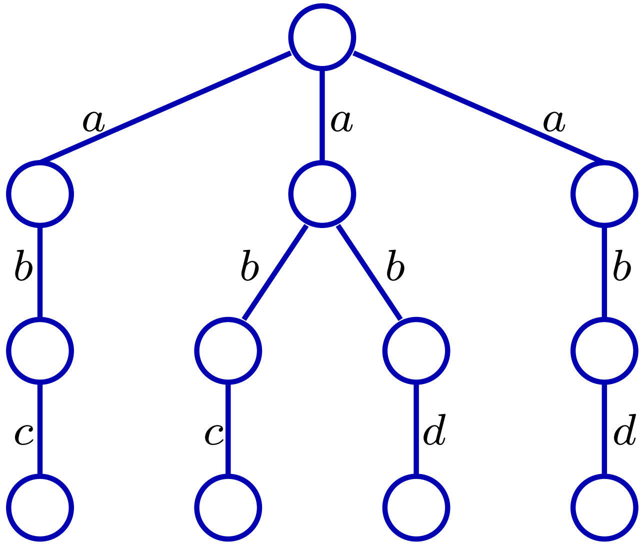
$$\simeq_S \qquad \sqsubseteq_{RS} \qquad \not\sqsupseteq_{RS} \qquad \not\underline{\leftrightarrow}$$



Ready Simulation equivalent but not Bisimilar:



$$\simeq_S \qquad \simeq_{RS} \qquad \not\underline{\leftrightarrow}$$



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SOS Glue Operators

A *glue operator* is a set of derivation rules of the form

$$r \quad = \quad \frac{\left\{ q_i \xrightarrow{a_i} q_i' \right\}_{i \in I} \qquad \left\{ q_j \overset{b_{j_k}}{\not\longrightarrow} \;\Big|\; k \in [1, m_j] \right\}_{j \in J}}{q_1 \dots q_n \xrightarrow{a} \widetilde{q}_1 \dots \widetilde{q}_n}$$

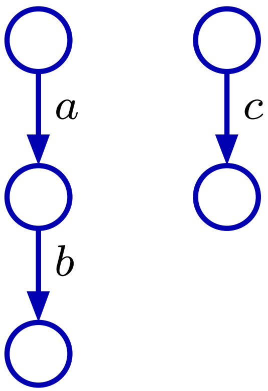
1. $a = \bigcup_{i \in I} a_i$.
2. For each $i \in [1, n]$, r has **at most one positive premise** involving the i-th argument.
3. r has at least one positive premise.
4. A label can appear either in positive or in negative premises, but not in both.

Example

$$g = \left\{ \frac{q_1 \xrightarrow{a} q_1'}{q_1 q_2 \xrightarrow{a} q_1' q_2}, \quad \frac{q_1 \xrightarrow{a} q_1' \qquad q_2 \xrightarrow{c} q_2'}{q_1 q_2 \xrightarrow{ac} q_1' q_2'}, \quad \frac{q_1 \xrightarrow{b} q_1' \qquad q_2 \stackrel{c}{\not\rightarrow}}{q_1 q_2 \xrightarrow{b} q_1' q_2} \right\}$$

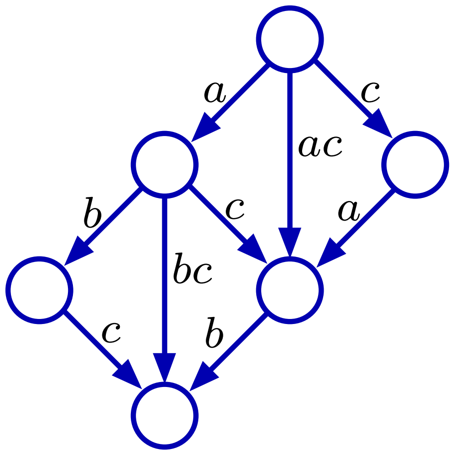
Behaviours

B_1, B_2



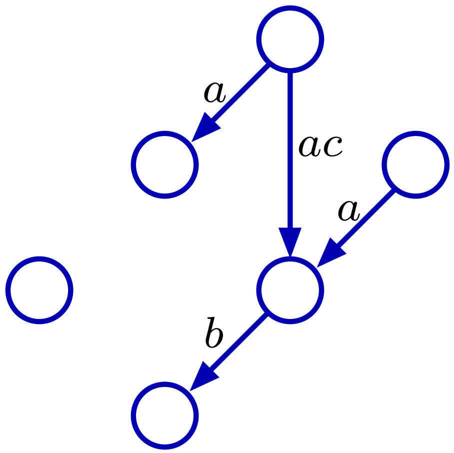
Parallel product

$B_1 \parallel B_2$



Application of glue

$g(B_1, B_2)$



Main result 1

Assumption: No redundant rules in a glue operator, i.e. $r_1, r_2 \in g$, such that

$$Pos(r_1) \ = \ Pos(r_2), \quad Neg(r_1) \ \subseteq Neg(r_2)\,.$$

Th 1 *Bisimulation, ready simulation preorder and equivalence, and simulation equivalence on glue operators coincide:*

$$\underline{\leftrightarrow} \quad = \quad \simeq_{RS} \quad = \quad \simeq_S \quad = \quad \sqsubseteq_{RS} \quad .$$

All these relations coincide with the equality of operators as sets of rules.

Very Strong Comparison too strong.

Pseudo-boolean encoding of glue operators

$$r \quad = \quad \frac{\left\{q_i \xrightarrow{a_i} q_i'\right\}_{i\in I} \qquad \left\{q_j \stackrel{b_{j_k}}{\not\longrightarrow} \,\middle|\, k \in K_j\right\}_{j\in J}}{q_1 \dots q_n \xrightarrow{a} \widetilde{q}_1 \dots \widetilde{q}_n}$$

$$C(r) \quad = \quad \bigwedge_{i\in I} a_i \;\wedge\; \bigwedge_{j\in J} \bigwedge_{k\in K_j} \neg b_{j_k}\,, \qquad\qquad g \quad \rightsquigarrow \quad \bigvee_{r\in g} C(r)\,.$$

Syntax:

$$\begin{array}{rcl} f & ::= & f \vee f \;\mid f \wedge t \mid e\,, \\ t & ::= & (t \vee t) \mid \;\; \neg e \;\; \mid e\,, \\ e & ::= & e \vee e \;\mid e \wedge e \mid (e) \mid a \in 2^P \mid 0 \mid 1\,, \end{array}$$

Axioms:

1. $\neg 0 = 1$ and $\neg 1 = 0$,
2. $f \wedge \neg f = 0$,
3. $\neg f_1 \wedge \neg f_2 = \neg(f_1 \vee f_2)$,
4. $\neg f_1 \vee \neg f_2 = \neg(f_1 \wedge f_2)$.

What does not hold? Essentially $f \vee \neg f \neq 1$, i.e.
$ab \vee a\neg b \neq a$ — a is only allowed alone, when b is not possible.

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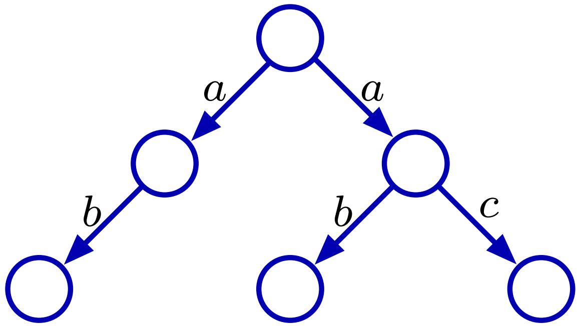
BIP

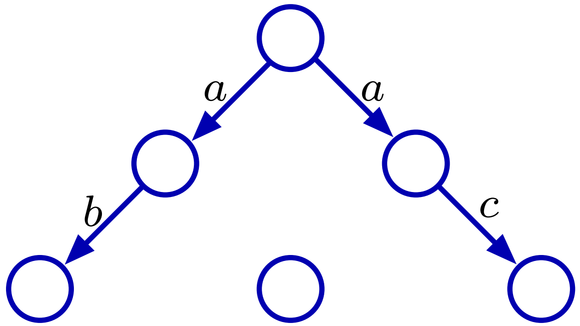
$int_\gamma = \bigvee \gamma$ — an *interaction model*, defined by a set of interactions $\gamma \subseteq 2^P$.

$pr_\pi = \bigvee_{a \in 2^P} \left(a \wedge \bigwedge_{a \prec a'} \neg a'\right)$ — a *priority model* π is a strict partial order on 2^P.

B:



$pr_{a \prec b}(B)$:



Prop 1 *IM is strongly equivalent to the set of all positive glue operators, whereas BIP is strongly equivalent to the set of all glue operators.*

Prop 2 *BIP is strongly more expressive than IM w.r.t.* $\simeq_S$ *(*a fortiori $\simeq_{RS}$ *and* $\underline{\leftrightarrow}$*). That is* $IM \preccurlyeq_S BIP$ *and* $BIP \not\preccurlyeq_W IM$*.*

Parallel composition in CCS, SCCS, and CSP

$L = A \cup \overline{A} \cup \{\tau\}$ is the set of labels. C is the set of channels.

CCS: Binary synchronisation of complementary actions $a, \overline{a} \in L$:

$$par_{CCS} \quad = \quad \bigvee_{a \in A} \bigvee_{i,j=1}^{n} B_i.a\, B_j.\overline{a} \quad \vee \quad \bigvee_{a \in A} \bigvee_{i=1}^{n} (B_i.a \vee B_i.\overline{a} \vee B_i.\tau)\,.$$

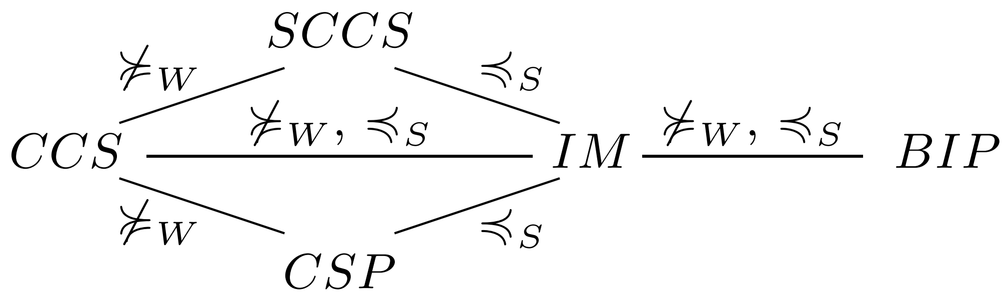
SCCS: All components must synchronise:

$$par_{SCCS} \quad = \quad \bigwedge_{i=1}^{n} \left(B_i.\tau \vee \bigvee_{a \in A} B_i.a \right)\,.$$

CSP: Processes communicate over a set of channels common to the system:

$$par_{CSP} \quad = \quad \bigvee_{c \in C'} \bigwedge_{i=1}^{n} B_i.c \;\vee\; \bigvee_{c \notin C'} \bigvee_{i=1}^{n} (B_i.\tau \vee B_i.c)\,.$$

Conclusion



- Three preorders for comparing glue expressiveness.
- For LTS and SOS glue operators,
  - classical equivalence relations coincide,
  - first results for comparison of classical glues according to strong and weak expressiveness preorders.
- Pseudo-boolean encoding of glue operators.

To Do

1. Complete the diagram in the previous slide.
2. Characterisation of the strong expressiveness preorder for all SOS glues (not only positive).
3. Operators with influences (positive premises not participating in the conclusion).
4. What is a glue operator in the general case?

Labelled Transition Systems

Labelled Transition System (LTS): $B = (Q, P, \rightarrow)$, where

- Q is the set of states,
- P is the set of ports,
- $\rightarrow \subseteq Q \times 2^P \times Q$ is the set of transitions.

Let $B_1 = (Q_1, P_1, \rightarrow)$ and $B_2 = (Q_2, P_2, \rightarrow)$ be two LTS, and let $\mathcal{R} \subseteq Q_1 \times Q_2$ be a binary relation. $\mathcal{R}$ is

1. a *simulation* iff, for all $q_1 \mathcal{R}\, q_2$, $q_1 \xrightarrow{a} q_1'$ implies $q_2 \xrightarrow{a} q_2'$, for some $q_2' \in Q_2$ such that $q_1' \mathcal{R}\, q_2'$.
2. a *ready simulation* iff it is a simulation and, for $q_1 \mathcal{R}\, q_2$, $q_1 \stackrel{a}{\not\rightarrow}$ implies $q_2 \stackrel{a}{\not\rightarrow}$.
3. a *bisimulation* iff both $\mathcal{R}$ and $\mathcal{R}^{-1}$ are simulations.

A proof

Lemma 1 *Let g_1, g_2 be glue operators, and g_1 be without redundancy. $g_1 \sqsubseteq_S g_2$ implies that, for each rule $r_1 \in g_1$, there is a rule $r_2 \in g_2$ having $Pos(r_2) = Pos(r_1)$ and $Neg(r_2) \subseteq Neg(r_1)$.*

Proof — Consider the rule

$$r_1 \;=\; \frac{\{q_i \xrightarrow{a_i} q_i'\}_{i\in I} \qquad \{q_j \overset{b_{j_k}}{\not\longrightarrow} \mid k \in [1, m_j]\}_{j\in J}}{q_1 \ldots q_n \xrightarrow{a} \widetilde{q_1} \ldots \widetilde{q_n}} \in g_1\,,$$

and, for $i \in [1, n]$, $B_i^1 = (Q_i, P, \rightarrow_i)$ having $Q_i = \{q^i\}$ and $\rightarrow_i$ defined by

$$\rightarrow_i \;=\; \begin{cases} \{q^i \xrightarrow{a} q^i \mid a \in 2^P\}, & \text{for } i \notin J,\\ \{q^i \xrightarrow{a} q^i \mid a \in 2^P\} \setminus \{q^i \xrightarrow{b_{i_k}} q^i \mid k \in [1, m_i]\}, & \text{for } i \in J\,. \end{cases}$$

Both $g_1(B_1^1, \ldots, B_n^1)$ and $g_2(B_1^1, \ldots, B_n^1)$ have exactly one state: q' and q''.

All the premises of r_1 are satisfied in q'. Hence $q' \xrightarrow{a} q'$ in $g_1(B_1^1, \ldots, B_n^1)$. By simulation $g_1 \sqsubseteq_S g_2$, we also have $g_1(B_1^1, \ldots, B_n^1) \sqsubseteq_S g_2(B_1^1, \ldots, B_n^1)$. Hence, $q'' \xrightarrow{a} q''$ in $g_2(B_1^1, \ldots, B_n^1)$, and there exists a rule $r_2 \in g_2$ enabling this transition. Thus, $Pos(r_2) = Pos(r_1)$ and $Neg(r_2) \subseteq Neg(r_1)$. ■